Solution

ARITHMETIC PROGRESSION WS 3

Class 10 - Mathematics

Section A

1.
   (d) A is false but R is true.
   Explanation: We have, common difference of an AP
   $d = a_n - a_{n-1}$ is independent of n or constant.
   So, A is false but R is true.

2.
   (d) A is false but R is true.
   Explanation: We have,
   $a_n = a + (n - 1)d$
   $a_{21} - a_7 = \{a + (21 - 1)d\} - \{a + (7 - 1)d\} = 84$
   $a + 20d - a - 6d = 84$
   $14d = 84$
   $d = \frac{18}{14} = 6$
   $d = 6$
   So, A is false but R is true.

Section B

3. 1st figure contains 12 matchsticks and 4 triangles,
   2nd figure contains 19 matchsticks that form 6 triangles,
   3rd figure contains 26 matchsticks that form 8 triangles
   AP for triangle is
   4, 6, 8, ...
   $a = 4, d = 6 - 4 = 2$
   $a_n = a + (n - 1)d$
   $a_n = 4 + (n - 1)2$
   $a_n = 2 + 2n$
4. For number of matchstick
   AP : 12, 19, 26, ...
   Given- $a_n = 61$
   $a_n = a + (n - 1)d$
   $61 = 12 + (n - 1)7$
   $49 = (n - 1)7$
   $n - 1 = 7$
   $n = 8$
   Figure number 8 will have 61 matchstic.
5. The Given pattern 6, 12, 18, 24,... forms an AP.
   Common difference (d) = 12 - 6 = 6
   And next term $a_5 = a_4 + d$
   $a_5 = 24 + 6 = 30$
6. n^{th} term of AP is
   $a_n = a + (n - 1)d$
   $a_n = 6 + (n - 1) 6$
   $a_n = 6 + 6n - 6$
   $a_n = 6n$

Number of stitches in 10^{th} circular row

$a_{10} = 6 \times 10 = 60$

Section C

7. Fill in the blanks:
   (i) 1. -1
   (ii) 1. 5
   (iii) 1. -5

Section D

8. Let production in a 1st year be 'a' unit and increase in production (every year) be 'd' units.
   Increase in production is constant, therefore unit produced every year forms an AP.
   Now, $a_3 = 6000$
   $a + 2d = 6000 \Rightarrow a = 6000 - 2d$ ...(i)
   and $a_7 = 7000 \Rightarrow a + 6d = 7000$
   $\Rightarrow (6000 - 2d) + 6d = 7000 \Rightarrow 4d = 1000$ [using eq. (i)]
   $\Rightarrow d = 250$
   When d = 250, eq. (i) becomes
   $a = 6000 - 2(250) = 5500$
   $\therefore$ Production in 1st year = 5500
9. Production in fifth year
   $a_5 = a + 4d = 5500 + 4(250) = 6500$
10. Total production in 7 years = $\frac{7}{2}(5500 + 7000) = 43750$
11. $a_n = 1000$ units
    $a_n = 1000$
    $\Rightarrow 10000 = a + (n - 1)d$
    $\Rightarrow 1000 = 5500 + 250n - 250$
    $\Rightarrow 10000 - 5500 + 250 = 250n$
    $\Rightarrow 4750 = 250n$
    $\Rightarrow n = \frac{4750}{250} = 19$
12. Let 1^{st} year production of TV = x
    Production in 6^{th} year = 16000
    $t_6 = 16000$
    $t_9 = 22,600$
    $t_6 = a + 5d$
    $t_9 = a + 8d$
    $16000 = x + 5d$ ...(i)
    $22600 = x + 8d$ ...(ii)
    $-\quad\quad -\quad -$
    $-6600 = -3d$
    d = 2200
    Putting d = 2200 in equation ...(i)
    $16000 = x + 5 \times (2200)$
    $16000 = x + 11000$
    $x = 16000 - 11000$
    $x = 5000$
    $\therefore$ Production during 1^{st} year = 5000
13. Production during 8th year is $(a + 7d) = 5000 + 7(2200) = 20400$
14. Production during first 3 year = Production in ($1^{st} + 2^{nd} + 3^{rd}$) year
    Production in 1^{st} year = 5000
    Production in 2^{nd} year = 5000 + 2200

= 7200

Production in 3^{rd} year = 7200 + 2200

= 9400

$\therefore$ Production in first 3 year = 5000 + 7200 + 9400

= 21,600

15. Let in n^{th} year production was = 29,200

$t_n = a + (n - 1)d$

29,200 = 5000 + (n - 1) 2200

29,200 = 5000 + 2200n - 2200

29200 - 2800 = 2200n

26,400 = 2200n

$\therefore n = \frac{26400}{2200}$

n = 12

i.e., in 12^{th} year, the production is 29,200

16. The number of rose plants in the 1^{st}, 2^{nd}, ... are 23, 21, 19, ... 5

a = 23, d = 21 - 23 = - 2, $a_n = 5$

$\therefore a_n = a + (n - 1)d$

or, 5 = 23 + (n - 1)(-2)

or, 5 = 23 - 2n + 2

or, 5 = 25 - 2n

or, 2n = 20

or, n = 10

17. Total number of rose plants in the flower bed,

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{10} = \frac{10}{2}[2(23) + (10 - 1)(-2)]$

$S_{10} = 5[46 - 20 + 2]$

$S_{10} = 5(46 - 18)$

$S_{10} = 5(28)$

$S_{10} = 140$

18. $a_n = a + (n - 1)d$

$\Rightarrow a_6 = 23 + 5 \times (-2)$

$\Rightarrow a_6 = 13$

19. $S_n = 80$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow 80 = \frac{n}{2}[2 \times 23 + (n - 1) \times -2]$

$\Rightarrow 80 = 23n - n^2 + n$

$\Rightarrow n^2 - 24n + 80 = 0$

$\Rightarrow (n - 4)(n - 20) = 0$

$\Rightarrow n = 4$ or $n = 20$

n = 20 not possible

$a_{20} = 23 + 19 \times (-2) = -15$

Number of plants cannot be negative.

n = 4

20. Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month

therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100

Now, amount paid in the 30th installment,

$a_{30} = 1000 + (30 - 1)100 = 3900$ $\{a_n = a + (n - 1)d\}$

21. Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month
therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month=1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
Amount paid in 30 instalments,
$S_{30} = \frac{30}{2}[2 \times 1000 + (30 - 1)100]$ = 73500
Hence, remaining amount of loan that he has to pay = 118000 - 73500 = 44500 Rupees
22. Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month
therefore, installment amount in second month = ₹ 1100, third month = ₹ 1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
Amount paid in 100 instalments
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_n = \frac{100}{2}[2 \times 1000 + (100 - 1)100]$
$\Rightarrow S_n = 100000 + 9900$
$\Rightarrow$ 109900
23. Clearly, the amount of installment in the first month = ₹ 1000, which increases by ₹ 100 every month
therefore, installment amount in second month = ₹ 1100, third month = ₹1200, fourth month = 1300 ..... which forms an AP, with first term, a = 1000 and common difference, d = 1100 - 1000 = 100
If he increases the instalment by 200 rupees every month, amount would he pay in 40th instalment
Then a = 1000, d = 200 and n = 40
$a_{40} = a + (n - 1)d$
$\Rightarrow a_{40} = 1000 + (40 - 1)200$
$\Rightarrow a_{40} = 880$
24. Let there be 'n' number of rows
Given 3, 5, 7... are in AP
First term a = 3 and common difference d = 2
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow 360 = \frac{n}{2}[2 \times 3 + (n - 1) \times 2]$
$\Rightarrow 360 = n[3 + (n - 1) \times 1]$
$\Rightarrow n^2 + 2n - 360 = 0$
$\Rightarrow (n + 20)(n - 18) = 0$
$\Rightarrow n = -20$ reject
n = 18 accept
25. Since there are 18 rows number of candies placed in last row (18^{th} row) is
$a_n = a + (n - 1)d$
$\Rightarrow a_{18} = 3 + (18 - 1)2$
$\Rightarrow a_{18} = 3 + 17 \times 2$
$\Rightarrow a_{18} = 37$
26. If there are 15 rows with same arrangement
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow S_{15} = \frac{15}{2}[2 \times 3 + (15 - 1) \times 2]$
$\Rightarrow S_{15} = 15[3 + 14 \times 1]$
$\Rightarrow S_{15} = 255$
There are 255 candies in 15 rows.
27. The number of candies in 12th row.
$a_n = a + (n - 1)d$
$\Rightarrow a_{12} = 3 + (12 - 1)2$
$\Rightarrow a_{12} = 3 + 11 \times 2$
$\Rightarrow a_{12} = 25$

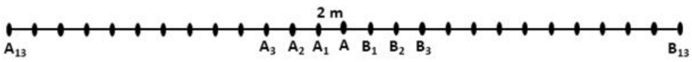
28. 1st installment = ₹3425
2nd installment = ₹3225
3rd installment = ₹3025
and so on
Now, 3425, 3225, 3025, ... are in AP, with
a = 3425, d = 3225 - 3425 = -200
Now 6th installment = a_n = a + 5d = 3425 + 5(-200) = ₹2425

29. Total amount paid = $\frac{15}{2}$(2a + 14d)
= $\frac{15}{2}$[2(3425) + 14(-200)] = $\frac{15}{2}$(6850 - 2800)
= $\frac{15}{2}$(4050) = ₹30375

30. a_n = a + (n - 1)d
$\Rightarrow a_{10} = 3425 + 9 \times (-200)$ = 1625
$\Rightarrow a_{11}$ = 3425 + 10 $\times$ (-200) = 1425
$a_{10} + a_{11}$ = 1625 + 1425 = 3050

31. a_n = a + (n - 1)d given a_n = 2625
2625 = 3425 + (n - 1) $\times$ - 200
$\Rightarrow$ - 800 = (n - 1) $\times$ - 200
$\Rightarrow$ 4 = n - 1
$\Rightarrow$ n = 5
So, in 5th installment, she pays ₹2625.

32. Distance covered in placing 6 flags on either side of center point is 84 + 84 = 168 m
$S_n = \frac{n}{2}$[2a + (n - 1)d]
$\Rightarrow S_6 = \frac{6}{2}[2 \times 4 + (6 - 1) \times 4]$
$\Rightarrow S_6$ = 3[8 + 20]
$\Rightarrow S_6$ = 84

33. 

Let A be the position of the middle-most flag.
Now, there are 13 flags (A_1, A_2 ... A_{12}) to the left of A and 13 flags (B_1, B_2, B_3 ... B_{13}) to the right of A.
Distance covered in fixing flag to A_1 = 2 + 2 = 4 m
Distance covered in fixing flag to A_2 = 4 + 4 = 8 m
Distance covered in fixing flag to A_3 = 6 + 6 = 12 m
...
Distance covered in fixing flag to A_{13} = 26 + 26 = 52 m
This forms an A.P. with,
First term, a = 4
Common difference, d = 4
and n = 13

34. $\therefore$ Distance covered in fixing 13 flags to the left of A = S_{13}
$S_n = \frac{n}{2}$[2a + (n - 1)d]
$\Rightarrow S_{13} = \frac{13}{2}[2 \times 4 + 12 \times 4]$
= $\frac{13}{2} \times$ [8 + 48]
= $\frac{13}{2} \times$ 56
= 364
Similarly, distance covered in fixing 13 flags to the right of A = 364
Total distance covered by Ruchi in completing the task
= 364 + 364 = 728 m

35. Maximum distance travelled by Ruchi in carrying a flag
= Distance from A_{13} to A or B_{13} to A = 26 m

36. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

i.e. $S_n = 360$

$\Rightarrow \frac{n}{2}[2 \times 30 + (n-1)(-1)] = 360$ $\{S_n = \frac{n}{2}(2a + (n-1)d)\}$

$\Rightarrow 720 = n(60 - n + 1)$

$\Rightarrow 720 = 60\text{n - n}^2 + n$

$\Rightarrow n^2 - 61n + 720 = 0$

$\Rightarrow n^2 - 16n - 45n + 720 = 0$ [by factorization]

$\Rightarrow n(n-16) - 45(n-16) = 0$

$\Rightarrow (n-16)(n-45) = 0$

$\Rightarrow (n-16) = 0$ or $(n-45) = 0$

$\Rightarrow n = 16$ or $n = 45$

Hence, number of rows is either 45 or 16.

n = 45 not possible so n = 16

$a_{45} = 30 + (45-1)(-1)$ $\{a_n = a + (n-1)d\}$

$= 30 - 44 = -14$ [$\because$ The number of logs cannot be negative]

Hence the number of rows is 16.

37. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

Number of bricks on top row are $n = 16$,

a_{16} = 30 + (16 - 1) (-1) {a_n = a + (n - 1)d}

= 30 - 15 = 15

Hence, and number of bricks in the top row is 15.

38. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27, ..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.

Suppose number of rows is n, then sum of number of bricks in n rows should be 360

Number of bricks in 10th row a = 30, d = -1, n = 10

a_n = a + (n - 1)d

$\Rightarrow$ a_{10} = 30 + 9 $\times$ -1

$\Rightarrow$ a_{10} = 30 - 9 = 21

Therefore, number of bricks in 10th row are 21.

39. Number of bricks in the bottom row = 30. in the next row = 29, and so on.

Therefore, Number of bricks stacked in each row form a sequence 30, 29, 28, 27,..., which is an AP with first term, a = 30 and common difference, d = 29 - 30 = -1.

Suppose number of rows is n, then sum of number of bricks in n rows should be 360.

a_n = 26, a = 30, d = -1

a_n = a + (n - 1)d

$\Rightarrow$ 26 = 30 + (n - 1) $\times$ -1

$\Rightarrow$ 26 - 30 = -n + 1

$\Rightarrow$ n = 5

Hence 26 bricks are in 5th row.

40. 51, 49, 47, ... 31 AP

d = -2

First 4 terms of AP are: 51, 49, 47, 45 ...

41. 51, 49, 47, ... 31 AP

d = -2

t_n = a +(n - 1)d

31 = 51 + (n - 1) (-2)
31 = 51 - 2n + 2
31 = 53 - 2n
31 - 53 = -2n
-22 = -2n
n = 11
i.e., he acheived his goal in 11 days.

42. 51, 49, 47, ... 31 AP
d = -2
t_6 = a + (n - 1)d
= 51 + (6 - 1)(-2)
= 51 + (-10)
= 41 sec

43. The given AP is
51, 49, 47, 45, 43, 41, 39, 37, 35, 33, 31, 29
$\therefore$ 30 is not in the AP.

44. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
We have, a_3 = 600 and
a_3 = 600
$\Rightarrow$ 600 = a + 2d
$\Rightarrow$ a = 600 - 2d ...(i)
$\Rightarrow$ a_7 = 700
$\Rightarrow$ a_7 = 700
$\Rightarrow$ 700 = a + 6d
$\Rightarrow$ a = 700 - 6d ...(ii)
From (i) and (ii)
600 - 2d = 700 - 6d
$\Rightarrow$ 4d = 100
$\Rightarrow$ d = 25

45. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
We know that first term = a = 550 and common difference = d = 25
a_n = 1000
$\Rightarrow$ 1000 = a + (n - 1)d
$\Rightarrow$ 1000 = 550 + 25n - 25
$\Rightarrow$ 1000 - 550 + 25 = 25n
$\Rightarrow$ 475 = 25n
$\Rightarrow$ n = $\frac{475}{25}$ = 19

46. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
The production in the 10th term is given by a_{10}. Therefore, production in the 10th year = a_{10} = a + 9d = 550 + 9 $\times$ 25 = 775. So, production in 10th year is of 775 TV sets.

47. Since the production increases uniformly by a fixed number every year. Therefore, the sequence formed by the production in different years is an A.P. Let a be the first term and d be the common difference of the A.P. formed i.e., 'a' denotes the production in the first year and d denotes the number of units by which the production increases every year.
Total production in 7 years = Sum of 7 terms of the A.P. with first term a (= 550) and d (= 25).
$S_n = \frac{n}{2}[2a + (n-1)d]$
$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (7-1)25]$

$\Rightarrow S_7 = \frac{7}{2}[2 \times 550 + (6) \times 25]$
$\Rightarrow S_7 = \frac{7}{2}[1100 + 150]$
$\Rightarrow$ S_7 = 4375

Section E

48. State True or False:

(i) **(b)** False
Explanation: False

(ii) **(b)** False
Explanation: False

(iii) **(a)** True
Explanation: True

(iv) **(a)** True
Explanation: True

(v) **(a)** True
Explanation: True

49. Given the seventh term of an A.P. is $\frac{1}{9}$ and its ninth term is $\frac{1}{7}$.Now,
A_7 = a + 6d = $\frac{1}{9}$
a = $\left(\frac{1}{9}\right)$ - 6d
A_9 = a + 8d = $\frac{1}{7}$
Putting value of a, $\frac{1}{9}$- 6d + 8d = $\frac{1}{7}$
2d = $\frac{1}{7} - \frac{1}{9}$
2d = $\frac{(9-7)}{63}$
2d = $\frac{2}{63}$
d = $\frac{1}{63}$
a = $\frac{1}{9} - 6d = \frac{1}{9} - 6 \times \left(\frac{1}{63}\right)$
a = $\frac{(7-6)}{63}$
a =$\frac{1}{63}$
Its's 63rd term,
A_{63}= a + $\frac{62}{63}$
A_{63} = $\frac{1}{63} + \frac{62}{63}$
A_{63} = $\frac{63}{63}$
A_{63}= 1

50. Cost of equipment = ₹6,00,000
Depreciation in first year = 15% of 6,00,000
$= \frac{15}{100} \times 600,000$
= ₹90,000
Depreciation in second year = 13.5% of 6,00,000
$= \frac{13.5}{100} \times 600,000$
= ₹81,000
Depreciation in third year = 12% of 6,00,000
$= \frac{12}{100} \times 600,000$
= ₹72,000
Change in Depreciation = 81000 - 90000 = -9000
Total Depreciation in 10 years = $\frac{n}{2}[2a + (n - l)d]$
$= \frac{10}{2}[2 \times 90000 + (10 - 1) \times (-9000)]$
= 5[180000 - 81000]
= 5(99000)
= ₹4,95,000

Cost of equipment after 10 years = 6,00,000 - 4,95,000

= ₹1,05,000

51. Given A.P is: a, a + d, a + 2d .....

Here, we first need to write the expression for $a_n - a_k$

Now, as we know,

$a_n = a + (n - 1)d$

So for the nth term

$a_n = a + (n - 1)d$

Similarly for k^{th} term

$a_k = a + (k - 1)d$

So,

$a_n - a_k = (a + nd - d) - (a + kd - d)$

= a + nd - d - a - kd + d

= nd - kd

= (n - k)d

So $a_n - a_k = (n - k)d$

We are given $a_{10} - a_5 = 200$

Here

Let us take the first term as 'a' and the common difference as 'd'

Now, as we know,

$a_n = a + (n - 1)d$

Here we find a_{30} and a_{20}

So, for 10th term,

$a_{10} = a + (10 - 1)d$

= a + 9d

Also for 5th term

$a_5 = a + (5 - 1)d$

= a + 4c

So, $a_{10} - a_5 = (a + 9d) - (a + 4d)$

200 = a = 9d - a - 4d

200 = 5d

$d = \frac{200}{5}$

d = 40

Therefore the common difference for the A.P is d = 40

52. Given,a man saved ₹32 during the first year, ₹36 in the second year and in this way he increases his savings by ₹4 every year.

Now

Saving in first year(a_1) = ₹32

Saving in second year(a_2) = ₹36

Increase in salary every year (d) = ₹4

Let in n years, his saving will be ₹200

$\Rightarrow S_n = 200$

$\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = 200$

$\Rightarrow \quad \frac{n}{2}[2 \times 32 + (n-1) \times 4] = 200$

$\Rightarrow \quad \frac{n}{2}[64 + 4n - 4] = 200$

$\Rightarrow \quad \frac{n}{2}[4n + 60] = 200$

$\Rightarrow 2n^2 + 30n = 200$

$\Rightarrow n^2 + 15n - 100 = 0$

$\Rightarrow n^2 + 20n - 5n + 100 = 0$

$\Rightarrow n(n + 20) - 5(n + 20) = 0$

⇒n + 20(n - 5) = 0

If, n + 20 = 0

n = -20 (rejected as n cannot be negative)

or,n - 5 = 0

n = 5

Therefore, in 5 years his saving will be ₹ 200.

53. The general term of an AP is given by

a_n=a+(n-1)d

Given that a_{16}=5a_3

⇒a+15d=5(a+2d)

⇒a+15d=5a+10d

⇒4a=5d...(i)

Now,

a_{10}=41

⇒a+9d=41

⇒4a+36d=164

⇒5d+36d=164....(from (i))

⇒41d=164

⇒d=4

Substituting in (i), we get a=5.

We know that, $S_n = \frac{n}{2}[2a + (n-1)d]$

Sum of the first 15 terms=S_{15}

$= \frac{15}{2}[2(5) + 14(4)]$

$= \frac{15}{2}[10 + 56]$

=495

54. Let the four numbers be $a - 3d, a - d, a + d, a + 3d$

$\therefore a - 3d + a - d + a + d + a + 3d$ = 56

or, $4a = 56$

or, $a = 14$

Hence numbers are $14 - 3d, 14 - d, 14 + d, 14 + 3d$

Now, according to question

$\frac{(14-3d)(14+3d)}{(14-d)(14+d)} = \frac{5}{6}$

or, $\frac{196-9d^2}{196-d^2} = \frac{5}{6}$

or, $6(196 - 9d^2) = 5(196 - d^2)$

or, $6 \times 196 - 54d^2 = 5 \times 196 - 5d^2$

or, $6 \times 196 - 5 \times 196 = 54d^2 - 5d^2$

or, $(6 - 5) \times 196 = 49d^2$

or, $196 = 49d^2$

or, $d^2 = \frac{196}{49} = 4$

or, $d = \pm 2$

Therefore, Numbers are

$14 - 3 \times 2, \quad 14 - 2, \quad 14 + 2, \quad 14 + 3 \times 2$

= $14 - 6, 12, 16, 14 + 6$

= $8, 12, 16, 20$

55. The man arranges to pay off a debt of ₹36000 by 40 monthly installments.

So, n=40 and S_{40}=36000

Let the first installment be ₹ a, and let d be the common difference.

One-third debt is unpaid, that means two-third is paid.

$\frac{2}{3}$(36000)=₹24000

Sn=$\frac{n}{2}$[2a+(n-1)d]

⇒S_{30}=$\frac{30}{2}$[2a+29d]

$\Rightarrow$24000=15[2a+29d]
$\Rightarrow$1600=2a+29d
$\Rightarrow$2a+29d=1600...(i)
$S_{40}=\frac{40}{2}$ [2a+39d]
$\Rightarrow$36000=20[2a+39d]
$\Rightarrow$1800=2a+39d
$\Rightarrow$2a+39d=1800...(ii)
Subtracting (i) from (ii), we get
10d=200
$\Rightarrow$d=20
Substituting in (i), we get
2a+29(20)=1600
$\Rightarrow$2a=1020
$\Rightarrow$a=510
Hence, the first installment he paid was ₹ 510.

56. Here Sum of the n terms of an Arithmetic progression, $S_n = 5n^2 + 3n$
$S_1 = 5 \times 1^2 + 3 \times 1 = 8 = t_1$ ....(i)
$S_2 = 5 \times 2^2 + 3 \times 2 = 26 = t_1 + t_2$ ...(ii)
$S_3 = 5 \times 3^2 + 3 \times 3 = 54 = t_1 + t_2 + t_3$ ...(iii)
From (i), (ii) and (iii),
$t_1 = 8, t_2 = 18, t_3 = 28$
So Common difference, d = 18 - 8 = 10 and first term a=8.
Now $t_m = 168$ (given)
We know that $T_m = a + (m-1)d$
$\Rightarrow$ a + (m - 1)d = 168
$\Rightarrow$ 8 + (m - 1) $\times$ 10 = 168
$\Rightarrow$ (m - 1) $\times$ 10 = 160
$\implies$ m-1 = $\frac{160}{10}$
$\Rightarrow$ m - 1 = 16
Therefore, m = 17
So, t_{20} = a + (20 - 1)d
= 8 + 19 $\times$ 10
= 8 + 190
= 198

57. We are given that,
$a_5 + a_9 = 72$
$\therefore$ a + 4d + a + 8d = 72
$\Rightarrow$ 2a + 12d = 72 ..(i)
and $a_7 + a_{12} = 97$
$\Rightarrow$ 2a + 17d = 97 ..(ii)
on equating (i) and (ii), we get
5d = 25
d = 5
put d = 5 in eq. (i)
$\Rightarrow$ 2a + 12(5) = 72
$\Rightarrow$ 2a + 60 = 72
$\Rightarrow$ 2a = 72 - 60
$\Rightarrow$ 2a = $\frac{12}{2}$
$\Rightarrow$ a = 6
then,

$a_1 = 6$, $a_2 = 11$, $a_3 = 16$, $a_4 = 21$,. so on

$\therefore$ AP is 6, 11, 16, 21, ...

58. Let the first term = a and common difference = d

Given, $a_{17} = 5 + 2 a_8$

a + 16d = 5 + 2(a + 7d)

a + 16d = 2a + 14d + 5

-a + 2d = 5

a - 2d = -5 ...(1)

Also, $a_{11} = 43$

a + 10d = 43 ...(2)

Subtracting (1) from (2) we get

12d = 48

d = 4

Now from (1)

a-2(4) = -5

a = 8 - 5 = 3

Thus, n^{th} term

$a_n = a + (n - 1)d$

$a_n = 3 + (n - 1) \times 4$

$a_n = 3 + 4n - 4$

$a_n = 4n - 1$

59. We have,

2 + 6 + 10 + ... x = 1800

Here ; 2, 6, 10 , ...x are in arithematic progression where, a = 2 is first term and d = 4 is common difference.

Using formula to find the number of terms in AP.

x = a + (n-1)d

x = 2 + (n - 1) .4

x = 2 + 4n - 4

x = 4n - 2

x + 2 = 4n

n = $\frac{(x+2)}{4}$

Now, using formula, $S_n = \frac{n}{2}(a + T_n)$

Here, $S_n = 1800$, n = $\frac{x+2}{4}$, a = 2, T_n = x

1800 = $\frac{\left(\frac{x+2}{4}\right)}{2}$ [2 + x]

1800 = $\frac{x+2}{8} \times (x + 2)$

$1800 \times 8 = (x + 2)^2$

$14400 = (x + 2)^2$

$(120)^2 = (x + 2)^2$

x + 2 = 120 $\Rightarrow$ x = 118

Hence, value of x = 118

60. Given

a = 22 , a_n= -6 ,S_n = 64

a_n = -6

a + (n - 1) = -6

22 + (n - 1)d = -6

(n - 1)d = -28 ...(i)

S_n = 64

$\frac{n}{2}(a + a_n)$ = 64

$\frac{n}{2}(22 - 6)$ = 64

$n = \frac{64\times2}{16}$ = 8

∴ Number of terms is 8.

From equation (i)

(n - 1)d = -28

7d = -28

∴ d = -4

Common difference = -4.

61. Let us suppose that the first term of the A.P. be 'a' and the common difference be 'd'.

Since, we know that in general ,5^{th} term of the A.P., t_5 = a + (5 - 1)d = a + 4d

Similarly, in general, 9^{th} term of the A.P., t_9 = a + (9 - 1)d = a + 8d

Now, according to question it is given that

$t_5 + t_9 = 30$

⇒ a + 4d + a + 8d = 30

⇒ 2a + 12d = 30...........................(i)

Also, we know that

t_{25} = a + (25 - 1)d = a + 24d

and

t_8 = a + (8 - 1)d = a + 7d

It is given that, $t_{25} = 3t_8$

⇒a + 24d = 3(a + 7d)

⇒a + 24d = 3a + 21d

.

⇒2a - 3d = 0

$\Rightarrow a = \frac{3d}{2}$ ....(ii)

Now, Substitute the value of 'a' in equation (i), we get

$2\left(\frac{3d}{2}\right) + 12d = 30$

⇒ 3d + 12d = 30

⇒15d = 30

$\Rightarrow d = \frac{30}{15} = 2$

Substituting the value of d in equation(ii), we get

$a = \frac{3\times2}{2} = 3$

t_2 = 3 + (2 - 1)2 = 5

t_3 = 3 + (3 - 1)2 = 7

Therefore, the required A.P. is 3, 5, 7,9,11,13 ....

62. The multiples of 4 that lie between 10 and 250 are:

12, 16, 20, 24, ..., 248

$a_2 - a_1$ = 16 - 12 = 4

$a_3 - a_2$ = 20 - 16 = 4

$a_4 - a_3$ = 24 - 20 = 4

As $a_{k+1} - a_k$ is the same for k = 1, 2, 3, etc.

The above list of numbers forms an AP with the first term a = 12

and the common difference d = 4

Last term (l) = 248

Let there be n term s in this AP. Then, nth term = l

⇒ a + (n - 1)d = 248

⇒ 12 + (n - 1)4 = 248

⇒ (n - 1)d = 248 - 12

⇒ (n - 1) = 236

$\Rightarrow n - 1 = \frac{236}{4}$

⇒ n - 1 = 59

$\Rightarrow$ n = 59 + 1

$\Rightarrow$ n = 60

Hence, 60 multiples of 4 lie between 10 and 250.

63. Consider the first term and the common difference as a and d respectively.

Now $a_8 = \frac{1}{2} a_2$ [Given]

$\Rightarrow a + (8 - 1)d = \frac{1}{2} [a + (2 - 1)d$ [$\because a_n = a + (n - 1)d$]

$\Rightarrow 2(a + 7d) = a + d$

$\Rightarrow 2a + 14d - a - d = 0$

$\Rightarrow a + 13d = 0$ …(i).

Now, $a_{11} = \frac{1}{3} a_4$ [Given]

$\Rightarrow a + (11 - 1)d = \frac{1}{3} [a + (4 - 1)d] + 1$

$\Rightarrow (a + 10d) = \frac{1}{3} (a + 3d) + 1$

$\Rightarrow 3(a + 10d) = a + 3d + 3$

$\Rightarrow 3a + 30d - a - 3d = 3$

$\Rightarrow 2a + 27d = 3$.......(ii).

Multiplying (i) by 2, we have

$2a + 26d = 0$........ (iii).

Now, subtracting (iii) from (ii), we get.

$$\begin{array}{ll} 2a + 27d = 3 & ...(ii) \\ 2a + 26d = 0 & (iii) \\ - \quad - \quad\quad - & \\ \hline \quad\quad d = 3 & \end{array}$$

Now, $a + 13d = 0$ [From (i)].

$\Rightarrow a + 13 \times 3 = 0 \Rightarrow a = -39$.

Now, we know that $a_n = a + (n - 1)d$

$\Rightarrow a_{15} = -39 + (15 - 1)3$

$= -39 + 14 \times 3$

$= -39 + 42$

$\Rightarrow a_{15} = 3$.

Thus 15th term is 3

64. Total amount of price(S_n) = ₹ 700

Number of prizes (n) = 7

Let the value of first prize = ₹ a

Depreciation in next prize (d) = - ₹ 20

We have,

$\therefore \quad (S_n) = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow \quad 700 = \frac{7}{2}[2a + (7 - 1) \times -20]$

$\Rightarrow \quad \frac{700 \times 2}{7} = 2a + 6 \times (-20)$

$\Rightarrow$ 200 = 2a - 120

$\Rightarrow$200 + 120 = 2a

$\Rightarrow$320 = 2a

$\Rightarrow \quad a = \frac{320}{2} = ₹160$

Therefore, value of first prize = 160

Value of second prize = 160 - 20 = ₹ 140

Value of third prize = 140 - 20 = ₹ 120

Value of fourth prize = 120 - 20 = ₹ 100

Value of fifth prize = 100 - 20 = ₹ 80

Value of sixth prize = 80 - 20 = ₹ 60

Value of seventh prize = 60 - 20 = ₹ 40.

65. Let first instalment(a) = Rs a

Let common difference = Rs d

Given,

Amount of 40 instalments = Rs 3600

$\Rightarrow \frac{40}{2}[2a + (40-1)d] = 3600$

$\Rightarrow$20[2a + 39d] = 3600

$\Rightarrow$2a + 39d = 180..............(i)

and, Amount of 30 instalments = Rs 2400

$\Rightarrow \frac{30}{2}[2a + (30-1)d] = 2400$

$\Rightarrow$15[2a + 29d] = 2400

$\Rightarrow$2a + 29d = 160..............(ii)

Subtracting (ii) from (i),

2a + 39d - 2a - 29d = 180 - 160

$\Rightarrow$ 10d = 20

$\Rightarrow \quad d = \frac{20}{10} = 2$

Putting value of d in eq(i),

2a + 39$\times$ 2 = 180

$\Rightarrow$2a + 78 = 180

$\Rightarrow$2a = 180 - 78

$\Rightarrow$2a = 102

$\Rightarrow a = \frac{102}{2} = 51$

Therefore, Value of first instalment = Rs 51

66. $a_n = \frac{1}{m}$

$a + (n - 1)d = \frac{1}{m}$

$a_m = \frac{1}{n}$

$a + (m - 1)d = \frac{1}{n}$

On solving,

$a = \frac{1}{mn}$

$d = \frac{1}{mn}$

i. $a_{mn} = \frac{1}{mn} + (mn - 1) \times \frac{1}{mn}$

$= \frac{1+mn-1}{mn} = 1$

ii. $S_{mn} = \frac{mn}{2}\left(\frac{1}{mn} + 1\right)$

$= \frac{1+mn}{2}$

67. Let a be the first term and d be the common difference of the given AP. Then,

S_m = sum of first m terms of the given AP;

S_n = sum of first n terms of the given AP.

$\frac{S_m}{S_n} = \frac{m^2}{n^2} \Rightarrow \frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

$\Rightarrow \frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

$\Rightarrow$ 2an+mnd-nd= 2am+mnd-md

$\Rightarrow$ 2an-2am=nd-md

$\Rightarrow$ 2a(n - m) = d(n - m) $\Rightarrow$2a=d...(i)

$\therefore \quad \frac{T_m}{T_n} = \frac{a+(m-1)d}{a+(n-1)d} = \frac{a+(m-1)\cdot 2a}{a+(n-1)\cdot 2a}$ [from (i)]

$= \frac{a+2am-2a}{a+2an-2a} = \frac{2am-a}{2an-a} = \frac{a(2m-1)}{a(2n-1)} = \frac{2m-1}{2n-1}$.

68. Let the four parts be (a-3d), (a-d), (a+d) and (a+3d).

Then, (a-3d)+(a-d)+(a+d)+(a+3d)=32

$\Rightarrow$4a=32

$\Rightarrow$a=8

It is given that

$\frac{(a-3d)(a+3d)}{(a-d)(a+d)} = \frac{7}{15}$

$\Rightarrow \frac{a^2-9d^2}{a^2-d^2} = \frac{7}{15}$

$\Rightarrow \frac{64-9d^2}{64-d^2} = \frac{7}{15}$

$\Rightarrow 960-135d^2=448-7d^2$

$\Rightarrow 128d^2=512$

$\Rightarrow d^2=4$

$\Rightarrow d=\pm 2$

When d=2,

a-3d=8-3(2)=2

a-d=8-2=6

a+d=8+2=10

a+3d=8+3(2)=14

when d=-2,

a-3d=8-3(-2)=14

a-d=8-(-2)=10

a+d=8+(-2)=6

a+3d=8+3(-2)=2

Thus, the four parts are 2,6,10,14 or 14,10,6,2

69. Amount to be counted = Rs 10710

Man counts Rs 180 per minute for half an hour

Amount counted in 30 min = 30 × Rs 180 = Rs 5400

Amount to be counted = Rs 10710 – Rs 5400 = Rs 5310

It is also given that after half an hour Rs 3 gets less every minute than the preceding minute.

The AP becomes, 177 + 174 + 171 … + 3 = 5310

Here a = 177, d = 174 – 177 = - 3, S_n = 5310

Sum of n terms of AP is $S_n = \frac{n}{2}[a+l]$

$5310 = \frac{n}{2}[177+3]$

$\Rightarrow 5310 = \frac{n}{2} \times 180$

⇒ 5310 = 90n

∴ n = 59

Hence the total time taken by the man to count the entire amount = 30 + 59

= 89 minutes or 1 hour 29 minutes