Solution

INTRODUCTION TO TRIGONOMETRY WS 4

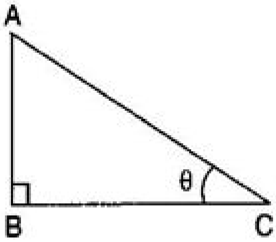
Class 10 - Mathematics

Section A

1.

(d) Base = Perpendicular

Explanation:



Given: in triangle ABC, $\angle C = 45^\circ$, and $\angle B = 90^\circ$,
Since, $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}}$
$\Rightarrow \tan 45^\circ = \frac{\text{Perpendicular}}{\text{Base}}$
$\Rightarrow 1 = \frac{\text{Perpendicular}}{\text{Base}}$
$\Rightarrow$ Base = perpendicular

2. **(a)** 9

Explanation: $2 \sin^2 30^o + 3 \tan^2 60^o - \cos^2 45^o$
$= 2 \times \left(\frac{1}{4}\right) + 3 \times 3 - \frac{1}{2} = 9$

3.

(b) $\sin 60^o$

Explanation: $\frac{2tan30^o}{1+tan^2 30^o} = \frac{2\times\frac{1}{\sqrt{3}}}{1+(\frac{1}{\sqrt{3}})^2}$
$\frac{\frac{2}{\sqrt{3}}}{1+\frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4} = \frac{\sqrt{3}}{2}$
$= \sin 60^o$

4.

(c) $\frac{83}{8}$

Explanation: $\cos^2 30^\circ \cos^2 45^\circ + 4\sec^2 60^\circ + \frac{1}{2}\cos^2 90^\circ - 2\tan^2 60^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)^2 \cdot \left(\frac{1}{\sqrt{2}}\right)^2 + \left(4 \times 2^2\right) + \left(\frac{1}{2} \times 0^2\right) - 2 \times (\sqrt{3})^2$
$= \left(\frac{3}{4} \times \frac{1}{2}\right) + 16 + 0 - 6 = \frac{3}{8} + 10 = \frac{83}{8}$

5.

(b) 30^o

Explanation: 30^o

6. **(a)** 1

Explanation: Given: $\frac{\tan 30^\circ}{\cot 60^\circ}$
$= \frac{\frac{1}{\sqrt{3}}}{\frac{1}{\sqrt{3}}} = 1$

7.

(b) $\frac{7}{4}$

Explanation: $(\cos 0^o + \sin 30^o + \sin 45^o)(\sin 90^o + \cos 60^o - \cos 45^o) = ?$
$= \left(1 + \frac{1}{2} + \frac{1}{\sqrt{2}}\right)\left(1 + \frac{1}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{3}{2} + \frac{1}{\sqrt{2}}\right)\left(\frac{3}{2} - \frac{1}{\sqrt{2}}\right) = \left(\frac{9}{4} - \frac{1}{2}\right) = \frac{7}{4}$

8.

(d) cos 60^o

Explanation: $\frac{1-\tan^2 30^o}{1+\tan^2 30^o}$ = $\frac{1-(\frac{1}{\sqrt{3}})^2}{1+(\frac{1}{\sqrt{3}})^2}$

= $\frac{1-\frac{1}{3}}{1+\frac{1}{3}}$

= $\frac{\frac{2}{3}}{\frac{4}{3}}$

= $\frac{1}{2}$

= cos 60^o

9.

(d) 1

Explanation: We have, $\frac{x \csc^2 30^\circ \sec^2 45^\circ}{8 \cos^2 45^\circ \sin^2 60^\circ} = \tan^2 60^\circ - \tan^2 30^\circ$

$\Rightarrow \frac{x(2)^2(\sqrt{2})^2}{8\left(\frac{1}{\sqrt{2}}\right)^2\left(\frac{\sqrt{3}}{2}\right)^2} = (\sqrt{3})^2 - \left(\frac{1}{\sqrt{3}}\right)^2$

$\Rightarrow \frac{x \times 4 \times 2}{8 \times \frac{1}{2} \times \frac{3}{4}} = 3 - \frac{1}{3} \Rightarrow \frac{8x}{3} = \frac{8}{3}$

$\Rightarrow x = \frac{8}{3} \times \frac{3}{8} = 1$

10.

(d) -1

Explanation: $\frac{3}{4}$tan^{2}30^o - sec^{2}45^o + sin^{2}60^o

= $\frac{3}{4}(\frac{1}{\sqrt{3}})^2$ = $(\sqrt{2})^2$ + $(\frac{\sqrt{3}}{2})^2$

= $\frac{1}{4}$ - 2 + $\frac{3}{4}$

= $\frac{1-8+3}{4}$ = $\frac{4-8}{4}$ = -1

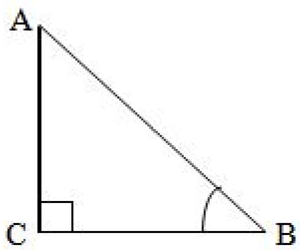
11. **(a)** 3

Explanation: 5 sin^2 90^o - 2 cos^2 0^o

5 × (1)^2 - 2 × (1)^2 = 5 - 2 = 3

12.

(b) 3 m

Explanation: 

Here, Height of the slide = AC = 1.5 m,

Angle of elevation = $\theta = 30^\circ$ To find: Length of slide = AB

$\therefore \sin 30^\circ = \frac{\text{AC}}{\text{AB}}$

$\Rightarrow \frac{1}{2} = \frac{1.5}{\text{AB}}$

$\Rightarrow \text{AB} = 3$ m

13.

(d) 20°

Explanation: $2\cos 3\theta = 1 \Rightarrow \cos 3\theta = \frac{1}{2} = \cos 60^\circ \quad \Rightarrow 3\theta = 60^\circ \Rightarrow \theta = 20^\circ$

14. **(a)** 30^o

Explanation: $\sqrt{3}\tan 2\theta - 3 = 0$

$\Rightarrow \sqrt{3}\tan 2\theta = 3$

$\Rightarrow \tan 2\theta = \frac{3}{\sqrt{3}}$

$\Rightarrow \tan 2\theta = \sqrt{3}$
$\Rightarrow \tan 2\theta = \tan 60^\circ$
$\Rightarrow 2\theta = 60^\circ$
$\Rightarrow \theta = 30^\circ$

15.

(c) $\frac{3}{4}$
Explanation: sin θ = $\frac{3}{4}$,
$\frac{(\sec^2\theta - 1)\cos^2\theta}{\sin\theta}$ = $\frac{tan^2\theta \times cos^2\theta}{sin\theta}$
= tan$^2\theta$ $\times$cotθ $\times$ cosθ
= tanθ $\times$ cosθ
= $\frac{sin\theta}{cos\theta}$ $\times$ cosθ
= sinθ
= $\frac{3}{4}$

16.

(b) tan 60^{o}

Explanation: We have $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$= $\frac{2\times\frac{1}{\sqrt{3}}}{1-(\frac{1}{\sqrt{3}})^2}$
= $\frac{2}{\sqrt{3}}$ $\times$ $\frac{3}{2}$
= $\frac{3}{\sqrt{3}} = \sqrt{3}$ = $\tan 60^\circ$

17.

(c) 45^{o}

Explanation: At A = 45^{o}

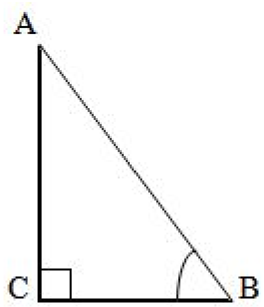
sin A = sin 45^{o} = $\frac{1}{\sqrt{2}}$

cos A = cos 45^{o} = $\frac{1}{\sqrt{2}}$

18. **(a)** $\frac{1}{2}$
Explanation: $\tan^2 45^\circ - \cos^2 30^\circ = x\sin 45^\circ \cos 45^\circ$
$\Rightarrow (1)^2 - \left(\frac{\sqrt{3}}{2}\right)^2 = x\frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \Rightarrow 1 - \frac{3}{4} = x \times \frac{1}{2}$
$\Rightarrow \frac{1}{4} = \frac{1}{2}x \Rightarrow x = \frac{1}{4} \times \frac{2}{1} = \frac{1}{2}$

19.

(d) 60°

Explanation: 

Given: $\angle C = 90^\circ$. If AC = $\sqrt{3}$ BC and$\angle B = \phi$,
$\therefore \tan\phi = \frac{\text{AC}}{\text{BC}}$
$\Rightarrow \tan\phi = \frac{\sqrt{3}\text{BC}}{\text{BC}}$ = $\sqrt{3}$
$\Rightarrow \tan\phi = \tan 60^\circ\ \phi$
$\Rightarrow \phi = 60^\circ$

20.

(b) 1

Explanation: We have, $\frac{\sec 30^\circ}{\csc 60^\circ} = \frac{\frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}}} = 1$

21. **(a)** $\sqrt{2}$

Explanation: Given: $\sin 45^\circ + \cos 45^\circ$

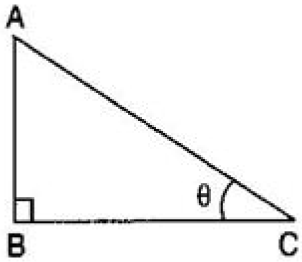
= $\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}$

= $\frac{2}{\sqrt{2}} = \sqrt{2}$

22.

(b) 60°

Explanation:



Given: $2\mathrm{AB} = \sqrt{3}\mathrm{AC}$

Let DC be θ

$\Rightarrow \mathrm{AB} = \frac{\sqrt{3}}{2}\mathrm{AC}$

$\therefore \sin\theta = \frac{\mathrm{AB}}{\mathrm{AC}}$

$\Rightarrow \sin\theta = \frac{\frac{\sqrt{3}}{2}\mathrm{AC}}{\mathrm{AC}}$

$\Rightarrow \sin\theta = \frac{\sqrt{3}}{2}$

$\Rightarrow \sin\theta = \sin 60^\circ \Rightarrow \theta = 60^\circ$

23.

(d) $\frac{1}{3}$

Explanation: Given: $\sqrt{3}\tan\theta = 3\sin\theta$

$\Rightarrow \sqrt{3}\frac{\sin\theta}{\cos\theta} = 3\sin\theta$

$\Rightarrow \frac{\sqrt{3}}{3} = \cos\theta$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{3}}$

And $\sin\theta = \sqrt{1-\cos^2\theta} = \sqrt{1-\frac{1}{3}} = \sqrt{\frac{2}{3}}$

$\therefore \sin^2\theta - \cos^2\theta = \frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

24.

(d) $sin 60^\circ$

Explanation: Given: $\frac{2\tan 30^\circ}{1+\tan^2 30^\circ}$

= $\frac{2\times\frac{1}{\sqrt{3}}}{1+\left(\frac{1}{\sqrt{3}}\right)^2}$

= $\frac{2}{\sqrt{3}\left(\frac{3+1}{3}\right)}$

= $\frac{6}{4\sqrt{3}} = \frac{3}{2\sqrt{3}}$

= $\frac{\sqrt{3}}{2}$

= $\sin 60^\circ$

25.

(b) 2

Explanation: Since $\sec\theta = \sqrt{1+\tan^2\theta}$

$\therefore \sec\theta = \sqrt{1+\left(\sqrt{3}\right)^2}$

= $\sqrt{1+3} = \sqrt{4}$ = 2

26.

(b) 0

Explanation: [$\frac{5}{8}$ sec^2 60^o - tan^2 60^o + cos^2 45^o]
= $\frac{5}{8}(2)^2$ - $(\sqrt{3})^2$ + $(\frac{1}{\sqrt{2}})^2$
= $\frac{5}{8}$ - 3 + $\frac{1}{2}$
= 0

27. **(a)** Both A and R are true and R is the correct explanation of A.

    Explanation: As ∠A + ∠B + ∠C = 180^o and ∠B = 90^o

    So, ∠A + ∠C = 90^o

    Hence, sin(A + C) = sin 90^o = 1

28. **(d)** A is false but R is true.

    Explanation: A is false but R is true.

29. **(a)** Both A and R are true and R is the correct explanation of A.

    Explanation: If sin θ = cos θ then θ = 45^o, so tan 45^o = 1

30. **(c)** 0°

    Explanation: sin 2A = 2 sin A is true when A =0°
    $\because \sin 2\mathrm{A} = 2 \sin \mathrm{A}$
    $\Rightarrow \sin(2 \times 0^\circ) = \sin 0^\circ$
    $\Rightarrow \sin 0^\circ = \sin 0^\circ$

31. **(d)** tan60^o

    Explanation: $\frac{2tan\ 30^0}{1-tan^2\ 30^0}$
    = $\frac{2\times\frac{1}{\sqrt{3}}}{1-(\frac{1}{\sqrt{3}})^2} = \frac{\frac{2}{\sqrt{3}}}{1-\frac{1}{3}}$
    = $\frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}}$ = $\frac{2}{\sqrt{3}} \times \frac{3}{2}$
    = $\sqrt{3}$
    = tan60^o

32. We know that, Sin30° =(1/2) =Cos60° , Cos45° = (1/√2), Sin90° = 1 & Sin60° =(√3/2), putting these values in the given expression , we get :-
    4(sin^{4}30^o + cos^{2}60^o) -3(cos^{2}45^o - sin^{2}90^o) - sin^{2}60^o
    $= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right] - \left(\frac{\sqrt{3}}{2}\right)^2$
    $= 4\left[\frac{1}{16} + \frac{1}{4}\right] - 3\left[\frac{1}{2} - 1\right] - \frac{3}{4}$
    $= 4 \times \frac{5}{16} - 3\left(\frac{-1}{2}\right) - \frac{3}{4}$
    $= \frac{5}{4} + \frac{3}{2} - \frac{3}{4}$
    $= \frac{5+6-3}{4} = \frac{8}{4} = 2$

33. We know that, cosec45°=√2, sec30°=(2/√3), sin30°=(1/2), cot45°=1 & sec60°=2, putting these values in the given expression, we get:-
    $\left(cosec^2 45^\circ \sec^2 30^\circ\right)\left(\sin^2 30^\circ + 4\cot^2 45^\circ - \sec^2 60^\circ\right)$
    $= \left[\left(\sqrt{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2\right]\left[\left(\frac{1}{2}\right)^2 + 4 \times (1)^2 - (2)^2\right]$
    $= \left[2 \times \frac{4}{3}\right]\left[\frac{1}{4} + 4 - 4\right]$
    $= \frac{8}{3} \times \frac{1}{4}$
    $= \frac{2}{3}$

34. 2 cos^{2}60° + 3sin^{2}45° - 3sin^{2}30° + 2cos^{2}90°
    $= 2 \times \left(\frac{1}{2}\right)^2 + 3 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 3 \times \left(\frac{1}{2}\right)^2 + 2(0)^2$

$= \frac{1}{2} + \frac{3}{2} - \frac{3}{4} \Rightarrow \frac{2+6-3}{4} = \frac{5}{4}$

35. Given, $\tan 5\theta = 1$ .Where, $0^\circ < \theta < 90$.

We have,

$\tan 5\theta = 1$

$\Rightarrow 5\theta = 45^\circ$

$\Rightarrow \theta = 9^\circ$

36. Given ,

$\sqrt{3}\tan 2x = \cos 60^\circ + \sin 45^\circ \cos 45^\circ$

$\Rightarrow \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$\Rightarrow \sqrt{3}\tan 2x = \frac{1}{2} + \frac{1}{2}$

$\Rightarrow \sqrt{3}\tan 2x = 1$

$\Rightarrow \tan 2x = \frac{1}{\sqrt{3}}$

$\Rightarrow$ tan 2x = tan30 ($Since, \tan 30^\circ = \sqrt{\frac{1}{3}}$)

$\Rightarrow 2x = 30^\circ$

$\Rightarrow x = 15^\circ$

37. To show:- tan2A =$\frac{2\tan A}{1-\tan^2 A}$

A = 30°

$\therefore$ To show:- tan 60° = $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$

Consider R.H.S. = $\frac{2\tan 30^\circ}{1-\tan^2 30^\circ}$ = $\frac{2\times\frac{1}{\sqrt{3}}}{1-\frac{1}{3}}$ = $\frac{2/\sqrt{3}}{2/3}$

= $\frac{3}{\sqrt{3}}$ = $\sqrt{3}$

= tan 60° = L.H.S.

$\therefore$ R.H.S. = L.H.S.

Hence, verified.

38. Given $2\cos^2\theta = \frac{1}{2}$ when $0^\circ < \theta < 90^\circ$.

Now we have,

$2\cos^2\theta = \frac{1}{2}$

$\Rightarrow \quad \cos^2\theta = \frac{1}{4}$

$\Rightarrow \quad \cos\theta = \frac{1}{2}$ $[\because \cos\theta > 0 \text{ for } 0^\circ < \theta < 90^\circ]$

$\Rightarrow \quad \theta = 60^\circ$

39. We know that, cot30°=√3, cos60°=(1/2), sec45°=√2 & sec30°=(2/√3), putting these values in the given expression, we get:-

$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$

$= (\sqrt{3})^2 - 2\left(\frac{1}{2}\right)^2 - \frac{3}{4}(\sqrt{2})^2 - 4\left(\frac{2}{\sqrt{3}}\right)^2$

$= 3 - 2 \times \frac{1}{4} - \frac{3}{4} \times 2 - 4 \times \frac{4}{3}$

$= \frac{3}{1} - \frac{1}{2} - \frac{3}{2} - \frac{16}{3}$

$= \frac{18-3-9-32}{6}$

$= \frac{18-44}{6}$

$= \frac{-26}{6}$

$= \frac{-13}{3}$

40. Put cos 30^{o} = $\frac{\sqrt{3}}{2}$, cos 60^{o} = $\frac{1}{2}$ & cos 90^{0} = 0

$\therefore$ cos 30^{0}cos60^{0}cos90^{0} = $\frac{\sqrt{3}}{2} \times \frac{1}{2} \times 0$ = 0.

41. We know that, Sin30° =(1/2), Cos45°=(1/√2), tan60°=√3, putting these values in the given expression, we get :-

$2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ$

$= 2\left(\frac{1}{2}\right)^2 - 3\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2$

$= \frac{2}{4} - \frac{3}{2} + 3$

$= \frac{2-6+12}{4}$

$= \frac{8}{4} = 2$

42. We know that, cos0°=1 =sin90°, sin45°=(1/√2)= cos45° & sin30°=(1/2)=cos60°, putting these values in the given expression, we get:-

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$

$= \left(\frac{2\sqrt{2}+2+\sqrt{2}}{2\sqrt{2}}\right)\left(\frac{2\sqrt{2}-2+\sqrt{2}}{2\sqrt{2}}\right)$

$= \left(\frac{3\sqrt{2}+2}{2\sqrt{2}}\right)\left(\frac{3\sqrt{2}-2}{2\sqrt{2}}\right)$

$= \frac{(3\sqrt{2})^2-(2)^2}{8}$ [Identity (a + b)(a - b) = a^2 - b^2]

$= \frac{18-4}{8}$

$= \frac{14}{8} = \frac{7}{4}$

43. According to question

$\frac{\sin 30^\circ - \sin 90^\circ + 2\cos 0^\circ}{\tan 30^\circ \tan 60^\circ}$

$= \frac{\frac{1}{2}-1+2\times 1}{\frac{1}{\sqrt{3}}\times\sqrt{3}}$

$= \frac{\frac{1}{2}-1+2}{1}$

$= \frac{3}{2}$

44. Given, cos 2x = cos60^ocos30^o + sin60^osin30^o

$\Rightarrow \cos 2x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$ [∵ Cos60°=Sin30°=(1/2), Sin60° = Cos30°= (√3/2)]

$\Rightarrow \cos 2x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$

$\Rightarrow \cos 2x = \frac{2\sqrt{3}}{4}$

$\Rightarrow \cos 2x = \frac{\sqrt{3}}{2}$

$\Rightarrow \cos 2x = \cos 30^\circ \quad \left[\because \cos 30^\circ = \frac{\sqrt{3}}{2}\right]$

$\Rightarrow 2x = 30^\circ$

$\Rightarrow x = \frac{30^\circ}{2}$

$\Rightarrow x = 15^\circ$

45. $\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

= $\frac{1}{2}$

46. Given;

sinθ - cosθ = 0

$\Rightarrow$ sinθ = cosθ

$\Rightarrow$ $\frac{\sin\theta}{\cos\theta}$ = 1

$\Rightarrow$ tanθ = 1

$\Rightarrow$ tanθ = tan 45^o

$\Rightarrow$ θ = 45^o

$\therefore$ $\sin^4\theta$ + $\cos^4\theta$

= $\sin^4 45^o$ + $\cos^4 45^o$

$= \left(\frac{1}{\sqrt{2}}\right)^4 + \left(\frac{1}{\sqrt{2}}\right)^4$

$= \frac{1}{4} + \frac{1}{4}$

= $\frac{1}{2}$

47. Given: tan (A-B) = $\frac{1}{\sqrt{3}}$ = tan 30^o

$\therefore$ A - B = 30^o ...(i)

tan (A + B) = $\sqrt{3}$ = tan 60^o ...(∵ tan 60^o = $\sqrt{3}$)

$\therefore$ A + B = 60^o ...(ii)

Adding (i) and (ii)

2A = 90^o

$\Rightarrow$ A = $\frac{90^\circ}{2}$ = 45^o

Subtracting (i) from (ii) gives

2B = 30^o

$\Rightarrow \angle B = \frac{30^\circ}{2} = 15^o$

Hence $\angle A = 45^o$, and $\angle B = 15^o$

48. Given ;

$2\cos^2(\frac{A}{2}) = 1$

$\Rightarrow \cos^2(\frac{A}{2}) = \frac{1}{2}$

$\Rightarrow \cos \frac{A}{2} = (\sqrt{\frac{1}{2}})$ (taking square root both sides)

$\Rightarrow \cos \frac{A}{2} = \frac{1}{\sqrt{2}}$

$\Rightarrow \cos \frac{A}{2} = \cos 45^0$

$\Rightarrow \frac{A}{2} = 45^o$

$\therefore A = 45^o \times 2 = 90^o$

49. We have, cosec 30° + cot 45° = 2 + 1 = 3

50. We have, $\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + (0)^2$

$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4} + 0$

$= \frac{3+2+1+0}{4}$

$= \frac{6}{4} = \frac{3}{2}$

51. According to question we have ,

$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$

$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$

$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$

$= \frac{1+2+3+4}{4}$

$= \frac{10}{4}$

$= \frac{5}{2}$

52. We know that , tan60° =√3 =Cot30° , Cos45° =(1/√2), Sec 30° =(2/√3), Cos90° =0, Cosec30° = 2 =Sec60°, putting these values in the given expression , we get :-

$\frac{\tan^2 60^\circ + 4\cos^2 45^\circ + 3\sec^2 30^\circ + 5\cos^2 90^\circ}{\cos ec 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$

$= \frac{(\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3\times\left(\frac{2}{\sqrt{3}}\right)^2 + 5(0)^2}{2+2-(\sqrt{3})^2}$

$= \frac{3+4\times\frac{1}{2}+3\times\frac{4}{3}+5\times 0}{2+2-3}$

$= \frac{3+2+4+0}{4-3}$

$= \frac{9}{1} = 9$

Section B

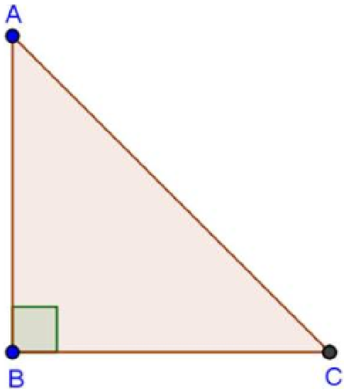
53. Fill in the blanks:

(i) 1. 2

(ii) 1. 1

Section C

54.



Given $\angle B = 90^\circ$

and $\angle A = \angle C = x^\circ (Let)$

In ΔABC, by angle sum property

$\angle A + \angle B + \angle C = 180^\circ$

$\Rightarrow x + 90^\circ + x + 180^\circ$

$\Rightarrow 2x = 180^\circ - 90^\circ$

$\Rightarrow x = \frac{90}{2} = 45^\circ$

$\therefore \angle A = \angle C = 45^\circ$

i. We have

sin A cos C + cos A sin C

$= \sin 45^\circ \cos 45^\circ + \cos 45^\circ \sin 45^\circ$

$= \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$

$= \frac{1}{2} + \frac{1}{2} = 1$

ii. We have

sin A sin B + cos A cos B

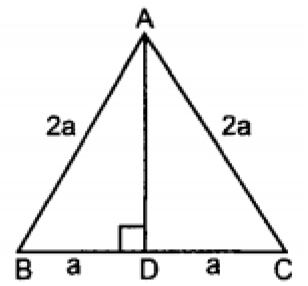
$= \sin 45^\circ \sin 90^\circ + \cos 45^\circ \cos 90^\circ$

$= \frac{1}{\sqrt{2}} \times 1 + \frac{1}{\sqrt{2}} \times 0$

$= \frac{1}{\sqrt{2}} + 0$

$= \frac{1}{\sqrt{2}}$

55.



Let $\triangle$ABC is an equilateral A with each side = 2a units. Draw $\mathrm{AD} \perp \mathrm{BC}$

$\therefore$ D is mid-point of BC

$\Rightarrow$ BD = a

In right $\triangle$ADB

$AB^2 = BD^2 + AD^2$

$\Rightarrow (2a)^2 = a^2 + AD^2$

$\Rightarrow 4a^2 - a^2 = AD^2$

$\Rightarrow$ AD = $\sqrt{3a^2} = \sqrt{3}a$

Now in right $\triangle$ADB

tan B $= \frac{\mathrm{AD}}{\mathrm{BD}}$

$\Rightarrow$ tan $60^\circ = \frac{\sqrt{3}a}{a}$ $(\because \angle B = 60^\circ)$

$\Rightarrow$ tan $60^\circ = \sqrt{3}$

56. We have,

$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45^\circ$ …(1)

Now,

$\sin 60^\circ = \cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 45^\circ = \frac{1}{\sqrt{2}}, \tan 60^\circ = \sqrt{3}, \tan 45^\circ = 1$

So by substituting above values in equation (1)

We get,

$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45^\circ$

$= 4\left(\left(\frac{\sqrt{3}}{2}\right)^4 + \left(\frac{\sqrt{3}}{2}\right)^4\right) - 3\left((\sqrt{3})^2 - 1^2\right) + 5 \times \left(\frac{1}{\sqrt{2}}\right)^2$

$= 4\left(\frac{(\sqrt{3})^4}{2^4} + \frac{(\sqrt{3})^4}{2^4}\right) - 3(3-1) + 5 \times \frac{1^2}{(\sqrt{2})^2}$

$= 4\left(\frac{9}{16} + \frac{9}{16}\right) - 3(2) + 5 \times \frac{1}{2}$

$= 4\left(\frac{9+9}{16}\right) - 6 + \frac{5}{2}$

$= 4\left(\frac{18}{16}\right) - 6 + \frac{5}{2}$

Now, $\frac{18}{16}$ gets reduced to $\frac{9}{8}$

Therefore,

$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45$

$= 4\left(\frac{9}{8}\right) - 6 + \frac{5}{2}$

$= \frac{36}{8} - 6 + \frac{5}{2}$

Now, $\frac{36}{8}$ gets reduced to $\frac{9}{2}$

Therefore,

$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45$

$= \frac{9}{2} - 6 + \frac{5}{2}$

Now by taking LCM

We get,

$-4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45$

$= \frac{9}{2} - \frac{6\times 2}{1\times 2} + \frac{5}{2}$

$= \frac{9}{2} - \frac{12}{2} + \frac{5}{2}$

$= \frac{9-12+5}{2}$

$= \frac{14-12}{2}$

$= \frac{2}{2}$

=1

Therefore,

$4\left(\sin^4 60^\circ + \cos^4 30^\circ\right) - 3\left(\tan^2 60^\circ - \tan^2 45^\circ\right) + 5\cos^2 45^\circ = 1$

57. $\sin^2 30° \cos^2 45° + 4 \tan^2 30° + \frac{1}{2}\sin^2 90° - 2 \cos^2 90° + \frac{1}{24}$

put $sin 30^\circ = \frac{1}{2}, cos 45^\circ = \frac{1}{\sqrt{2}}, tan 30^\circ = \frac{1}{\sqrt{3}}, sin 90^\circ = 1, cos 90^\circ = 0$

$= \left(\frac{1}{2}\right)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 + 4\left(\frac{1}{\sqrt{3}}\right)^2 + \frac{1}{2}(1)^2 - 2(0) + \frac{1}{24}$

$= \frac{1}{4} \times \frac{1}{2} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

$= \frac{1}{8} + \frac{4}{3} + \frac{1}{2} + \frac{1}{24}$

$= \frac{3+32+12+1}{24}$

$= \frac{48}{24}$

= 2

58. $\tan^2 30° \sin 30° + \cos 60° \sin^2 90° \tan^2 60° - 2 \tan 45° \cos^2 0° \sin 90°$

put $tan 30^\circ = \frac{1}{\sqrt{3}}, sin 30^\circ = \frac{1}{2}, cos 60^\circ = \frac{1}{2}, sin 90^\circ = 1, tan 60^\circ = \sqrt{3}, tan 45^\circ = 1, cos 0^\circ = 1$

$= \left(\frac{1}{\sqrt{3}}\right)^2 \times \frac{1}{2} + \frac{1}{2} \times (1)^2 \times (\sqrt{3})^2 - 2 \times 1 \times 1^2 \times 1$

= $\frac{1}{3} \times \frac{1}{2} + \frac{1}{2} \times 1 \times 3 \times$ - $2 \times 1 \times 1 \times 1$

$= \frac{1}{6} + \frac{3}{2} - 2$

$= \frac{1+9-12}{6}$

$= \frac{10-12}{6} = -\frac{2}{6} = -\frac{1}{3}$

59. We have,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90 - \cos 45 + \cos 60^\circ)$ ...... (1)

Now,

$\sin 90^\circ = \cos 0^\circ = 1, \sin 45^\circ = \cos 45^\circ = \frac{1}{\sqrt{2}}, \sin 30^\circ = \cos 60^\circ = \frac{1}{2}$

So by substituting above values in equation (1)

We get, $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \left(1 + \frac{1}{\sqrt{2}} + \frac{1}{2}\right)\left(1 - \frac{1}{\sqrt{2}} + \frac{1}{2}\right)$

Now, LCM of both the product terms in the above expression is $2\sqrt{2}$

Therefore we get,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \left(\frac{1\times 2\sqrt{2}}{1\times 2\sqrt{2}} + \frac{1\times 2}{\sqrt{2}\times 2} + \frac{1\times\sqrt{2}}{2\times\sqrt{2}}\right) \times \left(\frac{1\times 2\sqrt{2}}{1\times 2\sqrt{2}} - \frac{1\times 2}{\sqrt{2}\times 2} + \frac{1\times\sqrt{2}}{2\times\sqrt{2}}\right)$

$= \left(\frac{2\sqrt{2}}{2\sqrt{2}} + \frac{2}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}}\right) \times \left(\frac{2\sqrt{2}}{2\sqrt{2}} - \frac{2}{2\sqrt{2}} + \frac{\sqrt{2}}{2\sqrt{2}}\right)$

$= \left(\frac{2\sqrt{2}+2+\sqrt{2}}{2\sqrt{2}}\right) \times \left(\frac{2\sqrt{2}-2+\sqrt{2}}{2\sqrt{2}}\right)$

Now by rearranging terms in the numerator of above expression

We get,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \left(\frac{2\sqrt{2}+\sqrt{2}+2}{2\sqrt{2}}\right) \times \left(\frac{2\sqrt{2}+\sqrt{2}-2}{2\sqrt{2}}\right)$

$= \frac{(2\sqrt{2}+\sqrt{2}+2)\times(2\sqrt{2}+\sqrt{2}-2)}{(2\sqrt{2})\times(2\sqrt{2})}$

Now, by applying formula $\left[(a+b)(a-b) = a^2 - b^2\right]$ in the numerator of the above expression we get,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \frac{(2\sqrt{2}+\sqrt{2})^2 - 2^2}{2\times 2\times\sqrt{2}\times\sqrt{2}}$

$= \frac{(2\sqrt{2}+\sqrt{2})^2 - 2^2}{4\times 2}$ ...... (2)

Now, we know that $(a+b)^2 = a^2 + 2ab + b^2$

Therefore, $(2\sqrt{2}+\sqrt{2})^2 = (2\sqrt{2})^2 + 2(2\sqrt{2})(\sqrt{2}) + (\sqrt{2})^2$

Now, by substituting the above value of $(2\sqrt{2}+\sqrt{2})^2$ in equation (2)

We get, $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45 + \cos 60^\circ)$

$= \frac{\left[(2\sqrt{2})^2 + 2(2\sqrt{2})(\sqrt{2}) + (\sqrt{2})^2\right] - 2^2}{4\times 2}$

$= \frac{[8+8+2]-4}{8}$

$= \frac{18-4}{8}$

$= \frac{14}{8}$

Now $\frac{14}{8}$ gets reduced to $\frac{7}{4}$

Therefore,

$(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ)$

$= \frac{7}{4}$

Hence, $(\cos 0^\circ + \sin 45^\circ + \sin 30^\circ)(\sin 90^\circ - \cos 45^\circ + \cos 60^\circ) = \frac{7}{4}$

60. $\frac{(\sqrt{3})^2 + 4\left(\frac{1}{\sqrt{2}}\right)^2 + 3(2)^2 + 5(0)^2}{2+2-(\sqrt{3})^2}$

$= \frac{3+2+12+0}{4-3}$

= 17

61. We have,

$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30$ ...(1)

Now,

$\cot 30^\circ = \sqrt{3}, \cos 60^\circ = \frac{1}{2}, \sec 45^\circ = \sqrt{2}, \sec 30^\circ = \frac{2}{\sqrt{3}}$

So by substituting above values in equation (1)

We get,

$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45^\circ - 4\sec^2 30^\circ$

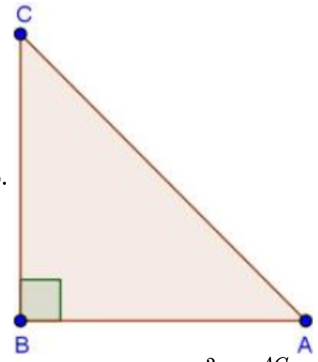
$= (\sqrt{3})^2 - 2\left(\frac{1}{2}\right)^2 - \frac{3}{4}(\sqrt{2})^2 - 4\left(\frac{2}{\sqrt{3}}\right)^2$
$= 3 - 2 \times \frac{1^2}{2^2} - \frac{3}{4} \times 2 - 4 \times \frac{2^2}{(\sqrt{3})^2}$
$= 3 - 2 \times \frac{1}{4} - \frac{3}{2} - 4 \times \frac{4}{3}$
$= 3 - \frac{1}{2} - \frac{3}{2} - \frac{16}{3}$
$= \frac{3\times6}{1\times6} - \frac{1\times3}{2\times3} - \frac{3\times3}{2\times3} - \frac{16\times2}{3\times2}$
$= \frac{18}{6} - \frac{3}{6} - \frac{9}{6} - \frac{32}{6}$
$= \frac{18-3-9-32}{6}$
$= \frac{18-12-32}{6}$
$= \frac{18-44}{6}$
$= \frac{-26}{6}$
Therefore,
$\cot^2 30^\circ - 2\cos^2 60^\circ - \frac{3}{4}\sec^2 45 - 4\sec^2 30^\circ = \frac{-13}{3}$

62. Given sec (B + C - A) = 2 ⇒ B + C - A = 60^o ...(1)
and tan (C + A - B) = $\frac{1}{\sqrt{3}}$ ⇒ C + A - B = 30^o ...(2)
Also, A + B + C = 180^o ...(3)
Solving (1), (2) and (3), we get
A = 60^o, B = 75^o and C = 45^o

63. 4($\sin^4 30°$ + $\cos^4 60°$) - 3($\cos^2 45°$- $\sin^2 90°$)
$sin30^\circ = \frac{1}{2}, cos60^\circ = \frac{1}{2}, sin90^\circ = 1, cos45^\circ = \frac{1}{\sqrt{2}}$
= $4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right]$
= $4\left[\frac{1}{16} + \frac{1}{16}\right] - 3\left[\frac{1}{2} - 1\right]$
= $4 \times \frac{2}{16} - 3 \times -\frac{1}{2}$
= $\frac{1}{2} + \frac{3}{2}$
= $\frac{4}{2}$
$= 2$

64. $\frac{2\cos^2 60^\circ + 3\sec^2 30^\circ - 2\tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 45^\circ}$
$cos60^\circ = \frac{1}{2}, sec30^\circ = \frac{2}{\sqrt{3}}, tan45^\circ = 1, sin30^\circ = \frac{1}{2}, cos45^\circ = \frac{1}{\sqrt{2}}$
= $\frac{2\left(\frac{1}{2}\right)^2 + 3\left(\frac{2}{\sqrt{3}}\right)^2 - 2(1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2}$
$= \frac{\frac{2}{4} + 4 - 2}{\frac{1}{4} + \frac{1}{2}}$
$= \frac{\frac{2+16-8}{4}}{\frac{1+2}{4}}$
$= \frac{10}{3}$

65. We have,
$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$
$= \left(\frac{1}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 + (1)^2$
$= \frac{1^2}{2^2} + \frac{1^2}{(\sqrt{2})^2} + \frac{(\sqrt{3})^2}{2^2} + 1$
$= \frac{1}{4} + \frac{1}{2} + \frac{3}{4} + 1$
Now by taking denominator 4 together and simplifying
We get,
$\sin^2 30^\circ + \sin^2 45^\circ + \sin^2 60^\circ + \sin^2 90^\circ$
= $\frac{10}{4} = \frac{5}{2}$

66. 

Given $\cos ec A = \frac{2}{1} = \frac{AC}{BC}$

Let AC = 2K

and, BC = 1K

In ΔABC, by Pythagoras theorem

$BC^2 + AB^2 = AC^2$

$(1K)^2 + AB^2 = (2K)^2$

$K^2 + AB^2 = 4K^2$

$AB^2 = 4K^2 - K^2 = 3K^2$

$AB = \sqrt{3K^2} = \sqrt{3}K$

$\therefore \tan A = \frac{BC}{AB} = \frac{1K}{\sqrt{3}K} = \frac{1}{\sqrt{3}}$

$\sin A = \frac{BC}{AC} = \frac{1K}{2K} = \frac{1}{2}$

$\cos A = \frac{AB}{AC} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$

Now, $\frac{1}{\tan A} + \frac{\sin A}{1+\cos A}$

$= \frac{1}{\frac{1}{\sqrt{3}}} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}$

$= \frac{\sqrt{3}}{1} + \frac{\frac{1}{2}}{\frac{2+\sqrt{3}}{2}}$

$= \frac{\sqrt{3}}{1} + \frac{1}{2} \times \frac{2}{2+\sqrt{3}}$

$= \frac{\sqrt{3}}{1} + \frac{1}{2+\sqrt{3}}$

$= \frac{2\sqrt{3}+3+1}{2+\sqrt{3}}$

$= \frac{2\sqrt{3}+4}{2+\sqrt{3}}$

$= \frac{2(\sqrt{3}+2)}{2+\sqrt{3}} = 2$

Section D

67. State True or False:

(i) **(a)** True

Explanation: True

(ii) **(b)** False

Explanation: False

(iii) **(b)** False

Explanation: False. The value of $(\sin\theta + \cos\theta)$ $\theta = 0^{o}$ is 1

(iv) **(a)** True

Explanation: True

(v) **(b)** False

Explanation: False

(vi) **(b)** False

Explanation: False

(vii) **(b)** False

Explanation: False

(viii) **(a)** True

Explanation: True