Solution

ARITHMETIC PROGRESSION WS 1

Class 10 - Mathematics

Section A

1. **(a)** 3
   Explanation: If the numbers x, 2x + k, 3x + 6 are in A.P.,
   then, 2x + k - x = 3x + 6 - 2x - k
   $\Rightarrow$ x + k = x + 6 - k
   $\Rightarrow$ 2k = 6
   $\Rightarrow$ k = 3
2.
   (b) 5
   Explanation: 2 (3y + 5) = 3y – 1 + 5y + 1
   (If a, b, c are in A.P., b – a = c – b $\Rightarrow$ 2b = a + c)
   $\Rightarrow$ 6y + 10 = 8y
   $\Rightarrow$ 10 = 2y
   $\Rightarrow$ y = 5
3.
   (d) 3
   Explanation: 3
4.
   (b) 30^o, 60^o, 90^o
   Explanation: Let the three angles of a triangle be a - d, a and a + d
   $\therefore$ a - d + a + a + d = 180^o
   $\Rightarrow$ 3a = 180^o
   $\Rightarrow$ a = 60^o
   Therefore, one angle is of 60^o and other is 90^o (given).
   Let the third angle be x^o, then
   $60^o + 90^o + x^o = 180^o$
   $\Rightarrow 150^o + x^o = 180^o$
   $\Rightarrow x^o = 180^o - 150^o = 30^o$
   Therefore, the angles of the right-angled triangle are 30^o, 60^o, 90^o
5. **(a)** 0
   Explanation: Given: a, b, c, l, m are in A.P.
   Therefore,
   a + m = 2c ... (i)
   b + l = 2c ... (ii)
   a - 4b + 6c - 4l + m
   = a + m + 6c - 4b - 4l
   = a + m + 6c - 4(b + l)
   substituting from (i) and (ii)
   = 2c + 6c - 8c
   = 0
6.
   (d) -b
   Explanation: A.P. is $\frac{1}{3}, \frac{1-3b}{3}, \frac{1-6b}{3}, \ldots$
   $\Rightarrow \frac{1}{3}, \frac{1}{3} - \frac{3b}{3}, \frac{1}{3} - \frac{6b}{3}, \ldots$

$\Rightarrow \frac{1}{3}, \frac{1}{3} - b, \frac{1}{3} - 2b, \ldots$

$\therefore d = \left(\frac{1}{3} - b\right) - \frac{1}{3} = \frac{1}{3} - b - \frac{1}{3} = -b$

7.

(c) Arithmetic Progression

Explanation: Progressions with an equal common difference are known as Arithmetic Progression. i.e the difference between any two consecutive terms is constant throughout the series, this constant difference is called common difference usually denoted by the letter d

if a is the 1st term, d is a common difference, then the AP is represented by a, a+d, a+2d, a+3d ... a + (n - 1)d

8. **(a)** 5k + 4 and 6k + 5

Explanation: Given: k, 2k + 1, 3k + 2, 4k + 3, ...

Here, d = 2k + 1 - k = k + 1

Therefore, the next two terms are

4k + 3 + k + 1 = 5k + 4 and 5k + 4 + k + 1 = 6k + 5

9. **(a)** 0

Explanation: 0

10. **(a)** 3

Explanation: (2k – 1) – k = (2k + 1) – (2k - 1)

2k – 1 – k = 2

⇒ k = 3

11. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

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14. Clearly, (6 -11) = (1 - 6) = (-4 -1) = (-9 + 4) = -5 (constant).

Thus, each term differs from its preceding term by -5.

So, the given progression is an A.P.

Its first term = 11 and common difference = -5.

15. Given AP = $\frac{3}{2}, \frac{1}{2}, \frac{-1}{2}, \frac{-3}{2}$ ...

1^{st} term = $\frac{3}{2}$

Common difference = $\frac{1}{2} - \frac{3}{2}$ = -1

16. For consecutive terms of AP,

$2(2k^2 + 3k + 6) = (3k^2 + 4k + 4) + (4k + 8)$

$\Rightarrow 4k^2 + 6k + 12 = 3k^2 + 8k + 12$

⇒ k^2 - 2k = 0

⇒ k(k - 2) = 0 ⇒ k =0 or k = 2

17. Given AP: 75, 67, 59, 51,...

Common difference = 67 - 75 = -8

5th term = 51 - 8 = 43

6th term = 43 - 8 = 35

18. Given progression is: 51, 59, 67, 75,...

Common difference being the difference between the two consecutive terms,

Common difference = 59 - 51 = 8

5th term = 75 + 8 = 83

6th term = 83 + 8 = 91

19. Given 18, a, b-3 are in A.P

$d = a - 18 = (b - 3) - a$

Now $a - 18 = b - 3 - a$.

$\Rightarrow 2a - b = 18 - 3$

∴ 2a - b = 15.

20. ∵ $\frac{3}{5}$, a, 4 are three consecutive terms of AP

    So, d = a_2 - a_1 = a - $\frac{3}{5}$ ...(i)

    Also d = a_3 - a_2 = 4 - a ...(ii)

    From equation (i) & (ii)

    a - $\frac{3}{5}$ = 4 - a

    2a = $4 + \frac{3}{5} = \frac{20+3}{5}$

    a = $\frac{23}{10}$ = 2.3

21. We have a_1 = 1 , a_2 = 1, a_3 = 2 and a_4 = 2

    a_2 - a_1 = 0

    a_3 - a_2 = 1

    Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

22. Given progression is: -1.1, - 3.1, -5.1, - 7.1,...

    First term (a) = -1.1

    We know that common difference is difference between any two consecutive terms of an A.P.

    So, common difference(d) = (-3.1) - (-1.1)

    = -3.1 + 1.1

    = -2

23. Here,

    $a = \frac{1}{3q}$, $a + d = \frac{1-6q}{3q}$

    d = $\frac{1-6q}{3q} - a$

    $\therefore \quad d = \frac{1-6q}{3q} - \frac{1}{3q}$

    $= \frac{1-6q-1}{3q}$

    $= \frac{-6q}{3q}$

    $= -2$

24. Here the given series is:

    $\sqrt{2}$,$\sqrt{8}$, $\sqrt{18}$,....

    This series can be written as:

    $\sqrt{2}$, $2\sqrt{2}$, $3\sqrt{2}$,,.....

    Therefore an A.P is formed.

    Here a = $\sqrt{2}$ and d = $\sqrt{2}$

    ∴ Next term, T_4 = a + 3d = $\sqrt{2}$ + $3\sqrt{2}$ = $4\sqrt{2}$ = $\sqrt{32}$

25. $a_1 = a - b$,

    $a_2 = a$,

    $a_3 = a + b$

    Now, $a_2 - a_1 = a - (a - b) = b$

    and a_3 - a_2 = (a + b) - a = b

    ∵ a_2 - a_1 = a_3 - a_2

    ∴ Given terms are the consecutive terms in AP.

26. Here a_1 = $\frac{1}{5}$, a_2 = $\frac{4}{5}$

    Common difference (d) = a_2 - a_1 = $\frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

    d = $\frac{3}{5}$

27. Given progression is: $0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$

    We know that common difference of an A.P is a difference between the consecutive terms of an A.P. So,

    Common difference = $\frac{1}{4} - 0 = \frac{1}{4}$

    5th term = $\frac{3}{4} + \frac{1}{4} = \frac{4}{4} = 1$

    6th term = $1 + \frac{1}{4} = \frac{5}{4}$

28. Given A.P is

119, 136, 153, 170........

We know that common difference is the difference between any two consecutive terms of an A.P.

So, common difference = 136 - 119 = 17

5th term = 170 + 17 = 187 (a_5 = a + 4d)

6th term = 187 + 17 = 204. (a_6 = a + 5d)

29. If T_1, T_2, T_3 are consecutive terms of an AP, then

$T_2 - T_1 = T_3 - T_2 \; or \; 2T_2 = T_1 + T_3$

$\therefore x + 2, 2x, 2x + 3$ are in AP, if

$2(2x) = x + 2 + 2x + 3$

⇒ 4x = 3x + 5 ⇒ x = 5

30. $a_1 = 1$

$d = a_2 - a_1 = -1 - 1 = -2$

$a_5 = a_1 + 4d$

= 1 + (4)(-2) = 1 - 8 = -7

a_6 = a_5 + d = -7 - 2 = -9

Next two terms are -7 and -9.

31. a = -1.5 , d = -0.5

The first five terms of the arithmetic progression are

= a, a + d, a + 2d, a + 3d, a + 4d...

= (-1.5), (-1.5) + (-0.5), (-1.5) + 2(-0.5), (-1.5) + 3(-0.5)...

= -1.5, -2, -2.5, -3....

32. So, $(2k - 1) - k = (2k + 1) - (2k - 1)$

$k - 1 = 2$

$k = 2 + 1$

$k = 3$

33. Since $(5x + 2), (4x - 1) \; and \; (x + 2)$ are in AP, we have

$(4x - 1) - (5x + 2) = (x + 2) - (4x - 1)$

$\Rightarrow 4x - 1 - 5x - 2 = x + 2 - 4x + 1$

$\Rightarrow -x - 3 = -3x + 3$

⇒ 2x=6

⇒ x=3.

34. 0.2, 0.22, 0.222, 0.2222, 000

a_2 - a_1 = 0.22 - 0.2 = 0.02

a_3 - a_2 = 0.222 - 0.22 = 0.002

As a_2 -a_1 ≠ a_3 - a_2, the given list of numbers does not form an AP.

35. If $2k + 1, 3k + 3, 5k - 1$ are in A.P.

then $(5k - 1) - (3k + 3) = (3k + 3) - (2k + 1)$

or, $5k - 1 - 3k - 3 = 3k + 3 - 2k - 1$

or, $2k - 4 = k + 2$

or, $2k - k = 4 + 2$

or $k = 6$

36. We have a_1 = 11, a_2 = 22 and a_3 = 33

a_2 - a_1 = 11

a_3 - a_2 = 11

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

37. We know that if a is the first term and d is the common difference, then the arithmetic progression is

a,a + d, a + 2d, a + 3d,...

Here, a = 10 and d = 3.

a = 10

a + d = 10 + 3 = 13

a + 2d = 10 + 2(3) = 10 + 6 = 16

a + 3d = 10 + 3(3) = 10 + 9 = 19

....

So, the arithmetic progression is 10,13,16,19,22,...

38. We have $a_1 = \frac{1}{2}$, $a_2 = \frac{1}{3}$ and $a_3 = \frac{1}{4}$

$a_2 - a_1 = \frac{-1}{6}$

$a_3 - a_2 = \frac{-1}{12}$

Clearly, the difference of successive terms is not same, therefore given list of numbers does not form an AP.

39. 3, 1, -1, -3, ...

First term (a) = 3

Common difference (d) = 1 - 3 = -2

40. Since $(2n-1), (3n+2)\ and\ (6n-1)$ are in AP, we have

$(3n+2)-(2n-1)=(6n-1)-(3n+2)$

$\Rightarrow n+3=3n-3$

$\Rightarrow$ 2n = 6 $\Rightarrow$ n= 3.

Hence, n = 3 and these numbers are 5,11 and 17.

41. Let (2p - 1), 7 and 3p be three consecutive terms of an A.P.

Then 7 - (2p - 1) = 3p - 7

5p = 15

p = 3

42. Middle term of an AP is arithmetic mean of AP.

$\Rightarrow$ a is the arithmetic mean.

Arithmetic mean of an AP

$=(4/5+2)/2=7/5$

$\Rightarrow a = \frac{7}{5}$.

43. Since $(3y-1), (3y+5)\ and\ (5y+1)$ are in AP, we have

$(3y+5)-(3y-1)=(5y+1)-(3y+5)$

$\Rightarrow 3y+5-3y+1=5y+1-3y-5$

$\Rightarrow$ 6 = 2y - 4

$\Rightarrow$ 2y = 10

$\Rightarrow$ y = 5

44. We have a_1 = - 1 , a_2 = - 1, a_3 = - 1 and a_4 = - 1

$a_2 - a_1 = 0$

$a_3 - a_2 = 0$

$a_4 - a_3 = 0$

Clearly, the difference of successive terms is same, therefore given list of numbers form an AP.

45. The fee charged from a student every month by a school for the whole session is

400, 400, 400, 400, …

which from an AP, with common difference , d = 400 – 400 = 0

46. The given progression is 8,11,14,17, 20,....

Clearly, (11-8) = (14-11) = (17 -14) = (20 -17) = 3 (constant).

Thus, each term differs from its preceding term by 3.

So, the given progression is an A.P.

Its first term= 8 and common difference = 3.

47. 0.6, 1.7, 2.8, 3.9...

First term = a= 0.6

Common difference (d) = Second term -First term

= Third term - Second term and so on

Therefore, Common difference (d) = 1.7−0.6=1.1

48. $\frac{1}{3}, \frac{5}{3}, \frac{9}{3}, \frac{13}{3}, \ldots\ldots$

First term (a) $= \frac{1}{3}$

Common difference (d) $= \frac{5}{3} - \frac{1}{3} = \frac{4}{3}$

49. Taxi fare for 1 km = Rs 15 = a_1

Taxi Fare for 2 kms
$= Rs15 + Rs8 = Rs23 = a_2$
Taxi fare for 3 km s
$= Rs23 + Rs8 = Rs31 = a_3$
Taxi fare for 4 kms
$= RS31 + Rs8 = RS39 = a_4$
and so on
$a_2 - a_1 = Rs.23 - Rs.15 = Rs8$
$a_3 - a_2 = Rs31 - Rs \cdot 23 = Rs8$
$a_4 - a_3 = Rs39 - Rs31 = Rs8$
So, the arithmetic progression formed is:-
i.e.,a_{k+1}-a_k is the same every time.

So, this list of numbers form an arithmetic
Progression with the first term a = Rs 15 and
the common difference d = Rs 8.

50. Given A.P is: -5, - 1, 3, 7, ...
First term (a) = -5
Common difference (d) = -1 - (-5) = -1 + 5 = 4

51. a = -1, $d = \frac{1}{2}$
First term = a = -1
Second term = -1 + d $= -1 + \frac{1}{2} = -\frac{1}{2}$
Third term $= -\frac{1}{2} + d = -\frac{1}{2} + \frac{1}{2} = 0$
Fourth term = 0 + d $= 0 + \frac{1}{2} = \frac{1}{2}$
Hence, the first four terms of the given AP are -1, $-\frac{1}{2}, 0, \frac{1}{2}$.

Section B

52. Cost of digging the well after 1 metre of digging = Rs 150 = a_1

Cost digging the well after 2 metres of digging
= Rs 150 + Rs 50 = Rs 200 = a_2

Cost of digging the well after 3 metres of digging
= Rs 200 + Rs 50 = Rs 250 = a_3

Cost of digging the well after 4 metres of digging
= Rs 250 + Rs 50 = Rs 300 = a_4

and so on.

$a_2 – a_1$ = Rs 200 – Rs 150 = Rs 50

$a_3 – a_2$ = RS 250 – Rs 200 = Rs 50

$a_4 – a_3$ = Rs 300 – Rs 250 = Rs 50

i.e. $a_{k + 1}$ – a is the same everytime.

So this list of numbers forms an AP with the first term a = Rs 150
and the common difference d = Rs 50.

53. Amount of money after 1 year $= Rs10000\left(1 + \frac{8}{100}\right) = a_1$

Amount of money after 2 year $= Rs10000\left(1 + \frac{8}{100}\right)^2 = a_2$

Amount of money after 3 year $= Rs10000\left(1 + \frac{8}{100}\right)^3 = a_3$

Amount of money after 4 year $= Rs10000\left(1 + \frac{8}{100}\right)^4 = a_4$

$a_2 - a_1 = Rs10000\left(1 + \frac{8}{100}\right)^2 - Rs10000\left(1 + \frac{8}{100}\right)$
$= Rs10000\left(1 + \frac{8}{100}\right)\left(1 + \frac{8}{100} - 1\right)$
$= 10000(1 + \frac{8}{100})(\frac{8}{100})$

a_3-a_2

$= 10000\left(1+\frac{8}{100}\right)^2 - 10000\left(1+\frac{8}{100}\right)$

$= 10000\left(1+\frac{8}{100}\right)\left(1+\frac{8}{100}-1\right)$

$= 10000\left(1+\frac{8}{100}\right)\left(\frac{8}{100}\right)$

Since.$a_3 - a_2 \neq a_2 - a_1$ .It does not form AP.

54. $a, 7, b, 23$ and c are in A.P.

Let the common difference be d.

$a + d = 7$ ....(i)

$a + 3d = 23$ ....(ii)

From (i) and (ii), we get

$2d = 16$

$d = 8$

Put d = 8 in (1) we get

$a + 8 = 7$

$a = -1$

$b = a + 2d$

$b = -1$ + 2 $\times$ 8

or,$b = -1 + 16$

or, $b = 15$

$c = a + 4d$

= - 1 + 4 $\times$ 8

= - 1 + 32

$c = 31$

$\therefore a = -1, b = 15, c = 31$

55. Let the volume of the cylinder be 16 litres(a_1).

Air removed by pump = $\frac{1}{4} \times 16 = 4\ litres$

Air present after first removal = 16 - 4 = 12 litres(a_2)

Air again removed = $\frac{1}{4} \times 12 = 3 litres$

Air present after second removal = 12 - 3 = 9 litres(a_3)

The amount of air present in the cyinder is the series

16,12,9.....

a_2 - a_1 = 12-16 = -4

a_3 - a_2 = 9-12 = -3

Since the difference is not same. This is not A.P.

56. Given sequence is,

3, 6, 12, 24,......

First term of this A.P is a_1 = 3a_1 = 3

Second term of this A.P is a_2 = 6a_2 = 6

Third term of this A.P is a_3 = 12a_3 = 12

The condition for an sequence to be an A.P is their must be a common difference (i.e., d = a_{n+1} - a_n)

putting n = 1 in above equation

d = a_2 - a_1 = 6 - 3 = 3

putting n = 2 in above equation

d = a_3 - a_2 = 12 - 6 = 6d = a_3 - a_2 = 12 - 6 = 6

as we can see we get two values of d,

But in an A.P their must be a single value of d which means for this sequence we can't define a common difference.

Hence this sequence does not form an A.P.

Section C

57. In the given problem,
Cost of digging a well for the first meter = ₹150
Cost of digging a well for the subsequent meter is increased by ₹20
So,
Cost of digging a well of depth one meter = ₹150
Cost of digging a well of depth two meters = ₹150 + 20 = ₹170
Cost of digging a well of depth three meters = ₹150 + 20 + 20 = ₹ 190
Cost of digging a well of depth four meters = ₹120 + 20 + 20 + 20 + 20 = ₹210
Thus, the costs of digging a well of different depths are 150, 170, 190, 210, ...
Now, for a sequence to be an A.P., the difference between adjacent terms should be equal.
Here
$a_2 - a_1 = 170 - 150 = 20$
Also
$a_3 - a_2 = 190 - 170 = 20$
Therefore $a_1 - a = a_2 - a_1$
Since the terms of the sequence are at a common difference of 20, the above sequence is an A.P. with the first terr as a = 150 and common difference d = 20
58. Here,
P = 1000
R = 10% per annum
Amount at the end of 1st year,
$A_1 = 1000\left(1 + \frac{10}{100}\right)^1$
$A_1 = 1100$
For 2nd year, the amount is
$A_2 = 1100\left(1 + \frac{10}{100}\right)^1$
$A_2 = 1210$
For 3rd year compound interest,
$A_3 = 1210\left(1 + \frac{10}{100}\right)^1$
$A_3 = 1331$
For 1st year, 2nd year and 3rd year, the respective amounts are
1100, 1210 and 1331
If any sequence is in A.P. then common difference between any two consecutive terms is constant.
So, 1210 - 1100 = 110
1331 - 1210 = 121
Since it is not constant, so it is not in A.P.
59. Let first three terms be a - d, a and a + d
a - d + a + a + d = 18
So a = 6
(a - d) (a + d) = 5d
$\Rightarrow 6^2 - d^2 = 5d$
or $d^2 + 5d - 36 = 0$
(d + 9)(d - 4) = 0
So d = -9 or 4
For d = -9 three numbers are 15, 6 and -3
For d = 4 three numbers are 2, 6 and 10.

Section D

60. Fill in the blanks:
(i) 1. a + 2d
(ii) 1. 16, 25

(iii) 1. -0.3

(iv) 1. a + d

(v) 1. a + 3d

(vi) 1. Arithmetic

Section E

61. State True or False:

(i) **(b)** False

Explanation: False, because the total fare after each km is

15, 15 + 8, 15 + 8(2), 15 + 8(3) …….

15, 23, 31, 39….

$a_1 = 15$, $a_2 = 23$, $a_3 = 31$ and $a_4 = 39$

$a_2 - a_1 = 23 - 15 = 8$

$a_3 - a_2 = 31 - 23 = 8$

$a_4 - a_3 = 39 - 31 = 8$

Since, all the successive terms of the given list have same difference i.e., common difference = 8.

Hence, the total fare after each km forms an AP.

(ii) **(b)** False

Explanation: False

(iii) **(a)** True

Explanation: True

(iv) **(a)** True

Explanation: True

(v) **(b)** False

Explanation: False

(vi) **(b)** False

Explanation: False

(vii) **(b)** False

Explanation: False

(viii) **(b)** False

Explanation: False

(ix) **(a)** True

Explanation: True

(x) **(a)** True

Explanation: True