Solution

ARITHMETIC PROGRESSION WS 4

Class 10 - Mathematics

1. According to question we observe that 56 is the first integer between 50 and 500 which is divisible by 7.
   Also, when we divide 500 by 7 the remainder is 3. Therefore, 500 - 3 = 497 is the largest integer divisible by 7 and lying between 50 and 500. Thus, we have to find the number of terms in an A.P. with first term = 56,
   last term = 497 and common difference = 7 (as the numbers are divisible by 7).
   Let there be n terms in the A.P. Then,
   a_n = 497
   ⇒ a + (n - 1) d = 497
   ⇒ $56 + (n - 1) \times 7 = 497$ ...[Because,a = 56 and d = 7]
   ⇒ 7n + 49 = 497
   ⇒ 7n = 448
   ⇒ n = 64
   Threfore, there are 64 integers between 50 and 500 which are divisible by 7.
2. Let the first term and common difference of an AP are a and d, respectively.
   Given
   $a_3 + a_8 = 7$
   As we know, nth term of an AP is
   $a_{n =}$ a + (n - 1)d
   where a = first term
   a_n is nth term
   d is the common difference
   a + 2d + a + 7d = 7
   2a + 9d = 7
   2a = 7 - 9d ...(1)
   $a_7 + a_{14}$ = - 3
   a + 6d + a + 13d = - 3
   2a + 19d = - 3
   7 - 9d + 19d = - 3 [using 1]
   7 + 10d = - 3
   10d = - 10
   d = - 1
   using this value in (1)
   2a = 7 - 9 (- 1)
   2a = 16
   a = 8
   Now,
   a_{10} = a + 9d
   = 8 + 9 (- 1)
   = 8 - 9 = - 1
3. Let the digits of the required 3 - digit number at hundreds, tens and ones places be $a - d,\ a,\ a + d$ respectively
   Then their sum = 15
   i.e, $a - d + a + a + d$ = 15
   ⇒ $3a$ = 15
   ⇒ a = 5
   Required three digit number = $100(a - d) + 10a + a + d$
   = $100a - 100d + 10a + a + d$
   = $111a - 99d$
   Number obtained by reversing the digits = $100(a + d) + 10a + a - d$

= $100a + 100d + 10a + a - d$

= $111a + 99d$

According to the question,

$111a + 99d = 111a - 99d - 594$

$\Rightarrow 594 = 111a - 99d - 111a - 99d$

$\Rightarrow 594 = -198d$

$\Rightarrow \frac{-594}{198} = d$

$d = -3$

The number is 111a - 99d

$111 \times 5 - 99 \times -3$= 852

555 + 297 = 852

Therefore, Number is 852.

4. a_3 = 16

   $\Rightarrow$ a + 2d = 16 ..... (i)

   $a_7 = a_5 + 12$

   $\Rightarrow$ a + 6d = a + 4d + 12

   $\Rightarrow$ 2d = 12

   $\Rightarrow$ d = 6

   Put the value of d in eq. (i)

   $a + 2 \times 6 = 16$

   $\Rightarrow$ a = 16 - 12

   $\Rightarrow$ a = 4

   4, 10, 16....

5. The given AP is:

   9,12, 15, 18.........

   Here a = 9 and d = 12 - 9 = 3

   Let n^{th} term of this A.P. is 39 more than 26^{th} term

   $a_n = a_{36} + 39$

   a + (n - 1)d = a + (36 - 1)d + 39

   9 + (n - 1)$\times$ 3 = 9 + 35$\times$ 3 + 39

   9 + 3n - 3 = 9 + 105 + 39

   3n=9+144-9+3

   3n=147

   $n = \frac{147}{3} = 49$

   Therefore, 49^{th} term of A.P. will be 39 more than its 36^{th} term.

6. Let a be the first term and d be the common difference of the given AP.

   According to the question,we are given that,

   $\frac{T_{11}}{T_{18}} = \frac{2}{3} \Rightarrow \frac{a+(11-1)d}{a+(18-1)d} = \frac{2}{3}$

   $\Rightarrow \frac{a+10d}{a+17d} = \frac{2}{3}$ $\Rightarrow$ 3a+30d= 2a +34d

   $\Rightarrow$a=4d. .....(i)

   Ratio of 5th term to 21st term$= \frac{T_5}{T_{21}} = \frac{a+(5-1)d}{a+(21-1)d}$

   $= \frac{a+4d}{a+20d} = \frac{4d+4d}{4d+20d}$ { from (i)}

   $= \frac{8d}{24d} = \frac{1}{3} = 1:3$

   Ratio of sum of first 5 terms to sum of first 21 terms

   $= \frac{S_5}{S_{21}} = \frac{\frac{5}{2}[2a+(5-1)d]}{\frac{21}{2}[2a+(21-1)d]} = \frac{5(2a+4d)}{21(2a+20d)}$

   $= \frac{10(a+2d)}{42(a+10d)} = \frac{10(4d+2d)}{42(4d+10d)}$ [from(i)]

   $= \frac{60d}{588d} = \frac{60}{588} = \frac{5}{49} = 5:49$.

7. The general term of an AP is given by a_n=a+(n-1)d

   and in general the sum to n terms of an A.P is given by $S_n=\frac{n}{2}$[2a+(n-1)d].

Let the common difference of the AP be d.
Now, given that a=5
Since sum of first four terms is half the sum of the next four terms,therefore,
$S_4=\frac{1}{2}(S_8-S_4)$
$\Rightarrow \frac{4}{2}[2(5)+3d]=\frac{1}{2}[\frac{8}{2}[2(5)+7d]-\frac{4}{2}[2(5)+3d]]$
$\Rightarrow$4[10+3d]=[4(10+7d)-2(10+3d)]
$\Rightarrow$40+12d=40+28d-20-6d
$\Rightarrow$-10d=-20
$\Rightarrow$d=2
thus, the common difference is 2.

8. Yes.
nth term of an A.P., a_n = a + (n - 1)d
a_{30} = a + (30 - 1)d = a + 29d
and a_{20} = a + (20 - 1)d = a + 19d .....(i)
Now, a_{30} - a_{20} = (a + 29d) - (a + 19d) = 10d.
From given A.P. we have
Common difference, d = -7 - (-3) = -7 + 3 = -4
a_{30} - a_{20} = 10(-4) = -40
So, a = -3, d = -4
Therefore, nth term is, an = a + (n - 1)d = -3 + (n - 1)(-4)
= -3 - 4n + 4
= 1 - 4n
Therefore, a_{20} = 1 - 4(20) = -79
a_{30} = 1 - 4(30) = -119

9. Let the first term and the common difference of the AP be a and d respectively.
Given that, a_{17} = a_{10}+7
$\Rightarrow$ a + (17 - 1) d = a + (10 - 1)d + 7 [$\because$ a_n = a +(n - 1)d]
$\Rightarrow$ a + 16d = a + 9d + 7
$\Rightarrow$ 16d - 9d = 7
$\Rightarrow$ 7d = 7
$\Rightarrow d = \frac{7}{7} = 1$
Hence, the common difference is 1.

10. Let a be the first term and d be the common difference of the given AP. Now, we know that in general mth and nth terms of the given A.P can be written as
T_m = a + (m-l)d and T_n = a + (n-1)d respectively.
Now, $T_m = \frac{1}{n}$ and $T_n = \frac{1}{m}$ (given).
$\therefore$ a + (m-1)d = $\frac{1}{n}$ ....(i)
and a + (n-1)d =$\frac{1}{m}$................ ... (ii)
On subtracting (ii) from (i), we get
(m-n)d = $(\frac{1}{n}-\frac{1}{m}) = (\frac{m-n}{mn}) \Rightarrow$ d = $\frac{1}{mn}$
Putting d =$\frac{1}{mn}$ in (i), we get
a + $\frac{(m-1)}{mn} \Rightarrow$ a = $\{\frac{1}{n}-\frac{(m-1)}{mn}\} = \frac{1}{mn}$
Thus, a =$\frac{1}{mn}$ and d =$\frac{1}{mn}$
$\therefore$Now, in general (mn)th term can be written as T_{mn} = a +(mn-1)d
= $\{\frac{1}{mn}+\frac{(mn-1)}{mn}\}$ ...[$\because$a=$\frac{1}{mn}$]
= 1.
Hence, the (mn)th term of the given AP is 1.

11. The three digit numbers divisible by 7 are; 105, 112, 119, 126, ..., 994
a_2 - a_1 = 112 - 105 = 7

$a_3 - a_2 = 119 - 112 = 7$

$a_4 - a_3 = 126 - 119 = 7$

i.e. $a_{k+1} - a_k$ is the same every time.

So, the above list of numbers forms an AP with the first term a = 105 and
the common difference d = 7
Last term(l) = 994
Let there be n terms in this AP,
Then, nth term = l
$\Rightarrow$ a + (n - 1) d = 994
$\Rightarrow$ 105 + (n - 1) d = 994
$\Rightarrow$ (n - 1) 7 = 994 - 105
$\Rightarrow$ (n - 1) 7 = 889
$\Rightarrow n - 1 = \frac{889}{7}$
$\Rightarrow$ n - 1 = 127
$\Rightarrow$ n = 127 + 1
$\Rightarrow$ n = 127 + 1
$\Rightarrow$ n = 128
Hence, there are 128 three digit numbers divisible by 7.

12. Let nth terms of the given arithmetic progressions be t_n and T_n respectively.

The first AP is 13,19,25,....
Let its first term be 'a' and common difference be 'd'. Then,
a = 13 and d = (19 -13) = 6.
We know that in general nth term is given by
t_n = a + (n-1)d
$\Rightarrow \quad t_n = 13 + (n-1) \times 6$
$\Rightarrow t_n = 6n + 7$ ....(i)

The second AP is 69,68,67,....
Let its first term be A and common difference be D. Then,
A = 69 and D = (68 - 69) = -1.
Now, we know that in general nth term is given by
$T_n = A + (n - 1) \times D$

$\Rightarrow$ Tn = 69 + (n -1) $\times$(-1)
$\Rightarrow T_n = 70 - n$ ....(ii)

Now, it is given that the two nth terms of the two arithmetic progressions are equal for a value of n, we have,
$t_n = T_n \Rightarrow 6n + 7 = 70 - n$

$\Rightarrow 7n = 63 \Rightarrow n = 9$.
Hence, the 9th term of each AP is the same.
This term = 70 - 9 = 61 [$\because T_n = (70-n)$].

13. $a_4 + a_8 = 24$ (Given)

$\Rightarrow$ a + 3d + a + 7d = 24
$\Rightarrow$ 2a + 10d = 24
$\Rightarrow$ a + 5d = 12 ..... (i)
$a_6 + a_{10} = 44$

$\Rightarrow$ a + 5d + a + 9d = 44
$\Rightarrow$ 2a + 14d = 44
$\Rightarrow$ a + 7d = 22 ...... (ii)
On solving equation (i) and (ii)
d = 5, a = -13
First three terms are -13, -8, -3.

14. Given, Sn = $\frac{n}{2}$(3n + 5)

$\therefore$ S_{n-1} = $\frac{n-1}{2}$[3(n - 1) + 5]

or S_{n-1} = $\frac{n-1}{2}$(3n + 2)

Since, $a_n = S_n - S_{n-1}$

$= \frac{n}{2}(3n+5) - \frac{n-1}{2}(3n+2)$

$= \frac{3n^2}{2} + \frac{5n}{2} - \frac{3n(n-1)}{2} - \frac{2(n-1)}{2}$

$= \frac{3n^2}{2} + \frac{5n}{2} - \frac{3n^2}{2} + \frac{3n}{2} - n + 1$

= $\frac{8n}{2}$ - n + 1

= 4n - n + 1

= 3n + 1

Now, a_{25} = 3(25) + 1

or, a_{25} = 75 + 1 = 76

Thus, 25^{th} term of AP. is 76

15. Let n^{th} term of A.P.

$a_n = n^2 + 1$

Putting the value of n = 1, 2 ,3 ,.....we get

$a_1 = 1^2 + 1 = 2$

$a_2 = 2^2 + 1 = 5$

$a_3 = 3^2 + 1 = 10$

The obtained sequence = $2, 5, 10, 17 \ldots\ldots$

Its common difference

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3$

or, 5 - 2 $\neq$ 10 - 5 $\neq$ 17 - 10

$\therefore 3 \neq 5 \neq 7$

Since the sequence has no. common difference

Hence, $n^2 + 1$ is not a form of n^{th} term of an A.P.

16. The given list of numbers is 11, 8, 5, 2,.....

$a_2 - a_1$= 8 - 11 = -3

$a_3 - a_2$ = 5 - 8 = -3

$a_4 - a_3$ = 2 - 5 = - 3

i.e. $a_{k+1} - a_k$ is the same every time.

So, the given list of numbers forms an AP with first term a = 11 and the common difference d = -3.

Let -150 be the nth term of the given AP

Then, a_n = -150

$\Rightarrow$ a + (n - 1) d = -150

$\Rightarrow$ 11+ (n - 1)(-3) = -150

$\Rightarrow$ (-3) (n - 1) = -150 - 11

$\Rightarrow$ (-3) (n - 1) = -161

$\Rightarrow$ 3(n - 1) = 161

$\Rightarrow n - 1 = \frac{161}{3}$

$\Rightarrow n = \frac{161}{3} + 1$

$\Rightarrow n = \frac{164}{3}$

But n should be a positive integer. So, -150 is not a term of 11, 8, 5, 2,....

17. Let the first term be a and the common difference be d

18. Let the first term and the common difference of the given AP be a and d respectively.

Second term = 13

$\Rightarrow$ a + (2 - 1)d = 13

$\Rightarrow$ a + d = 13 .... (1)

Fourth term = 3

$\Rightarrow$ a + (4 - 1) d = 3

$\Rightarrow$ a + 3d = 3 .... (2)

Solving (1) and (2), we get

a = 18

d = -5

Therefore,

Third term = a + (3 - 1) d

= a + 2d

= 18 + 2(-5)

= 18 - 10

= 8

Hence, the missing terms are 18 and 8.

19. Let the first term and the common difference of the AP be a and d respectively.

Then,

11th term = 38 .... Given

$\Rightarrow$ a + (11 - 1)d = 38 $\because a_n = a + (n - 1)d$

$\Rightarrow$ a + 10d = 38 .... (1)

and, 16th term = 73

$\Rightarrow$ a + (16 - 1)d = 73 $\because a_n = a + (n - 1)d$

$\Rightarrow$ a + 15d = 73 .... (2)

Solving (1) and (2), we get

a = -32

d = 7

Therefore, 31^{st} term

= a + (31 - 1)d

= a + 30d

= -32 + (30) (7)

= - 32 + 210 = 178

Hence, the 31^{st} term of the AP is 178.

20. Let the three terms of an AP be (a-d), a and (a+d).

It is given that the sum of first terms of an AP is 48.

Then, (a-d)+a+(a+d)=48

$\Rightarrow$3a=48

$\Rightarrow$a=16

it is given that, the product of the first and second terms exceeds 4 times the third term by 12. Therefore,

(a-d)$\times$a-4$\times$(a+d)=12

$\Rightarrow$$a^2$ -ad-4a-4d=12

$\Rightarrow$$(16)^2$-16d-4(16)-4d=12

$\Rightarrow$256-16d-64-4d=12

$\Rightarrow$-20d+192=12

$\Rightarrow$20d=180

$\Rightarrow$d=9

Thus, we have

a-d=16-9=7

a=16

a+d=16+9=25

Hence, the first three terms of given AP are 7,16,25.

21. Let the required angles be (a - 3d)°, (a - d) °, (a + d) ° and (a + 3d) °

Common difference = (a - d) - (a- 3d) = a - d - a + 3d = 2d

We are given that Common difference = 10°

$\therefore$2d = 10° = d = 5°

We know that sum of four angles of quadrilateral = 360°

$\Rightarrow$ (a-3d)^{o} +(a-d)^{o}+(a+d)^{o}+(a+3d)^{o}=360^{o}

4a=360^{o}

a= $\frac{360}{4}$ =90^{o}

$\therefore$a=90^{o} and d=5^{o}

First angle = (a - 3d)° = (90 - 3 × 5) ° = 75°

Second angle = (a - d)° = (90 - 5) ° = 85°

Third angle = (a + d)° = (90 + 5°) = 95°

Fourth angle = (a + 3d)° = (90 + 3 × 5)° = 105°

22. Annual salary received by Suba Rao in 1995, 1996, 1997,... is

₹5000, ₹5200, ₹5400,.......,......7000.

Clearly, it is an arithmetic progression with first term a = 5000 and common difference d = 200.

Suppose Suba Rao's annual salary reaches to ₹7000 in nth years. Then,

nth term of the above A.P. = ₹7000

$\Rightarrow$ a + (n -1) d = 7000

$\Rightarrow 5000 + (n-1) \times 200 = 7000$

$\Rightarrow (n-1) \times 200 = 2000$

$\Rightarrow \quad n-1 = \frac{2000}{200} \Rightarrow n-1 = 10 \Rightarrow n = 11$

Thus, 11th annual salary received by Suba Rao will be ₹7000. This means that after 10 years i.e., in the year 2005 his annual salary will reach to ₹7000.

23. First APs

63, 65, 67, ...

Here, a = 63

d = 65 - 63 = 2

$\therefore$ nth term = 63 + (n - 1)2 $\because$ a_n = a + (n - 1)d

Second APs

3, 10, 17, ...

Here, a = 3

d = 10 - 3 = 7

$\therefore$ nth term = 3 + (n - 1)7 $\because$ a_n = a+(n - 1)d

If the n th terms of two APs are equal then

63 + (n - 1)2 = 3 + (n - 1)7

$\Rightarrow$ (n - 1)2 - (n - 1)7 = 3 - 63

$\Rightarrow$ (n - 1) (2 - 7) = -60

$\Rightarrow$ (n - 1) (-5) = -60

$\Rightarrow n - 1 = \frac{-60}{-5}$

$\Rightarrow$ n - 1 = 12

$\Rightarrow$ n = 12 + 1

$\Rightarrow$ n = 13

Hence, for n = 13th terms of the two APs are equal

24. Nth term

a_n = 5n - 3

a_1 = (5 $\times$ 1) - 3 = 2

a_2 = (5 $\times$ 2) - 3 = 7

a_{16} = (5 $\times$ 16) - 3 = 77

Common difference, d = 7 - 2 = 5

S_n = $\frac{n}{2}$(2a + (n - 1)d)

S_{16} = $\frac{16}{2}$(2a + (16 - 1)d)

S_{16} = $\frac{16}{2}$(2a (16 - 1)d)

S_{16} = 8(2 $\times$ 2 + 15 $\times$ 5)

$S_{16} = 8(4 + 75)$

$S_{16} = 8(79)$

$S_{16} = 632$

25. We have,
$a_9 = 0$

$\Rightarrow a + (9 - 1)d = 0$

$\Rightarrow a + 8d = 0$

$\Rightarrow a = -8d$

To prove: $a_{29} = 2a_{19}$

Proof:

LHS = a_{29}

$= a + (29 - 1)d$

$= a + 28d$

$= -8d + 28d$

$= 20d$

RHS = $2a_{19}$

$= 2\ a + (19 - 1)d]$

$= 2[-8d + 18d]$

$= 2 \times 10d$

$= 20d$

$\therefore$ LHS = RHS

Hence, 29^{th} term is double the 19^{th} term.

26. Let the required number be (a - 3d), (a - d), (a + d) and (a + 3d)
Sum of these numbers = (a - 3d) + (a - d)+ (a + d) + (a + 3d)

According to the question, sum of the numbers=28

$\therefore 4a = 28 \Rightarrow a = 7$

Sum of the squares of these numbers

$=(a-3d)^2+(a-d)^2+(a+d)^2+(a+3d)^2=4(a^2+5d^2)$

Now, sum of the squares of numbers=216

$\therefore 4(a^2+5d^2)=216$

$\Rightarrow a^2+5d^2=54$ [$\because a=7$]

$\Rightarrow 5d^2= 54-49$

$\Rightarrow 5d^2=5$

$\Rightarrow d^2=1$

$\Rightarrow d=\pm1$

Hence, the required numbers (4, 6, 8, 10).

27. It is given that the sum of 5^{th} and 9^{th} terms of an A.P. is 72 and the sum of 7th and 12th terms is 97.
Let "a "be the first term and 'd' be the common difference of the Arithmetic progression .

It is given that $a_5 + a_9 = 72$ and, $a_7 + a_{12} = 97$

$\Rightarrow (a + 4d) + (a + 8d) = 72$ and, $(a + 6d) + (a + 11 d) = 97$.

Therefore, we have

$\Rightarrow 2a + 12d = 72$ ...(1)

$\Rightarrow 2a + 17d = 97$...(2)

Subtracting (1) from (2), we get

$2a+17d-2a-12d =97-72$

$\implies 5d = 25$

$\Rightarrow d = 5$

Putting d = 5 in (1), we get

$2a + 60 = 72 \Rightarrow 2a = 12 \Rightarrow a = 6$

Therefore,a = 6 and d = 5

Hence, the Arithmetic Progression a,a+d,a+2d ..... is 6, 11,16, 21, 26,......

28. Clearly, 7, 13, 19, ..., 241 is an A.P. with first term a = 7 and common difference d = 13 - 7 = 6.

Let there be n terms in the A.P.

Then, a_n = 241

⇒ a + (n - 1) d = 241

⇒ 7 + 6 (n - 1) = 241

⇒ 7 + 6n - 6 = 241

⇒ 6n + 1 = 241

⇒ 6n = 241 - 1

⇒ 6n = 240

⇒ n = 40

Clearly, n is even. So, $\left(\frac{n}{2}\right)^{th}$ = 20th and $\left(\frac{n}{2}+1\right)^{th} = 21^{\text{th}}$ are middle terms and are given by

a_{20} = a + (20 - 1) d

= a + 19d

= 7 + 19 × 6

= 7 + 114

= 121

and, a_{21} = a + (21 - 1) d

= a + 20d

= 7 + 20 × 6

= 7 + 120

= 127

So, the middle term of A.P. are 121 and 127.

29. Let 'a' be the first term and d be the common difference of the given A.P.

Then,according to question we have

T_p = a + (p - 1)d and T_q = a + (q - 1)d.(where T_p and T_q are p^{th} and q^{th} terms of given A.P)

Now, T_p = q and $T_{q=}$ p (given).

a + (p -1)d = q ...(i)

and a + (q - 1) d = p ...(ii)

On subtracting (i) from (ii), we get

$\Longrightarrow$ (q-1)d-(p-1)d=p-q

$\Longrightarrow$ qd-d-pd+d =p-q

$\Longrightarrow$ (q - p)d = (p - q)

$\Longrightarrow d = \frac{p-q}{q-p}$

⇒d = -1.

Putting d = -1 in (i), we get a = (p + q -1).

Thus, a = (p + q - 1) and d = -1.

Therefore, n^{th} term = a+ (n- 1)d = (p + q -1) + (n -1) $\times$ (-1)

=p+q-1-n+1

= p + q - n.

Hence, n^{th} term = (p + q - n). Which is required answer.

30. Let a and d be the first term and common difference respectively of the given A.P. Then

a_n = a + (n - 1)d

$\frac{1}{n} =$ m^{th} term

⇒ $\frac{1}{n}$ = a + (m - 1) d ...(i)

$\frac{1}{m}$= nth term

⇒ $\frac{1}{m}$ = a + (n - 1) d ...(ii)

On subtracting equation (ii) from equation (i), we get

$\frac{1}{n} - \frac{1}{m} =$ [a+ (m-1) d] -[a+ (n -1)d]

= a + md - d - a - nd + d

$= (m - n)d$
$\Rightarrow \frac{m-n}{mn} = (m - n)d$
$\Rightarrow d = \frac{1}{mn}$
Putting d = $\frac{1}{mn}$ in equation (i), we get
$\frac{1}{n} = a + \frac{(m-1)}{mn}$
$\Rightarrow \frac{1}{n} = a + \frac{1}{n} - \frac{1}{mn}$
$\Rightarrow a = \frac{1}{mn}$
$\therefore$ $(mn)^{th}$ term = a + (mn - 1) d
= $\frac{1}{mn} + (mn - 1)\frac{1}{mn}\left[\because a = \frac{1}{mn} = d\right]$
= $\frac{1}{mn} + \frac{mn}{mn} - \frac{1}{mn}$
= 1

31. Initial monthly salary of Tanvy in 2015 = Rs 40000
Annual increment = Rs 2500
Therefore an AP is formed
and here a = 40000 and d = 2500
Let the monthly salary of Tanvy becomes Rs 65000 in n^{th} year
So, a_n = a + (n-1)d
$\Rightarrow$ 65000=40000+(n-1)$\times$ 2500
$\Rightarrow$ 40000 + (n -1) $\times$ 2500 = 65000
$\Rightarrow$ (n -1) $\times$ 2500 = 65000 - 40000 = 25000
$\Rightarrow$ (n-1)=$\frac{25000}{2500}$ = 10
Hence n = 11
Thus, the 11th annual salary received by Tanvy will be Rs.65000. Thus, after 10 years, i.e., in the year 2025, her annual salary will be Rs.65000.

32. Since each prize is Rs 20 less than its preceding prize,
therefore, the value of the seven successive cash prizes will form an AP.
Let the first prize be Rs a.
Then the winner prizes, in succession,
will be Rs (a - 20), Rs (a - 40), Rs (a - 60), etc.
Here, A = a
d = (a - 20) - a = -20
n = 7
S_n = 700
We know that
$S_n = \frac{n}{2}[2A(n-1)d]$
$\Rightarrow 700 = \frac{7}{2}[2a + (7-1)(-20)]$
$\Rightarrow 700 = \frac{7}{2}[2a - 120]$
$\Rightarrow 700 = 7(a - 60)$
$\Rightarrow a - 60 = \frac{700}{7}$
$\Rightarrow$ a - 60 = 100
$\Rightarrow$ a = 100 + 60
$\Rightarrow$ a = 160
$\Rightarrow$ Value of first prize = Rs 160
Value of second prize = Rs 160 - Rs 20 = Rs 140
Value of third prize = Rs 140 - Rs 20 = Rs 120
Value of fourth prize = Rs 120 - Rs 20 = Rs 100
Value of fifth prize = Rs 100 - Rs 20= Rs 80
Value of sixth prize = Rs 80 - Rs 20 = Rs 60
Value of seventh prize = Rs 60 - Rs 20= Rs 40

33. Let a be the first term and d be the common difference of the given AP. Then, in general, the mth and nth terms can be written as T_m = a + (m - 1) d and T_n = a + (n - 1)d respectively.

According to the question,we are given that,

$(m.T_m) = (n.T_n)$

$\Rightarrow$ m.{a + (m - 1)d} = n.{a + (n - 1)d}

$\Rightarrow$ a.(m - n) + {$(m^2 - n^2)$ - (m - n)} . d = 0

$\Rightarrow$ (m - n).{a + (m + n - 1)}d.

$\Rightarrow$ (m - n).T_{m+n} = 0

$\Rightarrow T_{m+n} = 0$ [$\because$ (m-n)$\neq$0].

Hence, the (m + n)th term is zero.

34. Here a_{11} =38 and a_{16} =73

Using formula a_n =a +(n−1)d, to find n^{th} term of arithmetic progression,

38=a +(11−1) (d) And 73=a +(16−1)(d)

⇒ 38=a +10d And 73=a +15d

These are equations consisting of two variables.

We have, 38=a +10d ⇒ a =38−10d

Let us put value of a in equation (73=a +15d),

73=38−10d +15d ⇒ 35=5d

Therefore, Common difference =d =7

Putting value of d in equation 38=a +10d ,

38=a +70 ⇒ a =−32

Therefore, common difference = d = 7 and First term = a = -32

Using formula a_n =a +(n−1)d , to find n^{th} term of arithmetic progression,

a_{31}=-32+(31−1)(7) =−32+210=178

Therefore, 31^{st} term of AP is 178.

35. Let the first term be a and the common difference be d.

a_n = a + (n - 1)d

Here given, a_3 =9

or, a + 2d = 9 ....(i)

$a_8 - a_5 = 6$

or, (a + 7d) - (a + 4d) = 6

a + 7d - a - 4d = 6

or, 3d = 6

or, d = 2 ....(ii)

Substituting this value of d from (ii) in (i), we get

or, a + 2(2) = 9

or, a + 4 = 9

or a = 9 - 4

or, a = 5

a = 5 and d = 2

So, A.P. is 5,7,9,11,....

36. __, 38, __, __, __, -22

We are given 2^{nd} and 6^{th} term.

Using formula a_n =a +(n−1)d , to find n^{th} term of arithmetic progression,

a_2 =a +(2−1)d And a_6 =a +(6−1)d

⇒ 38=a +d And −22=a +5d

These are equations in two variables, we can solve them using any method.

Using equation (38=a +d), we can say that a =38−d.

Putting value of a in equation (−22=a +5d),

−22=38- d +5d ⇒ 4d =−60 ⇒ d =−15

Using this value of d and putting this in equation 38=a +d ,

38=a -15 ⇒ a =53

Therefore, we get a =53 and d =−15
First term = a = 53
Third term = second term + d = 38 - 15= 23
Fourth term = third term + d = 23 - 15= 8
Fifth term = fourth term + d = 8 - 15= -7
Therefore, missing terms are 53, 23, 8 and -7.

37. Let the first price be ₹ a.
Since each prize after the first is ₹200 less than the preceding prize,therefore,the prizes are ₹ a, ₹(a-200), ₹(a-400), ₹(a-600).
common difference d =(a-200)-a=-200. Then,
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$\Rightarrow 2800 = \frac{4}{2}[2a+3(-200)]$
$\Rightarrow 1400 = 2a-600$
$\Rightarrow 2a = 2000$
$\Rightarrow a = 1000$
So, the prizes are ₹ 1000, ₹ 800, ₹ 600 and ₹400.

38. In order to find an A.P we are required to find the first term and common difference.
In the given AP, let the first term = a and common difference =d
Now, in general n^{th} term of an A.P is given as $T_n = a + (n - 1)d$
$\Rightarrow T_7 = a + (7 - 1)d$, and $T_{13} = a + (13 - 1)d$
$\Rightarrow T_7 = a + 6d$, and $T_{13} = a + 12d$
$\Rightarrow T_7 = -4 \Rightarrow a + 6d = -4$ ....(1)
$\Rightarrow T_{13} = -16 \Rightarrow a + 12d = -16$ ....(2)
Subtracting (1) from (2), we get
$6d = -12 \Rightarrow d = -2$
Putting d = -2 in (1), we get
$a + 6\times(-2) = -4 \Rightarrow a - 12 = -4 \Rightarrow a = 8$
Thus, a = 8, d = -2
So the required AP is 8,6,4,2,0.....

39. Let the same common difference of two A.P's is d.
Given that, the first term of first A.P. and second A.P. are 2 and 7 respectively, then the A.P's are 2, 2 + d, 2+ 2d, 2 + 3d, ... and 7, 7 + d, 7 + 2d, 7 + 3d, ...
Now, 10th terms of first and second A.P's are 2 + 9d and 7 + 9d, respectively.
So, their difference is 7 + 9d - (2 + 9d) = 5
Also, 21st terms of first and second A.P.'s are 2 + 20d and 7 + 20d, respectively.
So, their difference is 7 + 20d - (2 + 9d) = 5
Also, if the a_n and b_n are the nth terms of first and second A.p.
Then $b_n - a_n = [7+ (n - 1)d] - [2 - (n - 1)d] = 5$
Hence, the difference between any two corresponding terms of such A.P.'s is the same as the difference between their first terms.

40. Given, $a_n = 7 - 3n$
Put n = 1, $a_1 = 7 - 3 \times 1 = 7 - 3 = 4$
Put n = 2, $a_2 = 7 - 3 \times 2 = 7 - 6 = 1$
Common difference(d) = 1 - 4 = -3
$S_n = \frac{n}{2}[2a + (n - 1)d]$
$S_{25} = \frac{25}{2}[2 \times 4 + (25 - 1)(-3)]$
$= \frac{25}{2}[8 - 72]$
$= \frac{25}{2} \times -64$
= -800

41. $S_n = 3n^2 - 4n$
$S_1 = 3(1)^2 - 4(1) = 3 - 4 = - 1$

$S_2 = 3(2)^2 - 4(2) = 12 - 8 = 4$

$a_1 = S_1 = -1$

$a_2 = S_2 - S_1 = 4 - (-1) = 4 + 1 = 5$

$d = d_2 - d_1 = 5 - (-1) = 5 + 1 = 6$

A.P. is -1, 5, 11.....

$a_{12} = a + 11d$

$= -1 + 11 \times 6$

$= -1 + 66$

$= 65$

42. Let a be the first term and d be the common difference of the given A.P.

Let the A.P. be $a_1, a_2, a_3, ..., a_n, ...$

$a_n = a + (n - 1)d$

It is given that

$a_7 = -1$

$\Rightarrow a + (7 - 1) d = -1$

$\Rightarrow a + 6d = -1$ ..... (i)

and $a_{16} = 17$

$\Rightarrow a + (16 - 1)d = 17$

$\Rightarrow a + 15d = 17$ ..... (ii)

Subtracting equation (i) from equation (ii), we get

$15d - 6d = 17 - (-1)$

$9d = 18$

$\Rightarrow d = 2$

Putting d = 2 in equation (i), we get

$a + 6(2) = -1$

$a + 12 = -1$

$\Rightarrow a = -13$

Hence, General term = $a_n = a + (n - 1) d$

$= -13 + (n - 1) 2$

$= -13 + 2n - 2$

$= 2n - 15$

43. In the given AP let the first term = a,

And common difference = d

Now, in general n^{th} term of an A.P is given by $T_n = a + (n-1)d$

$T_{10} = a + 9d = 52$--------------(1) [given]

$a + 16d = 20 + a + 12d$ [given]

$4d = 20$

$d = 5$

Substitute it in equation 1

$a + 9(5) = 52$

$a = 7$

Therefore, the required AP is

7, 12, 17,22.....

44. We know that the A.M. between a and b is $\frac{a+b}{2}$.

It is given that $\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M . between a and b.

$\therefore \frac{a^{n+1}+b^{n+1}}{a^n+b^n} = \frac{a+b}{2}$

$\Rightarrow 2(a^{n+1} + b^{n+1}) = (a^n + b^n)(a + b)$

$\Rightarrow 2a^{n+1} + 2b^{n+1} = a^{n+1} + ab^n + a^n b + b^{n+1}$

$\Rightarrow a^{n+1} + b^{n+1} = ab^n + a^n b$

$\Rightarrow a^{n+1} - a^n b = ab^n - b^{n+1}$

$\Rightarrow a^n\{a - b) = b^n(a - b)$

$\Rightarrow a^n = b^n \Rightarrow \frac{a^n}{b^n} = 1 \Rightarrow \left(\frac{a}{b}\right)^n = \left(\frac{a}{b}\right)^0 \Rightarrow n = 0$

45. Given that,

Yasmeen, during the first month, saves = 32 Rs

During the second month, saves = 36 Rs

During the third month, saves = 40 Rs

Let Yasmeen saves Rs 2000 during the n months.

Here, we have arithmetic progression 32, 36, 40, …

First term, a = 32

common difference, d = 36 – 32 = 4

Total money save by her in n months = Sum of this AP upto n terms

$2000 = \frac{n}{2}[2a + (n-1)d] \left(\text{using } S_n = \frac{n}{2}[2a + (n-1)d]\right)$

4000 = n[2(32) + (n - 1)4]

4000 = n[64 + 4n - 4)

4000 = n[4n + 60]

$4n^2 + 60n - 4000 = 0$

$n^2 + 15n - 1000 = 0$

$n^2 + 40n - 25n - 1000 = 0$

(n - 25)(n + 40) = 0

n = 25 or n = - 40

but n = - 40 as no of terms and months cannot be negative

So it would take 25 months to save Rs. 2000

46. Given,

Number of terms = n

First term = a

n^{th} term = l

To prove: Sum of m^{th} term from beginning and m^{th} term from end = a + l

Proof:

m^{th} term from beginning

= a + (m - 1)d

= a + md - d

m^{th} term form the end

= l - (m - 1)d

= l - md + d

Required Sum = a + md - d + l - md +d

= a + l

47. Let the 1st term of AP be a and common difference be d.

Now, three middle terms of this AP are a_{10}, a_{11} and a_{12}

from question,we have,

$a_{10} + a_{11} + a_{12} = 129$

$\Rightarrow$ (a + 9d) + (a + 10d) + (a + 11d) = 129

$\Rightarrow$ 3a + 30d = 129

$\Rightarrow$ a + 10d = 43 $\Rightarrow$ a = 43 - 10d ....(i)

Also, last three terms are a_{19}, a_{20} and a_{21}

$\therefore a_{19} + a_{20} + a_{21} = 129$

$\Rightarrow$ (a + 18d) + (a + 19d) + (a + 20d) = 237

$\Rightarrow$ 3a + 57d = 237

$\Rightarrow$ a + 19d = 79 $\Rightarrow$ 43 - 10d + 19d = 79 [using eq. (i)]

$\Rightarrow$ 9d = 36 $\Rightarrow$ d = 4

When d = 4, equation (i) becomes

a = 43 - 10(4) = 3

∴ AP is 3, 7, 11, 15, ...

48. Here, A.P is $-1, -\frac{5}{6}, -\frac{2}{3}, -\frac{1}{2}, \ldots, \frac{10}{3}$

The first term (*a*) = -1

The last term (a_n) = $\frac{10}{3}$

Now,

common difference (d) = a_1 - a

= $-\frac{5}{6} - (-1)$

= $-\frac{5}{6} + 1$

= $\frac{-5+6}{6}$

= $\frac{1}{6}$

Thus, using the formula for nth term, viz. a_n = a + (n-1) b, we get

$\frac{10}{3} = -1 + (n-1)\frac{1}{6}$

$\frac{10}{3} + 1 = \frac{1}{6}n - \frac{1}{6}$

$\frac{13}{3} + \frac{1}{6} = \frac{1}{6}n$

Further solving for *n*, we get

$\frac{26+1}{6} = \frac{1}{6}n$

$n = \frac{27}{6}(6)$

Thus, n = 27

Therefore, the number of terms present in the given A.P is 27

49. The given AP is 121, 117, 113, ...

Here, a = 121

d = 117 - 121 = -4

Let the nth term of the AP be the first negative term.

Then, a_n < 0

$\Rightarrow a + (n-1)d < 0$

$\Rightarrow a + (n-1)(-4) < 0$

$\Rightarrow 121 < (n-1)4$

$\Rightarrow (n-1)4 > \frac{121}{4}$

$\Rightarrow (n-1) > \frac{121}{4}$

$\Rightarrow n > \frac{121}{4} + 1$

$\Rightarrow n > \frac{125}{4}$

$\Rightarrow n > 31\frac{1}{4}$

Least integral value of n = 32. Hence, 32nd term

of the given AP is the first negative term 9.

50. Clearly, 1, 7, 13, 19,.... forms an A.P. with first term 1 and common difference 6.

Therefore, its nth term is given by

a_n = 1 + (n - 1) × 6 = 6n - 5

Also, 69, 68, 67, 66,... forms an A.P. with first term 69 and common difference -1.

a_n' = 69 + (n - 1) × (-1) = - n + 70

The two A.Ps will have identical ftth terms, if

$a_n = a_n'$

⇒ 6n - 5 = -n + 70

⇒ 7n = 75

⇒ n = $\frac{75}{7}$, which is not a natural number.

Hence, there is no value of n for which the two A.Ps will have identical terms.

51. Let the production during first year be a and let d be the increase in production every year. Then,

T_6 = 16000 ⇒ a + 5d = 16000 ... (i)

and T_9 = 22600 ⇒ a + 8d = 22600. ... (ii)

On subtracting (i) from (ii), we get 3d = 6600 $\Rightarrow$ d = 2200.
Putting d = 2200 in (i), we get
a + 5 $\times$ 2200 = 16000 $\Rightarrow$ a + 11000 = 16000 $\Rightarrow$ a = 16000-11000 = 5000.
Thus, a = 5000 and d = 2200.
Production during first year, a = 5000.

52. Let the first term and the common difference of the AP be a and d respectively.
Then,
Third term = 16
$\Rightarrow$ a + (3 - 1)d = 16 $\because a_n = a + (n-1)d$
$\Rightarrow$ a + 2d = 16 ...... (1)
and, 7th term = 5th term + 12
$\Rightarrow$ a + (7 - 1)d = a + (5 - 1)d + 12 $\because a_n = a + (n-1)d$
$\Rightarrow$ a + 6d = a + 4d + 12
$\Rightarrow$ 6d - 4d = 12
$\Rightarrow$ 2d = 12
$\Rightarrow d = \frac{12}{2} = 6$
Put d = 6 in (1), we get
a + 2(6) = 16
$\Rightarrow$ a + 12 = 16
$\Rightarrow$ a = 16 - 12
$\Rightarrow$ a = 4
Hence, the required APs also
4, 4 + 6, 4 + 6 + 6, 4 + 6 + 6 + 6, ...
i.e. 4, 10, 16, 22, ...

53. Here, first term = a = 3 and common difference = d = 15 – 3 = 12
Let nth term of the given AP be 132 more than its 54^{th} term, then
$a_n = a_{54} + 132$
$\Rightarrow$ a + (n – 1)d = a + (54 – 1)d + 132
$\Rightarrow$ (n – 1) d = (54 – 1)d + 132
$\Rightarrow$(n – 1) 12 = (53)12 + 132
$\Rightarrow$(n – 1) 12 = 768
$\Rightarrow$(n – 1) = 64
$\Rightarrow$ n = 65
Hence, the 65th term will be 132 more than the 54th term.

54. Let the first term be "a" and the common difference be "d" of the Arithmetic progression. According to the given information,
$a_4 = 9$
$\Rightarrow$ a+(4-1)d=9
$\Rightarrow$ a+3d=9...(1)
Also, $a_6 + a_{13} = 40$
$\Rightarrow$ a+(6-1)d+a+(13-1)d=40
$\Rightarrow$ a+5d+a+12d=40
$\Rightarrow$ 2a+17d=40...(2)
Multiplying equation (1) by 2,we get,
2a+6d=18..... (3)
Subtracting (3) from (2),
11d = 22
$\implies$ d=$\frac{22}{11}$
$\Rightarrow$ d = 2
from (1),we have,
a+3d =9
$\implies$ a = 9 - 3d (substitute value of d=2)
$\implies$ a= 9-6=3

So, we get "a" and "d" as 3 ,2.
Therefore,the AP is a, a+d, a+2d, a+3d,..., that is 3,5,7,9,......

55. Let a be the first term and d be the common difference of the given A.P.
Then, 8^{th} term = a_8 = a + 7d
and 2^{nd} term = a_2 = a + d
According to given information,
$a_8 = \frac{a_2}{2}$
$\Rightarrow$ $2a_8 = a_2$
$\Rightarrow$ 2(a + 7d) = a + d
$\Rightarrow$ a + 13d = 0...(i)
Now, 11^{th} term = a_{11} = a + 10d and 4^{th} term = a_4 = a + 3d
According to the given information,
$a_{11} - \frac{a_4}{3} = 1$
$\Rightarrow$ $3a_{11} - a_4 = 3$
$\Rightarrow$ 3(a + 10d) - (a + 3d) = 3
$\Rightarrow$ 3a + 30d - a - 3d = 3
$\Rightarrow$ 2a + 27d = 3...(ii)
Multiplying equation (i) by 2, we get
2a + 26d = 0...(iii)
Subtracting (iii) from (ii), we get
d = 3
$\Rightarrow$ a + 13(3) = 0
$\Rightarrow$ a = -39
Now, 15^{th} term = a_{15} = a + 14d = -39 + 14(3) = -39 + 42 = 3
Hence, 15^{th} term is 3 .

56. The 4^{th} term of an A.P. is three times the first term "a" and the 7^{th} term exceeds twice the third term by 1.
Now we have,
a_4 = 3a
$\Rightarrow$ a + (4 - 1)d = 3a
$\Rightarrow$a + 3d - 3a = 0
$\Rightarrow$-2a + 3d = 0............(i)
and,$a_7=2a_3+1$
$\implies$ $a_7 - 2a_3 = 1$
$\Rightarrow$ a+ (7 - 1)d -2[a + (3 - 1)d] = 1
$\Rightarrow$ a + 6d - 2[a + 2d] = 1
$\Rightarrow$ a + 6d - 2a - 4d = 1
$\Rightarrow$ -a + 2d = 1 (multiplying by 2 both sides)
$\Rightarrow$ -2a + 4d = 2..........(ii)
Subtracting equation (i) from (ii),
(-2a + 4d) - (-2a + 3d) = 2 - 0
$\Rightarrow$ -2a + 4d + 2a - 3d = 2
$\Rightarrow$ d = 2
Put the value of d in (i)
-2a + 3$\times$ 2 = 0
$\Rightarrow$ -2a = -6
$\Rightarrow$ $a = \frac{-6}{-2} = 3$
Therefore, First term a = 3 and Common difference d = 2

57. Let a be the first term and d be the common difference of the AP.
Given, $T_2 = 7\frac{3}{4}$
$\Rightarrow$ a + d = $\frac{31}{4}$ ....(i)

and $T_{31} = \frac{1}{2}$

$\Rightarrow$ a + 30d = $\frac{1}{2}$ ..(ii)

Subtracting (i) from (ii), we get

29d = $\frac{1}{2} - \frac{31}{4} = -\frac{29}{4} \Rightarrow$ d = $-\frac{1}{4}$

Putting the value of d in (i), we get

a $-\frac{1}{4}$ = $\frac{31}{4} \Rightarrow$ a = $\frac{31}{4} + \frac{1}{4} = \frac{32}{4}$ = 8

Let the number of terms be n, so that

$T_n = -\frac{13}{2}$

i.e., a + (n - 1)d = $-\frac{13}{2}$

$\Rightarrow$ 8 + (n - 1)($-\frac{1}{4}$) = $-\frac{13}{2} \Rightarrow$ 8 - $\frac{n}{4} + \frac{1}{4}$ = $-\frac{13}{2}$

$\Rightarrow$ 32 - n + 1 = -26 $\Rightarrow$ n = 59

Hence, first term = 8 and number of terms = 59.

58. Let the first term of the Arithmetic progression be 'a'.

and the common difference be 'd'.

24^{th} term of the Arithmetic progression, t_{24} = a + (24 - 1)d = a + 23d

10^{th} term of the A.P., t_{10} = a + (10 - 1)d = a + 9d

72^{nd} term of the A.P., t_{72} = a + (72 - 1)d = a + 71d

15^{th} term of the A.P., t_{15} = a + (15 - 1)d = a + 14d

$t_{24} = 2 \times t_{10}$

$\Rightarrow$ a + 23d = 2(a + 9d)

$\Rightarrow$ a + 23d = 2a + 18d

$\Rightarrow$ 23d - 18d = 2a - a

$\Rightarrow$ 5d = a

And, t_{72} = a + 71d (substitute value of a)

= 5d + 71d

= 76d

= 20d + 56d

= $4 \times 5d + 4 \times 14d$

= 4(5d + 14d)

= 4(a + 14d)

= $4t_{15}$

Therefore, $t_{72} = 4t_{15}$

Hence proved.

59. Let P be the principle, R rate of interest and I_n be the interest at the end of n year

We know that

$I_n = \frac{PRn}{100}$ $\left[\text{Using : Interest } = \frac{PRT}{100}\right]$

A sum of ₹1000 is invested at 8% simple interest per annum.

Here, we have

P = ₹1000, and R = 8% per annum

$\therefore$ I_n = ₹$\left(\frac{1000 \times 8 \times n}{100}\right)$= ₹ 80n

Putting n = 1,2,3,..., we have

l_n = 80n

$I_1 = 80 \times 1$ = ₹80

$I_2 = 80 \times 2$ = ₹160

$I_3 = 80 \times 3$ = ₹240

$I_4 = 80 \times 4$ = ₹320 and so on.

Since, I_n is a linear expression in n.

Therefore, the sequence of interest forms an A.P. with common difference 80.

Hence, the sequence of interests is an A.P.

Also, Interest at the end of 30 years = I_{30} = 80n = ₹ (80 $\times$ 30) = ₹ 2400

60. Let a be the first term and d be the common difference of the given A.P.

a_{10} = 52 and, a_{16} = 82

$\Rightarrow a+(10-1)d=52$ and, $a+(16-1)d=82$

$\Rightarrow a+9d=52$ .... (i) and, $a+15d=82$ ....(ii)

Subtracting equation (ii) from equation (i), we get

$-6d=-30$

$\Rightarrow d=5$

Putting d = 5 in (i),

$a+45=52$

$\Rightarrow a=7$

$\therefore a_{32}=a+(32-1)d=7+31\times 5=162$

$a_n=a+(n-1)d=7+(n-1)\times 5=5n+2$

Hence, a_{32} = 162 and a_n = 5n + 2

61. Let three terms of A.P be a - d, d, a + d

According to first condition,

a - d + a + a + d = 21

$\Rightarrow$ 3a = 21

$\Rightarrow$ a = 7

According to the second condition,

(a - d)(a + d) - a = 6

$\Rightarrow$(7 - d)(7 + d) - 7 = 6

$\Rightarrow$ 49 - d^2 - 7 = 6

$\Rightarrow$ -d^2 = 6 + 7 - 49

$\Rightarrow$ - d^2 = -36

$\Rightarrow d=\pm 6$

For, a = 7, d = 6

First term, a - d = 7 - 6 = 1

Second term, = a = 7

Third term, = a + d = 7 + 6 = 13

For, a = 7 and d = -6

First term = a - d

= 7 -(-6)

= 7 + 6 = 13

Second term = a= 7

Third term = a + d

= 7 - 6 = 1

62. Let the required numbers be (a - d), a, (a + d)

Sum of these numbers = (a - d) + a + (a + d) = 3a

$\therefore$sum of these squares = $(a-d)^2+a^2+(a+d)^2=3a^2+2d^2$

According to the question,we are given that,

Sum of three numbers = 21

$\therefore$3a = 21

a = 7

Also,sum of the squares of three numbers=165

$\Rightarrow 3a^2 + 2d^2=165$

$\Rightarrow 3(7)^2+2d^2=165$

$\Rightarrow 2d^2 =18$

$\Rightarrow d^2=9$

$\Rightarrow d=\pm 3$

Thus, a=7 and d=$\pm$3

Hence, the required numbers are (4, 7, 10) or (10, 7, 4)

63. The A.P's have same common difference. So, value of d is same.

Let the two A.P's be $a_1,a_2,a_3,...a_n,...$ and $b_1,b_2,b_3,...b_n,...$

Also, let d be the common difference of two A.P's. Then,

$a_n = a_1 + (n-1)d$ and $b_n = b_1 + (n-1)d$

$\Rightarrow a_n - b_n = [a_1 + (n-1)d] - [b_1 + (n-1)d]$

$\Rightarrow a_n - b_n = a_1 + nd - d - b_1 - nd + d$

$\Rightarrow a_n - b_n = a_1 - b_1$ .... (i)

Clearly, $a_n - b_n$ is independent of n and is equal to $a_1 - b_1$. In other words

$a_n - b_n = a_1 - b_1$ for all $n \in N$.

$\Rightarrow a_{100} - b_{100} = a_1 - b_1$ [From (i)] .... (ii)

and, $a_k = b_k = a_1 - b_1$ where k = 10,00,000.

But, $a_{100} - b_{100} = 111222333$ .... (iii)

From (ii) and (iii)

$\therefore a_1 - b_1 = 111\ 222\ 333$

$\Rightarrow a_k - b_k = a_1 - b_1 = 111222333$, where k = 10,00,000.

Hence, the difference between millionth terms is same as the difference between 100th terms.

So, the difference between their Millionth terms is 111222333.

64. Let the first term of given AP =a and common difference =d

Now, we know that in general the n^{th} term of an A.P is given by $T_n=a+(n-1)d$

$\Rightarrow T_4 = a + (4-1)d$, $T_{25} = a + (25-1)d$, and $T_{11} = a + (11-1)d$

$\Rightarrow T_4=a+3d$, $T_{25}=a+24d$, and $T_{11}=a+10d$

Now, $T_4=0$ [given]

$\Rightarrow a+3d=0$

$\Rightarrow a=-3d$ ....(1)

$\therefore T_{25}=a+24d=(-3d+24d)=21d$ [using (1)]

And $T_{11}=a+10d=-3d+10d=7d$ (substituting a from equation 1)

$\therefore T_{25}=21d=3\times 7d=3\times T_{11}$