Solution

ARITHMETIC PROGRESSION WS 5

Class 10 - Mathematics

1. Let the given AP be a,a+d,a+2d,a+3d,......
   4^{th} term = 18 (given)
   a+3d=18 ....(1)
   Difference between 15^{th} term and 9^{th} term =30 (given)
   (a + 14d) - (a + 8d) = 30
   a + 14d - a -8d = 30
   6d = 30
   d = $\frac{30}{6}$
   d = 5
   put d=5 in eq. (1),we get,
   a + 3 $\times$5=18
   a + 15 = 18
   a = 18 - 15
   a = 3
   A.P. is a, a + d, a + 2d,....
   i.e 3, 3 + 5, 3 + 10
   i.e 3, 8, 13.
2. Let the first term, common difference and the number of terms of an AP are a, d and n respectively.
   Given that, first term, a = - 5
   last term, a_n = 45
   Sum of the terms of the AP , S_n = 120
   We know that, if last term of an AP is known, then sum of n terms of an AP is,
   $S_n = \frac{n}{2}(a + a_n)$
   $120 = \frac{n}{2}(-5 + 45)$
   240 = 40n
   n = 6
   Also we know the nth term formula
   a_n = a + (n - 1)d
   45 = - 5 + (6 - 1)d
   50 = 5d
   d = 10
   Hence, number of terms and the common difference of an AP are 6 and 10 respectively.
3. The given AP is $-3, \frac{1}{2}, 2, \ldots\ldots$
   Here, a = -3
   $d = -\frac{1}{2} - (-3) = -\frac{1}{2} + 3 = \frac{5}{2}$
   and n = 11
   a_{11} = ?
   We have, a_n = a + (n - 1)d
   So, $a_{11} = -3 + (11 - 1)\left(\frac{5}{2}\right)$
   $\Rightarrow a_{11} = -3 + 25$
   $\Rightarrow a_{11} = 22$
4. According to the question,
   Ramkali saves ₹100 in the first week and increased her weekly savings by ₹ 20 every week.
   Therefore, the amount saved by Ramkali in successive weeks are ₹100, ₹120, ₹140, ₹160,... up to 12 terms.
   Clearly these amounts form an Arithmetic progression in which first term a = 100, and common difference d= 20 and n = 12.
   Using S_n= $\frac{n}{2}$[2a + (n-1)d], we get

$S_{12} = \frac{12}{2}$ [2×100+(12-1)d]

$\Rightarrow S_{12} = \frac{12}{2}$ [2× 100 +11 × 20]

= (6 × 420)

= 2520 > 2500.

Therefore, the total amount generated is equal to 2520.

Thus, Ramkali will be able to deposit her daughter's fees, and so she can send her to school.

The above situation shows that saving is a good habit as it helps preserve and collect money for a good cause.

5. Here, a = 21, d = 18 – 21 = – 3 and a_n = – 81, and we have to find n.

   As a_n = a + (n – 1) d,

   we have – 81 = 21 + (n – 1)(– 3)

   – 81 = 24 – 3n

   – 105 = – 3n

   So, n = 35

   Therefore, the 35th term of the given A.P is – 81.

   Next, we want to know if there is any n for which an = 0. If such an n is there, then

   21 + (n – 1) (–3) = 0,

   i.e., 3(n – 1) = 21

   i.e., n = 8

   So, the eighth term is 0.

6. p.

   Explanation:

   t(n) = p.n + q

   We know, Common difference is the difference in any two arbitrary consecutive terms in A.P.

   So Common difference = $T_{m+1} - T_m$

   = [p(m + 1) + q] – [p.m + q]

   = pm + p + q – pm – q

   = p

7. The given Arithmetic progression is 7,11,15,..., 139.

   We have first term a = 7 and common difference d = (11 - 7) = 4.

   Suppose there are n terms in the given Arithmetic progression .Then,

   T_n = 139

   ⇒ a+ (n - 1)d = 139

   ⇒ 7 + (n -1) × 4 = 139

   ⇒ 4n = 136

   ⇒n = 34.

   Hence, there are 34 terms in the given Arithmetic progression.

8. Here, we are given two A.P. sequences whose n^{th} terms are equal. We need to find n.

   So let us first find the n^{th} term for both the A.P.

   First A.P. is 9, 7, 5...

   Here

   First term (a) = 9

   Common difference of the A.P. (d) = 7 - 9

   = -2

   Now as we know

   a_n = a + (n - 1)d

   So for nth term

   a_n = a + (n - 1) d

   So for nth term

   an = 9 + (n - 1)(-2)

   = 9 - 2n + 2

   = 11 - 2n ...(i)

Second A.P. is 15, 12, 9...
Here,
First term (a) = 15
Common difference of the A.P. (d) = 12 - 15 = -3
Now as we know
$a_n = a + (n - 1)d$
So for nth term
$a_n = 15 + (n - 1)(-3)$
= 15 - 3n + 3
= 18 - 3n ...(ii)
Now, we are given that the nth terms for both the A.P. sequences are equal, we equate (i) and (ii),
11 - 2n = 18 - 3n
3n - 2n = 18 - 11
n = 7
Therefore n = 7

9. Here we have, $A_n = n^2 - n + 1$

   Put n = 1

   $A_1 = (1)^2 - 1 + 1 = 1$

   Put n = 2

   $A_2 = (2)^2 - 2 + 1 = 3$

   Put n = 3

   $A_3 = (3)^2 - 3 + 1 = 9 - 2 = 7$

   Put n = 4

   $A_4 = (4)^2 - 4 + 1 = 16 - 3 = 13$

   Put n = 5

   $A_5 = (5)^2 - 5 + 1 = 25 - 4 = 21$

10. The given term of arithmatic progression is17, 12, 7, 2, ...
    Here, a = 17, d = 12 - 17 = - 5 where a is first term and d is common difference
    Suppose $a_n = -150$
    $a + (n - 1)d = -150 \Rightarrow 17 + (n - 1)(-5) = -150$
    $\Rightarrow (n - 1)(-5) = -150 - 17 \Rightarrow (n - 1)(-5) = -167$
    $\Rightarrow n - 1 = \frac{167}{5} \Rightarrow n = \frac{167}{5} + 1$
    $\Rightarrow n = \frac{167+5}{5} = \frac{172}{5} = 34\frac{2}{5}$ where
    n is not a whole number.
    $\therefore$ -150 is not a term of the A.P.

11. Here we have, $A_n = 2n^2 - 3n + 1$

    Put n = 1

    $A_1 = 2(1)^2 - 3(1) + 1 = 2 - 3 + 1 = 0$

    Put n = 2

    $A_2 = 2(2)^2 - 3(2) + 1 = 8 - 6 + 1 = 3$

    Put n = 3

    $A_3 = 2(3)^2 - 3(3) + 1 = 18 - 9 + 1 = 10$

    Put n = 4

    $A_4 = 2(4)^2 - 3(4) + 1 = 32 - 12 + 1 = 21$

    Put n = 5

    $A_5 = 2(5)^2 - 3(5) + 1 = 50 - 15 + 1 = 36$

12. Here, $a = 6, l = 216, d = 13 - 6 = 7$

Let the number of terms be n

$l = a + (n-1)d$

216 = $6 + (n-1)(7)$

216 -6 = $7(n-1)$

$7(n-1)$= 210

$n-1$= $\frac{210}{7}$= 30

n = 30 + 1 = 31

The middle term will be = $\frac{31+1}{2}$= 16th term

$\therefore a_{16}$ = 6 +(16 - 1)(7)

$= 6 + 15 \times 7$

= 6 + 105

= 111

Middle term will be 111.

13. Let the three angles of triangle which are in A.P. be a - d, a, a + d

Now, sum of the angles = 180°

⇒ a - d + a+ a + d = 180°

⇒ 3a = 180°

⇒ a = 60°

It is given that,

a + d= 2(a - d)

⇒ a + d = 2a - 2d

⇒ a = 3d

⇒ 60° = 3d

⇒ d = 20°

Thus, the angles of triangle are 40°, 60°, 80°.

14. We have, a = 5 and d = 10

Let n^{th} term of the given A.P. be 130 more than its 31^{st} term. Then,

$a_n = 130 + a_{31}$

⇒a + (n-1)d = 130 + a + 30d

⇒ 5 + 10 (n -1) = 130 + 5 + 30(10)

⇒ 10 (n - 1) = 130 + 300

⇒10 (n - 1) = 430

⇒ n - 1 = 43

⇒ n = 44

Hence, 44^{th} term of the given A.P. is 130 more than its 31st term.

15. We have, $a_n = 4n + 5$

Putting n=1,2,3 and 4 we get

a_1= 4×1+5=9

a_2 =4×2+5=13

a_3 =4×3+5=17

a_4=4×4+5=21

Hence we get the series;

9,13,17,21 which is an A.P. and the common difference=17-13=13-9=4

16. Here we have, $A_n = \frac{2n-3}{6}$

Put n = 1

$A_1 = \frac{2(1)-3}{6} = \frac{-1}{6}$

Put n = 2

$A_2 = \frac{2(2)-3}{6} = \frac{1}{6}$

Put n = 3

$A_3 = \frac{2(3)-3}{6} = \frac{3}{6} = \frac{1}{2}$

Put n = 4

$A_4 = \frac{2(4)-3}{6} = \frac{5}{6}$

Put n = 5

$A_5 = \frac{2(5)-3}{6} = \frac{7}{6}$

17. Here, $a = 213, d = 205 - 213 = -8, l = 37$

    Let the number of terms be n

    $l = a + (n-1)d$

    $37 = 213 + (n-1)(-8)$

    or, $37 - 213 = -8(n-1)$

    or,$-8(n-1) = -178$

    or, n -1 = $\frac{-176}{-8}$

    or $n - 1 = 22$

    or,$n = 22 + 1 = 23$

    The middle term will be = $\frac{23+1}{2}$ = 12th term

    $\therefore a_{12}$ = a + (n - 1)d

    = 213 + (12 - 1)(-8)

    = 213 + (11)(-8)

    = 213 - 88

    = 125

    Middle term will be 125.

18. Given AP is: $17, 16\frac{1}{5}, 15\frac{2}{5}, 14\frac{3}{5}, \dots$

    Here, first term, a = 17

    and common difference, d = $16\frac{1}{5} - 17$ = $\frac{81}{5} - 17 = -\frac{4}{5}$

    $\because a_n = a + (n-1)d$

    $\therefore a_n$ = 17 + (n - 1)($-\frac{4}{5}$)

    $\Rightarrow a_n = 17 - \frac{4}{5}n + \frac{4}{5}$

    $\Rightarrow a_n = \frac{85+4}{5} - \frac{4}{5}n$

    $\Rightarrow a_n = \frac{89}{5} - \frac{4}{5}n$

    Let nth term be the first negative term, therefore, $a_n < 0$

    $\Rightarrow \quad \frac{89}{5} - \frac{4}{5}n < 0$

    $\Rightarrow \quad -\frac{4}{5}n < -\frac{89}{5}$

    $\Rightarrow \quad \frac{4}{5}n > \frac{89}{5}$

    $\Rightarrow \quad n > \frac{89}{5} \times \frac{5}{4}$

    $\Rightarrow \quad n > \frac{89}{4} \Rightarrow n > 22\frac{1}{4}$

    $\therefore$ n = 23.

    That is, 23rd term is the 1st negative term.

19. Here we are having

    a=5 and d=9-5=4

    Let its nth term be 81. Then,

    T_n = 81

    a + (n - 1)d = 81

    5 + (n - 1) $\times$ 4 = 81

    4n = 80

    n = 20.

    Hence 81 is $20^{th}\ term.$

20. a, 10, b, c, 31 are in A.P.

    Common difference = d

    a_1 = a

$a_2 = a + d = 10$ ...... (i)

$a_5 = a + 4d = 31$ ...... (ii)

Subtracting (i) and (ii)

3d = 21

d = 7

Put d = 7 in (i)

a + 7 = 10

a = 10 - 7

a = 3

or, d = 7 and a = 3

or, a = 3, b =3 + 14 = 17, c = 3 + 21 = 24

$\therefore$ a = 3, b = 17, c = 24

21. The given AP is 6,13,20,...,216

    Clearly, a = 6 and l = 216

    So, the middle term will be average of a and l

    Hence middle term = $\frac{\mathrm{a+l}}{2}$

    $= \frac{6+216}{2} = \frac{222}{2}$

    = 111

    So the middle term of this AP is 111

22. Given,

    AP: 65, 61, 57, 53

    a = 65 and d = 61 - 65 = -4

    Consider the n^{th} term of the A.P. as the first negative term

    i.e., $a_n < 0$

    We know that n^{th} term of an AP,

    $a^n = a + (n - 1)d$

    Here, [a + (n - 1)d] < 0

    $\Rightarrow$ 65 + (n - 1) (-4) < 0

    $\Rightarrow$ 65 - 4n + 4 < 0

    $\Rightarrow$ 69 - 4n < 0

    $\Rightarrow$ 4n > 69

    $\Rightarrow$ n > $\frac{69}{4}$

    $\Rightarrow$ n > 17.25

    $\Rightarrow$ n = 18

    So the 18^{th} term is the first negative term of the given AP.

23. Let the first term be a and the common difference be d.

    we know, $a_n = a + (n-1)d$

    $a_p = a + (p-1)d$

    $a_{p+2q} = a + (p+2q-1)d$

    $\therefore a_p + a_{p+2q} = a + (p-1)d + a + (p+2q-1)d$

    $= a + pd - d + a + pd + 2qd - d$

    $= 2a + 2pd + 2qd - 2d$

    $= 2[a + (p+q-1)d]$ ....(i)

    $2a_{p+q} = 2[a + (p+q-1)d]$ ....(ii)

    From (i) and (ii), we get

    $a_p + a_{p+2q} = 2a_{p+q}$

24. The given A.P can be rewritten as 6, $\frac{31}{4}$, $\frac{19}{2}$, $\frac{45}{4}$, $\ldots$

    Here, First term = a = 6

    Common difference = d = ($\frac{31}{4}$) - 6 = $\frac{7}{4}$

    To find = 37^{th} term,

    $\therefore$ n = 37

Using the formula for finding nth term of an A.P.,

$a_n = a + (n - 1) \times d$

$\therefore a_n = 6 + (37 - 1) \times (\frac{7}{4})$

$\Rightarrow a_n = 6 + 36 \times (\frac{7}{4}) = 6 + 63 = 69$

$\therefore$ 37th term of the given A.P is 69

25. The given A.P is 1, 2,...., 31

Here a = 1, d =1 and n = 31

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{31}{2}[2(1) + (31 - 1)1]$

$= \frac{31}{2}[2 + 30] = \frac{31}{2}(32)$

$\therefore$ Piggy bank amount = Rs. 496

Amount spent = Rs. 204

Amount left = Rs. 100

Total pocket money = Piggy bank amount + amount spent + amount left = 496 + 204 + 100 = 800 Rs.

26. $a_n = n^2 - 1$

Put n = 1, 2, 3, 4, ... we get,

$a_1 = 1^2 - 1 = 1 - 1 = 0$

$a_2 = 2^2 - 1 = 4 - 1 = 3$

$a_3 = 3^2 - 1 = 9 - 1 = 8$

$a_4 = 4^2 - 1 = 16 - 1 = 15$, and so on.

Hence, the given sequence is 0, 3, 8, 15

$a_2 - a_1 = 3 - 0 = 3$

$a_3 - a_2 = 8 - 3 = 5$

As $a_2 - a_1 \neq a_3 - a_2$, the given sequence is not an AP.

27. First term (a) = 7

Last term $(a_n) = 125$

Number of terms (n) = 60

$\therefore a_n = a + (n - 1)d$

$\Rightarrow 125 = 7 + (60 - 1)(d)$

$\Rightarrow 125 - 7 = 59d$

$\Rightarrow 118 = 59d$

$\Rightarrow \quad d = \frac{118}{59} = 2$

Therefore, Common difference = 2

$\therefore 32^{nd}$ term$(a_{32}) = a + (32 - 1)d$

$= 7 + (31)(2)$

$= 7 + 62$

$= 69$

28. We are given that $a_n = 5 - 6n$

Put, n = 1.

$a_1 = 5 - 6 \times 1$

$= 5 - 6 = -1$

Put n = 2,

$a_2 = 5 - 6 \times 2$

$= 5 - 12 = -7$

First term(a) = $a_1 = -1$

$d = a_2 - a_1 = -7 - (-1) = -6$

Sum of n terms = $\frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[2(-1) + (n - 1)(-6)]$

$= \frac{n}{2}[-2 -6n + 6]$
$= \frac{n}{2}[4 - 6n]$
$= \frac{n}{2} \times 2[2 - 3n]$
$= n (2- 3n) = 2n-3n^2$

29. $a_n = a + (n - 1)d$

$\Rightarrow a_n = 7 + (8 - 1)3$

$\Rightarrow a_n = 7 + (7)3$

$\Rightarrow a_n = 7 + 21$

$\Rightarrow a_n = 28$

30. Here, a = -7, and d = -5

We know that
$a_n = a + (n - 1)d$
$\Rightarrow -82 = -7 + (n - 1) - 5$
$\Rightarrow -82 = -7 + 5 - 5n$
$\Rightarrow 80 = 5n$
$\Rightarrow n = 16$
So, -82 is the 16th term of the given A.P
Now, let $a_n = -100$
$\Rightarrow -100 = -7 + 5 - 5n$
$\Rightarrow -98 = -5n$
$\Rightarrow n = \frac{98}{5}$
This is impossible as n cannot be a fraction.
So, n cannot be the term of the given A.P.

31. According to question we are given that the sum of three numbers of an AP is 27 and their product is 405.

Suppose three numbers in AP are a - d, a and a + d.
$\therefore (a - d) + a + (a + d) = 27$
$\Rightarrow 3a = 27$
$\Rightarrow a = 9$
Also, $(a - d)(a)(a + d) = 405$
$\Rightarrow (9 - d)(9)(9 + d) = 405$
$\Rightarrow (9 - d)(9 + d) = 45$
$\Rightarrow 81 - d^2 = 45$
$\Rightarrow d^2 = 36$
$\Rightarrow d = 6, -6$
When d = 6, numbers are 3, 9, 15
When d = -6, numbers are 15, 9, 3.

32. Let the first terms and the common difference of the given AP be a and d respectively.

$\Rightarrow a + (2 - 1)d = 38 \because a_n = a + (n-1)d$
$\Rightarrow a + d = 38$ .... (1)
Sixth term = -22
$\Rightarrow a + (6 - 1) d = -22$
$\Rightarrow a + 5d = -22$ .... (2)
Solving (1) and (2), we get
a = 53
d = -15
Therefore,
Third term = $53 + (3 - 1) (-15) \because a_n = a + (n-1)d$
= 53 - 30
= 23
Fourth term = $53 + (4 - 1) (-15) \because a_n = a + (n-1)d$
= 8

Fifth = 53 + (5 - 1) (-15) $\because a_n = a + (n-1)d$

= -7

Hence, the missing terms in the boxes are 53, 23, 8, -7

33. Let the first term of the A.P. be 'a'.

and the common difference be 'd'.

19^{th} term of the A.P., t_{19} = a + (19 - 1)d = a + 18d

6^{th} term of the A.P., t_6 = a + (6 - 1)d = a + 5d

9^{th} term of the A.P., t_9 = a + (9 - 1)d = a + 8d

$t_{19} = 3t_6$

$\Rightarrow$a + 18d = 3(a + 5d)

$\Rightarrow$a + 18d = 3a + 15d

$\Rightarrow$18d - 15d = 3a - a

$\Rightarrow$3d = 2a

$\therefore \mathrm{a} = \frac{3\mathrm{d}}{2}$

t_9 = 19

$\Rightarrow \frac{3\mathrm{d}}{2} + 8\mathrm{d} = 19$

$\Rightarrow \frac{3d+16d}{2} = 19$

$\Rightarrow \frac{19\mathrm{d}}{2} = 19$

$\Rightarrow$ d = 2

$\Rightarrow$a = 3

t_2 = 3 + (2 - 1)2 = 5

t_3 = 3 + (3 - 1)2 = 7

The series will be 3, 5, 7......

34. Given:-

T_n = 2n + 3

For 6 th term,

n = 6

Therefore,

T_6 = 2$\times$6 + 3 = 15

For 20 th term,

n = 20

T20 = 2$\times$20 + 3 = 43

35. a_n = a + (n - 1)d

$\Rightarrow$ 3.6 = - 18.9 + (n - 1) (2.5)

$\Rightarrow$ 3.6 + 18.9 = (n - 1) (2.5)

$\Rightarrow$ 22.5 = (n - 1) (2.5)

$\Rightarrow n - 1 = \frac{22.5}{2.5}$

$\Rightarrow$ n - 1 = 9

$\Rightarrow$ n = 10

36. $a_5 = a + 4d = 26$ ......(i)

$a_{10} = a + 9d = 51$ ...(ii)

Subtract (i) and (ii) we get,

$5d = 25$

$d = 5$

Put $d = 5$ in (i)

$a + 4(5) = 26$

$a + 20 = 26$

$a = 26 - 20$

and $a = 6$

Hence, the A.P. is 6,11,17.......

37. The natural numbers between 1 and 1000, which are divisible by 5 but not by 2, are:

5, 15, 25, 35,... 995

The above sequence is an A.P. with a common difference of 10.

Using formula, I = a + (n - 1)d

995 = 5 + (n - 1)10

$\Rightarrow \frac{990}{10}$ = n - 1

$\Rightarrow$ n - 1 = 99

$\Rightarrow$ n = 100

Thus, there are 100 terms between 1 and 1000, which are divisible by 5 but not by 2.

38. Here we have, $A_n = \frac{n-2}{3}$

Put n = 1

$A_1 = \frac{1-2}{3} = \frac{-1}{3}$

Put n = 2

$A_2 = \frac{2-2}{3} = 0$

Put n = 3

$A_3 = \frac{3-2}{3} = \frac{1}{3}$

Put n = 4

$A_4 = \frac{4-2}{3} = \frac{2}{3}$

Put n = 5

$A_5 = \frac{5-2}{3} = \frac{3}{3} = 1$

39. a+13d=44

13d = 44 - 17

d = $\frac{27}{13}$

a_{15} = a + 14d = 17 + 14 $\times$ $\frac{27}{13}$

a_{15} = 46.07

40. The given Arithmetic progression is $-5, \frac{-5}{2}, 0, \frac{5}{2}, ...$

Its first term a = -5 and common difference d = $\left(\frac{5}{2} - 0\right) = \frac{5}{2}.$

Therefore, a = -5 and $d = \frac{5}{2}.$

$\Rightarrow T_{25}$ = a + (25-1)d

$= a + 24d$

$= (-5) + \left(24 \times \frac{5}{2}\right)$

$= -5 + 60 = 55.$

Hence, 25th term = 55.

41. Let the production during first year be a and let d be the increase in production every year. Then,

T_6 = 16000 $\Rightarrow$ a + 5d = 16000 ... (i)

and T_9 = 22600 $\Rightarrow$ a + 8d = 22600. ... (ii)

On subtracting (i) from (ii), we get 3d = 6600 $\Rightarrow$ d = 2200

Putting d = 2200 in (i), we get

a + 5 $\times$ 2200 = 16000 $\Rightarrow$ a + 11000 = 16000 $\Rightarrow$ a = 16000-11000 = 5000

Thus, a = 5000 and d = 2200

Production during 8th year is given by

T_8 = (a + 7d) = (5000 + 7 $\times$ 2200) = (5000 + 15400) = 20400

42. Here we have, $a_1 = 4$, $a_n = 4a_{n-1} + 3$, n > 1

Put n = 2

$A_2 = 4a_{2-1} + 3 = 4a_1 + 3 = 19$

Put n = 3

$A_3 = 4a_{3-1} + 3 = 4a_2 + 3 = 79$

Put n = 4

$A_4 = 4a_{4-1} + 3 = 4a_3 + 3 = 316 + 3 = 319$

Put n = 5

$A_5 = 4a_{5-1} + 3 = 4a_4 + 3 = 1276 + 3 = 1279$

Put n = 6

$A_6 = 4a_{6-1} + 3 = 4a_5 + 3 = 5116 + 3 = 5119$

43. Here, a = ₹ 5

d = ₹ 1.75

a_n = ₹ 20.75

We know that

$a_n = a + (n - 1)d$

$\Rightarrow 20.75 = 5 + (n - 1)d$

$\Rightarrow (n - 1) (1.75) = 20.75 - 5$

$\Rightarrow (n - 1) (1.75) = 15.75$

$\Rightarrow n - 1 = \frac{15.75}{1.75}$

$\Rightarrow n - 1 = 9$

$\Rightarrow n = 10$

Hence, the required value of n is 10.

44. The smallest 3 digit number divisible by 9 =108

.and the biggest 3 digit number divisible by 9 is =999

Here an AP is formed where

a=108,d=9

Let n three-digit natural numbers are divisible by 9

Then n^{th} term $T_n = 999$

So $999 = 108 + (n - 1) \times 9$ ..

999 =108+9n-9

999-108+9 = 9n

900=9n

$n = \frac{900}{9} = 100$

Hence 100 three-digit natural numbers are divisible by 9.

45. First term = a = 114

Common difference = $a_2 - a_1$

d = 109 - 114

d = -5

Let n^{th} term of A.P = 0

a + (n - 1)d = 0

114 + (n - 1)(-5) = 0

114 - 5n + 5 = 0

119 - 5n = 0

-5n = - 119

$n = \frac{119}{5}$

n = 23.4

Therefore, 24th term of given A.P. is first negative term.

46. Given sequence is defined by $a_n = (-1)^{n-1} \cdot 2^{n}$.

We know that $a_n = (-1)^{n-1} \cdot 2^n$.

Therefore, $a_1 = (-1)^{1-1} \times 2^1 = (-1)^0 \times 2 = 1 \times 2 = 2$

$a_2 = (-1)^{2-1} \times 2^2 = (-1)^1 \times 4 = -1 \times 4 = -4$

$a_3 = (-1)^{3-1} \times 2^3 = (-1)^2 \times 8 = 1 \times 8 = 8$

$a_4 = (-1)^{4-1} \times 2^4 = (-1)^3 \times 16 = -1 \times 16 = -16$

$a_5 = (-1)^{5-1} \times 2^5 = (-1)^4 \times 32 = 1 \times 32 = 32$.

Hence ,first five terms are 2,-4,8,-16,32.

47. Given AP is 53, 48, 43, ….

Whose, first term, a = 53

common difference, d = 48 – 53 = - 5

Let nth term of the AP be the first negative term.

Then, we have to find the least value for which

$a_n < 0$

a + (n - 1)d < 0

53 + (n - 1) (- 5) < 0

53 - 5(n - 1) < 0

5(n - 1) > 53

$\mathrm{n}-1>\frac{53}{5}$

$\mathrm{n}>1+\frac{53}{5}$

$n>\frac{58}{5}$

So n will be the least natural number greater than $\frac{58}{5}=11.6$ i.e. 12

i.e., 12th term is the first negative term of the given AP.

48. According to question the given arithmetic progression: - 6, - 2, 2, ...,58.

Here, a = -6, d = - 2 + 6 = 4 and a_n = 58 where a is first term and d is common difference

⇒ a + (n - 1)d = 58 ⇒ - 6 + (n - 1)4 = 58

⇒ (n - 1)4 = 64 ⇒ n - 1 = 16 ⇒ n = 17 (odd)

∴ MIddle term = $\frac{17+1}{2}=\frac{18}{2}$ = 9th term

∴ 9th term is the middle term.

Now, a_9 = a + 8d = -6 + 8 × 4 = -6 + 32 = 26

49. $a_4 + a_8$ = 24 (Given)

⇒ a + 3d + a + 7d = 24

⇒ 2a + 10d = 24

⇒ a + 5d = 12 ..... (i)

$a_6 + a_{10}$ = 44

⇒ a + 5d + a + 9d = 44

⇒ 2a + 14d = 44

⇒ a + 7d = 22 ...... (ii)

On solving equation (i) and (ii)

d = 5, a = -13

A.P. -13, -8, -3, 2...........

50. We have, a_n = 3n - 2.

a_1=3×1-2=1

a_2=3×2-2=4

So d=4-1=3 and a=1

Hence, 10^{th} term=1+9×3

=1+27=28

51. Here a = 12,d = 6.

Let no. of terms = n

∴ a_n = 252

⇒ a + (n - 1)d = 252

⇒ 12 + (n -1)6 = 252 ⇒ (n - 1)6 = 240

⇒ n - 1 = 40 ⇒ n = 41

Now middle term = $\frac{41+1}{2}$ = 21st term

∴ a_{21} = a + 20d

= 12 + 20 × 6 = 132.

52. Let the common difference of the two A.P's be d. Then, their n^{th} terms are

a_n = 3 + (n - 1) d and b_n = 8 + (n - 1) d

$\Rightarrow a_n - b_n = [3 + (n - 1) d] - [8 + (n - 1)d]$

$\Rightarrow a_n - b_n = -5$ for all $n \in N$.

Hence, $a_2 - b_2 = -5$, $a_4 - b_4 = -5$, $a_{10} - b_{10} = -5$ and $a_{30} - b_{30} = -5$.

53. Given;

$a = 12x$

$t_n = -2x$

$t_n = a + (n - 1)d$

$-2x = 12x + (n - 1) - 2x$

$-2x - 12x = (n - 1) - 2x$

$\frac{-14x}{-2x} = (n - 1)$

$n - 1 = 7$

$n = 8$

hence, no. of terms in AP is 8.

54. Here, First term = a = 5

Common difference = $d = \frac{9}{2} - 5 = -(\frac{1}{2})$

we need to find = 25^{th} term,

$\therefore n = 25$

Using the formula for finding n^{th} term of an A.P.,

$a_n = a + (n - 1) \times d$

$\therefore a_n = 5 + (25 - 1) \times (\frac{-1}{2})$

$\Rightarrow a_n = 5 + 24 \times (\frac{-1}{2}) = 5 - 12 = -7$

$\therefore$ 25th term of the given AP is - 7.

55. We have, $a = -1$, $a_n = \frac{a_{n-1}}{n}, n \geq 2$

Put n = 2

$A_2 = \frac{a_{2-1}}{2} = \frac{-1}{2}$

Put n = 3

$A_3 = \frac{a_{3-1}}{3} = \frac{-1}{6}$

Put n = 4

$A_4 = \frac{a_{4-1}}{4} = \frac{-1}{24}$

Put n = 5

$A_5 = \frac{a_{5-1}}{5} = \frac{-1}{120}$

Put n = 6

$A_6 = \frac{a_{6-1}}{6} = \frac{-1}{720}$

56. Given A.P. is 3, 8, 13, ..., 253

As the 15^{th} term is considered from last, so a = 253

Common difference, d = 3 - 8 = - 5 (Considered in reverse order)

We know that the n^{th} term of an A.P. is $a_n = a + (n - 1) d$

So, 15^{th} Term is $a_{15} = a + (15 - 1)d$

$a_{20} = 253 + (15 - 1)(- 5)$

$= 253 - 14 \times 5$

$= 253 - 70$

$= 183$

15^{th} term from the last term is 183.

57. Let a be the first term and d be the common difference of the A.P.

We have, $a_{10} = 52$

$\Rightarrow a + 9d = 52$ ----(1)

Also $a_{17}=a_{13}+20$

$a+16d=a+12d+20$

4d=20. or d=5
From (1)
a+9×5=52
a+45=52
so a =7
Hence the AP formed is
7,12,17,22,27.....

58. we have
$a_1 = -7$ and $d = 5$
$a_n = a + (n-1)d$
$a_n = -7 + (n-1)5 = -7 + 5n - 5 = 5n - 12$
$a_{18} = 5(18) - 12 = 78$
Therefore, 18^{th} term is 78 and n^{th} term is 5n -12

59. n^{th} term of first A.P = n^{th} term of second A.P
23 + (n - 1)2 = 5 + (n - 1)3
21 + 2n = 2 + 3n
n = 19

60. We have :
$a_2 - a_1 = 11 - 5 = 6$, $a_3 - a_2 = 17 - 11 = 6$, $a_4 - a_3 = 23 - 17 = 6$
As $a_{k+1} - a_k$ is the same for k = 1, 2, 3, etc., so the given list of numbers are in AP.
Now, a = 5 and d = 6.
Let 301 be a term, say, the n^{th} term of this AP.
We know that
$a_n = a + (n - 1) d$
So, 301 = 5 + (n – 1) $\times$ 6
i.e., 301 = 6n – 1
So,$n = \frac{302}{6} = \frac{151}{3}$, since n is in the form of fraction ,thus 301 is not the term of given AP

61. Let first term = a and common difference = d
Then as per given
$10 \times a_{10} = 15 \times a_{15}$
10[a + (10 - 1)d] = 15[a + (15 - 1)d]
10(a + 9d) = 15(a + 14d)
10a+90d=15a+210d
5a+120d=0
or a + 24d=0 ....(1)
To prove**:** $a_{25} = 0$
a_{25}= a + (25 - 1)d
= a + 24d
= 0 [From (1)]
Hence $a_{25} = 0$

62. Let the first term and common difference of the AP be A and D
p^{th} term = a ……… (given)
= A + (p – 1)D = a ....(1)
q^{th} term = b
⇒ A + (q - 1) D = b .... (2)
rth term = c .... Given
⇒ A + (r - 1) D = c .... (3)
Multiplying equations (1), (2) and (3) by q - r, r - p and p - q respecitvely, we get
a(q - r) + b(r - p) + c(p - q)
= [A + (p - 1) D] (q - r) + [A + (q - 1)D] (r - p) + [A + (r - 1) D] (p - q)]
= A [q - r + r - p + p - q] + D [(p - q) (q - r) + (q - 1) (r- p) + (r - 1) (p - q)]

= A (0) + D(0)
= 0

63. Common difference d, of the AP = 9 - 5 = 4
Last term l, of the AP = 185
We know that in general the nth term from the end of an AP is given by l - (n - 1) d.
Thus, the 9th term from the end is
= 185 - (9 - 1)4
= 185 - 4 $\times$ 8
= 185 - 32
= 153
Therefore required 9th term from the end = 153

64. Here we have, $A_n = 3^n$
Put n = 1,
$A_1 = 3^1 = 3$
Put n = 2,
$A_2 = 3^2 = 9$
Put n = 3,
$A_3 = 3^3 = 27$
Put n = 4,
$A_4 = 3^4 = 81$
Put n = 5,
$A_5 = 3^5 = 243$

65. Here we are having a=3
and d=15-3=12
Let nth term $T_n = T_{21} + 120$
so a+(n-1)d = a+20d+120
or (n-1)$\times$12 = 20 $\times$ 12+120
12n -12 = 240+120
12n = 372
n$=\frac{372}{12} = 31$
Hence, 31st term is the required term.

66. $T_n = (4_n - 10)$ [Given]
$T_1 = (4\times 1 - 10) = -6$
$T_2 = (4\times 2 - 10) = -2$
$T_3 = (4\times 3 - 10) = 2$
$T_4 = (4\times 4 - 10) = 6$
Clearly, [-2-(-6)] = [2-(-2)] = [6 - 2] = 4
So, the terms -6, -2, 2, 6,... forms an AP.
Thus we have
T_{16} = a + (n - 1)d = a + 15d = -6 + 15$\times$4 = 54

67. The given A.P is $\sqrt{2}, \sqrt{18}, \sqrt{50}, \sqrt{98}, \ldots\ldots$
This can be re-written as
$\sqrt{2}, 3\sqrt{2}, 5\sqrt{2}, 7\sqrt{2}, \ldots\ldots$
First term, a = $\sqrt{2}$
Common difference, d = 3
$3\sqrt{2} - \sqrt{2} = 2\sqrt{2}$
nth term of the A.P an = a + (n - 1) d = $\sqrt{2} + (n-1) \times 2\sqrt{2}$
$\therefore$ 18th term of the A.P $a_{18} = \sqrt{2} + (18-1) \times 2\sqrt{2} = \sqrt{2} + 34\sqrt{2} = \sqrt{2450}$

68. Given,

$a_{24} = 2\times a_{10}$

$\Rightarrow$ a + (24 - 1)d = 2$\times$[a + (10 - 1)d]

$\Rightarrow$a + 23d = 2[a + 9d]

$\Rightarrow$a + 23d = 2a + 18d

$\Rightarrow$a - 2a = 18d - 23d

$\Rightarrow$-a = -5d

$\Rightarrow$a = 5d...........(i)

To prove: $a_{72} = 2a_{34}$

Proof:

LHS = a_{72}

= a + (72 - 1)d

= 5d + 71d [From (i)]

= 76d

RHS = $2a_{34}$

= 2[a + (34 - 1)d]

= 2[5d + 33d]

= 2 $\times$38d

= 76d

$\therefore$ LHS = RHS

69. The given AP is :

40, 37, 34, 31,....

Here a = 40 and d = 37 - 40 = -3

Let the n^{th} term of this AP be 0.

$a_n = a + (n - 1)d$

0 = 40 + (n - 1)(-3)

0 = 40 - 3(n - 1)

3(n - 1) = 40

3n-3=40

3n=43

$n = \frac{43}{3} = 14.33$

As n is not a natural number so 0 can not be term of this AP

70. Let 1^{st} term of AP = a and common difference = d.

$\therefore$ t_n = a + (n - 1)d

Now t_8 = a + 7d

$\Rightarrow$ 37 = a + 7d ..(i)

and t_{12} = a + 11d

$\Rightarrow$ 57 = a + 11d ..(ii)

Solving equations (i) and (ii), we get

d = 5 and a = 2

$\therefore$ AP is 2, 7, 12, 17,...

71. Here the given AP is 2,7,12.........47.

Let us re-write the given AP in reverse order then we get the AP

47, 42, .., 12, 7, 2.

Here a=47 d=42-47= -5

5th term of this AP

= 47 + (5 - 1)$\times$(-5)

= 47 - 20

= 27

Hence, the 5th term from the end of the given AP is 27.

72. Let the three numbers in A.P be a - d, a and a + d

Then the sum of the numbers = a - d + a + a + d = 27

$\Rightarrow$ 3a = 27

$\Rightarrow$ a = $\frac{27}{3}$ = 9

The product of these three numbers = (a - d)(a)(a + d) = 648

$\Rightarrow$ $a(a^2 - d^2) = 648$

Putting a=9 we get

$\Rightarrow$ $9(9^2 - d^2) = 648$

$\Rightarrow$ $81 - d^2 = \frac{648}{9}$

$\Rightarrow$ $81 - d^2 = 72$

$\Rightarrow$ $-d^2 = 72 - 81$

$\Rightarrow$ $-d^2 = -9$

$\Rightarrow$ $d^2 = 9$

$\Rightarrow$ d = $\pm$ 3

For, a = 9, d = 3

the numbers are 6, 9, 12

For a = 9, d = -3

The numbers are 12, 9, 6

Hence the numbers are 6, 9 and 12

73. Let the first term be a and common difference d.

$a_n = a + (n - 1)d$

As per given condition

$5a_5 = 8a_8$

$\Rightarrow$5(a + 4d) = 8 (a + 7d)

$\Rightarrow$ 5a + 20d = 8a + 56d

$\Rightarrow$5a - 8a = 56d - 20d

$\Rightarrow$-3a = 36d

$\Rightarrow$3a + 36d = 0

$\Rightarrow$3(a + 12d) = 0

$\Rightarrow$a + 12d = 0

$\therefore a_{13} = 0$

74. Here we have, $A_n = \frac{n(n-2)}{2}$

put n = 1

$A_1 = \frac{1(1-2)}{2} = \frac{-1}{2}$

Put n = 2

$A_2 = \frac{2(2-2)}{2} = 0$

Put n = 3

$A_3 = \frac{3(3-2)}{2} = \frac{3}{2}$

Put n = 4

$A_4 = \frac{4(4-2)}{2} = \frac{8}{2} = 4$

Put n = 5

$A_5 = \frac{5(5-2)}{2} = \frac{15}{2}$

75. The given three numbers (2p-1), (3p+1) and 11 are in AP.

Then,

2(3p+1)=(2p-1)+11 ;twice the middle term is equal to sum of first and last term

$\Rightarrow$ 6p+2 = 2p-1+11

$\Rightarrow$ 6p+2=2p+10

$\Rightarrow$ 6p-2p=10-2

$\Rightarrow$ 4p=8

$\Rightarrow$ p=8/4

$\Rightarrow$ p=2

So, the value of p is 2.

Then, the given numbers be

$(2 \times 2 - 1), (3 \times 2 + 1)$ and 11

i.e., 3, 7 and 11.

So, we get an Arithmetic progression

3, 7, 11.