Solution

ARITHMETIC PROGRESSION WS 8

Class 10 - Mathematics

Section A

1. First even natural numbers divisible by 5 are:
   10,20,30,...................................100 terms
   This is an AP in which a = 10, d = (20 – 10) = 10 and n = 100
   The sum of n terms of an AP is given by
   $S_n = \frac{n}{2}[2a+(n-1)d]$
   Now on substitution of values of a and d we get
   $S_{100} = (\frac{100}{2}) \times [\ 2 \times 10+(100-1) \times 10\ ]$
   $= 50 \times [20+990]=50 \times 1010=50500$
   Hence, the sum of the first 100 even natural numbers which are divisible by 5 is 50500.
2. The A.P as per the given condition is: 12, 17, ... 97
   Clearly, a = 12, and d = 5
   Now, $a_n = a + (n - 1)d$
   $\Rightarrow 97 = 12 + (n - 1)5$
   $\Rightarrow \frac{85}{5} = n - 1$
   $\Rightarrow n = 18$
   Now, sum of an A.P is given by,
   $S_n = \frac{n}{2}(a + l)$, l is last term
   $\Rightarrow S_{18} = \frac{18}{2}(12 + 97)$
   $\Rightarrow S_{18} = 981$
3. Given,
   n = 9
   and, nth term $(a_n) = 28$
   and, $S_n = 144$
   $\Rightarrow \quad \frac{n}{2}[a + a_n] = 144$
   $\Rightarrow \quad \frac{9}{2}[a + 28] = 144$
   $\Rightarrow \quad a + 28 = \frac{144 \times 2}{9}$
   $\Rightarrow a + 28 = 32$
   $\Rightarrow a = 4$
4. In the given AP
   a=2, l =29 and $S_n=155$
   We know that
   Sum of n terms = $S_n=\frac{n}{2}(a+l)=155$
   $\therefore \frac{n}{2}(2+29)=155$
   $n = \frac{155 \times 2}{31} = 10$
   Now $l=a+(n-1)d$
   or 29=2+(10-1)d
   29=2+9d
   29-2=9d
   27 = 9d
   d = 3
5. According to the question,we have to find the sum of the first 24 terms of the AP .
   Let 1st term be a and common difference be d.
   Now, $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 300$.
   $\Rightarrow a + (a + 4d) + (a + 9d) + (a + 14d) + (a + 19d) + (a + 23d) = 300$
   $\Rightarrow 6a + 69d = 300$

$\Rightarrow$ 3(2a + 23d) = 300
$\Rightarrow$ 2a + 23d = 100 ....(1)
Now,we have, $S_{24} = \frac{24}{2}[2a + (24 - 1)d]$
= 12[2a + 23d]
= 12(100) = 1200 [using (1)]

6. Here it is given that $S_n = 3n^2 + 5n$

   $S_{n-1} = 3(n - 1)^2 + 5(n - 1)$
   $= 3(n^2 - 2n + 1) + 5n - 5$
   $= 3n^2 - 6n + 3 + 5n - 5$
   $= 3n^2 - n - 2$
   $S_n - S_{n-1} = 3n^2 + 5n - (3n^2 - n - 2)$
   $\Rightarrow (a_1 + a_2 + a_3 + - - - + a_{n-1} + a_n) - (a_1 + a_2 + a_3 + - - - + a_{n-1}) = 6n + 2$
   $\Rightarrow a_n = 6n + 2$
   Then
   $a_k = 6k + 2 = 164$
   6k = 164 - 2 = 162
   k = 27
7. According to the question,we have,
   $b_n = 5 + 2n$
   Put n = 1,
   $b_1 = 5 + 2\times 1 = 5 + 2 = 7$
   $b_2 = 5 + 2\times 2 = 5 + 4 = 9$
   Common difference(d)= 9 - 7 = 2
   $S_n = \frac{n}{2}[a + (n - 1)d]$.
   Sum of 15 terms, $S_{15} = \frac{15}{2}[2 \times 7 + (15 - 1) \times 2]$
   $= \frac{15}{2}[14 + 28]$
   $= \frac{15}{2} \times 42$
   = 315
   Therefore, the sum of the first 15 terms of sequences having n^{th} term as $b_n = 5 + 2n$ is 315.
8. We have, $a_n = 5 - 2n$
   $\therefore a_1 = 5 - 2 = 3$
   and $a_2 = 5 - 4 = 1$
   $\therefore$ d(common difference) = 1 - 3 = -2
   Now, $S_{20} = \frac{20}{2}[2 \times 3 + (20 - 1)(-2)]$ ...$[\because S_n = \frac{n}{2}[2a + (n - 1)d]] = 10[6 - 19 \times 2]$
   =10[6 - 38]
   $= 10 \times (-32)$
   = -320
9. According to question we are given that the nth term of the given arithmetic progression is given by, $T_n = (5n - 1)$.
   Suppose a, be the first term and d, be the common difference of this AP. Then,
   $a = T_1 = (5\times 1 - 1) = 4$, $T_2 = (5\times 2 - 1) = 9$.
   $\therefore d = (T_2 - T_1) = (9 - 4) = 5$.
   $\therefore$ sum of first n terms = $\frac{n}{2}\cdot\{2a + (n - 1)d\}$
   $= \frac{n}{2}\times\{2\times 4 + (n-1)\times 5\} = \frac{1}{2}n(5n + 3)$.
   sum of first 20 terms = $\frac{1}{2}\times 20\times(5\times 20+3)=1030$, Therefore, Sn = $\frac{1}{2}n(5n+3)$ and $S_{20} = 1030$.Hence sum of first 20 terms=1030
10. Here, $a = \frac{a-b}{a+b}$, $d = \frac{3a-2b}{a+b} - \frac{a-b}{a+b} = \frac{2a-b}{a+b}$
    and n = 11
    $S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{11} = \frac{11}{2}\left[2\left(\frac{a-b}{a+b}\right) + (11-1)\left(\frac{2a-b}{a+b}\right)\right]$

$\Rightarrow S_{11} = \frac{11}{2} \times 2\left[\frac{a-b}{a+b} + \frac{5(2a-b)}{a+b}\right]$

$\Rightarrow S_{11} = 11\left[\frac{a-b+10a-5b}{a+b}\right]$

$\Rightarrow S_{11} = 11\left[\frac{11a-6b}{a+b}\right]$

11. Given: $S_n = 3n^2 + 5n$

    $\therefore S_1 = 3(1)^2 + 5(1) = 8$

    $\Rightarrow a_1 = 8$ ..(i)

    $S_2 = 3(2)^2 + 5(2) = 22$

    $\Rightarrow a_1 + a_2 = 22$

    $\Rightarrow 8 + a_2 = 22$ [using (i)]

    $\Rightarrow a_2 = 14$

    $d = a_2 - a_1 = 14 - 8 = 6$

    AP is 8, 14, 20, 26, ...

    Now, $a_{16} = a + 15d$

    $= 8 + 15(6) = 98$

12. Given series: 12, 18, 24, 30, 36, ...

    This is an Arithmetic Progression.

    $a_1 = 12$

    common difference = 6

    $\therefore S_{12} = \frac{n}{2}(2 \times a_1 + (n - 1) \times d)$

    $= \frac{n}{2}(2 \times 12 + (12 - 1) \times 6)$

    $= 540$

13. 1st three digit number leaving remainder 2 when divided by 3 is 101.

    $\therefore$ Required numbers are 101, 104, 107, ...

    last such no. = 999 - 1 = 998

    These numbers are in AP.

    Here a = 101, d = 104 - 101 = 3

    $a_n = 998$

    $\Rightarrow a + (n - 1)d = 998$

    $\Rightarrow 101 + (n - 1)3 = 998$

    $\Rightarrow (n - 1)3 = 897$

    $\Rightarrow n - 1 = 299 \Rightarrow n = 300$

    Now, $S_n = \frac{n}{2}(a + l)$

    $= \frac{300}{2}(101 + 998)$

    $= \frac{300}{2}(1099)$

    $= 164850$

    Therefore, the sum of all three-digit numbers each of which leaves the remainder 2, when divided by 3 is 164850.

14. According to question, the sum of first n terms of the AP,

    $S_n = 3n^2 - 4n$.....(i)

    We know that, $a_n = S_n - S_{n-1}$

    Now,

    Replace n by (n - 1) in (i), we get,

    $S_{n-1} = 3(n - 1)^2 - 4(n - 1)$

    $S_{n-1} = 3(n^2-2n+1) - 4n + 4$

    $S_{n-1} = 3n^2 - 6n + 3 - 4n + 4$

    $S_{n-1} = 3n^2 - 10n + 7$......(ii)

Now $a_n = S_n - S_{n-1}$

$\therefore a_n = (3n^2 - 4n) - (3n^2 - 10n + 7)$ [from (i) and (ii)]

$\Rightarrow a_n = 3n^2 - 4n - 3n^2 + 10n - 7$

$\Rightarrow a_n = 6n - 7$

Hence the required nth term of the Arithmetic progression i.e $a_n = 6n - 7$

15. The first 40 positive integers divisible by 5 are 5, 10, 15, 20.......
Here, a = 5
d = 5
n = 40
$\therefore$ Sum of the first 40 positive integers = S_{40}
$= \frac{40}{2}[2a + (40 - 1)d]$ ....$\because S_n = \frac{n}{2}[2a + (n - 1)d]$
= 20[2a + 39d]
$= (20)[2 \times 5 + 39 \times 5]$
= (20)(205)
= 4100
16. Given A.P. 8,10,12,14,.., 126.
Here, a = 2, d = 2 and l = 126
Now, 10th term from end
= l - 9d
= 126 - 9$\times$2
= 126 - 18
= 108
Now, for finding sum of last ten terms, we have
a = 108, n = 10 and l = 126
$\therefore S_{10} = \frac{10}{2}(108 + 126)$
= 5(234)
= 1170
Thus, the sum of last ten terms of the given A.P. is 1170.
17. Given, n^{th} term of an AP, $a_n = 7 - 3n$
$\therefore a = 7 - 3(1) = 4$
$a_{25} = 7 - 3(25) = -68$
Sum of n terms of an AP,
$S_n = \frac{n}{2}(a + l)$, where a is the first term and l is the last term ,here $l = a_{25}$
Clearly, sum of the first 25 terms, (S_{25})
$= \frac{25}{2}(a + a_{25})$
$= \frac{25}{2}[4 + (-68)]$
$= \frac{25}{2}(-64)$
$= 25 \times (-32)$
$= -800$
18. According to question we are given that
$a_n = 5 - 2n$, put n=1and 2 we get

$\Rightarrow a_1 = 5 - 2(1) = 3$

$\Rightarrow a_2 = 5 - 2(2) = 1$

$\Rightarrow d = 1 - 3 = -2$

$\therefore S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[2(3) + (n - 1)(-2)]$

$= \frac{n}{2}[6 - 2n + 2]$

$= \frac{n}{2}[8 - 2n] = 4n - n^2$

19. The consecutive numbers on the houses of a row are 1, 2, 3, ..., 49

Clearly this list of number forming an AP.

Here, a = 1

d = 2 - 1 = 1

$S_{x-1} = S_{49} - Sx$

$\Rightarrow \frac{x-1}{2}[2a+(x-1-1)d] - \frac{x}{2}[2a+(x-1)d]$

$\because S_n = \frac{n}{2}[2a+(n-1)d]$

$\Rightarrow \frac{x-1}{2}[2(1)+(x-2)(1)] = \frac{49}{2}[2(1)+(48)(1)] - \frac{x}{2}[2(1)+(x-1)(1)]$

$\Rightarrow \frac{x-1}{2}[x] = 1225 - \frac{x(x+1)}{2}$

$\Rightarrow \frac{(x-1)(x)}{2} + \frac{x(x+1)}{2} = 1225$

$\Rightarrow \frac{x}{2}(x-1+x+1) = 1225$

$\Rightarrow x^2 = 1225$

$\Rightarrow x = \sqrt{1225}$

$\Rightarrow x = 35$

Hence, the required value of x is 35.

20. Consider a and d as the first term and the common difference of an A.P. respectively.

n^th term of an A.P.,$a_n = a + (n–1)d$

Sum of n terms of an A.P. $S_n = \frac{n}{2}[2a+(n-1)d]$

Given that the sum of the first 10 terms is 210.

$\Rightarrow \frac{10}{2}\ [2a+9d] = 210$

$\Rightarrow 5[2a+9d] = 210$

$\Rightarrow 2a+9d = 42 ------------ (i)$

15 th term from the last = (50 – 15 + 1) th = 36 th term from the beginning

$\Rightarrow a_{36} = a + 35d$

Sum of last 15 terms = $\frac{15}{2}$(a + 35d + a + 49d)

$\Rightarrow$ $\frac{15}{2}$(2a + 84d) = 2565

$\Rightarrow a + 42d = 171 \ldots\ldots\ldots\ldots (ii)$

On solving (i) and (ii) we get a = 3 and d = 4

Hence, given A.P = 3, 7, 11......

21. The given AP is:

50,46,42,... to 10 terms.

here a= 50 and d=46-50 = -4

Number of terms(n) = 10

Hence sum of 10 terms of this AP

$\mathrm{S_n} = \frac{\mathrm{n}}{2} \times [2\mathrm{a} + (\mathrm{n}-1)\mathrm{d}]$

$\mathrm{S}_{10} = \frac{10}{2} \times [2 \times 50 + (10-1)(-4)]$

$= \frac{10}{2} \times [2 \times 50 + 9 \times (-4)]$

= 5[100 - 36]

$=5\times64=320$

Hence sum of this AP is 320.

22. We have a_n = 9 - 5n

Put n = 1, 2, 3, 4,.... in succession, we get

a_1 = 9 - 5(1) = 9 - 5 = 4

a_2 = 9 - 5(2) = 9 - 10 = - 1

a_3 = 9 - 5(3) = 9 - 15 = -6

a_4 = 9 - 5(4) = 9 - 20 = -11

: : :

$\therefore$ $a_2 - a_1$ = - 1 - 4 = - 5

$a_3 - a_2$ = -6 - (-1) = -6 + 1 = -5

$a_4 - a_5$ = -11 - (-6) = -11 + 6 = - 5 i.e. $a_{k+1} - a_k$ is the same everytime

So, $a_1, a_2, ..., an, ...$ form an AP

Here, $a = a_1 = 4$

$a = a_2 - a_1 = -5$

$\therefore$ Sum of the first 15 terms = S_{15}

$= \frac{15}{2}[2a + (n-1)d] \because S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{15}{2}[2 \times a + (15-1)d]$

$= \frac{15}{2}[2a + 14d]$

= 15(a + 7d)

= (15) [4 + 7(-5)]

= (15) (4 - 35)

= (15) (-31)

= -465

23. Here first term $a = 4 - \frac{1}{n}$

Common difference, d = $\left(4 - \frac{2}{n}\right) - \left(4 - \frac{1}{n}\right) = -\frac{1}{n}$

Sum of first n terms, $S_n = \frac{n}{2}(2a + (n-1)d)$

$S_n = \left(\frac{n}{2}\right)\left[2\left(4 - \frac{1}{n}\right) + \frac{(n-1)(-1)}{n}\right]$

$S_n = \left(\frac{n}{2}\right)\left[8 - \frac{2}{n} - 1 + \frac{1}{n}\right]$

$= \left(\frac{n}{2}\right)\left(7 - \frac{1}{n}\right)$

$= \left(\frac{n}{2}\right)\left[\frac{7n-1}{n}\right]$

$= \frac{7n-1}{2}$

24. Here, a = $\sqrt{2}, d = \sqrt{8} - \sqrt{2} = \sqrt{2}$

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$= \frac{n}{2}[2\sqrt{2} + (n-1)\sqrt{2}]$

$= \frac{n}{2}[\sqrt{2} + \sqrt{2}n]$

$= \frac{n(n+1)}{\sqrt{2}}$

25. Let the first term be a and the common difference be d,

$a_n = a + (n -1)d$

As per given condition

$a_4 = 11$

a + 3d = 11 ...(i)

And $a_5 + a_7 = 34$

a + 4d + a + 6d = 34

2a + 10d = 34

or, a + 5d = 17 ...(ii)

Solving equations (i) and (ii),

(a + 5d) - (a + 3d) = 17 -11

a + 5d - a - 3d = 6

2d = 6

d = 3

Common difference d = 3.

26. Given $S_n = 730$, l= 65, a = 8

$730 = \frac{n}{2}$ (8 + 65)

$730 = \frac{n}{2}$ (73)

$10 = \frac{n}{2}$

20 = n

Now 65 = l = a + (n - 1)d

65 - 8 = (20 - 1)d

65 - 8 = 19d

57 = 19d

d = $\frac{57}{19}$ = 3

The common difference is 3.

27. According to question the three-digit numbers which are divisible by 13 are 104, 117, 130, 143,....... 938.

This forms an AP in which a = 104, d = (117 – 104) = 13 and l = 938 (last term)

Let the number of terms be n

Then Tn = 938

$\Rightarrow$a+(n-1)d=988

$\Rightarrow$104+(n-1)$\times$13=988

$\Rightarrow$13n=897

$\Rightarrow$n=69

Therefore required sum=$\frac{n}{2}$(a+l)

=$\frac{69}{2}$[104+988]=69$\times$546=37674

Hence, the sum of all three digit numbers which are divisible by 13 is equal to 37674.

28. Multiple of 5 lying between 101 and 999 are 105,110,115,..., 995 which are in AP.

Here a = 105 and d = 5. where a is first term and d is common difference

a_n = a + (n - 1)d

$\Rightarrow$ 995 = 105 + (n - 1)5

$\Rightarrow$ 890 = 5n - 5

$\Rightarrow$ 895 = 5n

$\therefore$ n = 179

$\therefore$ S_n = $\frac{n}{2}[a+l]$ = $\frac{179}{2}$[105 + 995] = $\frac{179}{2}\times$1100 = 98450.

29. Here, $a_n = 3 - 2n$

Taking $n = 1, t_n = 3 - 2 = 1$

n =15,

t_{15} = $3 - 2 \times 15$

=$3 - 30$

= -27

S_n = $\frac{n}{2}(a + a_n)$

S_{15} = $\frac{n}{2}(a + a_{15})$

S_{15} = $\frac{15}{2}$[1 + (-27)]

= $\frac{15}{2}$[-26]

$= 15 \times (-13)$

= -195

30. Here, a = first term = 10

Let d be the common difference of the A.P.

we know that the sum of first n terms of an AP is given by:

$S_n = \frac{n}{2}[2a + (n-1)d]$

Given, S_{14} = 1505

$\therefore S_{14} = \frac{14}{2}[2 \times 10 + (14-1)d]$ = 1505

7[20 + 13d] = 1505

$20 + 13d = \frac{1505}{7} = 215$

13d = 195

$d = \frac{195}{13} = 15$

$\therefore$ 25^{th} term = a_{25} = a + (n - 1)d = 10 + 24 $\times$ 15 = 370

31. Let a be the first term and d be the common difference of the given A.P. Then,

.$S_6 = 42$

$\frac{6}{2}(2a + 5d) = 42$

$2a + 5d = 14 \ldots\ldots\ldots (i)$

It is given that

$a_{10} : a_{30}$ = 1: 3

$\Rightarrow \quad \frac{a+9d}{a+29d} = \frac{1}{3}$

$3a + 27d = a + 29d$

$2a = 2d$

$a = d.........(ii)$

From (i) and(ii) we get

a=d=2

Hence a_{13}=2+12×2= 26 and a_1 =2

32. let, the first term = a

common difference = d

as per question,

S_{10} = 4 (S_5)

5(2a+9d) = 4[2.5 (2a+4d)]

2a+9d = 2(2a+4d)

2a+9d = 4a+8d

d = 2a

a = $\frac{d}{2}$ = $\frac{6}{2} = 3$

33. The numbers lying between 10 and 300, which when divided by 4 leave a remainder 3 are

11, 15, 19...........,299

This is an A.P. with a = 11, d = 4 and l = 299

Let the number of terms be n.

then, a_n = 299

⇒ a + (n - 1)d = 299

⇒ 11 + (n - 1)4 = 299

⇒ (n - 1)4 = 288

⇒ n - 1 = 72

⇒ n = 73

Thus, the required number of terms are 73.

34. Natural numbers between 200 and 400 divisible by 7 are 203,210,217,...,399.Now first term a=203 and common difference d = 210-203 = 7

Here , T_n=399

⇒ a+(n-1)d=399

⇒ 203+(n-1)(7)=399

$\Longrightarrow$ (n-1)7=399-203

$\Longrightarrow$ 7n-7=196

$\Longrightarrow$ 7n=196+7

$\Longrightarrow$ 7n=203

$\Longrightarrow$ n=$\frac{203}{7}$

⇒ n=29

We know that, $S_n = \frac{n}{2}[2a + (n-1)d]$

Therefore ,$S_{29} = \frac{29}{2}[2 \times 203 + (29-1)7]$

$\Longrightarrow$ S_{29}=$\frac{29}{2}$[2×203+28×7]

= $\frac{29}{2}$[406+196]

= $\frac{29}{2}\times 602$

= 8729.

35. Let n number of terms are needed to make the sum - 55

Here, first term , a = - 15

common difference, d = - 13 + 15 = 2

By using the sum of n terms formula,

$S_n = \frac{n}{2}[2a + (n-1)d]$
$-55 = \frac{n}{2}[2(-15) + (n-1)2]$
- 110 = n (- 30 + 2n - 2)
- 110 = n(2n - 32)
$2n^2$ - 32n + 110 = 0
n^2 - 16n + 55 = 0
n^2 - 11n - 5n + 55 = 0
n(n - 11) - 5(n - 11) = 0
(n - 5)(n - 11) = 0
So n is either 5 or 11
Hence, either 5 or 11 terms are needed to make the sum - 55.

36. Let 'a' be first term and 'd' be common difference of an A.P
Given,
4^{th} term = - 15
a + 3d = - 15 [using a_n = a + (n - 1)d]
a = - 3d - 15 ...(1)
9^{th} term = - 30
a + 8d = - 30
- 3d - 15 + 8d = - 30 [using 1]
5d = - 15
d = - 3
Putting this value in (1) we get
a = - 3 (- 3) - 15
= 9 - 15 = - 6
Also we know,
Sum of n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$
$S_{17} = \frac{17}{2}[2(-6) + (17-1)(-3)]$
$= \frac{17}{2}[-12 - 48]$
$= \frac{17}{2}[-60]$
= 17 (- 30)
= - 510

37. a_1 = -30, a_2 = -24
d = a_2 - a_1 = -24 - (-30) = 6
$\therefore$ sum of first 30 terms is
S_{30} = $\frac{30}{2}$[2a + (30 - 1)]
= 15[2 $\times$ -30 + 29 $\times$ 6]
= 15[-60 + 174]
= 15 $\times$ 114
= 1710

38. According to question we are given that, a = 4, l = 49 and S_n = 265.we know that S_n = $\frac{n}{2}$(a+l)
$\therefore$ 265 = $\frac{n}{2}$(4 + 49) $\Rightarrow$ 530 = 53n $\Rightarrow$ n = 10
$\therefore l$ = a_{10} = a + 9d
$\Rightarrow$ 49 = 4 + 9d $\Rightarrow$ 9d = 45 $\Rightarrow$ d = 5
$\therefore$ Common difference of = 5.

39. S_n= $\frac{n}{2}$[2a + (n - 1)d]
S_7 = $\frac{7}{2}$(2a + 6d) = 49
or, $a + 3d = 7$ ....(i)
S_{17} = $\frac{17}{2}$(2a + 16d) = 289
or, $a + 8d = 17$ ....(ii)
On subtracting (i) from (ii), we get

or, 5d = 10 or, d = 2
Put d = 2 in (i)
a + 3d = 7
a + 2(3) = 7
a + 6 = 7
and a = 1
$S_n = \frac{n}{2}[2 \times 1 + (n-1)2]$
$= \frac{n}{2}[2 + 2n - 2]$
$= \frac{n}{2}[2n]$
$= n^2$
Hence, sum of n terms = n^2

40. Let d be the common difference of the AP.
Here, a = 10 and n = 14
Now,
S_{14} = 1505 (Given)
$\Rightarrow \frac{14}{2}[2 \times 10 + (14 - 1) \times d] = 1505$ {Sn = $\frac{n}{2}$[2a + (n - 1)d]}
$\Rightarrow$ 7(20 + 13d) = 1505
$\Rightarrow$ 20 + 13d = 215
$\Rightarrow$ 13d = 215 - 20 = 195
$\Rightarrow$ d = 15

41. We have First term = a
second term = b
Last term = c
Difference d = b − a
In an AP, nth term T_n = a + (n − 1)d
= a + (n − 1)(b − a)
c = a + (n − 1)(b − a)
$\therefore$ c − a = (n − 1)(b − a)
$\therefore$ n − 1 = (b − a)(c − a)
$\therefore n = \frac{c-a}{b-a} + 1$
Sum of n numbers = $\frac{n}{2}$ (first term + last term)
= $\frac{n}{2}$(a + c)
$= \frac{\left(\frac{c-a}{b-a}+1\right)(a+c)}{2}$
$= \frac{(c-a+b-a)(a+c)}{2(b-a)}$
$= \frac{(a+c)(b+c-2a)}{2(b-a)}$

42. Total loan = ₹ 1,18,000
First installment = ₹ 1000
Since he increases the installment by ₹ 100 every month.
$\therefore$ Monthly installments paid by Rohan are 1000, 1100, 1200, 1300, ... 30 tems
$\therefore$ a = 1,000, d = 100, n = 30
T_n = a + (n - 1)d
T_{30} = 1,000 + (30 - 1)100
= 1,000 + 2,900
= 3,900
So, amount paid by him in the 30^{th} installment = ₹ 3,900
and $S_n = \frac{n}{2}$[2a + (n - 1)d]
$S_{30} = \frac{30}{2}[2 \times 1,000 + (30 - 1)100]$
= 15[2,000 + 2,900]
= 15 $\times$ 4900

S_{30} = 73,500

So, total amount paid by Rohan in 30 installments = ₹ 73,500

Therefore amount left after the 30^{th} installment

= 1,18,000 - 73,500

= ₹ 44,500

Hence, he still has to pay ₹ 44500 after 30 installments.

43. Let S_1 be the sum of the first n even natural numbers.

Then, $S_1 = 2 + 4 + 6 + ... + 2n$

$\Rightarrow S_1 = \frac{n}{2}[2 \times 2 + (n - 1)2]$

$\Rightarrow S_1 = \frac{n}{2}[4 + 2n - 2] = n(n + 1)$ ....(i)

Let the S_2 be the sum of the first n odd natural numbers.

Then, $S_2 = \frac{n}{2}[2 \times 1 + (n - 1)2] = n^2$

From (i),we have, $S_1 = n^2(1 + \frac{1}{n}) = (1 + \frac{1}{n})n^2 = (1 + \frac{1}{n})S_2$

Therefore, the sum of first n even natural numbers is equal to $(1 + \frac{1}{n})$ times the sum of the first n odd natural numbers $= (1 + \frac{1}{n})S_2$

Section B

44. Each class has 3 section

class 1 plants = 3 trees

class 2 plants = 6 trees

class 3 plants = 9 trees

$\therefore$ 3, 6, 9, ...

The no of trees planted by each class is in AP.

$S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$S_5 = \frac{5}{2}\{2 \times 3 + (5 - 1)3\}$

$S_5 = \frac{5}{2}\{6 + 12\}$

$S_5 = \frac{5}{2} \times 18$

$S_5 = 45$

$\therefore$ class 1 to 5 students plant 45 trees.

45. $S_n = \frac{n}{2}\{2a + (n - 1)d\}$

$S_{12} = \frac{12}{2}\{2 \times 3 + (12 - 1)3\}$

$S_{12} = 6\{6 + 33\}$

$S_{12} = 6 \times 39$

$S_{12} = 234$

$\therefore$ total no of trees planted by school = 234

46. 30

47. $\because$ Class 12^{th} has 3 sections and each section plants 12 trees.

$\therefore$ total no of trees = 12×3

= 36 trees.

48. The distance covered by Dinesh to pick up the first flower plant and the second flower plant,

$= 2 \times 10 + 2 \times (10 + 5) = 20 + 30$

therefore, the distance covered for planting the first 5 plants

= 20 + 30 + 40 + ... 5 terms

This is in AP where the first term a = 20

and common difference d = 30 - 20 = 10

49. We know that a = 20, d = 10 and number of terms = n = 5 so,

$\mathrm{S_n} = \frac{\mathrm{n}}{2}[2\mathrm{a} + (\mathrm{n} - 1)\,\mathrm{d}]$

So, the sum of 5 terms

$\mathrm{S_5} = \frac{5}{2}[2 \times 20 + 4 \times 10] = \frac{5}{2} \times 80 = 200$ m

Hence, Dinesh will cover 200 m to plant the first 5 plants.

50. As a = 20, d = 10 and here n = 10

So, $S_{10} = \frac{10}{2}[2 \times 20 + 9 \times 10] = 5 \times 130$ = 650 m

So, hence Ramesh will cover 650 m to plant all 10 plants.

51. Total distance covered by Ramesh 650 m

Time = $\frac{\text{distance}}{\text{speed}} = \frac{650}{10}$ = 65 minutes

Time taken to plant all 10 plants = 15 $\times$ 10 = 150 minutes

Total time = 65 + 150 = 215 minutes = 3 hrs 35 minutes

52. a = 1000

d = 100

S_n = 1,18,000

t_{30} = a + 29 d

= 1000 + 29 $\times$ 100

= 1000 + 2900

t_{30} = 3900

i.e., he will pay ₹ 3900 in 30^{th} installment.

53. $S_n = \frac{n}{2}$ {2a + (n - 1)d}

$S_{30} = \frac{30}{2}$ {2 $\times$ 1000 + (30 - 1) $\times$ 100}

S_{30} = 15 {2000 + 2900}

S_{30} = 15 $\times$ 4900

S_{30} = 73,500

i.e., he will pay ₹ 73500 in 30 installments.

54. $S_n = \frac{n}{2}$ {a + l}

1,18,000 = $\frac{40}{2}$ {1000 + l}

1,18,000 = 20,000 + 20 l

98,000 = 20 l

l = 4900

i.e., the last installment will be of ₹ 4900.

55. t_{10} = a + 9d

= 2000 + 9 $\times$ 100

t_{10} = 2000 + 900

t_{10} = ₹ 2900

56. Distance travel by the competitor to pick up each potato form an AP

10, 16, 22 ...

57. $S_n = \frac{n}{2}$ {2a + (n - 1)d}

$S_{10} = \frac{10}{2}$ {2 $\times$10 + 9 $\times$ 6}

S_{10} = 5{20 + 54}

S_{10} = 5 $\times$ 74

S_{10} = 370 m

i.e., The competitor has to run 370 m.

58. $S_4 = \frac{4}{2}$ {2 $\times$ 10 + (4 - 1)6}

= 2 {20 + 18}

= 2 $\times$ 38

S_4 = 76

$\therefore$ Required distance = 370 - 76

= 294

59. t_n = a + (n - 1)d

t_5 = 10 + (5 - 1)6

$t_5 = 10 + 24$

$t_5 = 34$ m

60. Number of pots in the 10th row

$= a_{10} = a + 9d = 29$

61. $a_5 - a_2 = (a + 4d) - (a + d) = 3d = 9$

62. $S_n = 100 \Rightarrow \frac{n}{2}[2(2) + (n - 1)3 = 100$

$3n^2 + n - 200 = 0 \Rightarrow (3n + 25)(n - 8) = 0$

$\therefore n = 8$ ($n = -\frac{25}{3}$ rejected).

63. $S_{12} = \frac{12}{2}[2(2) + 11(3)]$

$= 222$

64. A.P. for the number of squares in each row is 1, 3, 5, 7, 9 …

65. A.P. for the number of triangles in each row is 2, 6, 10, 14 ...

66. Area of each square = $2 \times 2 = 4$ cm^2

Number of squares in 15 rows = $\frac{15}{2}(2 + 14 \times 2) = 225$

Shaded area = $225 \times 4 = 900$ cm^2

67. $S_n = \frac{n}{2}[4 + (n - 1)4] = 2n^2$

$\therefore S_{10} = 2 \times 10^2 = 200$

68. Child's Day wise are,

$\underbrace{5}_{1\text{ coin}}, \underbrace{10}_{2\text{ coins}}, \underbrace{15}_{3\text{ coins}}, \underbrace{20}_{4\text{ coins}}, \underbrace{25}_{5\text{ coins}}$ .... to $\underbrace{n\text{ days}}_{n\text{ coins}}$

We can have at most 190 coins

i.e., 1 + 2 + 3 + 4 + 5 + ... to n term = 190

$\Rightarrow \frac{n}{2}[2 \times 1 + (n - 1)1] = 190$

$\Rightarrow n(n + 1) = 380 \Rightarrow n^2 + n - 380 = 0$

$\Rightarrow (n + 20)(n - 19) = 0 \Rightarrow (n + 20)(n - 19) = 0$

$\Rightarrow n = -20$ or $n = 19 \Rightarrow n = -20$ or $n = 19$

But number of coins cannot be negative

$\therefore n = 19$ (rejecting $n = -20$)

So, number of days = 19

69. Total money she saved = 5 + 10 + 15 + 20 + ... = 5 + 10 + 15 + 20 + ... upto 19 terms

$= \frac{19}{2}[2 \times 5 + (19 - 1)5]$

$= \frac{19}{2}[100] = \frac{1900}{2} = 950$

and total money she shaved = ₹950

70. Money saved in 10 days

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{10} = \frac{10}{2}[2 \times 5 + (10 - 1) \times 5]$

$\Rightarrow S_{10} = 5[10 + 45\}]$

$\Rightarrow S_{10} = 275$

Money saved in 10 days = ₹275

71. Number of coins in piggy bank on 15th day

$S_n = \frac{n}{2}[2a + (n - 1)d]$

$\Rightarrow S_{15} = \frac{15}{2}[2 \times 5 + (15 - 1) \times 5]$

$\Rightarrow S_{15} = \frac{15}{2}$ [2 + 14]

$\Rightarrow S_{15} = 120$

So, there are 120 coins on 15th day.

72. Money saved on 1st day = ₹27.5

$\because$ Sehaj increases his saving by a fixed amount of ₹2.5

$\therefore$ His saving form an AP with a = 27.5 and d = 2.5

$\therefore$ Money saved on 10th day,

a_{10} = a + 9d = 27.5 + 9(2.5)

= 27.5 + 22.5 = ₹50

73. a_{25} = a = 24d

= 27.5 + 24(2.5)

= 27.5 + 60 = ₹ 87.5

74. Total amount saved by Sehaj in 30 days.

= $\frac{30}{2}[2 \times 27.5 + (30 - 1) \times 2.5]$

= 15(55 + 29(2.5)

= ₹1912.5

75. Let S_n = 387.5, a = 27.5 and d = 2.5

$S_n = \frac{n}{2}[2a + (n-1)d]$

$\Rightarrow 387.5 = \frac{n}{2}[2 \times 27.5 + (n-1)2.5]$

$\Rightarrow 387.5 = \frac{n}{2}[55 + (n-1) \times 2.5]$

$\Rightarrow$ 775 = 55n + 2.5n^2 - 2.5n

$\Rightarrow$ 25n^2 + 525n = 7750 = 0

$\Rightarrow$ n^2 + 21n - 310 = 0

$\Rightarrow$ (n + 31)(n - 10) = 0

$\Rightarrow$ n = -31 reject n = 10 accept

So in 10 years Sehaj saves ₹ 387.5.