

Solution

ARITHMETIC PROGRESSION WS 6

Class 10 - Mathematics

Section A

1. (a) 4920

Explanation: Here, $a = 6$, $d = 6$

Now, $a_{40} = a + 39d = 6 + 39 \times 6 = 240$

Hence, the required sum $= 6 + 12 + 18 + \dots + 240 = \frac{40}{2}(6 + 240) = 4920$.

2. (a) Both A and R are true and R is the correct explanation of A.

Explanation: nth term of an AP be $a_n = S_n - S_{n-1}$

$$a_n = 3n^2 - 4n - 3(n-1)^2 + 4(n-1)$$

$$a_n = 6n - 7$$

So, both A and R are true and R is the correct explanation of A.

3.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: For $2k + 1$, $3k + 3$ and $5k - 1$ to form an AP

$$(3k + 3) - (2k + 1) = (5k - 1) - (3k + 3)$$

$$k + 2 = 2k - 4$$

$$2 + 4 = 2k - k = k$$

$$k = 6$$

So, both assertion and reason are correct but reason does not explain assertion.

4.

(c) A is true but R is false.

Explanation: $a_{10} = a + 9d$

$$= 5 + 9(3) = 5 + 27 = 32$$

5. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

6. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

7.

(d) A is false but R is true.

Explanation: Assertion: Even natural numbers divisible by 5 are 10, 20, 30, 40, ...

They form an A.P. with,

$$a = 10, d = 10$$

$$S_{100} = \frac{100}{2}[2(10) + 99(10)] = 50500$$

So, reason is correct.

8. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both are correct. Reason is the correct reasoning for Assertion.

$$\text{Assertion, } S_{10} = \frac{10}{2}[2(-0.5) + (10 - 1)(-0.5)]$$

$$= 5[-1 - 4.5]$$

$$= 5(-5.5) = 27.5$$

9.

(c) A is true but R is false.

Explanation: A is true but R is false.

10. (a) Both A and R are true and R is the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

11.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true but R is not the correct explanation of A.

12. State True or False:

(i) **(b)** False

Explanation: False

13. Fill in the blanks:

(i) 1. 55

14. Here, $a = 1$, $l = 11$ and $S_n = 36$

$$\text{Now } S_n = \frac{n}{2}(a + l) \Rightarrow 36 = \frac{n}{2}(1 + 11)$$

$$\Rightarrow n = 6.$$

Section B

15. 5610

Explanation:

If $a_2 = 14$, $a_3 = 18$, then $S_{51} = ?$

$$a_2 = 14$$

$$\Rightarrow a + d = 14 \dots (i)$$

$$a_3 = 18$$

$$\Rightarrow a + 2d = 18 \dots (ii)$$

equation (i) - equation (ii),

$$\begin{array}{r}
 \mathbf{a + d = 14} \\
 \mathbf{a + 2d = 18} \\
 \hline
 \mathbf{- d = - 4}
 \end{array}$$

$$d = 4$$

Substituting the value of 'd' in eqn. (i)

$$a + d = 14$$

$$a + 4 = 14$$

$$a = 14 - 4$$

$$\therefore a = 10$$

$$a = 10, d = 4$$

$$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$S_{51} = \frac{51}{2}[2 \times 10 + (51 - 1)4]$$

$$= \frac{51}{2}[20 + 50 \times 4]$$

$$= \frac{51}{2}[20 + 200]$$

$$= \frac{51}{2} \times 220$$

$$= 51 \times 110$$

$$\therefore S_{51} = 5610$$

16. 6

Explanation:

Here, $a = 3$

$$n = 8$$

$$S = 192$$

We know that

$$S = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow 192 = \frac{8}{2}[2(3) + (8 - 1)d]$$

$$\Rightarrow 192 = 4[6 + 7d]$$

$$\Rightarrow \frac{192}{4} = 6 + 7d$$

$$\Rightarrow 48 = 6 + 7d$$

$$\Rightarrow 48 - 6 = 7d$$

$$\Rightarrow 42 = 7d$$

$$\Rightarrow 7d = 42$$

$$\Rightarrow d = \frac{42}{7}$$

$$\Rightarrow d = 6$$

Section C

17. $a_4 + a_8 = 24, \Rightarrow a + 3d + a + 7d = 24$

$$\Rightarrow 2a + 10d = 24 \text{ or } a + 5d = 12$$

$$a_6 + a_{10} = 44 \Rightarrow a + 5d + a + 9d = 44$$

$$2a + 14d = 44 \text{ or } a + 7d = 22$$

Solving (i) and (ii), $d = 5, a = -13$

\therefore AP is -13, -8, -3, 2, 7, ...

$$S_{25} = \frac{25}{2}[2a + 24d]$$

$$= \frac{25}{2}[-26 + 120]$$

$$= 1175$$

18. Natural numbers between 1 and 100, which are divisible by 3

= 3, 6, 9, ..., 99. As it forms Arithmetic progression, we can find its first term "a" and common difference "d"

Now first term (a) = 3

Common difference (d) = 6 - 3 = 3

Last term (a_n) = 99

$$\text{As } T_n = a + (n - 1)d$$

$$\Rightarrow a + (n - 1)d = 99$$

$$\Rightarrow 3 + (n - 1) \times 3 = 99$$

$$\Rightarrow 3 + 3n - 3 = 99$$

$$\Rightarrow 3n = 99$$

$$\Rightarrow n = \frac{99}{3} = 33$$

Since, $S_n = \frac{n}{2}[a + a_n]$

Therefore, Sum of 33 terms $S_{33} = \frac{n}{2}[a + a_n]$

$$= \frac{33}{2}[3 + 99]$$

$$= \frac{33}{2} \times 102$$

$$= 33 \times 51$$

$$= 1683$$

Hence, the sum of all natural numbers between 1 and 100 which are divisible by 3 is equal to 1683.

19. Let a be the first term and d be the common difference of the given AP.

Now $a_4 = a + (4 - 1)d$

$$\Rightarrow a_4 = a + 3d.$$

And, $a_{17} = a + (17 - 1)d$

$$\Rightarrow a_{17} = a + 16d.$$

Then,

$$T_4 = a + 3d \text{ and } T_{17} = a + 16d$$

$$\text{Now, } \frac{T_4}{T_{17}} = \frac{1}{5}$$

$$\Rightarrow \frac{a+3d}{a+16d} = \frac{1}{5}$$

$$\Rightarrow 5a + 15d = a + 16d$$

$$\Rightarrow 5a - a + 15d - 16d = 0$$

$$\Rightarrow 4a - d = 0$$

$$\Rightarrow 4a = d \dots (i)$$

Also, $S_7 = 182$

$$\text{Where, } S_7 = \frac{7}{2}[2a + (7 - 1)d]$$

$$\Rightarrow \frac{7}{2}[2a + 6d] = 182$$

$$\Rightarrow \frac{7 \times 2}{2}[a + 3d] = 182$$

$$\Rightarrow a + 3d = 26$$

$$\Rightarrow a + 3(4a) = 26 \dots [\text{from (i)}]$$

$$\Rightarrow 13a = 26$$

$$\Rightarrow a = 2$$

$$\Rightarrow d = 4(2) = 8$$

Thus, We have

$$T_1 = 2$$

$$T_2 = T_1 + d = 2 + 8 = 10$$

$$T_3 = T_1 + 2d = 2 + 2(8) = 2 + 16 = 18$$

$$T_4 = T_1 + 3d = 2 + 3(8) = 2 + 24 = 26$$

Thus, the required AP is 2, 10, 18, 26,...

20. Let S_m be the sum of the first m terms of the AP.

$$S_m = 4m^2 - m$$

$$\Rightarrow S_{m-1} = 4(m-1)^2 - (m-1)$$

$$\Rightarrow S_{m-1} = 4(m^2 - 2m + 1) - m + 1$$

$$\Rightarrow S_{m-1} = 4m^2 - 8m + 4 - m + 1$$

$$\Rightarrow S_{m-1} = 4m^2 - 9m + 5$$

Let a_m be the m th term of an AP.

$$\therefore a_m = S_m - S_{m-1}$$

$$\therefore a_m = (4m^2 - m) - (4m^2 - 9m + 5)$$

$$\therefore a_m = 4m^2 - m - 4m^2 + 9m - 5$$

$$\therefore a_m = 8m - 5 \dots (i)$$

Given that $a_n = 107$

$$\Rightarrow 8n - 5 = 107$$

$$\Rightarrow 8n = 112$$

$$\Rightarrow n = 14$$

The 21st term = $a_{21} = 8(21) - 5 = 163$.

21. The sequence formed by the given numbers is 103, 107, 111, 115, ..., 999.

This is an AP in which $a = 103$ and $d = (107 - 103) = 4$.

Let the total number of these terms be n . Then,

$$T_n = 999 \Rightarrow a + (n-1)d = 999$$

$$\Rightarrow 103 + (n-1) \times 4 = 999$$

$$\Rightarrow (n-1) \times 4 = 896 \Rightarrow (n-1) = 224 \Rightarrow n = 225.$$

$$\therefore \text{middle term} = \left(\frac{n+1}{2}\right)\text{th term} = \left(\frac{225+1}{2}\right)\text{th term} = 113\text{th term.}$$

$$T_{113} = (a + 112d) = (103 + 112 \times 4) = 551.$$

$$\therefore T_{112} = (551 - 4) = 547.$$

So, we have to find S_{112} and $(S_{225} - S_{113})$.

Using the formula $S_m = \frac{m}{2}(a + l)$ for each sum, we get

$$s_{112} = \frac{112}{2}(103 + 547) = (112 \times 325) = 36400$$

$$(S_{225} - S_{113}) = \frac{225}{2}(103 + 999) - \frac{113}{2}(103 + 551)$$

$$= (225 \times 551) - (113 \times 327)$$

$$= 123975 - 36951 = 87024.$$

Sum of all numbers on LHS of the middle term is 36400.

Sum of all numbers on RHS of the middle term is 87024.

22. According to the question,

All integers between 100 and 550, which are divisible by 9

= 108, 117, 126,, 549

First term (a) = 108

Common difference(d) = 117 - 108 = 9

$$\text{Last term}(a_n) = 549$$

$$\Rightarrow a + (n - 1)d = 549$$

$$\Rightarrow 108 + (n - 1)(9) = 549$$

$$\Rightarrow 108 + 9n - 9 = 549$$

$$\Rightarrow 9n = 549 + 9 - 108$$

$$\Rightarrow 9n = 450$$

$$\Rightarrow n = \frac{450}{9} = 50$$

$$\text{Sum of 50 terms} = \frac{n}{2}[a + a_n]$$

$$= \frac{50}{2}[108 + 549]$$

$$= 25 \times 657$$

$$= 16425$$

23. Since, the difference between the savings of two consecutive months is ₹20, therefore the series is an A.P.

Here, the savings of the first month is ₹50

First term, $a = 50$, Common difference, $d = 20$

No. of terms = no. of months

No. of terms, $n = 12$

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$= \frac{12}{2}[2 \times 50 + (12 - 1)20]$$

$$= 6[100 + 220]$$

$$= 6(320)$$

$$= 1920$$

After a year, Ramakali will save ₹1920.

Yes, Ramakali will be able to fulfill her dream of sending her daughter to school.

24. Given $\frac{a+10d}{a+16d} = \frac{3}{4}$

$$\Rightarrow 4a + 40d = 3a + 48d$$

$$\Rightarrow a = 8d$$

therefore $\frac{a_5}{a_{21}} = \frac{a+4d}{a+20d} = \frac{3}{7}$ using (i)

$$a_5 : a_{21} = 3 : 7$$

$$\frac{s_5}{s_{21}} = \frac{\frac{5}{2}(2a+4d)}{\frac{21}{2}(2a+20d)} = \frac{5 \times 20d}{21 \times 36d} = \frac{25}{189}$$

Therefore, $S_5 : S_{21} = 25 : 189$

25. 32, 36, 40, ..., 2000

$$36 - 32 = 40 - 36 = 4$$

So, This is an A.P.

$$a = 32$$

$$d = 4$$

According to question,

$$32 + 36 + 40 + \dots = 2000$$

$$S_n = \frac{n}{2}[2a + (n - 1)d] \quad [\because n = \text{No. of months}]$$

$$\Rightarrow 2000 = \frac{n}{2}[64 + (n - 1)4]$$

$$\Rightarrow 2000 = \frac{n}{2}[60 + 4n]$$

$$\Rightarrow 4n^2 + 60n - 4000 = 0$$

$$\Rightarrow n^2 + 15n - 1000 = 0$$

$$n^2 + 40n - 25n - 1000 = 0$$

$$n(n + 40) - 25(n + 40) = 0$$

$$(n - 25)(n + 40) = 0$$

$$n = 25, n = -40 \text{ [Not possible]}$$

Purna saves ₹4000 in 25 months.

26. $S_1 = \frac{n}{2}[2 \times 5 + (n - 1) \times 2]$

$$= \frac{n}{2}[10 + 2n - 2]$$

$$\begin{aligned}
&= \frac{n}{2}[8 + 2n] = n[4 + n] = 4n + n^2 \\
S_2 &= \frac{n}{2}[2 \times 5 + (n - 1) \times 4] \\
&= \frac{n}{2}[10 + 4n - 4] \\
&= \frac{n}{2}[6 + 4n] = n[3 + 2n] = 3n + 2n^2 \\
S_3 &= \frac{n}{2}[2 \times 5 + (n - 1) \times 6] \\
&= \frac{n}{2}[10 + 6n - 6] \\
&= \frac{n}{2}[4 + 6n] = n[2 + 3n] = 2n + 3n^2
\end{aligned}$$

Now,

$$\begin{aligned}
S_1 + S_3 &= (4n + n^2) + (2n + 3n^2) \\
&= 4n + n^2 + 2n + 3n^2 \\
&= 6n + 4n^2 \\
&= 2(3n + 2n^2) \\
&= 2S_2
\end{aligned}$$

27. According to the question,

$$\text{Given Sum of } n \text{ terms } (S_n) = \frac{3n^2}{2} + \frac{13}{2}n$$

$$\text{Put } n = 24, S_{24} = \frac{3 \times 24 \times 24}{2} + \frac{13 \times 24}{2}$$

$$= 864 + 156$$

$$= 1020$$

$$\text{Put } n = 25, S_{25} = \frac{3 \times 25 \times 25}{2} + \frac{13 \times 25}{2}$$

$$= \frac{1875}{2} + \frac{325}{2}$$

$$= \frac{2200}{2} = 1100$$

$$\therefore \text{25th term } (a_{25}) = S_{25} - S_{24}$$

$$= 1100 - 1020$$

$$= 80$$

28. According to the question, the sum of first 10 terms of an AP is -150 and the sum of its next 10 terms is -550

Let a be the first term and d be the common difference of the given A.P.

Then, we have

$$S_{10} = -150$$

$$\Rightarrow \frac{10}{2}[2a + 9d] = -150$$

$$\Rightarrow 5[2a + 9d] = -150$$

$$\Rightarrow 2a + 9d = -30 \dots (i)$$

Clearly, the sum of first 20 terms = $-150 + (-550) = -700$

$$\therefore S_{20} = -700$$

$$\Rightarrow \frac{20}{2}[2a + 19d] = -700$$

$$\Rightarrow 10[2a + 19d] = -700$$

$$\Rightarrow 2a + 19d = -70 \dots (iii)$$

Subtracting (i) from (ii), we get

$$10d = -40$$

$$\Rightarrow d = -4$$

$$\Rightarrow 2a = -30 - 9(-4) = -30 + 36 = 6$$

$$\Rightarrow a = 3$$

Thus, we have

First term, $a = 3$

Second term = $a + d = 3 + 2(-4) = -1$

Third term = $a + 2d = 3 + 2(-4) = 3 - 8 = -5$

Fourth term = $a + 3d = 3 + 3(-4) = 3 - 12 = -9$

Thus, the required A.P is 3, -1, -5, -9,

29. Let a_1 and a_2 be the first terms and d_1 and d_2 be the common difference of the two APs respectively.

Let S_n and S'_n be the sums of the first n terms of the two APs and T_n and T'_n be their n th terms respectively.

$$\begin{aligned} \text{Then, } \frac{S_n}{S'_n} &= \frac{7n+1}{4n+27} \Rightarrow \frac{\frac{n}{2}[2a_1+(n-1)d_1]}{\frac{n}{2}[2a_2+(n-1)d_2]} = \frac{7n+1}{4n+27} \\ \Rightarrow \frac{2a_1+(n-1)d_1}{2a_2+(n-1)d_2} &= \frac{7n+1}{4n+27} \dots\dots(i) \end{aligned}$$

To find the ratio of mth terms, we replace n by (2m -1) in the above expression.

Replacing n by (2 × 9 -1), i.e., 17 on both sides in (i), we get

$$\begin{aligned} \frac{2a_1+(17-1)d_1}{2a_2+(17-1)d_2} &= \frac{7 \times 17+1}{4 \times 17+27} \Rightarrow \frac{2a_1+16d_1}{2a_2+16d_2} = \frac{120}{95} \\ \Rightarrow \frac{a_1+8d_1}{a_2+8d_2} &= \frac{24}{19} \\ \Rightarrow \frac{a_1+(9-1)d_1}{a_2+(9-1)d_2} &= \frac{24}{19} \\ \Rightarrow \frac{a_1+(9-1)d_1}{a_2+(9-1)d_2} &= \frac{24}{19} \\ \Rightarrow \frac{T_9}{T'_9} &= \frac{24}{19} \end{aligned}$$

∴ required ratio = 24:19.

30. Given that,

$$\Rightarrow a_n = 5 + 6n$$

We have,

$$\Rightarrow a_1 = 5 + 6(1) = 11$$

$$\Rightarrow a_2 = 5 + 6(2) = 17$$

So, a = 11, d = 6

$$\begin{aligned} \text{Sum of first 25 terms} &= \frac{n}{2}(2a + (n - 1)d) \\ &= \frac{25}{2}[2(11) + (25 - 1)6] \\ &= \frac{25}{2}[22 + 144] \\ &= \frac{25}{2}[166] \\ &= 2075 \end{aligned}$$

Now, $a_{20} = a + 19d$

$$= 11 + 19(6)$$

$$= 125$$

$$\Rightarrow a_{45} = a + 19d$$

$$= 11 + 44(6)$$

$$= 275$$

$$\text{Required ratio} = \frac{a_{20}}{a_{45}}$$

$$= \frac{125}{275}$$

$$= \frac{5}{11}$$

Ratio is 5 : 11.

31. Odd numbers between 100 and 300 are 101, 103, 299

First term (a) = 101

Common difference (d) = 103 - 101 = 2

Last term (a_n) = 299

$$\Rightarrow a + (n - 1)d = 299$$

$$\Rightarrow 101 + (n - 1) \times 2 = 299$$

$$\Rightarrow 101 + 2n - 2 = 299$$

$$\Rightarrow 2n = 299 + 2 - 101$$

$$\Rightarrow 2n = 200$$

$$\Rightarrow n = \frac{200}{2} = 100$$

$$\therefore \text{Sum of 100 terms} = \frac{n}{2}[a + a_n]$$

$$= \frac{100}{2}[101 + 299]$$

$$= 50 \times 400$$

$$= 20,000$$

32. Here, (-4) + (-1) + 2 + 5 + ---- + x = 437.

Now,

$$-1 - (-4) = -1 + 4 = 3$$

$$2 - (-1) = 2 + 1 = 3$$

$$5 - 2 = 3$$

Thus, this forms an A.P. with $a = -4$, $d = 3$, $l = x$

Let there be n terms in this A.P.

Then,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow 437 = \frac{n}{2}[2 \times (-4) + (n - 1) \times 3]$$

$$\Rightarrow 874 = n[-8 + 3n - 3]$$

$$\Rightarrow 874 = n[3n - 11]$$

$$\Rightarrow 874 = 3n^2 - 11n$$

$$\Rightarrow 3n^2 - 11n - 874 = 0$$

$$\Rightarrow 3n^2 - 57n + 46n - 874 = 0$$

$$\Rightarrow 3n(n - 19) + 46(n - 19) = 0$$

$$\Rightarrow 3n + 46 = 0 \text{ or } n = 19$$

$$\Rightarrow n = -\frac{46}{3} \text{ or } n = 19$$

Numbers of terms cannot be negative or fraction.

$$\Rightarrow n = 19$$

$$\text{Now, } S_n = \frac{n}{2}[a + l]$$

$$\Rightarrow 437 = \frac{19}{2}[-4 + x]$$

$$\Rightarrow -4 + x = \frac{437 \times 2}{19}$$

$$\Rightarrow -4 + x = 46$$

$$\Rightarrow x = 50$$

33. Given that

$$\text{First term}(a) = 3$$

$$n = 8$$

$$\text{and, } S_n = 192$$

$$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 192$$

$$\Rightarrow \frac{8}{2}[2 \times 3 + (8 - 1)d] = 192$$

$$\Rightarrow 4[6 + 7d] = 192$$

$$\Rightarrow 24 + 28d = 192$$

$$\Rightarrow 28d = 192 - 24$$

$$\Rightarrow 28d = 168$$

$$\Rightarrow d = \frac{168}{28} = 6$$

34. $S_n = 3n^2 + 5n$

$$S_1 = 3(1)^2 + 5(1) = 3 + 5 = 8 = a_1$$

$$S_2 = 3(2)^2 + 5(2) = 12 + 10 = 22$$

$$\text{Now, } a_2 = S_2 - S_1 = 22 - 8 = 14$$

$$\text{and, } a_2 - a_1 = 14 - 8 = 6 = d$$

Thus, we have $a = 8$ and $d = 6$

$$\therefore a_n = a + (n - 1)d = 8 + (n - 1)(6)$$

$$\Rightarrow a_k = 8 + (k - 1)(6)$$

$$\Rightarrow 164 = 8 + 6k - 6$$

$$\Rightarrow 6k = 162$$

$$\Rightarrow k = 27$$

35. It is given that, Sum of first p terms of an AP = q

and Sum of the first q terms the same AP = p

Let us take the first term as a and the common difference d

$$\text{Therefore, the sum } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$q = \frac{p}{2}[2a + (p - 1)d]$$

$$p = \frac{q}{2}[2a + (q - 1)d]$$

Subtracting the sum of the q terms from the sum of p terms

we get

$$q - p = \left[\frac{p}{2}(2a + (p - 1)d) - \frac{q}{2}[2a + (q - 1)d] \right]$$

$$q - p = a(p - q) + \frac{d}{2}(p^2 - p - q^2 + q)$$

After solving the equation we get

$$d = -\frac{2(p+q)}{pq}$$

Now with $d = -\frac{2(p+q)}{pq}$, the first term of the series is a and the number of terms is (p + q)

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$Sp + q = \frac{p+q}{2}[2a + (p + q - 1)d] = \frac{p+q}{pq}(-pq)$$

Therefore, the sum is -(p + q).

36. Given that, $a = 2$, $d = 8$ and $S_n = 90$.

$$\text{As, } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$90 = \frac{n}{2}[4 + (n - 1)8]$$

$$90 = n[2 + (n - 1)4]$$

$$90 = n[2 + 4n - 4]$$

$$90 = n(4n - 2) = 4n^2 - 2n$$

$$4n^2 - 2n - 90 = 0$$

$$4n^2 - 20n + 18n - 90 = 0$$

$$4n(n - 5) + 18(n - 5) = 0$$

$$(n - 5)(4n + 18) = 0$$

$$\text{Either } n = 5 \text{ or } n = -\frac{18}{4} = \frac{-9}{2}$$

However, n can neither be negative nor fractional.

Therefore, $n = 5$

$$a_n = a + (n - 1)d$$

$$a_5 = 2 + (5 - 1)8$$

$$= 2 + 4(8)$$

$$= 2 + 32 = 34$$

37. Let the number of rows be n.

A.P. formed is 22, 21, 20, 19, ...

Here, $a = 22$, $d = -1$ $S_n = 250$

$$\therefore 250 = \frac{n}{2}[44 + (n - 1)(-1)]$$

$$\Rightarrow n^2 - 45n + 500 = 0$$

$$\Rightarrow (n - 25)(n - 20) = 0$$

$$n \neq 25 \therefore n = 20$$

$$\text{logs in top row} = a_{20} = 22 + 19(-1) = 3$$

38. $S_7 = \frac{7}{2}[2a + 6d] = -21$... (i)

$$S_{17} = \frac{17}{2}[2a + 16d] = -221$$
 ... (ii)

Solving (i) and (ii) } $d = -2$ and $a = 3$

$$\therefore S_n = \frac{n}{2}[6 + (n - 1)(-2)]$$

$$= \frac{n}{2}(8 - 2n) \text{ or } (4n - n^2)$$

39. According to the question, we have to find the sum of the integers between 100 and 200 that are not divisible by 9.

Numbers between 100 and 200 = 101, 102, 103, ... 199

Here, $a = 101$, $d = 1$, $a_n = 199$

$$\Rightarrow a + (n - 1)d = 199$$

$$\Rightarrow 101 + (n - 1)(1) = 199$$

$$\Rightarrow (n - 1) = 199 - 101 = 98$$

$$\Rightarrow n = 99$$

$$\text{Now, } S_{99} = \frac{99}{2}[2 \times 101 + (99 - 1)(1)] \left[\because S_n = \frac{n}{2}[2a + (n - 1)d] \right]$$

$$= \frac{99}{2} [202 + 98] = \frac{99}{2} \times 300 = 99 \times 150 = 14850$$

Now the sum of numbers between 100 and 200 which are divisible by 9 are given by 108, 117, 126.... 198 and so on.

So $a = 108$, $d = 9$, $l = 198$, $n = ?$

$$\text{nth term} = a + (n - 1)d$$

$$198 = 108 + (n - 1)9$$

$$198 = 108 + 9n - 9$$

$$198 - 99 = 9n$$

$$\frac{99}{9} = n$$

$$n = 11$$

$$\text{Now, Sum of first 11 terms, } S'_{11} = \frac{11}{2} [2 \times 108 + (11 - 1)9]$$

$$= \frac{11}{2} (306) = 1683$$

Therefore, the sum of integers between 100 and 200 which are not divisible by 9 = $S_{99} - S'_{11} = 14850 - 1683 = 13167$.

40. Given,

$$\text{Common difference}(d) = 2$$

$$\text{and, } n^{\text{th}} \text{ term } (a_n) = 4$$

$$a + (n - 1)d = 4$$

$$\Rightarrow a + (n - 1)(2) = 4$$

$$\Rightarrow a + 2n - 2 = 4$$

$$\Rightarrow a + 2n = 6$$

$$\text{and, } S_n = -14$$

$$\Rightarrow \frac{n}{2} [a + a_n] = -14$$

$$\Rightarrow \frac{n}{2} [a + 4] = -14$$

$$\Rightarrow \frac{n}{2} [6 - 2n + 4] = -14 \text{ [using } a=6-2n]$$

$$\Rightarrow \frac{n}{2} [10 - 2n] = -14$$

$$\Rightarrow 5n - n^2 = -14$$

$$\Rightarrow n^2 - 5n - 14 = 0$$

$$\Rightarrow n^2 - 7n + 2n - 14 = 0$$

$$\Rightarrow n(n - 7) + 2(n - 7) = 0$$

$$\text{If, } n - 7 = 0$$

$$n = 7$$

$$\text{or } n + 2 = 0$$

$$n = -2 \text{ (Rejected as } n \text{ cannot be negative)}$$

Put value of n in (i),

$$a + 2 \times 7 = 6$$

$$\Rightarrow a = 6 - 14$$

$$\Rightarrow a = -8$$

41. Consider the A.P. whose first term and common difference are 'a' and 'd' respectively.

If sum of first 6 terms of an A.P. is 36.

$$S_6 = 36$$

$$\therefore \frac{6}{2} [2a + (6 - 1)d] = 36 \text{ [}\because S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow 3[2a + 5d] = 36$$

$$\Rightarrow 2a + 5d = \frac{36}{3}$$

$$\Rightarrow 2a + 5d = 12 \dots \text{(i)}$$

If sum of first 16 terms is 256,

$$\text{So, } S_{16} = 256$$

$$\Rightarrow \frac{16}{2} [2a + (16 - 1)d] = 256$$

$$\Rightarrow 8[2a + 15d] = 256$$

$$\Rightarrow 2a + 15d = \frac{256}{8}$$

$$\Rightarrow 2a + 15d = 32 \dots \text{(ii)}$$

Subtracting (i) from (ii), we get

$$2a + 15d = 32 \quad \dots(i)$$

$$2a + 5d = 12 \quad [\text{From (i)}]$$

$$\begin{array}{r} - \quad - \quad - \\ \hline 10d = 20 \\ 2a + 15d = 32 \\ 2a + 5d = 12 \\ \hline 10d = 20 \end{array}$$

$$\Rightarrow d = 2$$

$$\text{Now, } 2a + 5d = 12 \quad [\text{From (i)}]$$

$$\Rightarrow 2a + 5(2) = 12$$

$$\Rightarrow 2a + 10 = 12$$

$$\Rightarrow 2a = 12 - 10$$

$$\Rightarrow a = \frac{2}{2}$$

$$\Rightarrow a = 1$$

$$\text{Hence, } a = 1 \text{ and } d = 2$$

$$\text{So, } S_{10} = \frac{10}{2} [2a + (10 - 1)d]$$

$$= 5[2(1) + 9(2)]$$

$$= 5[2 + 18]$$

$$= 5[20]$$

$$= 100$$

$$\Rightarrow S_{10} = 100$$

Hence, the sum of first 10 terms is 100.

42. All integers between 84 and 719, which are multiples of 5.

$$= 85, 90, 95, \dots, 715$$

$$\text{First term (a)} = 85$$

$$\text{Common difference} = 90 - 85 = 5$$

$$\text{Last term (a}_n) = 715$$

$$\Rightarrow a + (n - 1)d = 715$$

$$\Rightarrow 85 + (n - 1)(5) = 715$$

$$\Rightarrow 85 + 5n - 5 = 715$$

$$\Rightarrow 5n = 715 + 5 - 85$$

$$\Rightarrow 5n = 635$$

$$\Rightarrow n = \frac{635}{5} = 127$$

$$\text{Sum of 127 terms} = \frac{n}{2} [a + a_n]$$

$$= \frac{127}{2} [85 + 715]$$

$$= \frac{127}{2} \times 800$$

$$= 127 \times 400$$

$$= 50800$$

43. i. Number between 100–200 divisible by 9 are 108, 117, 126, ..., 198

$$\text{Here, } a = 108, d = 117 - 108 = 9 \text{ and } a_n = 198$$

$$= a + (n - 1)d = 198$$

$$\rightarrow 108 + (n - 1)9 = 198$$

$$\rightarrow 9[12 + n - 1] = 198 \Rightarrow n = 22 - 11 \Rightarrow n = 11$$

$$\text{Now, } S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow S_{11} = \frac{11}{2} [2(108) + (11 - 1)(9)]$$

$$= \frac{11}{2} [216 + 90]$$

$$= \frac{11}{2} \times 306$$

$$= 11 \times 153$$

$$S_{11} = 1683$$

ii. Numbers between 100 and 200 = 101, 102, 103, ..., 199

Here, $a = 101$, $l = 199$, $d = 1$

$$\Rightarrow a + (n - 1)d = 199$$

$$\Rightarrow 101 + (n - 1)(1) = 199$$

$$\Rightarrow (n - 1) = 199 - 101 = 98$$

$$\Rightarrow n = 99$$

$$\text{Now, } S_{99} = \frac{99}{2} [2 \times 101 + (99 - 1)(1)]$$

$$= \frac{99}{2} [202 + 98] = \frac{99}{2} \times 300 = 99 \times 150 = 14850$$

So, the sum of integers between 100 and 200 which are not divisible by 9 = $14850 - 1683 = 13167$.

44. $S_n = 3n^2 + 2n$

Taking $n = 1$, we get

$$S_1 = 3(1)^2 + 2(1)$$

$$\Rightarrow S_1 = 3 + 2$$

$$\Rightarrow S_1 = 5$$

$$\Rightarrow a_1 = 5$$

Taking $n = 2$, we get

$$S_2 = 3(2)^2 + 2(2)$$

$$\Rightarrow S_2 = 12 + 4$$

$$\Rightarrow S_2 = 16$$

$$\therefore a_2 = S_2 - S_1 = 16 - 5 = 11$$

Taking $n = 3$, we get

$$S_3 = 3(3)^2 + 2(3)$$

$$\Rightarrow S_3 = 27 + 6$$

$$\Rightarrow S_3 = 33$$

$$\therefore a_3 = S_3 - S_2 = 33 - 16 = 17$$

So, $a = 5$,

$$d = a_2 - a_1 = 11 - 5 = 6$$

Now, we have to find the 15th term

$$a_n = a + (n - 1)d$$

$$a_{15} = 5 + (15 - 1)6$$

$$a_{15} = 5 + 14 \times 6$$

$$a_{15} = 5 + 84$$

$$a_{15} = 89$$

Hence, the 15th term is 89 and AP is 5, 11, 17, 23, ...

45. Let a be the first term and d be the common difference of the given AP. Therefore,

$$T_4 + T_8 = 24 \Rightarrow (a+3d)+(a+7d)=24$$

$$\Rightarrow 2a+10d=24$$

$$\Rightarrow a+5d=12 \dots\dots(i)$$

$$\text{And, } T_6+T_{10}=44 \Rightarrow (a + 5d) + (a + 9d) = 44$$

$$\Rightarrow 2a+14d=44$$

$$\Rightarrow a+7d=22 \dots\dots(ii)$$

Therefore, on solving (i) and (ii), we get $a = -13$ and $d = 5$.

\therefore the sum of first 10 terms of the given AP is given by

$$S_{10} = \left(\frac{10}{2}\right) \cdot (2a + 9d) \text{ [using } S_n = \frac{n}{2} [(2a + (n - 1)d)]$$

$$= 5 \times \{2 \times (-13) + 9 \times 5\} = 5(-26 + 45) = 5 \times 19 = 95.$$

Hence, the sum of first 10 terms of the given AP is 95.

46. Here, $(-4) + (-1) + 2 + 5 + \dots + x = 437$.

Now,

$$-1 - (-4) = -1 + 4 = 3$$

$$2 - (-1) = 2 + 1 = 3$$

$$5 - 2 = 3$$

Thus, this forms an A.P. with $a = -4$, $d = 3$, $l = x$

$$S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow 437 = \frac{n}{2} [2 \times (-4) + (n - 1) \times 3]$$

$$\Rightarrow 874 = n[-8 + 3n - 3]$$

$$\Rightarrow 874 = n[3n - 11]$$

$$\Rightarrow 874 = 3n^2 - 11n$$

$$\Rightarrow 3n^2 - 11n - 874 = 0$$

$$\Rightarrow 3n^2 - 57n + 46n - 874 = 0$$

$$\Rightarrow 3n(n - 19) + 46(n - 19) = 0$$

$$\Rightarrow 3n + 46 = 0 \text{ or } n = 19$$

$$\Rightarrow n = -\frac{46}{3} \text{ or } n = 19$$

Numbers of terms cannot be negative or fraction.

$$\Rightarrow n = 19$$

Now, $S_n = \frac{n}{2} [a + l]$

$$\Rightarrow 437 = \frac{19}{2} [-4 + x]$$

$$\Rightarrow -4 + x = \frac{437 \times 2}{19}$$

$$\Rightarrow -4 + x = 46$$

$$\Rightarrow x = 50$$

47. Let a be the first term and d be the common difference of the given AP. Therefore, the sum of first n terms is given by

$$S_n = \frac{n}{2} \cdot \{2a + (n - 1)d\}$$

$$\therefore S_{10} = \frac{10}{2} \cdot (2a + 9d) \Rightarrow 5(2a + 9d) = 210$$

$$\Rightarrow 2a + 9d = 42. \dots(i)$$

Sum of last 15 terms = $(S_{50} - S_{35})$.

$$\therefore (S_{50} - S_{35}) = 2565$$

$$\Rightarrow \frac{50}{2} (2a + 49d) - \frac{35}{2} (2a + 34d) = 2565$$

$$\Rightarrow 25(2a + 49d) - 35(a + 17d) = 2565$$

$$\Rightarrow (50a - 35a) + (1225d - 595d) = 2565$$

$$\Rightarrow 15a + 630d = 2565 \Rightarrow a + 42d = 171 \dots\dots (ii)$$

Therefore, on solving (i) and (ii), we get $a = 3$ and $d = 4$.

Hence, the required AP is 3, 7, 11, 15, 19,

48. Given; $a = 5$, $l = 45$, $S_n = 400$

We know that

$$S_n = \frac{n}{2} (a + l)$$

$$\Rightarrow 400 = \frac{n}{2} (5 + 45)$$

$$\Rightarrow 400 = \frac{n}{2} (50)$$

$$\Rightarrow 400 = 25n$$

$$\Rightarrow n = \frac{400}{25}$$

$$\Rightarrow n = 16$$

Hence, the number of terms is 16.

Again, we know that

$$l = a + (n - 1)d$$

$$\Rightarrow 45 = 5 + (16 - 1)d$$

$$\Rightarrow 45 = 5 + 15d$$

$$\Rightarrow 40 = 15d$$

$$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$$

Hence, the common difference is $\frac{8}{3}$.

49. Given;

$$a = -5, a_n = 45, S_n = 120$$

$$S_n = \frac{n}{2}(a + a_n)$$

$$120 = \frac{n}{2}(-5 + 45)$$

$$240 = 40n$$

$$n = 6$$

Also we know the n^{th} term formula

$$a_n = a + (n - 1)d$$

$$45 = -5 + (6 - 1)d$$

$$50 = 5d$$

$$d = 10$$

Hence, number of terms and the common difference of an AP are 6 and 10 respectively.

50. Students of first section of class 1 will plant 2 trees.

Students of second section of class 1 will plant 2 trees.

Thus, students of class 1 will plant 4 trees.

Students of first section of class 2 will plant 4 trees.

Students of second section of class 2 will plant 4 trees.

Thus, students of class 2 will plant 8 trees.

Students of first section of class 3 will plant 6 trees.

Students of second section of class 3 will plant 6 trees.

Thus, students of class 3 will plant 12 trees.

Thus, the number of trees planted by the students, form an AP : 4,8,12,...

Thus, $a=4$ and $d=4$

Let us find the number of trees planted in total.

$$\Rightarrow S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\Rightarrow S_{12} = \frac{12}{2}[2 \times 4 + (12-1)4]$$

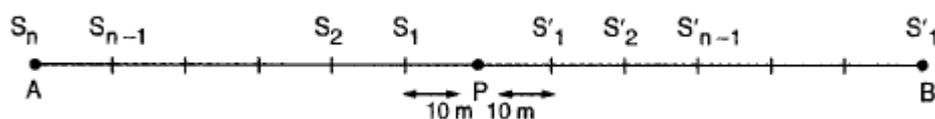
$$\Rightarrow S_{12} = 6[8 + 44]$$

$$\Rightarrow S_{12} = 312$$

Thus, the total number of trees is 312.

51. Let's suppose there are $(2n + 1)$ stones. Clearly, one stone lies in the middle and n stones on each side of it in a row.

Let P be the middle stone and let A and B be the end stones on the left and right of P respectively.



Clearly, there are n intervals each of length 10 meters on both the sides of P .

Now, suppose the man starts from A . He picks up the end stone on the left of mid-stone and goes to the mid-stone, drops it and goes to $(n - 1)$ th stone on left, picks it up, goes to the mid-stone and drops it. This process is repeated till he collects all stones on the left of the mid-stone at the mid-stone.

So, distance covered in collecting stones on the left of the mid-stones is

$$10 \times n + 2 [10 \times (n - 1) + 10 \times (n - 2) + \dots + 10 \times 2 + 10 \times 1].$$

After collecting all stones on left of the mid-stone the man goes to the stone B on the right side of the mid-stone, picks it up, goes to the mid-stone and drops it. Then, he goes to $(n - 1)^{\text{th}}$ stone on the right and the process is repeated till he collects all stones at the mid-stone.

So that distance covered in collecting the stones on the right side of the mid-stone is equal to

$$2 [10 \times n + 10 \times (n - 1) + 10 \times (n - 2) + \dots + 10 \times 2 + 10 \times 1].$$

Therefore, total distance covered

$$= 10 \times n + 2 [10 \times (n - 1) + 10 \times (n - 2) + \dots + 10 \times 2 + 10 \times 1]$$

$$+ 2 [10 \times n + 10 \times (n - 1) + \dots + 10 \times 2 + 10 \times 1]$$

$$= 4 [10 \times n + 10 \times (n - 1) + \dots + 10 \times 2 + 10 \times 1] - 10 \times n$$

$$= 40\{1 + 2 + 3 + \dots + n\} - 10n = 40 \left\{ \frac{n}{2}(1 + n) \right\} - 10n = 20n(n + 1) - 10n = 20n^2 + 10n$$

But, the total distance that a man covered in collecting stones is 3 km.i.e;3000m.

$$\text{Therefore, } 20n^2 + 10n = 3000$$

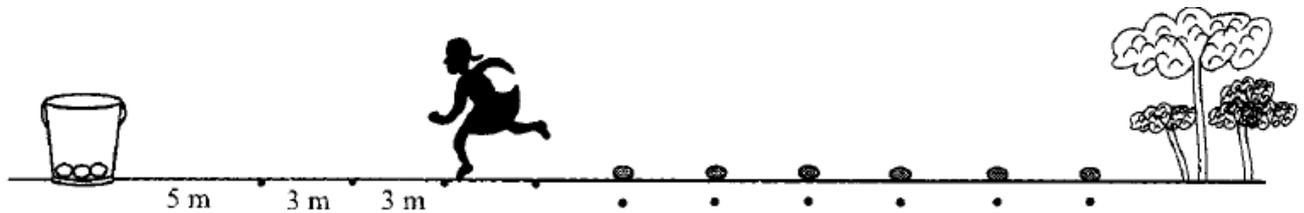
$$\Rightarrow 2n^2 + n - 300 = 0. \Rightarrow 2n^2 + 25n + 24n - 300 = 0$$

$$\Rightarrow (n - 12)(2n + 25) = 0 \dots [\text{Therefore, } 2n + 25 \neq 0] \Rightarrow n - 12 = 0$$

$$\Rightarrow n = 12. \text{Hence, the number of stones is equal to } 2n + 1 = 2(12) + 1 = 25$$

So, the total stones carried to the middle is 25.

52.



Let d_1 = Distance run by the competitor to pick up first potato = 2×5 m

d_2 = Distance run by the competitor to pick up second potato = $2(5 + 3)$ m

d_3 = Distance run by the competitor to pick up third potato = $2(5 + 2 \times 3)$ m

d_4 = Distance run by the competitor to pick up fourth potato = $2(5 + 3 \times 3)$ m

d_n = Distance run by the competitor to pick up n^{th} potato = $2\{5 + (n - 1) \times 3\}$ m

Therefore, total distance run by the competitor to pick up n potatoes

$$= d_1 + d_2 + d_3 + \dots + d_n$$

$$= 2 \times 5 + 2(5 + 3) + 2(5 + 2 \times 3) + 2(5 + 3 \times 3) + \dots + 2\{5 + (n - 1) \times 3\} \text{ metres}$$

$$= 2[5 + \{5 + 3\} + \{5 + (2 \times 3)\} + \{5 + (3 \times 3)\} + \dots + \{5 + (n - 1) \times 3\}]$$

$$= 2 \left[\underbrace{(5 + 5 + \dots + 5)}_{n\text{-times}} + \{3 + (2 \times 3) + (3 \times 3) + \dots + (n - 1) \times 3\} \right]$$

$$= 2[5n + 3\{1 + 2 + 3 + \dots + (n - 1)\}]$$

$$= 2 \left[5n + 3 \left(\frac{n-1}{2} \right) \{1 + (n - 1)\} \right] \text{ [Using : } S_n = \frac{n}{2}(a + l)]$$

$$= 2 \left\{ 5n + \frac{3n(n-1)}{2} \right\} = [10n + 3n(n - 1)] = 3n^2 + 7n = n(3n + 7) \text{ metres}$$

53. We have,

odd integers between 1 and 1000, which are divisible by 3

are 3, 9, 15, 999

First term (a) = 3

Common difference(d) = 9 - 3 = 6

Last term (a_n) = 999

$$\Rightarrow a + (n - 1)d = 999$$

$$\Rightarrow 3 + (n - 1) \times 6 = 999$$

$$\Rightarrow 3 + 6n - 6 = 999$$

$$\Rightarrow 6n = 999 + 6 - 3$$

$$\Rightarrow 6n = 1002$$

$$\Rightarrow n = \frac{1002}{6} = 167$$

$$\therefore \text{Sum of 167 terms} = \frac{n}{2}[a + a_n]$$

$$= \frac{167}{2}[3 + 999]$$

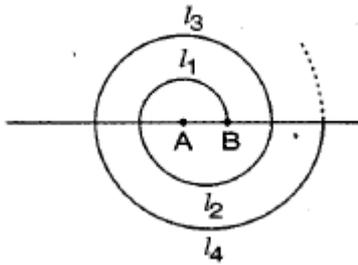
$$= \frac{167}{2} \times 1002$$

$$= 167 \times 501$$

$$= 83667$$

54. According to question we are given that a spiral is made up of successive semi-circles, with centres alternately at A and B, starting with centre at A, of radii 0.5 cm, 1.0 cm, 1.5 cm, 2.0 cm,as shown in Fig.

Let $l_1, l_2, l_3, l_4, \dots, l_{13}$ be the lengths (circumferences) of semi-circles of radii $r_1 = 0.5$ cm, $r_2 = 1.0$ cm, $r_3 = 1.5$ cm, $r_4 = 2.0$ cm, $r_5 = 2.5$ cm, ... respectively.



Now, Semi-perimeter of circle = $\pi \cdot r$

Therefore,

$$l_1 = \pi r_1 = \pi \times 0.5 = \frac{\pi}{2} \text{ cm}$$

$$l_2 = \pi r_2 = \pi \times 1 = 2 \left(\frac{\pi}{2} \right) \text{ cm}$$

$$l_3 = \pi r_3 = \pi \times \frac{3}{2} = 3 \left(\frac{\pi}{2} \right) \text{ cm}$$

$$l_4 = \pi r_4 = \pi \times 2 = 4 \left(\frac{\pi}{2} \right) \text{ cm}$$

and

$$l_{13} = \pi r_{13} = \pi \times \frac{13}{2} \text{ cm} = 13 \left(\frac{\pi}{2} \right) \text{ cm}$$

Therefore total length of the spiral = $l_1 + l_2 + l_3 + \dots + l_{13}$

$$= \left\{ \frac{\pi}{2} + 2 \left(\frac{\pi}{2} \right) + 3 \left(\frac{\pi}{2} \right) + \dots + 13 \left(\frac{\pi}{2} \right) \right\}$$

$$= \frac{\pi}{2} (1 + 2 + 3 + \dots + 13)$$

$$= \frac{\pi}{2} \times \frac{13}{2} (1 + 13) \quad [\text{Using } S_n = \frac{n}{2}(a + l)]$$

$$= \frac{\pi}{2} \times \frac{13}{2} \times 14 = \frac{1}{2} \times \frac{22}{7} \times 13 \times 7 = 143 \text{ cm}$$

which is required length of the spiral made up of thirteen consecutive semi-circles.

55. Let $S_n = -5 + (-8) + (-11) + \dots + (-230)$

Clearly, the terms of the sum form an A.P.

with, $a = -5$

$$d = -8 - (-5) = -8 + 5 = -3$$

$$l = -230$$

Let the number of terms of the AP be n

We know that

$$l = a + (n - 1)d$$

$$\Rightarrow -230 = -5 + (n - 1)(-3)$$

$$\Rightarrow (n - 1)(-3) = -230 + 5$$

$$\Rightarrow (n - 1)(-3) = -225$$

$$\Rightarrow n - 1 = \frac{-225}{-3} = 75$$

$$\Rightarrow n = 75 + 1$$

$$\Rightarrow n = 76$$

Again, we know that

$$S_n = \frac{n}{2}(a + l)$$

$$\Rightarrow S_{76} = \frac{76}{2}[(-5) + (-230)]$$

$$\Rightarrow S_{76} = 38(-235)$$

$$\Rightarrow S_{76} = -8930$$

Hence, the required sum is -8930.

56. The general term of an AP is given by

$$a_n = a + (n - 1)d$$

$$\text{Given that } a_{13} = 4a_3$$

$$\Rightarrow a + 12d = 4(a + 2d)$$

$$\Rightarrow a + 12d = 4a + 8d$$

$$\Rightarrow 3a = 4d \dots (i)$$

Now,

$$a_5 = 16$$

$$\Rightarrow a+4d=16$$

$$\Rightarrow 3a+12d=48$$

$$\Rightarrow 4d+12d=48\dots(\text{from(i)})$$

$$\Rightarrow 16d=48$$

$$\Rightarrow d=3$$

Substituting in (i), we get $a=4$.

$$\text{We know that, } S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\text{Sum of the first 10 terms} = S_{10}$$

$$= \frac{10}{2}[2(4) + 9(3)]$$

$$= 5[8+27]$$

$$= 175$$

57. Given,

$$\text{First term}(a) = 5$$

$$\text{Common difference}(d) = 3$$

$$\text{and, nth term } (a_n) = 50$$

$$\Rightarrow a + (n - 1)d = 50$$

$$\Rightarrow 5 + (n - 1)(3) = 50$$

$$\Rightarrow 5 + 3n - 3 = 50$$

$$\Rightarrow 3n = 50 - 5 + 3$$

$$\Rightarrow 3n = 48$$

$$\Rightarrow n = \frac{48}{3} = 16$$

$$\text{Therefore, } S_n = \frac{n}{2}[a + a_n]$$

$$= \frac{16}{2}[5 + 50]$$

$$= 8 \times 55$$

$$= 440$$

58. Given, $a = 1, d = 4 - 1 = 3$.

Let number of terms in the series be n .

$$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\therefore \frac{n}{2}[2 \times 1 + (n - 1)3] = 287$$

$$\text{or, } \frac{n}{2}[2 + (3n - 3)] = 287$$

$$\text{or, } n[3n - 1] = 574$$

$$\text{or, } 3n^2 - n - 574 = 0$$

$$3n^2 - n - 574 = 0$$

$$3n(n - 14) - 41(n - 14) = 0$$

$$(n - 14)(3n - 41) = 0$$

$$n - 14 = 0 \text{ or } 3n - 41 = 0$$

$$n = 14 \text{ or } 3n = 41 \text{ (reject it)}$$

The 14th term is x .

$$\therefore a + (n - 1)d = x$$

$$1 + 13 \times 3 = x$$

$$1 + 39 = x$$

$$x = 40$$

59. Given that,

$$S_n = 4n - n^2$$

$$\text{First term, } a = S_1 = 4(1) - (1)^2 = 4 - 1 = 3$$

$$\text{Sum of first two terms} = S_2$$

$$= 4(2) - (2)^2 = 8 - 4 = 4$$

$$\text{Second term, } a_2 = S_2 - S_1 = 4 - 3 = 1$$

$$d = a_2 - a = 1 - 3 = -2$$

$$a_n = a + (n - 1)d$$

$$= 3 + (n - 1)(-2)$$

$$= 3 - 2n + 2$$

$$= 5 - 2n$$

$$\text{Therefore, } a_3 = 5 - 2(3) = 5 - 6 = -1$$

$$a_{10} = 5 - 2(10) = 5 - 20 = -15$$

Hence, the sum of first two terms is 4. The second term is 1. 3rd, 10th and nth terms are -1, -15, and 5 - 2n respectively.

$$60. S_n = \frac{7}{2}(2a + 6d) = -14 \dots(i)$$

$$S_n = \frac{11}{2}(2a + 10d) = -55 \dots(ii)$$

$$\text{Solving (i) and (ii) } d = -\frac{3}{2}, a = \frac{5}{2}$$

$$S_n = \frac{n}{2} \left[5 + (n - 1) \left(-\frac{3}{2} \right) \right] = \frac{n}{4} [13 - 3n]$$

61. Let a be the First term and d be the common difference of given AP.

Then, we have

$$\Rightarrow \frac{9}{2}[2a + 8d] = 81$$

$$\Rightarrow \frac{9 \times 2}{2}[a + 4d] = 81$$

$$\Rightarrow a + 4d = 9 \dots(i)$$

$$\text{Also, } S_{20} = 400$$

$$\Rightarrow \frac{20}{2}[2a + 19d] = 400$$

$$\Rightarrow 10[2a + 19d] = 400$$

$$\Rightarrow 2a + 19d = 40 \dots(ii)$$

Multiplying equation (i) by 2, we get

$$2a + 8d = 18 \dots(iii)$$

Subtracting (iii) from (ii), we get

$$11d = 22$$

$$\Rightarrow d = 2$$

$$\Rightarrow a = 9 - 4(2) = 9 - 8 = 1$$

Thus, the first term is 1 and the common difference is 2.

62. Numbers between 100 – 200 divisible by 9 are 108, 117, 126, ... 198.

Here, a = 108, d = 117 – 108 = 9 and a_n = 198.

$$\Rightarrow a + (n - 1)d = 198 \text{ [}\therefore a_n = a + (n - 1)d\text{]}$$

$$\Rightarrow 108 + (n - 1)9 = 198.$$

$$\Rightarrow 108 + 9n - 9 = 198$$

$$\Rightarrow 9n + 99 = 198$$

$$\Rightarrow 9(n + 11) = 198$$

$$\Rightarrow 11 + n = \frac{198}{9}$$

$$\Rightarrow n = 22 - 11.$$

$$\Rightarrow n = 11$$

$$\text{Now, } S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow S_{11} = \frac{11}{2} [2(108) + (11 - 1)(9)]$$

$$= \frac{11}{2} [216 + 99 - 9]$$

$$= \frac{11}{2} [216 + 90]$$

$$= \frac{11}{2} \times 306$$

$$= 11 \times 153$$

$$\Rightarrow S_{11} = 1683.$$

63. Integers between 1 and 500 which are multiples of 2 as well as 5 are

10, 20, 30,, 490

This forms an A.P. with a = 10, d = 10, and l = 490

Let the number of these terms be n.

Then,

$$a_n = 490$$

$$\Rightarrow a + (n - 1)d = 490$$

$$\begin{aligned} \Rightarrow 10 + (n - 1)(10) &= 490 \\ \Rightarrow (n - 1)(10) &= 480 \\ \Rightarrow n - 1 &= 48 \\ \Rightarrow n &= 49 \\ \Rightarrow S_{49} &= \frac{49}{2} [2 \times 10 + 48 \times 10] \\ &= \frac{49}{2} \times [20 + 480] \\ &= \frac{49}{2} \times 500 \\ &= 12250 \end{aligned}$$

64. Odd numbers between 100 and 200

are 101, 103, 199

First term (a) = 101

Common difference (d) = 103 - 101 = 2

Last term (a_n) = 199

$$\Rightarrow a + (n - 1)d = 199$$

$$\Rightarrow 101 + (n - 1) \times 2 = 199$$

$$\Rightarrow 101 + 2n - 2 = 199$$

$$\Rightarrow 2n = 199 + 2 - 101$$

$$\Rightarrow 2n = 100$$

$$\Rightarrow n = \frac{100}{2} = 50$$

$$\therefore \text{Sum of 50 terms} = \frac{n}{2} [a + a_n]$$

$$= \frac{50}{2} [101 + 199]$$

$$= 25 \times 300$$

$$= 7500$$

65. Integers from 1 to 500 which are multiples of 2 are 2, 4, 6, 8.....

This forms an A.P., with a = 2, d = 2 and l = 500

Let the number of these terms be n.

Then,

$$a_n = 500$$

$$\Rightarrow a + (n - 1)d = 500$$

$$\Rightarrow 2 + (n - 1)(2) = 500$$

$$\Rightarrow (n - 1)(2) = 498$$

$$\Rightarrow n - 1 = 249$$

$$\Rightarrow n = 250$$

$$\Rightarrow S_{250} = \frac{250}{2} [2 \times 2 + 249 \times 2]$$

$$= 125 \times [4 + 498]$$

$$= 125 \times 502$$

$$= 62750$$

Integers from 1 to 500 which are multiples of 5 are 5, 10, 15, 20,.....

This forms an A.P., with a = 5, d = 5 and l = 500

Let the number of these terms be n.

Then,

$$a_n = 500$$

$$\Rightarrow a + (n - 1)d = 500$$

$$\Rightarrow 5 + (n - 1)(5) = 500$$

$$\Rightarrow (n - 1)(5) = 495$$

$$\Rightarrow n - 1 = 99$$

$$\Rightarrow n = 100$$

$$\Rightarrow S_{100} = \frac{100}{2} [2 \times 5 + 99 \times 5]$$

$$= 50 \times [10 + 495]$$

$$= 50 \times 505$$

$$= 25250$$

Again, multiples of 10 are included in both i.e., multiple of 2 and multiple of 5 also.

Integers from 1 to 500 which are multiples of 10 are 10, 20....., 500

This forms an A.P., with $a = 10$, $d = 10$ and $l = 500$

Let the number of these terms be n .

Then,

$$a_n = 500$$

$$\Rightarrow a + (n - 1)d = 500$$

$$\Rightarrow 10 + (n - 1)(10) = 500$$

$$\Rightarrow (n - 1)(10) = 490$$

$$\Rightarrow n - 1 = 49$$

$$\Rightarrow n = 50$$

$$\Rightarrow S_{50} = \frac{50}{2} [2 \times 10 + 49 \times 10]$$

$$= 25 \times [20 + 490]$$

$$= 25 \times 510$$

$$= 12750$$

Now, Sum of all integers from 1 to 500 which are multiples of 2 or 5

= (Sum of all integers from 1 to 500 which are multiples of 2 + Sum of all integers from 1 to 500 which are multiples of 5 - Sum of all integers from 1 to 500 which are multiples of 2 and 5)

$$= 62750 + 25250 - 12750$$

$$= 75250$$

66. Given A.P. is -2, -7, -12, ... -77. Here, first term $a = -2$, and (last term) $a_n = -77$.

And common difference, $d = (-7) - (-2) = (-7 + 2) = -5$.

Now $a_n = -77$

$$\Rightarrow a + (n - 1)d = -77 \quad [\because a_n = a + (n - 1)d]$$

$$\Rightarrow -2 + (n - 1)(-5) = -77$$

$$\Rightarrow -[2 + (n - 1)5] = -77$$

$$\Rightarrow (2 + 5n - 5) = 77$$

$$\Rightarrow 5n - 3 = 77$$

$$\Rightarrow 5n = 77 + 3$$

$$\Rightarrow n = \frac{80}{5}$$

$$\Rightarrow n = 16$$

So, the 16th term will be -77.

$$\text{Now, } S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\Rightarrow S_{16} = \frac{16}{2} [2(-2) + (16 - 1)(-5)]$$

$$= 8 [-4 + (15)(-5)]$$

$$= 8 [-4 - 75]$$

$$= 8 [-79] = -632$$

Hence, the sum of the given A.P. upto the term -77 is -632.

$$67. S_8 = \frac{8}{2}(2a + 7d) = 100 \dots(i)$$

$$\text{and } S_{19} = \frac{19}{2}(2a + 18d) = 551 \dots(ii)$$

Solving (i) and (ii), $d = 3$ and $a = 2$

$$\therefore S_n = \frac{n}{2}[4 + (n - 1) \times 3] = \frac{n}{2}(3n + 1)$$

$$68. S_n = \frac{n}{2}[2a + (n-1)d]$$

$$\text{Now, } S_5 + S_7 = 167$$

$$\Rightarrow \frac{5}{2}[2a + 4d] + \frac{7}{2}[2a + 6d] = 167$$

$$\Rightarrow \frac{5 \times 2}{2}[a + 2d] + \frac{7 \times 2}{2}[a + 3d] = 167$$

$$\Rightarrow 5a + 10d + 7a + 21d = 167$$

$$\Rightarrow 12a + 31d = 167 \dots(i)$$

$$\text{Also, } S_{10} = 235$$

$$\Rightarrow \frac{10}{2}[2a + 9d] = 235$$

$$\Rightarrow 5[2a + 9d] = 235$$

$$\Rightarrow 10a + 45d = 235$$

$$\Rightarrow 2a+9d=47...(ii)$$

Multiplying equation (ii) by 6, we get

$$12a+54d=282...(iii)$$

subtracting (i) from (iii), we get

$$23d = 115$$

$$\Rightarrow d = 5$$

$$\Rightarrow 2a+9(5)=47$$

$$\Rightarrow 2a = 2$$

$$\Rightarrow a = 1$$

$$\text{First term} = a = 1$$

$$\text{Second term} = a+d=1+5=6$$

$$\text{Third term} = a+2d=1+2(5)=11$$

Thus, the Ap is 1,6,11,...

69. The given AP is 54, 51, 48, ...

$$\text{Here } a = 54$$

$$d = 51 - 54 = -3$$

Let the sum of n terms of this A.P be 513.

We know that,

$$S_n = \frac{n}{2}[2a + (n - 1)d]$$

$$\Rightarrow 513 = \frac{n}{2}[2(54) + (n - 1)3]$$

$$\Rightarrow 513 = \frac{n}{2}[108 - 3n + 3]$$

$$\Rightarrow 513 = \frac{n}{2}[111 - 3n]$$

$$\Rightarrow 1026 = n[111 - 3n]$$

$$\Rightarrow 1026 = 111n - 3n^2$$

$$\Rightarrow 3n^2 - 111n + 1026 = 0$$

$$\Rightarrow n^2 - 37n + 342 = 0 \dots \text{Dividing throughout by 3}$$

$$\Rightarrow n^2 - 18n - 19n + 342 = 0$$

$$\Rightarrow n(n - 18) - 19(n - 18) = 0$$

$$\Rightarrow n - 18 = 0 \text{ or } n - 19 = 0$$

$$\Rightarrow n = 8 \text{ or } n = 19$$

$$\Rightarrow n = 18, 19$$

Hence, the sum of 18 terms or 19 terms of the given AP is 513.

$$= a + (19 - 1)d \because a_n = a + (n - 1)d$$

$$= a + 18d$$

$$= 54 + 19(-3)$$

$$= 54 - 54$$

$$= 0$$

70. Here it is given that sum of the 4th and the 8th terms of an AP is 24 and the sum of its 6th and 10th terms is 44.

We know that general term of an AP is as, $a_n = a + (n-1)d$.

$$\text{Now, } a_4 = a + (4-1)d$$

$$\implies a_4 = a + 3d$$

$$\text{And } a_8 = a + (8-1)d$$

$$\implies a_8 = a + 7d$$

$$\text{Also, } a_6 = a + (6-1)d$$

$$\implies a_6 = a + 5d.$$

$$\text{And, } a_{10} = a + (10-1)d$$

$$\implies a_{10} = a + 9d.$$

$$\text{Given that } a_4 + a_8 = 24$$

$$\implies a + 3d + a + 7d = 24$$

$$\implies 2a + 10d = 24...(i)$$

$$\text{Also, } a_6 + a_{10} = 44$$

$$\Rightarrow a + 5d + a + 9d = 44$$

$$\Rightarrow 2a + 14d = 44 \dots (ii)$$

Subtracting (i) from (ii), we get

$$4d = 20$$

$$\Rightarrow d = 5$$

Substituting in (i), we get $a = -13$

We get first term $a = -13$ and common difference $d = 5$.

So, the sum of the first 10 terms is

$$S_{10} = \frac{10}{2} [2(-13) + (10-1)(5)]$$

$$\Rightarrow S_{10} = 5 [-26 + 9(5)]$$

$$\Rightarrow S_{10} = 5[-26 + 45]$$

$$\Rightarrow S_{10} = 5(19)$$

$$\Rightarrow S_{10} = 95.$$

Thus, sum of first ten terms is 95

71. All integers between 100 and 550, which are divisible by 9

$$= 108, 117, 126, \dots, 549$$

$$\text{First term (a)} = 108$$

$$\text{Common difference (d)} = 117 - 108 = 9$$

$$\text{Last term (a}_n) = 549$$

$$\Rightarrow a + (n - 1)d = 549$$

$$\Rightarrow 108 + (n - 1)(9) = 549$$

$$\Rightarrow 108 + 9n - 9 = 549$$

$$\Rightarrow 9n = 549 + 9 - 108$$

$$\Rightarrow 9n = 450$$

$$\Rightarrow n = \frac{450}{9} = 50$$

$$\text{Sum of 50 terms} = \frac{n}{2} [a + a_n]$$

$$= \frac{50}{2} [108 + 549]$$

$$= 25 \times 657$$

$$= 16425$$

Now, sum of all integers between 100 and 550 which are not divisible by 9

$$= \text{Sum of all integers between 100 and 550} - \text{Sum of all integers between 100 and 550 which are divisible by 9}$$

$$= [101 + 102 + 130 + \dots + 549] - 16425$$

$$= \frac{549 \times 550}{2} - \frac{100 \times 101}{2} - 16425$$

$$= 150975 - 5050 - 16425$$

$$= 129500$$

72. Let a be the first term and d be the common difference of given AP.

Then, we have

$$S_7 = 49$$

$$\Rightarrow \frac{7}{2} [2a + 6d] = 49$$

$$\Rightarrow \frac{7 \times 2}{2} [a + 3d] = 49$$

$$\Rightarrow a + 3d = 7 \dots (i)$$

$$\text{Also, } S_{17} = 289$$

$$\Rightarrow \frac{17}{2} [2a + 16d] = 289$$

$$\Rightarrow \frac{17 \times 2}{2} [a + 8d] = 289$$

$$\Rightarrow a + 8d = 17 \dots (ii)$$

Subtracting (i) from (ii), we get,

$$5d = 10$$

$$\Rightarrow d = 2$$

$$\Rightarrow a = 7 - 3(2) = 7 - 6 = 1$$

$$\therefore S_n = \frac{n}{2} [2a + (n - 1)d]$$

$$\begin{aligned}
&= \frac{n}{2}[2(1) + (n-1)2] \\
&= \frac{n}{2}[2 + 2n - 2] \\
&= \frac{n}{2} \times 2n \\
&= n^2
\end{aligned}$$

73. The first term of an AP is $a = 3$.

The last term of an AP is $l = 83$.

The sum of all terms is $S_n = 903$.

The sum of n terms formula is given by,

$$S_n = \frac{n}{2}(a + l)$$

$$903 = \frac{n}{2}(3 + 83)$$

$$903 = \frac{n}{2}(86)$$

$$903 = 43n$$

$$n = \frac{903}{43}$$

$$n = 21$$

The number of terms are 21.

The last term formula of an A.P is

$$l = a + (n - 1)d$$

$$83 = 3 + (21 - 1)d$$

$$80 = 20d$$

$$d = \frac{80}{20}$$

$$d = 4$$

The common difference is 4.

74. According to the question, We have to find number of terms of the AP $20, 19\frac{1}{3}, 18\frac{2}{3}, \dots$ that must be taken so that their sum is 300.

$$\text{Here, } a=20 \text{ and } d=19\frac{1}{3}-20=\frac{58}{3}-20=\frac{58-60}{3}=\frac{-2}{3}$$

Let the required number of terms be n .

Then, $S_n=300$

$$\Rightarrow \frac{n}{2}[2a+(n-1)d]=300$$

$$\Rightarrow \frac{n}{2}[2(20)+(n-1)(\frac{-2}{3})]=300$$

$$\Rightarrow n[40-\frac{2}{3}n+\frac{2}{3}]=600$$

$$\Rightarrow n[\frac{122}{3}-\frac{2}{3}n]=600$$

$$\Rightarrow \frac{122}{3}n-\frac{2}{3}n^2=600$$

$$\Rightarrow 122n-2n^2=1800$$

$$\Rightarrow 61n-n^2=900$$

$$\Rightarrow n^2-61n+900=0$$

$$\Rightarrow n^2-25n-36n+900=0$$

$$\Rightarrow n(n-25)-36(n-25)=0$$

$$\Rightarrow (n-25)(n-36)=0$$

$$\Rightarrow n-25=0 \text{ or } n-36=0$$

$$\Rightarrow n=25 \text{ or } n=36$$

\therefore Sum of first 25 terms = sum of first 36 terms = 300

This means that the sum of all terms from 26th to 36th is zero.

75. According to the question, Integers from 1 to 500 which are multiplies 2 as well as of 5 are

10, 20, 30,....., 500

This forms an A.P. with $a = 10$, $d = 10$, and $l = 500$

Let the number of these terms be n .

Then,

$$a_n = 500$$

$$\Rightarrow a + (n - 1)d = 500$$

$$\Rightarrow 10 + (n - 1)(10) = 500$$

$$\Rightarrow (n - 1)(10) = 490$$

$$\Rightarrow n - 1 = 49$$

$$\Rightarrow n = 50$$

$$\Rightarrow S_{49} = \frac{50}{2} [2 \times 10 + 49 \times 10]$$

$$= 25 \times [20 + 490]$$

$$= 25 \times 510$$

$$= 12750.$$