## Solution

## COORDINATE GEOMETRY WS 1

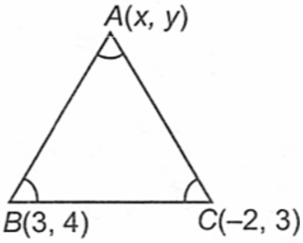
## Class 10 - Mathematics

### Section A

1.

**(d)** $\left(\frac{1+\sqrt{3}}{2}, \frac{7-5\sqrt{3}}{2}\right)$ or $\left(\frac{1-\sqrt{3}}{2}, \frac{7+5\sqrt{3}}{2}\right)$

**Explanation:** Since, $\triangle$ABC is an equilateral triangle.




$\therefore$ AB = AC $\Rightarrow$ $AB^2 = AC^2$

$\Rightarrow (x - 3)^2 + (y - 4)^2$

$= (x + 2)^2 + (y - 3)^2$

$\Rightarrow$ 5x + y - 6 = 0 ...(i)

Now, area of equilateral triangle

$= \frac{\sqrt{3}}{4} \times (\text{side})^2$

$\therefore$ Area of $\triangle$ABC = $\frac{\sqrt{3}}{4} \times (BC)^2 = \frac{13\sqrt{3}}{2}$ units

$\Rightarrow \frac{13\sqrt{3}}{2} = \frac{1}{2}|x(1) + 3(3 - y) - 2(y - 4)$

$\Rightarrow \pm 13\sqrt{3}$ = x + 9 - 3y - 2y + 8

$\Rightarrow$ x - 5y + 17 = $\pm 13\sqrt{3}$

$\Rightarrow$ x - 5y = $13\sqrt{3}$ - 17 ...(ii)

or x - 5y = -($13\sqrt{3}$ + 7) ...(iii)

Solving (i) and (ii), we get,

x = $\frac{1+\sqrt{3}}{2}$, y = $\frac{7-5\sqrt{3}}{2}$

Also, on solving (i) and (iii), we get

x = $\frac{1-\sqrt{3}}{2}$, y = $\frac{5\sqrt{3}+7}{2}$

2.

**(d)** 6

**Explanation:** Vertices of a square are A(5, p), B(1, 5), C(2, 1) and D(6, 2).

The diagonals bisect each other at O

O is the mid-point of AC and BD

O is mid-point of BD, then

CO-ordinates of O will be $\left(\frac{1+6}{2}, \frac{5+2}{2}\right)$

or $\left(\frac{7}{2}, \frac{7}{2}\right)$

$\because$ O is mid-point of AC also

$\therefore \frac{p+1}{2} = \frac{7}{2} \Rightarrow p + 1 = 7$

$\Rightarrow$ p = 7 - 1 = 6

3.

**(b)** $\sqrt{2a^2 + 2b^2}$

**Explanation:** distance between the point. (0, 0) and (a - b, a + b) is

$= \sqrt{(a-b-0)^2 + (a+b-0)^2}$

$= \sqrt{(a-b)^2 + (a+b)^2}$

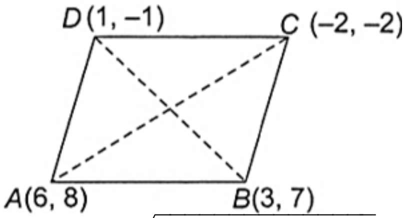
$= \sqrt{a^2 + b^2 - 2ab + a^2 + b^2 + 2ab}$

$= \sqrt{2\left(a^2 + b^2\right)} = \sqrt{2a^2 + 2b^2}$ units.

4.

**(d)** Parallelogram

**Explanation:** Let the points be A(6, 8), B(3, 7), C(-2, -2) and D(1, -1).




Now, AB = $\sqrt{(3-6)^2+(7-8)^2}$ = $\sqrt{10}$

BC = $\sqrt{(-2-3)^2+(-2-7)^2}$ = $\sqrt{106}$

CD = $\sqrt{(1+2)^2+(-1+2)^2}$ = $\sqrt{10}$

DA = $\sqrt{(6-1)^2+(8+1)^2}$ = $\sqrt{106}$

Also, AC = $\sqrt{(-8)^2+(-10)^2}$ = $\sqrt{64+100}$ = $\sqrt{164}$

BD = $\sqrt{(-2)^2+(-8)^2}$ = $\sqrt{4+64}$ = $\sqrt{68}$

Since, AB = DC and BC = DA and AC $\neq$ BD.

$\therefore$ ABCD is a parallelogram.

5.

**(d)** 7 units

**Explanation:** Given,

2x + 4 = 0 ...(i)

2x = -4

x = $-\frac{4}{2}$

x = -2

$\therefore$ coordinate is (-2, 0)

again, x - 5 = 0.

x = 5

coordinate is (5, 0)

distance between (-2, 0) and (5, 0) is

d = $\sqrt{(5+2)^2+(0-0)^2}$

= $\sqrt{7^2+0}$

= 7 units.

6.

**(c)** 3 units

**Explanation:** PQ = $\sqrt{\left(\frac{-2}{3}+\frac{11}{3}\right)^2+(5-5)^2}$

= $\sqrt{9+0}$

= 3 units

7.

**(b)** an isosceles triangle

**Explanation:** Let vertices of a triangle ABC are A (0, 3), B$(-4, 0)$ and C (4, 0).

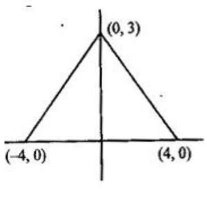
$\therefore$ AB = $\sqrt{(-4-0)^2+(0-3)^2}$

= $\sqrt{16+9}$ = $\sqrt{25}$ = 5 units

BC = $\sqrt{(4+4)^2+(0-0)^2}$ = $\sqrt{64+0}$ = $\sqrt{64}$ = 8 units

AC = $\sqrt{(4-0)^2+(0-3)^2}$ = $\sqrt{16+9}$ = $\sqrt{25}$ = 5 units

Since, two sides are equal, therefore, ABC is an isosceles triangle.



8.

**(b)** 5 units

**Explanation:** 5 units

9.

**(c)** 0

**Explanation:** Let the points A (2, 3), B(4, k) and C(6, -3) be collinear.

If the points are collinear then the area of triangle ABC formed by these three points is 0.

$\therefore$ ar($\triangle$ABC) = $\frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = 0$

$\Rightarrow 2 = \frac{1}{2}|(k + 3) +4(-3 - 3) + 6(3 - k)| = 0$

$\Rightarrow |2k + 6 - 24 + 18 - 6k| = 0$

$\Rightarrow |-4k| = 0$

$\Rightarrow k = 0$

10. **(a)** 7

**Explanation:** Distance of point (x, y) from x-axis is y-coordinate.

$\therefore$ Distance of P(-1, 7) from x-axis = 7 units

11.

**(d)** 10

**Explanation:** The distance of the point P( -6,8) from the origin (0, 0)

$= \sqrt{(-6)^2 + 8^2}$

$= \sqrt{36 + 64}$

$= \sqrt{100}$

= 10

12.

**(c)** -7

**Explanation:** Since y-coordinate of a point is called ordinate. Its distance from the x-axis measured parallel to the y-axis Therefore, the ordinate is -7.

13.

**(d)** ordinate

**Explanation:** The distance of a point from the x-axis is the y (vertical) coordinate of the point and is called ordinate.

14.

**(c)** A is true but R is false.

**Explanation:** Since A and B lie on the circle having centre O.

Therefore, OA = OB

$\sqrt{(4-2)^2 + (3-3)^2} = \sqrt{((x-2)^2 + (5-3)^2}$

$2 = \sqrt{(x-2)^2 + 4}$

$(x - 2)^2 + 4 = 4$

$(x - 2)^2 = 0$

x - 2 = 0

x = 2

A is true but R is false.

15. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Both A and R are true and R is the correct explanation of A.

16. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Both A and R are true and R is the correct explanation of A.

17. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Image of points of type (h, 0) is (–h, 0) only.

18.

**(d)** A is false but R is true.

**Explanation:** PQ = 10

$PQ^2 = 100$

$(10 - 2)^2 + (y + 3)^2 = 100$

$(y + 3)^2 = 100 - 64 = 36$

$y + 3 = \pm 6$

$y = -3 \pm 6$

$y = 3, -9$

19.

**(c)** A is true but R is false.

**Explanation:** Put (0, 2) in 3x + 2y = 4

We get LHS = RHS

Assertion is true.

Reason is also true. But it is not the correct explanation of Assertion (A).

Hence option B is the answer.

20.

**(d)** A is false but R is true.

**Explanation:** A is false but R is true.

21.

**(c)** A is true but R is false.

**Explanation:** Distance of (5, 12) from y-axis will be equal to x-coordinate to point. So, distance of (5, 12) from y-axis will be 5 units.

22. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Both A and R are true and R is the correct explanation of A.

23. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** The x coordinate of the point (0, 4) is zero.

Point (0, 4) lies on y-axis.

24. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Image of points of type (0, k) is (0, –k) only.

25.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:** It will be $\sqrt{(a-0)^2+(b-0)^2} = \sqrt{a^2+b^2}$

26. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Distance of point (5, 12) from 8 origin is given, d = $\sqrt{(5-0)^2+(12-0)^2}$

= $\sqrt{25+144} = \sqrt{169}$ = 13

27.

**(c)** A is true but R is false.

**Explanation:** Let, $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ are all rational coordinates,

$$\text{ar}(\Delta ABC) = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$= \frac{\sqrt{3}}{4}\left[(x_1 - x_2)^2 + (y_1 - y_2)^2\right]$

LHS = rational

RHS = irrational

Hence, $(x_1, y_1)$ $(x_2, y_2)$ and $(x_3, y_3)$ cannot be all rational.

28.

**(d)** A is false but R is true.

**Explanation:** Rule: Image of (x, y) under x-axis is given by (x, –y) and under y-axis given by (–x, y).

29.

**(d)** (x, y)

**Explanation:** AB = $\sqrt{(2x-0)^2+(0-2y)^2}$

= $\sqrt{4x^2+4y^2}$ = $2\sqrt{x^2+y^2}$ units

BO = $\sqrt{(0-2x)^2+(0-0)^2}$

= $\sqrt{4x^2}$ = 2x units

AO = $\sqrt{(0-0)^2+(0-2y)^2}$

= $\sqrt{4y^2}$ = $2y$ units

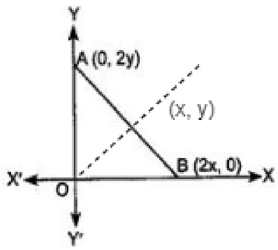
Now, $\mathrm{AB}^2=\mathrm{AO}^2+\mathrm{BO}^2 \Rightarrow \left(2\sqrt{x^2+y^2}\right)^2=(2x)^2+(2y)^2$

$\Rightarrow 4\left(x^2+y^2\right)=4\left(x^2+y^2\right)$

Therefore, triangle AOB is an isosceles right-angled triangle.

Since the coordinate of the point which is equidistant from the three vertices of a right-angled triangle is the coordinates of mid-point of its hypotenuse.

$\therefore$ Mid-point of AB = $\left(\frac{0+2x}{2}, \frac{2y+0}{2}\right)$ = (x,y)




30.

**(b)** 2nd

**Explanation:** Since x-coordinate is negative and y-coordinate is positive.

Therefore, the point (-3, 5) lies in II quadrant.

**Section B**

31. -2

Explanation:

Let the required point on y-axis be P(0, y) which is equidistant from A(5, -2) and B(-3, 2)

According to question, PA = PB

$\Rightarrow$ $PA^2 = PB^2$

$\Rightarrow$ $(5 - 0)^2 + (-2 - y)^2 = (-3 - 0)^2 + (2 - y)^2$ [By using distance formula]

$\Rightarrow$ $25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$

$\Rightarrow$ 29 - 13 = -8y

$\Rightarrow$ 16 = -8y

$\Rightarrow$ y = -2

$\therefore$ Required point is (0, -2)

32. 13

Explanation:

The given point is A(5, -12) and let O(0, 0) be the origin

Then, $AO=\sqrt{(5-0)^2+(-12-0)^2}$

$=\sqrt{5^2+(-12)^2}=\sqrt{25+144}=\sqrt{169}$

= 13 units

33. 4

Explanation:

Distance between (3, 0) and (0, y) is 5 units

$\therefore \sqrt{(0-3)^2+(y-0)^2}=5$

$\sqrt{9+y^2}=5$

$9 + y^2 = 25 \Rightarrow y^2=25-9=16=(\pm 4)^2$

$\therefore y=\pm 4$

But y is positive

$\therefore$ y = 4

34. 39

Explanation:

Required distance

$= \sqrt{(36-0)^2 + (15-0)^2}$

$= \sqrt{(36)^2 + (15)^2}$

$= \sqrt{1296 + 225} = \sqrt{1521}$

= 39

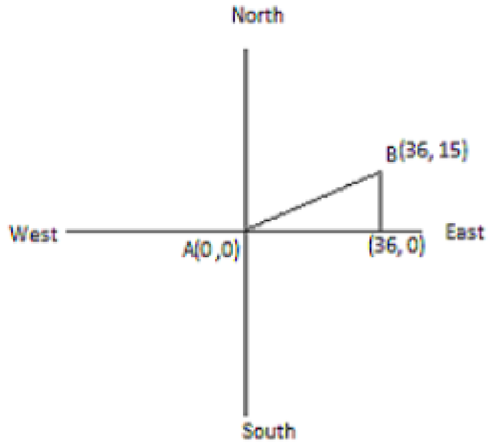
35. 39

Explanation:

Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

d = $\sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39$

Town B is located at 36 km east and 15 km north of town A. So, the location of town A and B can be shown as:




Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

To find the distance between them, we use Distance formula:

d = $\sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521}$ = 39 km

36. 5

Explanation:

Distance between the given points

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - 5\sin 60^\circ)^2 + (5\sin 30^\circ - 0)^2}$

$= \sqrt{\left(-5 \times \frac{\sqrt{3}}{2}\right)^2 + \left[5\left(\frac{1}{2}\right)\right]^2}$

$= \sqrt{\left(-5 \times \frac{\sqrt{3}}{2}\right)^2 + \left[5\left(\frac{1}{2}\right)\right]^2} \left\{\because \sin 60^\circ = \frac{\sqrt{3}}{2},\ \sin 30^\circ = \frac{1}{2}\ \right\}$

$= \sqrt{\frac{25\times3}{4} + \frac{25\times1}{4}} = \sqrt{\frac{75}{4} + \frac{25}{4}}$

$= \sqrt{\frac{100}{4}} = \sqrt{25} = 5 \text{ units}$

37. 10

Explanation:

The given points are A(9, 3) and B(15, 11)

Then, $x_1 = 9$, $y_1 = 3$, $x_2 = 15$, $y_2 = 11$

By distance formula,

$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(15-9)^2 + (11-3)^2} = \sqrt{6^2 + 8^2}$

$= \sqrt{36 + 64} = \sqrt{100}$ = 10 units

38. 2

Explanation:

Distance between the points $\left(\frac{-8}{5}, 2\right)$ and $\left(\frac{2}{5}, 2\right)$

$= \sqrt{\left(-\frac{8}{5} - \frac{2}{5}\right)^2 + (2-2)^2}$

$= \sqrt{\left(-\frac{10}{5}\right)^2 + 0}$

$= \sqrt{(-2)^2 + 0}$

= 2 units

39. 1

Explanation:

It is given that A(0, 2) is equidistant from the points B(3, p) and C(p, 5).

$\therefore$ AB = AC

$\Rightarrow \sqrt{(3-0)^2 + (p-2)^2} = \sqrt{(p-0)^2 + (5-2)^2}$ (Distance formula)

Squaring on both sides, we get

$9 + p^2 - 4p + 4 = p^2 + 9$

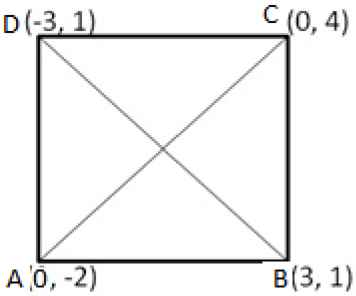
$\Rightarrow -4p + 4 = 0$

$\Rightarrow p = 1$

Thus, the values of p is 1.

**Section C**

40. Let A(0, -2), B(3,1), C(0,4) and D(-3,1) be the angular points of quad. ABCD




Now,

$AB = \sqrt{(3-0)^2 + (1+2)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\, units$

$BC = \sqrt{(0-3)^2 + (4-1)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\, unit$

$CD = \sqrt{(-3-0)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\, unit$

$DA = \sqrt{(0+3)^2 + (-2-1)^2} = \sqrt{(3)^2 + (-3)^2}\ \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\, unit$

Thus, AB = BC = CD = DA

$Diag.$ AC $= \sqrt{(0-0)^2 + (4+2)^2} = \sqrt{(6)^2} = 6\, units$

Diag. $BD = \sqrt{(-3-3)^2 + (1-1)^2} = \sqrt{(-6)^2} = 6 units$

$\therefore$ Diag. AC= Diag. BD

Thus, ABCD is a quadrilateral in which all sides are equal and the diagonals are equal

Hence, quadrilateral ABCD is a square

41. Let OABC be a rhombus such that OA is along x-axis. Let BL and CM be perpendiculars from B and C respectively on x-axis. Clearly, triangles ABL and OCM are congruent.

$\therefore$ OM = AL and CM = BL

Let the coordinates of A and C be ($x_1$, O) and ($x_2$, $y_2$) respectively. Then, OM = $x_2$ and OA = $x_1$

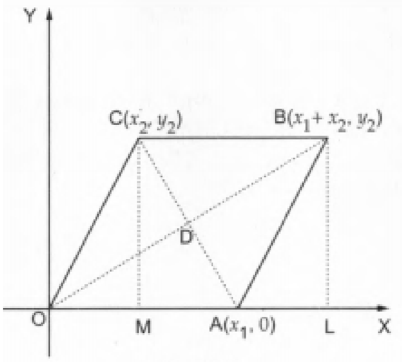
$\therefore$ OL = OA + AL = OA + OM = $x_1 + x_2$ and BL = CM = $y_2$

So, the coordinates of B are ($x_1 + x_2$, $y_2$).

Now OA = OC $\Rightarrow$ $OA^2 = OC^2 \Rightarrow x_1^2 = x_2^2 + y_2^2$ .... (i)

In order to prove that the diagonals OB and AC are mutually perpendicular, it is sufficient to show that $\angle ODA = \pi/2$.

Since the diagonals of a rhombus bisect each other. Therefore, coordinates of D are $\left(\frac{x_1+x_2}{2}, \frac{y_2}{2}\right)$




Now, $OD^2 = \left(\frac{x_1+x_2}{2} - 0\right)^2 + \left(\frac{y_2}{2} - 0\right)^2 = \left(\frac{x_1+x_2}{2}\right)^2 + \left(\frac{y_2}{2}\right)^2$

$AD^2 = \left(\frac{x_1+x_2}{2} - x_1\right)^2 + \left(\frac{y_2}{2} - 0\right)^2 \Rightarrow AD^2 = \left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2}{2}\right)^2$

and $OA^2 = (x_1 - 0)^2 + (0 - 0)^2 = x_1^2$

$\therefore \quad OD^2 + AD^2 = \left(\frac{x_1+x_2}{2}\right)^2 + \left(\frac{y_2}{2}\right)^2 + \left(\frac{x_2-x_1}{2}\right)^2 + \left(\frac{y_2}{y}\right)^2$

$\Rightarrow \quad OD^2 + AD^2 = \frac{1}{4}\left\{2x_1^2 + 2x_2^2 + 2y_2^2\right\}$

$\Rightarrow \quad OD^2 + AD^2 = \frac{1}{2}\left(x_1^2 + x_2^2 + y_2^2\right) = \frac{1}{2}\left(x_1^2 + x_1^2\right)$ [Using (i)]

$\Rightarrow \quad OD^2 + AD^2 = x_1^2 = OA^2$

$\therefore \Delta ODA$ is a right-angled triangle such that $\angle ODA = \pi/2$

Hence, the diagonals of a rhombus are at right angles.

42. According to the question, O(0, 0), A(3, $\sqrt{3}$) and B(3, $-\sqrt{3}$)

$OA = \sqrt{(3-0)^2 + (\sqrt{3}-0)^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}\, units$

$AB = \sqrt{(3-3)^2 + (-\sqrt{3}-\sqrt{3})^2} = \sqrt{0^2 + (-2\sqrt{3})^2} = \sqrt{12} = 2\sqrt{3}\, units$

$OB = \sqrt{(3-0)^2 + (-\sqrt{3}-0)^2} = \sqrt{(3)^2 + (-\sqrt{3})^2} = \sqrt{9+3} = \sqrt{12} = 2\sqrt{3}\, units$

$\therefore$ OA = AB = OB = $2\sqrt{3}\, units$

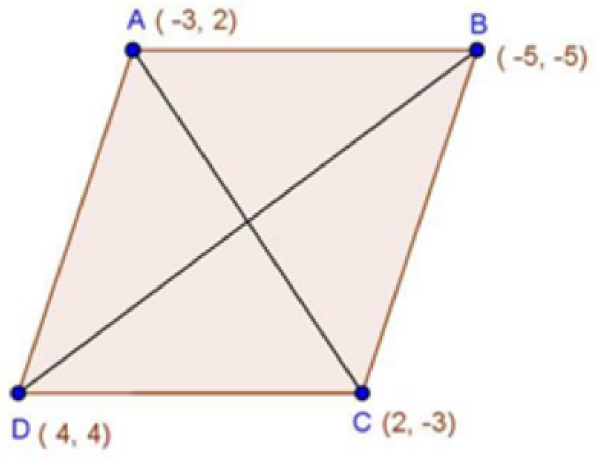
Hence, OAB is equilateral triangle and each of its sides is $2\sqrt{3}\, units$

$Area\, of \Delta ABC = \left[\frac{\sqrt{3}}{4} \times (\text{ side })^2\right]$

$= \left[\frac{\sqrt{3}}{4} \times (2\sqrt{3})^2\right]$

$= \left[\frac{\sqrt{3}}{4} \times 4 \times 3\right]$

$= 3\sqrt{3}\, sq.\ units$

43. 


Let A(-3, 2), B(-5, -5), C(2, -3) and D(4, 4) be the given points.

Coordinates of the mid-point of AC are $\left(\frac{-3+2}{2}, \frac{2-3}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

Coordinates of the mid-point of BD are $\left(\frac{-5+4}{2}, \frac{-5+4}{2}\right) = \left(\frac{-1}{2}, \frac{-1}{2}\right)$

Thus, AC and BD have the same mid-point

Hence, ABCD is a parallelogram.

Now, $AB = \sqrt{(-5+3)^2 + (-5-2)^2}$

$\Rightarrow \quad AB = \sqrt{4+49}$

$\Rightarrow \quad AB = \sqrt{53}$

$BC = \sqrt{(-5-2)^2 + (-5+3)^2}$

$\Rightarrow \quad BC = \sqrt{49+4}$

$\Rightarrow \quad BC = \sqrt{53}$

$\therefore$ AB = BC

So, ABCD is a parallelogram whose adjacent sides are equal.

Hence, ABCD is a rhombus.

44. Let B(-4, 3) and C(4, 3) be the given two vertices of an equilateral triangle.

Let A(x, y) be the third vertex.

Then, we have

AB = BC = AC

Consider AB = BC

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (-4 - x)^2 + (3 - y)^2 = (4 + 4)^2 + (3 - 3)^2$

$\Rightarrow 16 + x^2 + 8x + 9 + y^2 - 6y = 64$

$\Rightarrow x^2 + y^2 + 8x - 6y = 39$ .....(i)

Consider AB = AC

$\Rightarrow AB^2 = AC^2$

$\Rightarrow (-4 - x)^2 + (3 - y)^2 = (4 - x)^2 + (3 - y)^2$

$\Rightarrow 16 + x^2 + 8x = 16 + x^2 - 8x$

$\Rightarrow 16x = 0$

$\Rightarrow x = 0$

Consider BC = AC

$\Rightarrow BC^2 = AC^2$

$\Rightarrow (4 + 4)^2 + (3 - 3)^2 = (4 - x)^2 + (3 - y)^2$

$\Rightarrow 8^2 + 0 = (4 - 0)^2 + (3 - y)^2$

$\Rightarrow 64 = 16 + (3 - y)^2$

$(3 - y)^2 = 48$

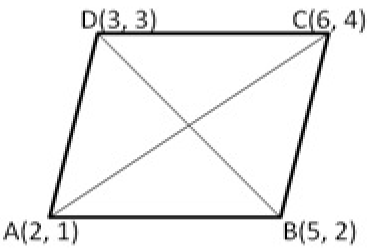
$3 - y = \pm 4\sqrt{3}$

$y = 3 \pm 4\sqrt{3}$

Thus, the coordinates of third vertex

i. when origin lies in the interior of a triangle = $(0, 3 - 4\sqrt{3})$
ii. when origin lies in the exterior of a triangle $= (0, 3 + 4\sqrt{3})$

45. Let A(2, 1), B(5, 2), C(6, 4) and D(3, 3) are the angular points of a parallelogram ABCD. Then




Now, distance between,

$AB = \sqrt{(5-2)^2 + (2-1)^2}$

$= \sqrt{(3)^2 + (1)^2}$

$= \sqrt{10} = \sqrt{10}\, units$

Distance between,

$BC = \sqrt{(6-5)^2 + (4-2)^2}$

$= \sqrt{(1)^2 + (2)^2}$

$= \sqrt{1+4} = \sqrt{5}\, units$

Distance between,

$DC = \sqrt{(6-3)^2 + (4-3)^2} = \sqrt{(3)^2 + (1)^2} = \sqrt{9+1} = \sqrt{10}\, units$

And distance between,

$AD = \sqrt{(3-2)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}\, units$

Therefore, AB = DC and AD = BC

Now,

$\text{D}iagonal\ AC = \sqrt{(6-2)^2 + (4-1)^2} = \sqrt{(4)^2 + (3)^2} = \sqrt{16+9}$

$= \sqrt{25} = 5\, units$

And,

$Diagonal\ BD = \sqrt{(3-5)^2 + (3-2)^2} = \sqrt{(-2)^2 + (1)^2} = \sqrt{5}\, unit\,.$

Since,

Diagonal AC $\neq$ Diagonal BD

Thus ABCD is not a rectangle but it is a parallelogram because its opposite sides are equal and diagonals are not equal.

46. According to the question,we are given that,

PA = QA

⇒ $PA^2 = QA^2$

⇒ $(3 – 2)^2 + (8 + 4)^2 = (–10 – 2)^2 + (y + 4)^2$

⇒ $1^2 + 12^2 = (–12)^2 + y^2 + 16 + 8y$

⇒ $y^2 + 8y + 16 – 1 = 0$

⇒ $y^2 + 8y + 15 = 0$

⇒ $y^2 + 5y + 3y + 15 = 0$

⇒ y(y + 5) + 3(y + 5) = 0

⇒ (y + 5) (y + 3) = 0

⇒ y + 5 = 0 or y + 3 = 0

⇒ y = –5 or y = –3

So, the co–ordinates are P(3, 8), $Q_1$(–10, –3), $Q_2$(–10, –5).

Now, $(PQ_1)^2 = (3 + 10)^2 + (8 + 3)^2 = 13^2 + 11^2$

⇒ $(PQ_1)^2 = 169 + 121$

⇒ $PQ_1 = \sqrt{290}$ units

and $(PQ_2)^2 = (3 + 10)^2 + (8 + 5)^2 = 13^2 + 13^2$

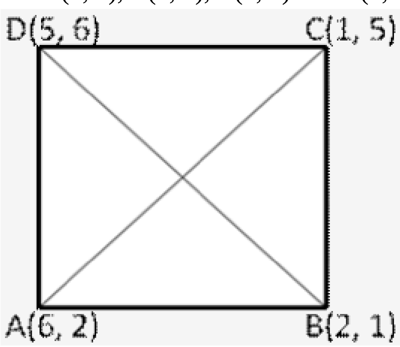
$= 13^2[1 + 1]$

⇒ $(PQ_2)^2 = 13^2 \times 2$

⇒ $PQ_2 = 13\sqrt{2}$ units

Hence, y = –3, –5 and PQ = $\sqrt{290}$ units and $13\sqrt{2}$units.

47. Let A(6, 2), B(2, 1), C(1, 5) and D(5, 6) be the angular points of quadrilateral ABCD. Join AC and BD




Now,

$AB = \sqrt{(2-6)^2 + (1-2)^2} = \sqrt{(-4)^2 + (-1)^2} = \sqrt{16+1} = \sqrt{17}\, units$

$BC = \sqrt{(1-2)^2 + (5-1)^2} = \sqrt{(-1)^2 + (4)^2} = \sqrt{1+16} = \sqrt{17}\, units$

$CD = \sqrt{(5-1)^2 + (6-5)^2} = \sqrt{(4)^2 + (1)^2} = \sqrt{16+1} = \sqrt{17}\, units$

$DA = \sqrt{(6-5)^2 + (2-6)^2} = \sqrt{(1)^2 + (-4)^2} = \sqrt{1+16} = \sqrt{17}\, units$

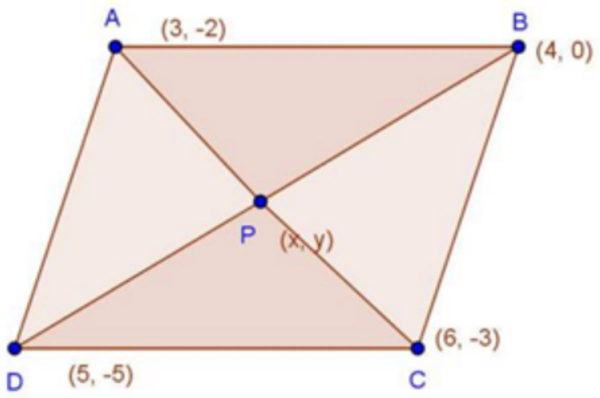
thus, AB = BC = CD = DA

$Diagonal\ AC = \sqrt{(1-6)^2 + (5-2)^2} = \sqrt{(-5)^2 + (3)^2} = \sqrt{25+9} = \sqrt{34}\, units$

$Diagonal\ BD = \sqrt{(5-2)^2 + (6-1)^2} = \sqrt{(3)^2 + (5)^2} = \sqrt{9+25} = \sqrt{34}\, units$

$\therefore$ Diagonal $AC =$ Diagonal $BD$

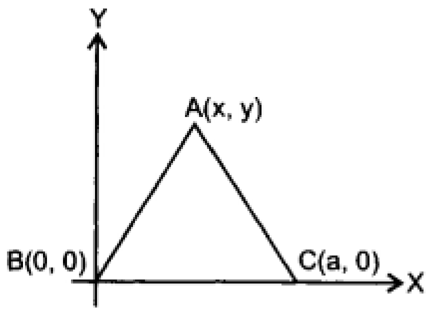
Thus, ABCD is a quadrilateral in which all sides are equal and the diagonal are equal.
Hence, quadrilateral ABCD is a square.

48. 


Let P(x, y) be the point of intersection of diagonals AC and BD of ABCD.
$x = \frac{3+6}{2} = \frac{9}{2}$
$y = \frac{-2-3}{2} = \frac{-5}{2}$
Mid-point of $AC = \left(\frac{9}{2}, \frac{-5}{2}\right)$
Again,
$x = \frac{5+4}{2} = \frac{9}{2}$
$y = \frac{-5+0}{2} = \frac{-5}{2}$
Mid-point of $BD = \left(\frac{9}{2}, -\frac{5}{2}\right)$
Here mid-point of AC = mid point of BD
i.e., diagonals AC and BD bisect each other.
We know that diagonals of a parallelogram bisect each other
$\therefore$ ABCD is a parallelogram.

49. Let the point of x-axis be P(x, 0)
Given A(2, -5) and B(-2, 9) are equidistant from P
That is PA = PB
Hence $PA^2 = PB^2 \rightarrow (1)$
Distance between two points is $\sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2]}$
PA = $\sqrt{[(2 - x)^2 + (-5 - 0)^2]}$
$PA^2 = 4 - 4x + x^2 + 25$
$= x^2 - 4x + 29$
Similarly, $PB^2 = x^2 + 4x + 85$
Equation (1) becomes
$x^2 - 4x + 29 = x^2 + 4x + 85$
$- 8x = 56$
$x = -7$
Hence the point on x-axis is (-7, 0)

50. Given: equilateral triangle of side a units.
As B is at origin, therefore, coordinates of B are (0,0).
As BC = a and C lies on x-axis, coordinates of C are (a, 0).
Let coordinates of A (x, y).



As, AB = BC = AC

$\sqrt{(x-0)^2+(y-0)^2}$ = a = $\sqrt{(x-a)^2+(y-0)^2}$

Squaring, we get

$x^2 + y^2 = a^2 = x^2 + a^2 - 2ax + y^2$

$\Rightarrow x^2 + y^2 = x^2 + y^2 + a^2 - 2ax$

$\Rightarrow 2ax - a^2 = 0$

$\Rightarrow$ a = 0 or 2x - a = 0, but $a \neq 0$

$\therefore$ x = $\frac{a}{2}$.

Substituting x = $\frac{a}{2}$ in (i), we get

$\left(\frac{a}{2}\right)^2 + y^2 = a^2$

$\Rightarrow y^2 = a^2 - \frac{a^2}{4} = \frac{3a^2}{4}$

$\Rightarrow$ y = $\frac{\sqrt{3}a}{2}$

$\therefore$ Coordinates of A are $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$

$\therefore$ Coordinates of vertices are A $\left(\frac{a}{2}, \frac{\sqrt{3}a}{2}\right)$, B(0, 0) and C(a, 0).

51. Let B(-4, 3) and C(4, 3) be the given two vertices of an equilateral triangle.

Let A(x, y) be the third vertex.

Then, we have

AB = BC = AC

Consider AB = BC

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (-4 - x)^2 + (3 - y)^2 = (4 + 4)^2 + (3 - 3)^2$

$\Rightarrow 16 + x^2 + 8x + 9 + y^2 - 6y = 64$

$\Rightarrow x^2 + y^2 + 8x - 6y = 39$ .....(i)

Consider AB = AC

$\Rightarrow AB^2 = AC^2$

$\Rightarrow (-4 - x)^2 + (3 - y)^2 = (4 - x)^2 + (3 - y)^2$

$\Rightarrow 16 + x^2 + 8x = 16 + x^2 - 8x$

$\Rightarrow$ 16x = 0

$\Rightarrow$ x = 0

Consider BC = AC

$\Rightarrow BC^2 = AC^2$

$\Rightarrow (4 + 4)^2 + (3 - 3)^2 = (4 - x)^2 + (3 - y)^2$

$\Rightarrow 8^2 + 0 = (4 - 0)^2 + (3 - y)^2$

$\Rightarrow 64 = 16 + (3 - y)^2$

$(3 - y)^2 = 48$

3 - y = $\pm 4\sqrt{3}$

y = 3 $\pm 4\sqrt{3}$

Thus, the coordinates of third vertex when origin lies in the interior of a triangle = $(0, 3-4\sqrt{3})$

52. We have P(3, a) and Q(4,1)

Here,

$x_1 = 3, y_1 = a$

$x_2 = 4, y_2 = 1$

PQ = $\sqrt{10}$

$PQ = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\Rightarrow \quad \sqrt{10} = \sqrt{(4-3)^2+(1-a)^2}$

$\Rightarrow \quad \sqrt{10} = \sqrt{(1)^2+(1-a)^2}$

$\Rightarrow \quad \sqrt{10} = \sqrt{1+1+a^2-2a}$

$\Rightarrow \quad \sqrt{10} = \sqrt{2+a^2-2a}$

Squaring both sides

$\Rightarrow (\sqrt{10})^2 = \left(\sqrt{2+a^2-2a}\right)^2$

$\Rightarrow$ 10 = 2 + $a^2$ - 2a

$\Rightarrow a^2 - 2a + 2 - 10 = 0$

$\Rightarrow a^2 - 2a - 8 = 0$

Splitting the middle term.

$\Rightarrow$ $a^2$ - 4a + 2a - 8 = 0

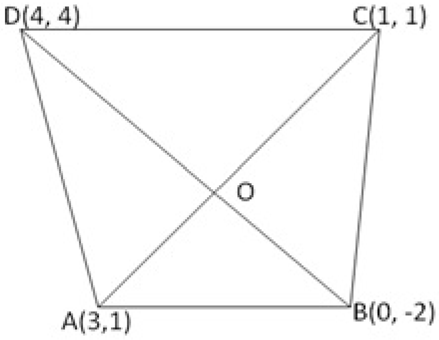
$\Rightarrow a(a-4) + 2(a-4) = 0$

$\Rightarrow (a-4)(a+2) = 0$

$\Rightarrow a = 4, a = -2$

53. Let A(3, 1), B(0, -2), C(1, 1) and D(4, 4) be the vertices of quadrilateral

Join AC, BD. AC and BD, intersect other at the point O.




We know that the diagonals of a parallelogram bisect each other

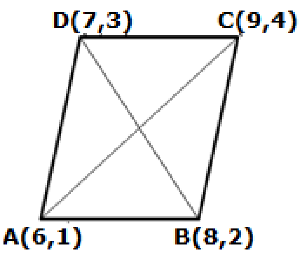
Therefore, O is midpoint of AC as well as that of BD

Now midpoint of AC is $\left(\frac{3+1}{2}, \frac{1+1}{2}\right)$ ie., $(2, 1)$

And midpoint of BD is $\left(\frac{0+4}{2}, \frac{-2+4}{2}\right)$ ie., $(2, 1)$

Mid point of AC is the same as midpoint of BD

Hence, A, B, C, D, are the vertices of a parallelogram ABCD

54. 


Let A(6, 1), B(8, 2), C(9, 4) and D(7, 3) be the angular points of a quadrilateral ABCD.

Join AC and BD.

Now,

$AB = \sqrt{(8-6)^2 + (2-1)^2} = \sqrt{(2)^2 + (1)^2} = \sqrt{4+1} = \sqrt{5}\, units$

$BC = \sqrt{(9-8)^2 + (4-2)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}\, units$

$CD = \sqrt{(7-9)^2 + (3-4)^2} \sqrt{(-2)^2 + (-1)^2} = \sqrt{4+1} = \sqrt{5}\, units$

$DA = \sqrt{(7-6)^2 + (3-1)^2} = \sqrt{(1)^2 + (2)^2} = \sqrt{1+4} = \sqrt{5}\, units$

$\therefore$ AB = BC = CD = DA = $\sqrt{5}\, units$

$Diagonal\ AC = \sqrt{(9-6)^2 + (4-1)^2} = \sqrt{(3)^2 + (3)^2} = \sqrt{9+9} = \sqrt{18} = 3\sqrt{2}\, units$

$Diagonal\ BD = \sqrt{(7-8)^2 + (3-2)^2} = \sqrt{(-1)^2 + (1)^2} = \sqrt{1+1} = \sqrt{2}\, units$

$\therefore$ Diagonal AC $\neq$ Diagonal BD

Thus, ABCD is a quadrilateral having all sides equal but diagonals unequal.

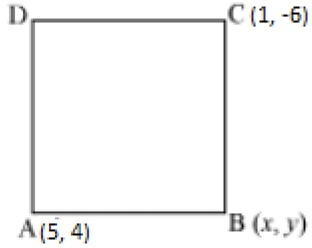
$\therefore$ ABCD is a rhombus but not a square.

$\therefore$ Area of rhombus $ABCD = \frac{1}{2} \times product\ of\ diagonals$

$= \frac{1}{2} \times AC \times BD$

$= \frac{1}{2} \times 3\sqrt{2} \times \sqrt{2}$

=3 sq. units

55. 


Let ABCD be a square and let A(5,4) and C(1,-6). Let (x, y) be the coordinates of B. Then,

AB = BC

$AB^2 = BC^2$

$(x - 5)^2 + (y - 4)^2 = (x - 1)^2 + (y + 6)^2$

$x^2 + 25 - 10x + y^2 + 16 - 8y = x^2 + 1 - 2x + y^2 + 36 + 12y$

$x^2 - 10x + y^2 - 8y - x^2 + 2x - y^2 - 12y = 1 + 36 - 25 - 16$

$-8x - 20y = -4$

$-8x = 20y - 4$

$x = \frac{20y-4}{-8}$

$x = \frac{4(5y-1)}{-8} = \frac{1-5y}{2}$ ..........(i)

In $\triangle$ABC, we have,

$AB^2 + BC^2 = AC^2$

$(x - 5)^2 + (y - 4)^2 + (x - 1)^2 + (y + 6)^2 = (5 - 1)^2 + (4 + 6)^2$

$x^2 + 25 - 10x + y^2 + 16 - 8y + x^2 + 1 - 2x + y^2 + 36 + 12y = 16 + 100$

$2x^2 + 2y^2 - 12x + 4y = 116 - 78$

$2x^2 + 2y^2 - 12x + 4y = 38$

$x^2 + y^2 - 6x + 2y = 19$

$x^2 + y^2 - 6x + 2y - 19 = 0$ ....... (ii)

Substituting the value of x from (i) in (ii), we get,

$(\frac{1-5y}{2})^2 + y^2 - 6(\frac{1-5y}{2}) + 2y - 19 = 0$

$\frac{(1-5y)^2}{4} + y^2 - 3(1-5y) + 2y - 19 = 0$

$\frac{1+25y^2-10y}{4} + y^2 - 3 + 15y + 2y - 19 = 0$

$\frac{1+25y^2-10y+4y^2-12+60y+8y-76}{4} = 0$

$29y^2 + 58y - 87 = 0$

$y^2 + 2y - 3 = 0$

$y^2 + 3y - y - 3 = 0$

$y(y + 3) -1 (y + 3) = 0$

$(y + 3) (y - 1) = 0$

$y = -3, 1$

From (i), For y = -3, we have,

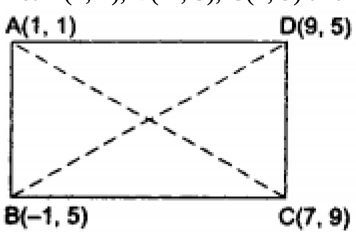
$x = \frac{1-5(-3)}{2} = \frac{1+15}{2} = 8$

For y = 1,we have,

$x = \frac{1-5(1)}{2} = \frac{-4}{2} = -2$

Hence, the required vertices of the square are, (-2, 1) and (8, -3).

56. Let A(1, 1), B(-1, 5), C(7, 9) and D(9, 5) be the vertices of quad. ABCD. Then,




$AB^2 = (-1 -1)^2 + (5 - 1)^2$ (by using distance formula)

= $(-2)^2 + 4^2 = 4 + 16 = 20$

$\therefore \quad AB = \sqrt{20}$ units $= \sqrt{4 \times 5}\, units \; = 2\sqrt{5}\, units$

$BC^2 = (7 + 1)^2 + (9 - 5)^2$ (by using distance formula)

$= 8^2 + 4^2 = 64 + 16 = 80.$

$\therefore \quad BC = \sqrt{80}$ units $= \sqrt{16 \times 5}$ units $= 4\sqrt{5}\, units.$

$CD^2 = (9 - 7)^2 + (5 - 9)^2$ (by using distance formula)

$= 2^2 + (-4)^2 = 4 + 16 = 20.$

$\therefore \quad CD = \sqrt{20}$ units $= \sqrt{4 \times 5}$ units $= 2\sqrt{5}\, units.$

$AD^2 = (9 - 1)^2 + (5 - 1)^2$ (by using distance formula)

$= 8^2 + 4^2 = 64 + 16 = 80$

$\therefore AD = \sqrt{80}$ units $= \sqrt{16 \times 5}$ units $= 4\sqrt{5}\, units$

Thus, AB = CD = $2\sqrt{5}$ units and BC = AD = $4\sqrt{5}\, units$

Also, $AC^2 = (7 - 1)^2 + (9 - 1)^2$ (by using distance formula)

$= 6^2 + 8^2 = 36 + 64 = 100$

$\therefore AC = \sqrt{100}$ units $= 10\, units.$

And, $BD^2 = (9 + 1)^2 + (5 - 5)^2 = 10^2 + 0^2 = 100$ ( by using distance formula)

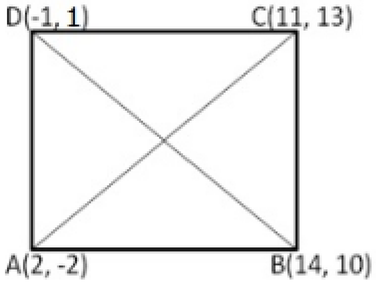
$\therefore \quad BD = \sqrt{100}$ units $= 10\, units.$

$\therefore$ diagonal AC = diagonal BD.

Thus, ABCD is a quadrilateral whose opposite sides are equal and the diagonals are equal.

Hence, quad. ABCD is a rectangle.

57. Let A(2, -2), B(14, 10), C(11, 13) and D(-1, 1) be the angular points of quad. ABCD, then




$AB = \sqrt{(14-2)^2 + (10+2)^2} = \sqrt{(12)^2 + (12)^2} = \sqrt{288} = 12\sqrt{2}\, units$

$BC = \sqrt{(11-14)^2 + (13-10)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{18} = 3\sqrt{2}$

$DC = \sqrt{(-1-11)^2 + (1-13)^2} = \sqrt{(-12)^2 + (-12)^2} = \sqrt{288} = 12\sqrt{2}\, units$

$AD = \sqrt{(-1-2)^2 + (1+2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{18} = 3\sqrt{2}\, units$

thus, AB = DC and AD = BC

Diagonal $AC = \sqrt{(11-2)^2 + (13+2)^2} = \sqrt{(9)^2 + (15)^2} = \sqrt{306}$

$= 3\sqrt{34}\, units$

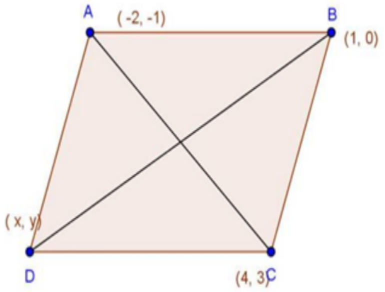
$Diagonal\ BD = \sqrt{(-1-14)^2 + (1-10)^2} = \sqrt{(-15)^2 + (-9)^2}$

$= \sqrt{225 + 81} = \sqrt{306} = 3\sqrt{34}\, units$

$\therefore$ Diagonal AC = Diagonal BD

Thus, ABCD is a quadrilateral whose opposite sides are equal and diagonals are equal.

Hence, quadrilateral ABCD is rectangle.

58. 


Let A(-2, -1), B(1, 0), C(4, 3) and D(x, y) be the vertices of a parallelogram ABCD taken in order.

Since the diagonals of a parallelogram bisect each other.

$\therefore$Coordinates of the mid point of AC = Coordinates of the mid point of BD.

$\Rightarrow \quad \frac{-2+4}{2} = \frac{1+x}{2}$

$\Rightarrow \quad \frac{2}{2} = \frac{x+1}{2}$

$\Rightarrow \quad 1 = \frac{x+1}{2}$

$\Rightarrow$x + 1 = 2

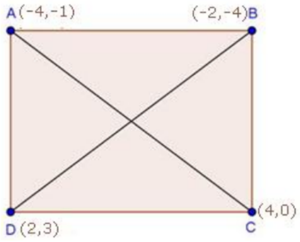
$\Rightarrow$x = 1

and, $\frac{-1+3}{2} = \frac{y+0}{2}$

$\Rightarrow \quad \frac{2}{2} = \frac{y}{2}$

$\Rightarrow$y = 2

Hence, fourth vertex of the parallelogram is (1, 2).

59. 


Let A(-4, -1), B(-2, -4), C(4, 0) and D(2, 3) be the given points.

Coordinates of the mid-point of AC are $\left(\frac{-4+4}{2}, \frac{-1+0}{2}\right) = \left(0, \frac{-1}{2}\right)$

Coordinates of the mid-point of BD are $\left(\frac{-2+2}{2}, \frac{-4+3}{2}\right) = \left(0, \frac{-1}{2}\right)$

Thus, AC and BD have the same mid-point

We have,

$AC = \sqrt{(4+4)^2 + (0+1)^2} = \sqrt{65}$

$BD = \sqrt{(-2-2)^2 + (-4-3)^2} = \sqrt{65}$

Hence, ABCD is a rectangle.

60. (-1, -2), (1, 0), (-1, 2), (-3, 0)

Let A $\rightarrow$ (-1, -2), B $\rightarrow$ (1, 0)

C $\rightarrow$ (-1, 2) and D $\rightarrow$ (-3, 0)

Then,

$AB = \sqrt{[1-(-1)]^2 + [0-(-2)]^2}$

$= \sqrt{(2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$BC = \sqrt{(-1-1)^2 + (2-0)^2}$

$= \sqrt{(-2)^2 + (2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$CD = \sqrt{[(-3)-(-1)]^2 + (0-2)^2}$

$= \sqrt{(-2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$DA = \sqrt{[(-1)-(-3)]^2 + (-2-0)^2}$

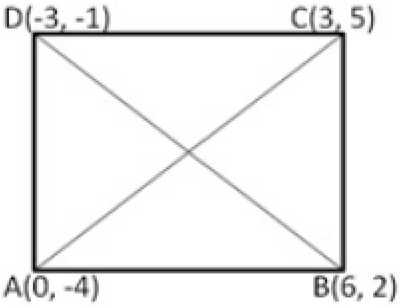
$= \sqrt{(2)^2 + (-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}$

$AC = \sqrt{[(-1)-(-1)]^2 + [(2)-(-2)]^2} = 4$

$BD = \sqrt{[(-3)-(1)]^2 + (0-0)^2} = 4$

Since AB = BC = CD = DA (i.e., all the four sides of the quadrilateral ABCD are equal) and AC = BD (i.e. diagonals of the quadrilateral ABCD are equal). Therefore, ABCD is a square.

61. Let A(0, -4), B(6,2), C(3,5) and D(-3,-1) are the vertices of quad. ABCD. Then




$AB = \sqrt{(6-0)^2 + (2+4)^2} = \sqrt{(6)^2 + (6)^2} = \sqrt{36+36} = \sqrt{72} = 6\sqrt{2}\, units$
$BC = \sqrt{(3-6)^2 + (5-2)^2} = \sqrt{(-3)^2 + (3)^2} = \sqrt{18} = 3\sqrt{2}\, units$
$DC = \sqrt{(-3-3)^2 + (-1-5)^2} = \sqrt{(-6)^2 + (-6)^2} = 6\sqrt{2}\, units$
$AD = \sqrt{(-3-0)^2 + (-1+4)^2}\sqrt{(-3)^2 + (3)^2} = \sqrt{18} = 3\sqrt{2}\, units$

Thus, AB = DC and AD = BC

Diagonal $AC = \sqrt{(3-0)^2 + (5+4)^2} = \sqrt{(3)^2 + (9)^2} = \sqrt{9+81} = \sqrt{90}$
$= 3\sqrt{10}\, units$

Diag $BD = \sqrt{(-3-6)^2 + (-1-2)^2} = \sqrt{(-9)^2 + (-3)^2} = \sqrt{81+9} = \sqrt{90}$
$= 3\sqrt{10} units$

Thus, ABCD is a quadrilateral whose opposite sides are equal and the diagonals are equal

Hence, quadrilateral ABCD is a rectangle.

**Section D**

62. Fill in the blanks:

(i) 1. +ve, -ve

(ii) 1. (-3, -9)

(iii) 1. equilateral

(iv) 1. 10 units

(v) 1. collinear

(vi) 1. 13

**Section E**

63. State True or False:

(i) **(a)** True
**Explanation:** True

(ii) **(a)** True
**Explanation:** True

(iii) **(a)** True
**Explanation:** True

(iv) **(a)** True
**Explanation:** True

(v) **(b)** False
**Explanation:** False

(vi) **(a)** True
**Explanation:** True

(vii) **(a)** True
**Explanation:** True

(viii) **(a)** True
**Explanation:** True