Solution

ARITHMETIC PROGRESSION WS 7

Class 10 - Mathematics

1. $S_n = \frac{3n^2}{2} + \frac{13n}{2}$
   $S_n = \frac{3n^2+13n}{2}$
   $a_n = S_n - S_{n-1}$
   or, $a_{25} = S_{25} - S_{24}$
   $= \frac{3(25)^2+13(25)}{2} - \frac{3(24)^2+13(24)}{2}$
   $= \frac{1}{2}[3(25^2 - 24^2) + 13(25 - 24)]$
   $= \frac{1}{2}[3(625 - 576) + 13(1)]$
   $= \frac{1}{2}(3 \times 49 + 13)$
   $= \frac{1}{2}(147 + 13)$
   $= \frac{1}{2}(160)$
   = 80
2. According to the question, man saved ₹33000 in 10 months.
   $\Rightarrow S_{10}$=₹33000, common difference=d=₹100
   Let the amount he saved in the first month be ₹ a.
   $S_n = \frac{n}{2}[2a+(n-1)d]$
   $\Rightarrow S_{10} = \frac{10}{2}[2a+(10-1)(100)]$
   $\Rightarrow 33000=5[2a+900]$
   $\Rightarrow 33000=10a+4500$
   $\Rightarrow 10a=28500$
   $\Rightarrow$a=₹2850
   Hence, he saved ₹ 2850 in the first month.
3. Here, a = 2
   d = 8
   $S_n = 90$
   We know that
   $S_n = \frac{n}{2}[2a + (n-1)d]$
   $\Rightarrow 90 = \frac{n}{2}[2(2) + (n-1)8]$
   $\Rightarrow 90 = n[2 + (n - 1) 4]$
   $\Rightarrow 90 = n[2 + 4n - 4]$
   $\Rightarrow 90 = n[4n - 2]$
   $\Rightarrow 90 = 2n[2n - 1]$
   $\Rightarrow 45 = n[2n - 1]$
   $\Rightarrow 45 = 2n^2 - n$
   $\Rightarrow 2n^2 - n - 45 = 0$
   $\Rightarrow 2n^2 - 10n + 9n - 45 = 0$
   $\Rightarrow 2n(n - 5) + 9(n - 5) = 0$
   $\Rightarrow (n - 5)(2n + 9) = 0$
   $\Rightarrow n - 5 = 0$ or $2n + 9 = 0$
   $\Rightarrow n = 5$ or $n = -\frac{9}{2}$
   $n = -\frac{9}{2}$ is in admissible as n, being the number of terms, is a natural number.
   $\therefore n = 5$
   Again, we know that
   $a_n = a + (n - 1)d$
   $\Rightarrow a_n = 2 (5 - 1)8$

$\Rightarrow a_n = 2 + (4)8$

$\Rightarrow a_n = 2 + 32$

$\Rightarrow a_n = 34$

4. Given,
   First term(a) = -14
   Fifth term (a_5) = 2

   We know that, n^{th} term of an A.P. is
   $a_n = a + (n - 1)d$

   Since, $a_5 = 2$

   $\Rightarrow a + 4d = 2$
   $\Rightarrow -14 + 4d = 2$
   $\Rightarrow 4d = 16$
   $\Rightarrow d = \frac{16}{4} = 4$
   So,Common difference (d) = 4
   Let the number of terms = n
   Given,
   Sum of n terms = 40
   $\Rightarrow \quad \frac{n}{2}[2a + (n-1)d] = 40$
   $\Rightarrow \quad \frac{n}{2}[2(-14) + (n-1) \times 4] = 40$
   $\Rightarrow \quad \frac{n}{2}[-28 + 4n - 4] = 40$
   $\Rightarrow \quad \frac{n}{2}[4n - 32] = 40$
   $\Rightarrow \quad n(4n - 32) = 80$
   $\Rightarrow \quad n^2 - 8n - 20 = 0$
   $\Rightarrow \quad n^2 - 10n + 2n - 20 = 0$
   $\Rightarrow \quad n(n - 10) + 2(n - 10) = 0$
   $\Rightarrow \quad (n - 10)(n + 2) = 0$
   Either, n - 10 = 0. Or, n + 2 =0
   $\Rightarrow$ n = 10 . Or, n = -2 (but no. of terms can't be negative)
   Therefore, number of terms = 10.
5. The first 40 positive integers divisible by 6 are 6, 12, 18, 24, ...
   Here, $a_2 - a_1 = 12 - 6 = 6$

   $a_3 - a_2 = 18 - 12 = 6$

   $a_4 - a_3 = 24 - 18 = 6$

   i.e.$a_{k+1} - a_k$ is the same every time.
   So, the above list of numbers form an AP.
   Here, a = 6
   d = 6
   n = 40
   $\therefore$ Sum of the first 40 positive integers = S_{40}
   $= \frac{40}{2}[2a + (40-1)d]$ ....$\{\because S_n = \frac{n}{2}[2a + (n-1)d]\}$
   = 20[2a + 39d]
   $= (20)[2 \times 6 + 39 \times 6]$
   = (20) (246)
   = 4920
6. Here, d = 7
   $a_{22} = 149$

   Let the first term of the AP be a.
   We know that
   $a_n = a + (n - 1)d$

   $\Rightarrow a_{22} = a + (22 - 1)d$

⇒ $a_{22} = a + 21d$

⇒ 149 = a + (21) (7)

⇒ 149 = a + 147

⇒ a = 2

Again, we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{22} = \frac{22}{2}[2(2) + (22-1)7]$

⇒ S_{22} = (11) [4 + 147]

⇒ S_{22} = (11) (151)

⇒ S_{22} = 1661

Hence, the sum of the first 22 terms of the AP is 1661.

7. All natural numbers between 100 and 200 which are divisible by 4 are

104, 108, 112, 116, ..., 196

Here, a_1 = 104

a_2 = 108

a_3 = 112

a_4 = 116

$\therefore a_2 - a_1 = 108 - 104 = 4$

$a_3 - a_2$ = 112 - 108 = 4

$a_4 - a_3$ = 116 - 112 = 4

∴ $a_2 - a_1 = a_3 - a_2 = a_4 - a_3$ = ...

∴ This sequence is an arithmetic progression whose common difference is 4

here, a = 104

d = 4

l = 196

Let the number of terms be n. Then

l = a + (n - 1)d

⇒ 196 = 104 + (n - 1)4

⇒ (n - 1)4 = 92

⇒ n - 1 = 23

⇒ n = 23 + 1

⇒ n = 24

$\therefore S_n = \frac{n}{2}(a+1)$

$= \frac{24}{2}(104 + 196)$

= (12) (300)

= 3600

8. All the natural numbers less than 100 which are divisible by 6 are

6, 12, 18, 24,................., 96

Here, a_1 = 6

a_2 = 12

a_3 = 18

a_4 = 24

: :

∴ $a_2 - a_1$ = 12 - 6 = 6

$a_3 - a_2$ = 18 - 12 = 6

$a_4 - a_3$ = 24 - 18 = 6

$a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = 6$ (=6 in each case)

∴ This sequence is an arithmetic progression whose difference is 6.

Here, a = 6

d = 6

l = 96

Let the number of terms be n. Then,

l = a + (n - 1)d

⇒ 96 = 6 + (n - 1)6

⇒ 96 - 6 = (n - 1)6

⇒ 90 = (n - 1)6

⇒ (n - 1)6 = 90

⇒ $n - 1 = \frac{90}{6}$

⇒ n - 1 = 15

⇒ n = 15 + 1

⇒ n = 16

$\therefore S_n = \frac{n}{2}(a + l)$

$= \left(\frac{16}{2}\right)(6 + 96)$

= (8) (102)

= 816

9. Let the three numbers in A.P. be $a - d, a, a + d$.

   $3a = 12$ or, $a = 4$.

   Also, $(4 - d)^3 + 4^3 + (4 + d)^3 = 288$

   or, $64 - 48d + 12d^2 - d^3 + 64 + 64 + 48d + 12d^2 + d^3 = 288$

   or, $24d^2 + 192 = 288$

   or, $24d^2 = 288 - 192$

   or, $24d^2 = 96$

   or, $d^2 = 96/24$

   or, $d^2 = 4$

   d =$\pm$2

   The numbers are 2,4, 6, or 6,4,2.

10. Three digits numbers that are divisible by 11 are :-

    110, 121, 132,.............., 990

    Clearly ,above sequence is an A.P.

    Here,

    a (First term) = 110

    d (common difference) = 11

    $a_n = 990$

    We know that, in A.P.

    $a_n = a + (n - 1)d$

    Or, 990 = 110 + (n - 1) $\times$ 11

    Or, 990 = 110 + 11n - 11

    Or, 990 = 99 + 11n

    Or, 891 = 11n

    Or, n = 81

    Also,

    $S_n = \frac{n}{2}(a + a_n)$

    Or, $S_{81} = \frac{81}{2}(110 + 990)$

    Or, $S_{81} = 81 \times 550$

    Or, $S_{81} = 44550$.

11. Let the first term be a and the common difference be d.

    $a_n = a + (n - 1)d$

    By given condition

    $a_{12} = a + 11d = -13$...(i)

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_4 = \frac{4}{2}[2a + (3 - 1)d]$

2[2a + 3d] = 24

2a + 3d = 12 ...(ii)

Multiplying (i) by 2 and subtracting (ii) from it

(2a + 22d) - (2a + 3d) = - 26 - 12

2a + 22d - 2a - 3d = - 38

19d = - 38

d = - 2

Putting the value of d in (i) we get

$a + 11d = -13$

$a + 11 \times -2 = -13$

$a = -13 + 22$

$a = 9$

Now, $S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{10} = \frac{10}{2}(2 \times 9 + 9 \times -2)$

$= 5 \times (18 - 18)$

= 0

Hence $S_{10} = 0$

12. Let the first term be a and common difference be d.

Given,

Second term is 2.

$\Rightarrow$ a + (2 - 1)d = 2

$\Rightarrow$ a + d = 2...........(i)

and, fourth term is 8.

$\Rightarrow$ a + (4 - 1)d = 8

$\Rightarrow$ a + 3d = 8........(ii)

Subtracting equation (i) from (ii),

a + 3d - a - d = 8 - 2

$\Rightarrow$ 2d = 6

$\Rightarrow d = \frac{6}{2} = 3$

Putting value of d in eq(i),

a + 3 = 2

$\Rightarrow$ a = 2 - 3 = -1

Sum of first 51 terms = $\frac{51}{2}[2 \times (-1) + (51 - 1) \times 3]$

$= \frac{51}{2}[-2 + 150]$

$= \frac{51}{2} \times 148$

$= 51 \times 74$

= 3774

13. According to the question,

First term of an AP, a=22

Last term=n^{th} term=-11

Sum of n terms = $S_n = \frac{n}{2}(a+l) = 66$

$\Rightarrow \frac{n}{2}(22-11) = 66$ or $\frac{n}{2} \times 11 = 66$

$\therefore n = \frac{66 \times 2}{11} = 12$

n^{th} term = l = a+(n-1)d

$\therefore -11 = 22 + (12-1) \times d$ or -11 = 22+11d

$\Rightarrow$ 11d = -22-11

$\Rightarrow$ 11d = - 33

$\therefore d = \frac{-33}{11} = -3$

Thus, n=12, d=-3.

14. Here a = 45, d = -6, S_n = 180

To find: n

We know,

$s_n = \frac{n}{2} \times 2a + (n-1)d$

Therefore,

$180 = \frac{n}{2} \times 2(45) + (n-1) - 6$

$180 = \frac{n}{2} \times 90 - 6n + 6$

180 × 2 = n(96 - 6n)

360 = 96n + 360 = 0

Dividing by 6

n^2 - 16n + 60 = 0 .... (factorizing)

n (n - 10) - 6 (n - 10) = 0 ....{ taking common out }

(n - 6) (n - 10) = 0

n - 6 = 0 or n - 10 = 0

n = 6. or. n = 10

Therefore 6 or 10 terms must be taken so that their sum is 180.

15. a_n = 3 + 2n

Put n = 1, 2, 3,...

a_1 = 5, a_2 = 7, a_3 = 9....

a = 5, d = 7 - 5 = 2

$S_{24} = \frac{24}{2}[2 \times 5 + (24-1) \times 2]$ = 12[10 + 46] = 672

16. Here, a_{12} = 37

d = 3

We know that

a_n = a + (n - 1)d

$\Rightarrow$ a_{12} = a + (12 - 1)d

$\Rightarrow$ a_{12} = a + 11d

$\Rightarrow$ 37 = a + 33

$\Rightarrow$ a = 37 - 33 = 4

Again, we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{12} = \frac{12}{2}[2a + (12-1)d]$

$\Rightarrow S_{12} = 6[2a + 11d]$

$\Rightarrow S_{12} = 6[2 \times 4 + 11 \times 3]$

$\Rightarrow S_{12} = 6[8 + 33]$

$\Rightarrow S_{12} = 6 \times 41$

$\Rightarrow S_{12} = 246$

17. The given n odd natural numbers are,

1, 3, 5, ... 2n - 1

Clearly, it is an AP with

First term (a) = 1

Common difference (d) = 3 - 1 = 2

Number of terms = n

Sum of n terms $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[2 \times 1 + (n-1) \times 2]$

$= \frac{n}{2}[2 + (2n-2)]$

$= n^2$

18. Here it is given that first car goes with uniform speed of 10 km/h. The second goes at a speed of 8 km/h in the first hour and increases the speed by 1/2 km in each succeeding hour.

Suppose the second car overtakes the first car after t hours. Then, the two cars travel the same distance in t hours.Distance travelled by the first car in t hours = 10×t km.

|Distance travelled by the second car in t hours= Sum of t terms of an A.P. with first term 8 and common difference 1/ 2

$= \frac{t}{2}\left\{2 \times 8 + (t-1) \times \frac{1}{2}\right\} = \frac{t(t+31)}{4}$

When the second car overtakes the first car, we have

$10t = \frac{t(t+31)}{4}$

$\Rightarrow 40t = t^2 + 31t$

$\Rightarrow t^2 - 9t = 0$

$\Rightarrow t(t-9) = 0$

$\Rightarrow t = 9$

Since t can not be equal to zero, so we get t=9. Hence, it is proved that the second car will overtake the first car in 9 hours.

19. Here, 12^{th} term, T_{12} = -13

$\Rightarrow$ a+11d = -13...(i)

S_4 = 24

$\Rightarrow \frac{4}{2}$ [2a+3d] = 24

$\Rightarrow$ 2[2a+3d] = 24

$\Rightarrow$ 2a+3d = 12...(ii)

Multiplying equation (i) by 2, we get

2a + 22d = -26...(iii)

Subtracting (ii) from (i), we get

19d = -38

$\Rightarrow$d = -2

$\Rightarrow$ a +11(-2) = -13 ...[from (i)]

$\Rightarrow$ a = 9

$\therefore$ Sum fo first 10 terms, $S_{10} = \frac{10}{2}$[2(9)+9(-2)] = 5[18-18] = 5$\times$0 = 0

20. A.P. is given as: 25, 22, 19, ...

Sum of first n(say) terms = 116

Here a = 25 and d = 22 - 25 = 3

But $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow$ 116 = $\frac{n}{2}[25 \times 2 + (n-1)(-3)]$

232 = n(50 - 3n + 3) $\Rightarrow$ 232 = (53 - 3n)n

$\Rightarrow$ 232 = 53n - $3n^2$ $\Rightarrow$ $3n^2$ - 53n + 232 = 0

$\Rightarrow$ $3n^2$ - 24n - 29n + 232 = 0

$$\left\{\begin{array}{c} \because 232 \times 3 = 696 \\ \therefore 696 = -24 \times (-29) \\ -53 = -24 - 29 \end{array}\right.$$

$\Rightarrow$ 3n(n - 8) - 29(n - 8) = 0

$\Rightarrow$ (n - 8)(3n - 29) = 0

Either n - 8 = 0, then n = 8

or 3n - 29 = 0 then 3n = 29 $\Rightarrow$ n = $\frac{29}{3}$

Which is not possible being a fraction

$\therefore$ Number of terms = 8

Now l = a_8 = a + (n - 1)d = 25 + (8 - 1) $\times$ (-3)

= 25 + 7(-3) = 25 - 21 = 4

$\therefore$ Last term = 4

21. All the numbers between 100 and 500 which are divisible by 8 are

104, 112, 120, 128, ..., 496

Here, a_1 = 104

a_2 = 112

a_3 = 120

a_4 = 128

$\therefore$ a_2 - a_1 = 112 - 104 = 8

a_3 - a_2 = 120 - 112 = 8

a_4 - a_3 = 128 - 120 = 8

∵ a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... (8 each)

∴ This sequence is an arithmetic progression whose difference is 8.

Here, a = 104

d = 8

l = 496

Let the number of term s be n . Then,

l = a + (n - 1)d

⇒ 496 = 104 + (n - 1)8

⇒ 392 = (n - 1)8

⇒ (n - 1)8 = 392

⇒ n - 1 = 49

⇒ n = 49 + 1

⇒ n = 50

$\therefore S_n = \frac{n}{2}(a + l)$

$= \left(\frac{50}{2}\right)(104 + 496)$

= (25) (600)

= 15000

22. Here, a = 5

l = 45

S = 400

We know that

$S = \frac{n}{2}(a + l)$

$\Rightarrow 400 = \frac{n}{2}(5 + 45)$

$\Rightarrow 400 = \frac{n}{2}(50)$

$\Rightarrow 400 - 25n$

$\Rightarrow n = \frac{400}{25}$

$\Rightarrow n = 16$

Hence, the number of terms is 16.

Again, we know that

l = a + (n - 1)d

⇒ 45 = 5 + (16 - 1)d

⇒ 45 = 5 + 15d

⇒ 40 = 15 d

$\Rightarrow d = \frac{40}{15} = \frac{8}{3}$

Hence, the common difference is $\frac{8}{3}$.

23. The given AP is 9, 17, 25,...

Here, a = 9

d = 17 - 9 = 8

Let n terms of the AP must be taken

Then, S_n = 636

$\Rightarrow \frac{n}{2}[2a + (n - 1)d] = 636$

$\Rightarrow \frac{n}{2}[2(9) + (n - 1)8] = 636$

⇒ n[9 + (n - 1)4] = 636

⇒ n[9 + 4n - 4] = 636

⇒ n[(4n + 5)] = 636

⇒ $4n^2$ + 5n - 636 = 0

⇒ $4n^2$ + 53n - 48n - 636 = 0

⇒ n(4n + 53) - 12(4n + 53) = 0

⇒ (4n + 53) (n - 12) = 0

$\Rightarrow$ 4n + 53 = 0 or n - 12 = 0

$\Rightarrow n = -\frac{53}{4}$ or n = 12

$n = -\frac{53}{4}$ is in admissible as n, being the number of terms, is a natural number

$\therefore$ n = 12

Hence, 12 terms of the AP must be taken.

24. $S_n = 3n^2 + 2n$

Taking n = 1, we get

S1 = $3(1)^2 + 2(1)$

$\Rightarrow S_1 = 3 + 2$

$\Rightarrow S_1 = 5$

$\Rightarrow a_1 = 5$

Taking n = 2, we get

$S_2 = 3(2)^2 + 2(2)$

$\Rightarrow S_2 = 12 + 4$

$\Rightarrow S_2 = 16$

$\therefore a_2 = S_2 - S_1 = 16 - 5 = 11$

Taking n = 3, we get

$S_3 = 3(3)^2 + 2(3)$

$\Rightarrow S_3 = 27 + 6$

$\Rightarrow S_3 = 33$

$\therefore a_3 = S_3 - S_2 = 33 - 16 = 17$

So, a = 5

$d = a_2 - a_1 = 11 - 5 = 6$

Hence the required AP is 5, 11, 17...

25. Given that, nth term of the series is $a_n = 3 - 4n$

For a_1,

Put n = 1 so $a_1 = 3 - 4(1) = -1$

For a_2,

Put n = 2, so $a_2 = 3 - 4(2) = -5$

For a_3,

Put n = 3 so $a_3 = 3 - 4(3) = -9$

For a_4,

Put n = 4 so $a_4 = 3 - 4(4) = -13$

So AP is - 1, - 5, - 9, - 13, …

$a_2 - a_1 = -5 - (-1) = -4$

$a_3 - a_2 = -9 - (-5) = -4$

$a_4 - a_3 = -13 - (-9) = -4$

Since, the each successive term of the series has the same difference. So, it forms an AP with common difference, d = - 4

We know that, sum of n terms of an AP is

$S_n = \frac{n}{2}(2a + (n-1)d)$

Where a = first term

d = common difference

and n = no of terms

$S_{20} = \frac{20}{2}[2(-1) + (20-1)(-4)]$

= 10[- 2 - 76]

= - 780

So Sum of first 20 terms of this AP is - 780.

26. It is given that the sum of seven cash prizes is equal to ₹ 700.

And, each prize is ₹ 20 less than its preceding term.

Let the value of first prize = ₹ a

Let the value of second prize =₹ (a−20)

Let the value of third prize = ₹ (a−40)

So, we have a sequence of the form:

a, a−20, a−40, ...

It is an arithmetic progression because the difference between consecutive terms is constant.

First term = a, Common difference = d = (a − 20) − a= −20

n = 7 (Because there are total of seven prizes)

S_7 = ₹ 700 {given}

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$S_7 = \frac{7}{2}[2a + (7-1)(-20)]$

$\Rightarrow 700 = \frac{7}{2}[2a - 120]$

⇒ 200 = 2a− 120

⇒ 320 = 2a

⇒ a =160

Therefore, value of first prize = ₹ 160

Value of second prize = 160 - 20= ₹ 140

Value of third prize = 140 - 20= ₹ 120

Value of fourth prize = 120 - 20 = ₹ 100

Value of fifth prize = 100 - 20 = ₹ 80

Value of sixth prize = 80 - 20 = ₹ 60

Value of seventh prize = 60 - 20 = ₹ 40

27. According to the question, $S_5 + S_7 = 167$

$\Rightarrow \frac{5}{2}[2a + 4d] + \frac{7}{2}$[2a + 6d] = 167

⇒ 5(a + 2d) + 7(a + 3d) = 167

⇒ 12a + 31d = 167 ......(i)

and S_{10} = 235

⇒ $\frac{10}{2}$[2a + 9d] = 235

⇒ 2a + 9d = $\frac{235}{5}$ = 47 ....(ii)

Multiplying eq. (ii) by 6 and then subtracting from (i), we have

$$\begin{array}{r} 12a + 31d = 167 \\ 12a + 54d = 282 \\ - \quad - \qquad - \\ \hline -23d = -115 \end{array}$$

d = $\frac{-115}{-23}$ = 5

From (ii), we get

2a + 9d = 47

⇒ 2a + 9(5) = 47

⇒ 2a = 47 - 45

⇒ 2a = 2 ⇒ a = 1

Hence, S_{20} = $\frac{20}{2}$[2a + 19d]

= 10[2(1) + 19(5)] = 10[2 + 95] = 970

28. Here a = 1 and d = 1

$S_{x-1} = \frac{x-1}{2}[2 \times 1 + (x-1-1) \times 1]$

$= \frac{x-1}{2}[2 + x - 2]$

$= \frac{x(x-1)}{2} = \frac{x^2-x}{2}$

$S_x = \frac{x}{2}[2 \times 1 + (x-1) \times 1]$

$= \frac{x}{2}(x+1) = \frac{x^2+x}{2}$

$S_{49} = \frac{49}{2}[2 \times 1 + (49-1) \times 1]$

$= \frac{49}{2}[2 + 48] = 49 \times 25$

According to question,

$S_{x-1} = S_{49} - S_x$

$\Rightarrow \frac{x^2-x}{2} = 49 \times 25 - \frac{x^2+x}{2}$

$\Rightarrow \frac{x^2-x+x^2+x}{2} = 49 \times 25$

$\Rightarrow \frac{2x^2}{2} = 49 \times 25$

$\Rightarrow x^2 = \pm 35$

Since, x is a counting number, so negative value will be neglected.

x = 35

29. Given, last term, l = 119

No. of terms in A.P. = 30

8th term from the end = 91

Let d be a common difference and assume that the first term of A.P. is 119 (from the end)

Since the n_{th} term of AP is

an = l + (n - 1)d

$\therefore$ a8 = 119 + (8 - 1)d

$\Rightarrow$ 91 = 119 + 7d

$\Rightarrow$ 7d = 91 - 119

$\Rightarrow$ 7d = -28

$\Rightarrow$ d = -4

Now, this common difference is from the end of A.P.

So, the common difference from the beginning = -d

= (-4) = 4

Thus, a common difference for the A.P. is 4.

Now, using the formula

I = a + (n - 1)d

$\Rightarrow$ 119 = a + (30 - 1)4

$\Rightarrow$ 119 = a + 29 $\times$ 4

$\Rightarrow$ 119 = a + 116

$\Rightarrow$ a = 119 - 116

$\Rightarrow$ a = 3

Hence, using the formula for the sum of n terms of an A.P.

i.e., $S_n = \frac{n}{2}[2a + (n - 1)d]$

S30 = $\frac{30}{2}[2 \times 3 + (30 - 1) \times 4]$

= 15(6 + 29 $\times$ 4)

= 15 $\times$ 122

= 1830

Therefore, the sum of 30 terms of an A.P. is 1830

30. Here, a_3 = 15

S_{10} = 125

We know that

a_n = a + (n - 1)d

$\Rightarrow a_3$ = a + (3 - 1)d

$\Rightarrow a_3$ = a + 2d

$\Rightarrow$ 15 = a + 2d

$\Rightarrow$ a + 2d = 15 ...... (1)

Again, we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2a + (10-1)d]$

$\Rightarrow S_{10}$ = 5(2a + 9d)

$\Rightarrow$ 125 = 5(2a + 9d)

$\Rightarrow$ 25 = 2a + 9d

$\Rightarrow$ 2a + 9d = 25 .... (2)

Solving equation (1) and equation (2), we get

a = 17

d = -1

Now an = a + (n - 1)d

$\Rightarrow a_{10}$ = a + (10 - 1)d

$\Rightarrow a_{10}$ = a + 9d

$\Rightarrow a_{10}$ = 17 + 9(-1)

$\Rightarrow a_{10}$ = 17 - 9

$\Rightarrow a_{10}$ = 8

31. Since the penalty for each succeeding day is ₹ 50 more than for the preceding day.

Therefore, amount of penalty for different days forms an A.P. with first term a = 200 and common difference d = 250 - 200= 50.

We have to find how much does a delay of 30 days cost the contractor.

In other words, we have to find the sum of 30 terms of the A.P.

n = 30, a = 200 and d= 50

$\therefore$ Required sum $= \frac{30}{2}\{2 \times 200 + (30 - 1) \times 50\} \left[\because S_n = \frac{n}{2}[2a + (n-1)d]\right]$

$\Rightarrow$ Required sum = 15 (400 + 29 $\times$ 50)

$\Rightarrow$ Required sum = 15 (400 + 1450)

$\Rightarrow$ Required sum = 15 $\times$ 1850 = 27750

Thus, a delay of 30 days will cost the contractor of ₹ 27750.

32. Let the first term be a and the common difference be d.

a_n = a + (n - 1)d

$S_n = \frac{n}{2}$[2a + (n - 1)d]

As per given condition

a_{10} = -37

a + 9d = - 37 .,.(i)

Sum of first 6 term is - 27

$S_6 = \frac{6}{2}$[2a + (6 - 1)d]

3(2a + 5d) = -27

or, 2a + 5d = - 9 ...(ii)

Multiplying (i) by 2 and subtracting (ii) from it, we get

(2a + 18d) - (2a + 5d) = - 74 + 9

2a + 18d - 2a - 5d = -65

13d = - 65

d = -65/13

d = - 5

Putting d = -5 in (i) we get

a + 9d = - 37

a + 9 $\times$ - 5 = -37

a = - 37 + 45

a = 8

$S_n = \frac{n}{2}$[2a + (n - 1)d]

$= \frac{8}{2}[2 \times 8 + (8-1)(-5)]$

$= 4[16 + (7)(-5)]$

= $4[16 - 35]$

= 4 $\times -19$

$= -76$

Hence, S_n = -76

33. Let the given AP contains n terms.

First term, a=5

Last term, l=45

S_n=400

$\Rightarrow \frac{n}{2}$ [a+l]=400

$\Rightarrow \frac{n}{2}$ [5+45]=400

$\Rightarrow$n$\times$50=800

$\Rightarrow$n=16

Thus, the given AP contains 16 terms.

Let d be the common difference of the given AP.

then,

T_{16}=45

$\Rightarrow$a+15d=45

$\Rightarrow$5+15d=45

$\Rightarrow$15d=40

$\Rightarrow$d=$\frac{40}{15}=\frac{8}{3}$.

Therefore, common difference of the given AP is $\frac{8}{3}$.

34. Here, we have the sum of first 20 terms of an A.P., in which 3rd term is 7 and 7^{th} term is two more than thrice of its 3rd term.

Let "a" be the first term and" d"be the common difference of the given A.P. Therefore, a_3 = 7 and a_7 = $3a_3$ + 2 [Given]

$\Rightarrow$a + 2d = 7 and a + 6d = 3(a + 2d) + 2

$\Rightarrow$ a + 2d = 7 and a + 6d = 3a + 6d + 2

$\Rightarrow$ a + 2d = 7 and a -3a= 6d-6d+2

$\implies$ a+2d=7 and -2a=2

$\implies$ a+2d=7 and a=-1

$\implies$ -1+2d=7

$\implies$ 2d=7+1=8

$\implies$ d=4

$\Rightarrow$ a = -1 and d = 4

Putting n = 20, a = - 1 and d = 4 in $S_n=\frac{n}{2}\{2a+(n-1)d\}$, we get

$S_{20}=\frac{20}{2}\{2\times -1+(20-1)\times 4\}=\frac{20}{2}(-2+76)=740$

35. A.P. is $-6,-\frac{11}{2},-5$ ......

$a=-6$

$d=-\frac{11}{2}+\frac{6}{1}=\frac{-11+12}{2}=\frac{1}{2}$

$S_n=-25$

or, $S_n=\frac{n}{2}[2a+(n-1)d]$

or, $-25=\frac{n}{2}\left[-12+(n-1)\times\frac{1}{2}\right]$

or, $-50=n\left[\frac{-24+(n-1)}{2}\right]$

or, -100 = n[n - 25]

or, $n^2-25n+100=0$

or, $(n-20)(n-5)=0$

or, $n-20=0, n-5=0$

or,$n=20,5$

or S_{20} = S_5

Two answers $\because$a is negative and d is positive and the sum of the terms from 6th to 20th is zero.

36. Since each section of each class plants the same number of trees as the class number and there are three sections of each class.

Three sections of class I will plant = 1 $\times$ 3 = 3

Three sections of class II will plant = 2 $\times$ 3 = 6

Three sections of class III will plant = 3 $\times$ 3 = 9 and so on.

Three sections of class XII will plant = 12 $\times$ 3 = 36

So, we get an A .P. 3, 6, 9, ..., 36.

Here a = 3 and d = 6 -3 = 3

a_n = 36

We know that, a_n = a +(n -1) d

3 + (n - 1) 3 = 36

(n - 1) 3 = 33

(n - 1) = 11

n = 11+ 1

n = 12

Sn $= \frac{n}{2}(a_1 + a_n) = \frac{12}{2}(3 + 36) = 234$

37. Here, a = 7

a_{13} = 35

a_n = a + (n - 1)d

⇒ a_{13} = a + (13 - 1)d

⇒ a_{13} = a + 12d

⇒ 35 = 7 + 12d

⇒ 12d = 35 - 7

⇒ 12d = 28

$\Rightarrow d = \frac{28}{12}$

$\Rightarrow d = \frac{7}{3}$

Again, we know that

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2a + (13 - 1)d]$

$\Rightarrow S_{13} = \frac{13}{2}[2a + 12d]$

$= S_{13} = \frac{13}{2}\left[2(7) + 12\left(\frac{7}{3}\right)\right]$

$\Rightarrow S_{13} = \frac{13}{2}(14 + 28)$

$\Rightarrow S_{13} = \frac{13}{2}(42)$

$\Rightarrow S_{13} = (13)(21)$

$\Rightarrow S_{13} = 273$

38. Put n = 1, 2, 3, 4, ... in succession, we get

a_1 = 3 + 4(1) = 3 + 4 = 7

a_2 = 3 + 4(2) = 3 + 8 = 11

a_3 = 3 + 4(3) = 3 + 12 = 15

a_4 = 3 + 4(4) = 3 + 16 = 19

: : :

∴ a_2 - a1 = 11 - 7 = 4

a_3 - a_2 = 15 - 11 = 4

a_4 - a_3 = 19 - 15 = 4

i.e. a_{k+1} - a_k is the same every time.

So, a_1, a_2, ..., an, ... from an AP.

Here, a = a_1 = 7

d = a_2 - a_1 = 4

∴ Sum of the first 15 terms = S_{15}

$= \frac{15}{2}[2a + (15 - 1)d] \because S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{15}{2}[2a + 14d]$

= 15(a + 7d)

$= (15)(7 + 7 \times 4)$

= (15) (7 + 28)

= (15) (35)

= 525

39. Given, a = 10, and S_{14} = 1050

Let the common difference of the A.P. be d

$\therefore \quad S_n = \frac{n}{2}[2a + (n-1)d]$

$\therefore \quad S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d]$

1050 = $7(20 + 13d)$

$20 + 13d$ = $\frac{1050}{7}$

$20 + 13d$ = 150

$13d$ = 150 - 20

$13d$ = 130

d = $\frac{130}{13}$ = 10

$a_{20} = a + (n-1)d$

= 10 + (20 - 1) 10

= 10 + 19 $\times$ 10

= 10 + 190

= 200

Hence, a_{20} = 200

40. The number of logs in the bottom row = 20

The number of logs in the next row = 19

The number of logs in the next to next row = 18

Therefore, we have sequence of the form 20, 19, 18 ...

First term = a = 20, Common difference = d = 19 – 20 = –1

We need to find that how many rows make total of 200 logs.

Applying formula, $S_n = \frac{n}{2}[2a + (n-1)d]$ to find sum of n terms of AP, we get

$200 = \frac{n}{2}[40 + (n-1)(-1)]$

⇒ 400 = n (40 – n + 1)

⇒ 400 = $40n - n^2 + n$

⇒ $n^2 - 41n + 400 = 0$

It is a quadratic equation, we can factorize to solve the equation.

⇒ $n^2 - 25n - 16n + 400 = 0$

⇒ n (n − 25) – 16 (n − 25) = 0

⇒ (n − 25) (n − 16)

⇒ n = 25, 16

We discard n = 25 because we cannot have more than 20 rows in the sequence. The sequence is of the form: 20, 19, 18 ...

At most, we can have 20 or less number of rows.

Therefore, n = 16 which means 16 rows make total number of logs equal to 200.

We also need to find number of logs in the 16th row.

Applying formula, $S_n = \frac{n}{2}(a + l)$ to find sum of n terms of AP, we get

200 = 8 (20 + l)

⇒ 200 = 160 + 8

⇒ 40 = 8l ⇒ l= 5

Therefore, there are 5 logs in the top most row and there are total of 16 rows.

41. The general term of an AP is given by a_n= a+(n-1)d and $S_n = \frac{n}{2}$[2a+(n-1)d].

The AP is 8,10,12,....

So, a=8 and d=2

Now, a_n=a+(n-1)d

⇒a_{60}=8+59(2)

⇒a_{60}=126

So, its last term is 126.

Sum of its last 10 terms= sum of 60 terms -sum of 50 terms

=$\frac{60}{2}$[2(8)+59(2)]-$\frac{50}{2}$[2(8)+49(2)]

=30[16+118]-25[16+98]

=4020-2850

=1170.

42. Let a be the first term and the common difference be d.

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$S_{12} = \frac{12}{2}[2a + (12 - 1)d]$

= 6[2a + 11d]

= 12a + 66d

$S_8 = \frac{8}{2}[2a + (8 - 1)d]$

= 4[2a + 7d]

= 8a + 28d

$S_4 = \frac{4}{2}[2a + (4- 1)d]$

= 2[2a + 3d]

= 4a + 6d

$3(S_8 - S_4) = 3[(8a + 28d) - (4a + 6d)]$

= 3[8a + 28d - 4a - 6d]

= 3[4a + 22d]

= 12a + 66d

$= S_{12}$

43. Savings of Resham in a year = ₹450 + ₹470 + ₹490 +..........

This forms an A.P. with a = 450, d = 20 and n = 12

Now, $S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_{12} = \frac{12}{2}[2 \times 450 + 11 \times 20]$

= 6[900 + 220]

= 6(1120)

= 6720

Thus, Resham saved ₹6720 in 12 months.

Since amount required for admission is less than the saving, Resham would be able to send her daughter to the school next year.

44. Let L_1, L_2, L_3, L_4, ...,L_{13} be the lengths of semicircles of radii 0.5 cm, 1 cm, 1.5 cm, 2 cm,... and $\frac{13}{2}$cm respectively.

Then, we have

$L_1 = (\pi \times 0.5)$ cm = $\frac{\pi}{2}$cm,

$L_2 = (\pi \times 1)$ cm = $2(\frac{\pi}{2})$cm,

$L_3 = (\pi \times 1.5)$cm = $3(\frac{\pi}{2})$cm,

$L_4 = (\pi \times 2)$cm = $4(\frac{\pi}{2})$cm,....

$L_{13} = (\pi \times \frac{13}{2})$cm = $3(\frac{\pi}{2})$cm.

∴ total length of the spiral

$= L_1 + L_2 + L_3 + L_4 + ... +_{13}$

$=\{\frac{\pi}{2} + 2(\frac{\pi}{2}) + 3(\frac{\pi}{2}) + 4(\frac{\pi}{2}) + ... + 13(\frac{\pi}{2})\}$cm

$=\frac{\pi}{2}(1+2+3+4+...+13)$cm

$= \frac{\pi}{2} \times \frac{13}{2} \times (1+13)$cm

$=(\frac{1}{2} \times \frac{22}{2} \times \frac{13}{2} \times 14)$cm=143cm.

Hence, the required length of the spiral is 143 cm.

45. The given AP is $-\frac{4}{3}, -1, \frac{-2}{3}, \ldots, 4\frac{1}{3}$

First term a = $-\frac{4}{3}$

Common difference d = $-1 - (-\frac{4}{3}) = -1 + \frac{4}{3} = \frac{1}{3}$

Suppose there are n terms in the given AP. Then,

$a_n = 4\frac{1}{3}$

$\Rightarrow -\frac{4}{3} + (n-1) \times \left(\frac{1}{3}\right) = \frac{13}{3}$ $[a_n = a + (n - 1)d]$

$\Rightarrow \frac{1}{3}(n-1) = \frac{13}{3} + \frac{4}{3} = \frac{17}{3}$

$\Rightarrow$ n - 1 = 17

$\Rightarrow$ n = 17 + 1 = 18

Thus, the given AP contains 18 terms. So, there are two middle terms in the given AP. The middle terms of the given AP are

(18/2)th terms and (18/2+1)th term i.e. 9th term and 10th term.

Sum of the middle most terms of the given AP.

= 9th term + 10th term

$= \left[-\frac{4}{3} + (9-1) \times \frac{1}{3}\right] + \left[-\frac{4}{3} + (10-1) \times \frac{1}{3}\right]$

$= -\frac{4}{3} + \frac{8}{3} - \frac{4}{3} + 3$

= 3

Hence, the sum of the middle most terms of the give A.P. is 3.

46. Let there be n terms in the given A.P.

We have, First term = a, second term = b

∴ Common difference d = b - a

It is given that the last term is c i.e. nth term a_n = c.

a_n = a + (n - 1)d

∴ c = a + (n - 1)d

⇒ c = a + (n - 1) (b - a)

⇒ (c - a) = (n - 1) (b - a)

⇒ $n - 1 = \frac{c-a}{b-a}$

⇒ $n = \frac{c-a}{b-a}$ + 1

⇒ $n = \frac{c-a+b-a}{b-a}$

⇒ $n = \frac{b+c-2a}{b-a}$ ...(1)

Let S_n be the sum of n terms of the A.P.

Then,

$S_n = \frac{n}{2}(a+c) = \frac{(b+c-2a)(a+c)}{2(b-a)}$ [Using (1)]

47. Given, in an arithmetic progression First term is "a "= 22

Common difference d = -4

Sum of n terms = 64

We know that S_n=$\frac{n}{2}[2a + (n-1)d]$

⇒ $\frac{n}{2}[2a + (n-1)d] = 64$

⇒ $\frac{n}{2}[2 \times 22 + (n-1)(-4)] = 64$

⇒ $\frac{n}{2}[44 - 4n + 4] = 64$

⇒ $\frac{n}{2}[48 - 4n] = 64$

⇒ 24n - $2n^2$ = 64

⇒ 12n - n^2 = 32 [Divide by 2]

⇒ n^2 - 12n + 32 = 0 (spliting middle term)

⇒ n^2 - 8n - 4n + 32 = 0

⇒ n(n - 8) - 4(n - 8) = 0

⇒ (n - 8)(n - 4) = 0

$\implies$ either n - 8 = 0 or n - 4 = 0

$\implies$ n = 8 or n = 4

Therefore, number of terms = 4 or 8

48. Let the numbers be (a - 3d), (a - d), (a + d), (a + 3d)

(a - 3d) + (a - d) + (a + d) + (a + 3d) = 50

⇒ 4a = 50

⇒ $a = \frac{50}{4} = \frac{25}{2}$

According to question,

(a + 3d) $= 4 \times (a - 3d)$

⇒ a + 3d = 4a - 12d

⇒ -3a = -15d

⇒ a = 5d

⇒ $\frac{25}{2} = 5d$

$\Rightarrow \frac{5}{2} = d$

Numbers be 5, 10, 15, 20

49. Given First term (a) = 8

and, n^{th} term (a_n) = 33

$\Rightarrow$ a + (n - 1)d = 33

$\Rightarrow$ 8 + (n - 1)d = 33

$\Rightarrow$ (n - 1)d = 33 - 8

$\Rightarrow$ (n - 1)d = 25 .....(i)

and, Sum of first n terms = 123

$\Rightarrow \frac{n}{2}[a + a_n] = 123$

$\Rightarrow \frac{n}{2}[8 + 33] = 123$

$\Rightarrow \frac{n}{2} \times 41 = 123$

$\Rightarrow n = \frac{123 \times 2}{41}$

$\Rightarrow$ n = 6

Put value of n in equation (i)

(6 - 1)d = 25

$\Rightarrow$ 5d = 25

$\Rightarrow d = \frac{25}{5} = 5$

50. All the two-digit natural numbers divisible by 4 are 12, 16, 20, 24, ..., 96

Here, a_1 = 12

a_2 = 16

a_3 = 20

a_4 = 24

$\therefore$ a_2 - a_1 = 16 - 12 = 4

a_3 - a_2 = 20 - 16 = 4

a_4 - a_3 = 24 - 20 = 4

$\because$ a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... (= 4 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 4.

Here, a = 12, d = 4, l = 96

Let the number of terms be n.

Then, l = a + (n - 1)d $\Rightarrow$ 96 = 12 + (n - 1)4

$\Rightarrow$ 96 - 12 = (n - 1) 4 $\Rightarrow$ 84 = (n - 1)4

$\Rightarrow$ (n - 1) 4 = 84 $\Rightarrow$ n - 1 $= \frac{84}{4}$

$\Rightarrow$ n - 1 = 21 $\Rightarrow$ n = 21 + 1 $\Rightarrow$ n = 22

$\therefore S_n = \frac{n}{2}(a + l) = \frac{22}{2}(12 + 96) = (11)(108) = 1188$

51. The first 15 multiples of 8 are 8, 16, 24, 32,....

Here, a_2 - a_4 = 16 - 8 = 8

a_3 - a_2 = 24 - 16 = 8

a_4 - a_3 = 32 - 24 = 8

i.e. a_{k-1} - a_k is the same everytime.

So, the above list of numbers forms an AP.

Here, a = 8

d = 8

n = 15

$\therefore$ Sum of first 15 multiples of 8 =S_{15}

$= \frac{15}{2}[2a + (15 - 1)d]$ .... $\because S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{15}{2}[2a + 14d]$

= 15 (a + 7d)

$= (15)(8 + 7 \times 8)$

= (15) (8 + 56)
= (15) (64)
= 960

52. S_1 = 1 + 2 + 3 + ...n

S_2 = 1 + 3 + 5 + ...upto n terms

S_3 = 1 + 4 + 7 + ...upto n terms

$S_n = \frac{n}{2}[2a + (n-1)d]$
$S_1 = \frac{n}{2}[2(1) + (n-1)1]$
$S_1 = \frac{n}{2}[2 + n - 1]$
or, $S_1 = \frac{n(n+1)}{2}$
Also, $S_2 = \frac{n}{2}[2 \times 1 + (n-1)2]$
$S_2 = \frac{n}{2}[2 + 2n - 2]$
$= \frac{n}{2}[2n] = n^2$
and $S_3 = \frac{n}{2}[2 \times 1 + (n-1)3]$
$S_3 = \frac{n}{2}[2 + 3n - 3]$
$= \frac{n(3n-1)}{2}$
Now, $S_1 + S_3 = \frac{n(n+1)}{2} + \frac{n(3n-1)}{2}$
$= \frac{n[n+1+3n-1]}{2}$
$= \frac{n[4n]}{2}$
= $2n^2$ = $2S_2$

Hence Proved.

53. The given A.P is: -12, - 9 , - 6 , ..., 21 is the given A.P, then

$a = -12, d = -9 - (-12) = 3$
$t_n = a + (n-1)d$ and t_n = 21
or, 21 = -12 +$(n-1) \times 3$
or, 21 + 12 =$(n-1) \times 3$
or, 33 = $(n-1) \times 3$
or, $(n-1)$ =11
or, n = 11 + 1
or, n = 12
Now, if 1 is added to each term we have a New A.P
-12 + 1, -a + 1, -6 + 1,..........., 21 + 1
-11, -8, -5,.........22
Now, we have $a = -11,\ d = 3$ and $l = 22$
and $n = 12$
$\therefore$ Sum of this obtained A.P.
or, S_{12} = $\frac{12}{2}$ [-11 + 22]
= 6 $\times$ 11 = 66
Hence the sum of new A.P = 66

54. Let saving in first year = a

Increased saving per year (d) = ₹100
Given, Saving in 10 years = ₹16500
$\Rightarrow S_{10}$ = 16500
$\Rightarrow \frac{10}{2}[2a + (10-1)d] = 16500$
$\Rightarrow$ 5[2a + (10 - 1)(100)] = 16500
$\Rightarrow$10a + 4500 = 16500
$\Rightarrow$10a = 16500 - 4500
$\Rightarrow$10a = 12000
$\Rightarrow a = \frac{12000}{10} = 1200$
Therefore, Saving in first year = ₹1200

55. Let first term of given A.P. be a and common difference be d also sum of first m and first n terms be S_m and S_n respectively

$\therefore \quad \frac{S_m}{S_n} = \frac{m^2}{n^2}$

or, $\frac{\frac{m}{2}[2a+(m-1)d]}{\frac{n}{2}[2a+(n-1)d]} = \frac{m^2}{n^2}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m^2}{n^2} \times \frac{n}{m}$

or, $\frac{2a+(m-1)d}{2a+(n-1)d} = \frac{m}{n}$

or, $m(2a+(n-1)d) = n[2a+(m-1)d]$

Now, $\frac{a_m}{a_n} = \frac{a+(m-1)d}{a+(n-1)d}$

$= \frac{a+(m-1)\times 2a}{a+(n-1)\times 2a}$

or, $= \frac{a+2ma-2a}{a+2na-2a}$

or, $= \frac{2ma-a}{2na-a}$

or, $= \frac{a(2m-1)}{a(2n-1)}$

or, $= \frac{(2m-1)}{(2n-1)}$

= $2m-1 : 2n-1$

The ratio of its m^{th} and n^{th} terms is $2m-1 : 2n-1$.

Hence proved

56. Given,$a = 100, d = -2, t_n = -10$

Using, $t_n = a + (n-1)d$

or, $-10 = 100 + (n-1)(-2)$

or, $-10 = 100 - 2n + 2$

or,$2n - 2 = 100 + 10$

or, $2n = 110 + 2$

or, $2n = 112$

or, $n = 56$

$\therefore$Here 56th term is - 10

$\therefore$Number of terms in A.P. are 56

$\therefore \quad S_n = \frac{n}{2}(a+l)$

$S_{56} = \frac{56}{2}(100-10)$

$= \frac{56}{2}(90)$

$= 56 \times 45$

or, S_n = 2520.

57. The general term of an AP is given by a_n=a+(n-1)d and $S_n=\frac{n}{2}$[2a+(n-1)d].

Now, d=-9-(-12)=3

So, 21=-12+(n-1)(3)

$\Rightarrow$33=3n-3

$\Rightarrow$36=3n

$\Rightarrow$n=12

If 1 is added to each term of this AP, then the AP becomes -11,-8,-5,...,20.

d=-8-(-11)=3

So,

$\Rightarrow S_{12}=\frac{12}{2}$[2(-11)+11(3)]

$\Rightarrow S_{12}$=6[-22+33]

$\Rightarrow S_{12}$=6(11)

$\Rightarrow S_{12}$=66.

58. According to the question,

for first AP - first term =1, common difference=1

$\therefore AP_1 = 1, 2, 3...$

$S_1 = \frac{n}{2}[2(1)+(n-1)1] = \frac{(n)(n+1)}{2}$

for first AP - first term =1, common difference=2

$\therefore AP_2 = 1, 3, 5\ldots$

$S_2 = \frac{n}{2}[2(1) + (n-1)2] = n^2$

for first AP - first term =1, common difference=3

$\therefore AP_3 = 1, 4, 7\ldots$

$S_3 = \frac{n}{2}[2(1) + (n-1)3] = \frac{(n)(3n-1)}{2}$

$S_3 + S_1 = \frac{(n)(3n-1)}{2} + \frac{(n)(n+1)}{2} = 2n^2 = 2S_2$

hence proved.

59. Here, a_n = 4

d = 2

S_n = -14

We know that

a_n = a + (n - 1)d

⇒ 4 = a + (n - 1)d

⇒ 4 = a + 2n - 2

⇒ 4 + 2 = a + 2n

⇒ 6 = a + 2n

⇒ a + 2n = 6 ...... (1)

Again, we know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow -14 = \frac{n}{2}[2a + (n-1)2]$

⇒ -14 = n[a + (n - 1)]

⇒ -14 = n (a + n - 1)

⇒ -14 = n (6 - n - 1) ....From (1), (a + 2n = 6 ⇒ a + n = 6 - n)

⇒ -14 = n(-n + 5)

⇒ -14 = $-n^2$ + 5n

⇒ n^2 - 7n + 2n - 14 = 0

⇒ n(n - 7) + 2(n - 7) = 0

⇒ (n - 7) (n + 2) = 0

⇒ n - 7 = 0 or n + 2 = 0

⇒ n = 7 or n = -2

⇒ n = - 2 is in admissible as n, being the number of terms, is a natural number.

∴ n = 7

Putting n = 7 in equation (1), we get

a + 2(7) = 6

⇒ a + 14 = 6

⇒ a = 6 - 14

⇒ a = -8

60. Here, a = 8

a_n = 62

S_n = 210

We know that

a_n = a + (n - 1)d

⇒ 62 = 8 + (n - 1)d

⇒ 62 - 8 = (n - 1)d

⇒ 54 = (n - 1)d

⇒ (n - 1)d = 54 .... (1)

Again we know that

$S_n = \frac{n}{2}[2n + (n-1)d]$

$\Rightarrow 210 = \frac{n}{2}[2(8) + (n-1)d]$

$\Rightarrow 210 = \frac{n}{2}[16 + (n-1)d]$

$\Rightarrow 210 = \frac{n}{2}[16 + 54]$ .... Using (1)
$\Rightarrow 210 = \frac{n}{2}(70)$
$\Rightarrow 210 = 35n$
$\Rightarrow n = \frac{210}{35}$
$\Rightarrow n = 6$
Putting n = 6 in equation (1), we get
(6 - 1)d = 54
$\Rightarrow 5d = 54$
$\Rightarrow d = \frac{54}{5}$

61. Here, according to question it is given that in an A.P., the sum of first n terms is $\frac{3n^2}{2} + \frac{5n}{2}$.
Let S_n denote the sum of n terms of an A.P. whose nth term is a_n.So we have,
$S_n = \frac{3n^2}{2} + \frac{5n}{2}$
$\Rightarrow \quad S_{n-1} = \frac{3}{2}(n-1)^2 + \frac{5}{2}(n-1)$ [Replacing n by (n-1)]
$\therefore \quad S_n - S_{n-1} = \left\{\frac{3n^2}{2} + \frac{5n}{2}\right\} - \left\{\frac{3}{2}(n-1)^2 + \frac{5}{2}(n-1)\right\}$
$\Rightarrow \quad a_n = \frac{3}{2}\{n^2 - (n-1)^2\} + \frac{5}{2}\{n - (n-1)\}$ [because,$a_n = S_n - S_{n-1}$]
$\Rightarrow \quad a_n = \frac{3}{2}(2n-1) + \frac{5}{2}$
$\Rightarrow \quad a_{25} = \frac{3}{2}(2 \times 25 - 1) + \frac{5}{2} = \frac{3}{2} \times 49 + \frac{5}{2} = 76$ [Replacing n by 25]
Hence 25th term is equal to 76

62. Sum of the first n terms is: $S_n = \frac{n}{2}[2a + (n-1)d]$
Now, $S_7 = 63$
or, $\frac{7}{2}[2a + 6d] = 63$
or, $7[2a + 6d] = 126$
or, $2a + 6d = 18$ ....(i)
Also, sum of next 7 terms is given,
$S_{14} = S_{\text{first }7} + S_{\text{next }7} = 63 + 161$
= 224
$\frac{14}{2}[2a + 13d]$ = 224
$14[2a + 13d]$ = 448
or, $2a + 13d = 32$ ....(ii)
On subtracting (i) from (ii), we get
(2a + 13d) - (2a + 6d) = 32 - 18
7d = 14
or, d = 2
Putting this value of d in (i), we get
2a + 6(2) = 18
2a + 12 = 18
2a = 18 - 12
a = 3
$t_n = a + (n - 1)d$
$t_{28} = 3 + 2\ (28 - 1)$
$t_{28} = 3 + 2 \times (27)$
= 57
Therefore, the 28th term is 57.

63. Given, a = 10, and $S_{14} = 1050$
Let the common difference of the A.P. be d
we know that $S_n = \frac{n}{2}[2a + (n-1)d]$
$\therefore S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d]$
1050 = 7(20 + 13d)
or 20 + 13d = $\frac{1050}{7}$
20 + 13d = 150

13d = 150 - 20

13d = 130

d = $\frac{130}{13}$

d = 10

Now, a_{21} = a + (n − 1)d

= 10 + (21 - 1) 10

= 10 + 20 $\times$ 10

= 10 + 190

= 210

Hence, a_{20} = 210

64. All natural numbers less than 200 which are divisible by 5 are

5, 10, 15, 20, ..., 195

Here, a_1 = 5

a_2 = 10

a_3 = 15

a_4 = 20

: :

$\therefore$ a_2 - a_1 = 10 - 5 = 5

a_3 - a_2 = 15 - 10 = 5

a_4 - a_3 = 20 - 15 = 5

$\because$ a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = ... (= 5 each)

$\therefore$ This sequence is an arithmetic progression whose common difference is 5.

Here, a = 5

d = 5

l = 195

Let the numbers of terms be n. Then,

l = a + (n - 1)d

$\Rightarrow$ 195 = 5 + (n - 1)d

$\Rightarrow$ (n - 1)5 = 190

$\Rightarrow$ n - 1 = 38

$\Rightarrow$ n = 39

$\therefore S_n = \frac{n}{2}(a + l)$

$= \left(\frac{39}{2}\right)(5 + 195)$

$= \left(\frac{39}{2}\right)(200)$

= (39) (100)

= 3900

65. Let first term be a and common difference be d

Given 5^{th} term = 30

$\Rightarrow$ a + (5 - 1)d = 30

$\Rightarrow$ a + 4d = 30 .... (i)

and, 12^{th} term = 65

$\Rightarrow$ a + (12 - 1)d = 65

$\Rightarrow$ a + 11d = 65 .... (ii)

Subtracting equation (i) from equation (ii)

a + 11d - a - 4d = 65 - 30

$\Rightarrow$ 7d = 35

$\Rightarrow d = \frac{35}{7} = 5$

Putting value of d in equation (i)

a + 4 $\times$ 5 = 30

$\Rightarrow$ a = 30 - 20 = 10

$\therefore$ Sum of first 20 terms $= \frac{n}{2}[2a + (n-1)d]$

$= \frac{20}{2}[2 \times 10 + (20-1) \times 5]$

= 10[20 + 95]

= 10 $\times$ 115

= 1150

66. Let 'a' be the first term and d be the common difference of the given AP. Then, using $S_n = \frac{n}{2} \cdot [2a+(n-1)d]$, we get,

$S_7 = \frac{7}{2}(2a + 6d)$

Since sum of the first 7 terms of AP is 63 . Therefore,

$S_7 = 63$

$\Rightarrow 7(a+3d)=63$

$\Rightarrow a+3d=9$.... (i)

Since sum of the first 7 terms of an AP is 63 and the sum of its next 7 terms is 161.

Therefore, the sum of first 14 terms = 63 +161 = 224.

$\therefore S_{14}=224 \Rightarrow \frac{14}{2}(2a+13d)=224$

$\Rightarrow 7(2a+13d)=224$

$\Rightarrow 2a+13d=32$....(ii)

Multiplying (i) by 2 and subtracting the result from (ii), we get 7d = 14 $\Rightarrow$ d = 2.

Putting d = 2 in (i), we get a=9-6=3.

Thus, a=3 and d=2.

$\Rightarrow$28th term of this AP is given by

$T_{28}=(a+27d)=(3+27\times 2)=57$.

Hence, the 28th term of the given AP is 57.

67. Suppose the work is completed in n days when the workers started dropping. Since 4 workers are dropped on every day except the first day. Therefore, the total number of workers who worked all the n days is the sum of n terms of an A.P. with first term 150 and common difference - 4

i.e. $\frac{n}{2}\{2 \times 150 + (n-1) \times -4\}$ = n(152 - 2n)

Had the workers not dropped then the work would have finished it in (n - 8) days with 150 workers working on each day.

Therefore, the total number of workers who would have worked all the n days is 150 (n - 8).

$\therefore$ n (152 - 2n) = 150 (n - 8)

$\Rightarrow 152n - 2n^2 = 150n - 1200$

$\Rightarrow 2n^2 - 2n - 1200 = 0$

$\Rightarrow n^2 - n - 600 = 0$

$\Rightarrow$ (n - 25) (n + 24) = 0 [$\because$ n > 0]

$\Rightarrow$ n = 25

Hence, the work is completed in 25 days.

68. Let r_1, r_2,.... be the radii of semicircles and l_1, l_2 .....be the lengths of circumferences of semi-circles, then

$l_1 = \pi r_1 = \pi(1) = \pi$cm

$l_2 = \pi r_2 = \pi(2) = 2\pi$cm

$l_3 = \pi r_3 = \pi(3) = 3\pi$cm

....

$l_{11} = \pi r_{11} = \pi(11) = 11\ \pi$cm

$\therefore$ Total length of spiral

$= l_1 + l_2 + \ldots + l_{11}$

$= \pi + 2\pi + 3\pi + \ldots + 11\pi$

$= \pi(1 + 2 + 3 + 4 \ldots + 11)$

$= \pi \times \frac{11 \times 12}{2}$

$= 66 \times 3.14$

$= 207.24cm$

69. The general term of an AP is given by $a_n=a+(n-1)d$ and $S_n=\frac{n}{2}[2a+(n-1)d]$.

Given that $a_2=14$ and $a_3=18$

So, $d=a_3-a_2=18-14=4$

Now, $a_2=14 \Rightarrow a+4=14 \Rightarrow a=10$

Also, $S_{51}=\frac{51}{2}[2(10)+(50)4]$

$\Rightarrow S_{51}=\frac{51}{2}[20+200]$

$\Rightarrow S_{51}=\frac{51}{2}[220]$

$\Rightarrow S_{51}=51\times 110$

$\Rightarrow S_{51}=5610$

70. For 1st AP, a = 5 and d = 36.

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow \quad S_n = \frac{n}{2}[2 \times 5 + (n - 1)36]$

$\Rightarrow S_n = n(18n - 13)$ ..(i)

For 2nd AP, A = 36 and D = 5.

$S_{2n} = \frac{2n}{2}[2A + (2n - 1)D]$

$\Rightarrow S_{2n} = n[2(36) + (2n - 1)5]$

$\Rightarrow S_{2n} = n(10n + 67)$ ..(ii)

ATQ., $S_n = S_{2n}$

$\Rightarrow n(18n - 13) = n(10n + 67)$

$\Rightarrow 18n^2 - 13n = 10n^2 + 67n$

$\Rightarrow 8n^2 - 80n = 0$

$\Rightarrow 8n(n - 10) = 0$

$\Rightarrow n = 0$ or $n = 10$

$\therefore n = 10$

71. Here, d = 5

$S_9 = 75$

We know that

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow S_9 = \frac{9}{2}[2a + (9-1)d]$

$\Rightarrow S_9 = \frac{9}{2}[2a + 8d]$

$\Rightarrow S_9 = 9\,[a + 4d]$

$\Rightarrow S_9 = 9\,[a + 4 \times 5]$

$\Rightarrow S_9 = 9[a + 20]$

$\Rightarrow 75 = 9a + 180$

$\Rightarrow 9a = 75 - 180$

$\Rightarrow 9a = -105$

$\Rightarrow a = -\frac{105}{9}$

$\Rightarrow a = -\frac{35}{3}$

Again, we know that

$a_n = a + (n - 1)d$

$\Rightarrow a_9 = a + (9 - 1)d$

$\Rightarrow a_9 = a + 8d$

$\Rightarrow a_9 = -\frac{35}{3} + 8(5)$

$\Rightarrow a_9 = -\frac{35}{3} + 40$

$\Rightarrow a_9 = \frac{-35+120}{3}$

$\Rightarrow a_9 = \frac{85}{3}$

72. Let 1st term of the AP be a and common difference be d.

Now $a_8 = 37$

$\Rightarrow a + 7d = 37$

$\Rightarrow a = 37 - 7d$ ..(i)

Also, $a_{15} = a_{12} + 15$

$\Rightarrow$ a + 14d = a + 11d + 15

$\Rightarrow$ 3d = 15 $\Rightarrow$ d = 5

Substituting in eq. (i), we get

a = 37 - 7(5) = 2

AP. is 2, 7, 12, 17, ...

$S_{15} = \frac{15}{2}[2 \times 2 + 14 \times 5]$

= 15(37) = 555

73. Given, 12^{th} term=213,

i.e. $a_{12} = 213$

$a_n = a + (n-1)d$

$\Rightarrow a + (12-1)d = 213$

$\Rightarrow a + 11d = 213 \ \ldots.(1)$

and $S_4 = 24$

$\Rightarrow \frac{4}{2}[2a + (4-1)d] = 24$

$\Rightarrow 2a + 3d = 12 \ \ldots..(2)$

On solving Eqs. (1) and (2), we get

$S_{10} = \frac{10}{2}\left[2 \times \left(\frac{-507}{19}\right) + (10-1) \times \frac{414}{19}\right] = \frac{10}{2}[\frac{-1014}{19} + 9 \times \frac{414}{19}] = \frac{10}{2}(\frac{-1014+3726}{19}) = \frac{27120}{38}$

74. Let the first term be a and common difference be d.

$a_n = a + (n-1)d$

a_4 = a + 3d = -15 .... (i)

a_9 = a + 8d = - 30 .... (ii)

Subtracting eqn (i) from eqn (ii), we get

(a + 8d)- (a + 3d) = -30 - (- 15)

a + 8d -a - 3d = -15

5d = -15

d = -$\frac{15}{5}$ = -3

Put d = -3 in (i) we get,

a + 3d = -15

a + 3(-3) = - 15

a - 9 = -15

a = -15 + 9

a = - 6

$S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{17} = \frac{17}{2}[2 \times (-6) + (17-1)(-3)]$

$= \frac{17}{2}[-12 + 16 \times (-3)]$

= $\frac{17}{2}$[-12 - 48]

= $\frac{17}{2}$[-60]

= $17 \times (-30)$

= -510

75. Let a, and A be the first terms and d and D be the common difference of two A.Ps

Then, according to the question,

$\frac{S_n}{S'_n} = \frac{\frac{n}{2}[2a+(n-1)d]}{\frac{n}{2}[2A+(n-1)D]} = \frac{7n+1}{4n+27}$

or, $\frac{2a+(n-1)d}{2A+(n-1)D} = \frac{7n+1}{4n+27}$

or,$\frac{a+\left(\frac{n-1}{2}\right)d}{A+\left(\frac{n-1}{2}\right)D} = \frac{7n+1}{4n+27}$

Putting, $\frac{n-1}{2} = m - 1$

$n - 1 = 2m - 2$

$n = 2m - 2 + 1$

or, $n = 2m - 1$

$$\frac{a+(m-1)d}{A+(m-1)D} = \frac{7(2m-1)+1}{4(2m-1)+27}$$
$$\frac{a+(m-1)d}{A+(m-1)D} = \frac{14m-7+1}{8m-4+27}$$
$$\frac{a+(m-1)d}{A+(m-1)D} = \frac{14m-6}{8m+23}$$
Hence, $\frac{a_m}{A_m} = \frac{14m-6}{8m+23}$