Solution

COORDINATE GEOMETRY WS 2

Class 10 - Mathematics

Section A

1. A(a, b), B(-b, a)
   $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-b - a)^2 + (a - b)^2}$
   $= \sqrt{b^2 + a^2 + 2ab + a^2 + b^2 - 2ab}$
   $= \sqrt{2a^2 + 2b^2} = \sqrt{2(a^2 + b^2)}\ units$
2. 5 unit,
   Length of Diagonal AB
   = $\sqrt{(0-4)^2 + (3-0)^2}$
   = $\sqrt{16 + 9}$
   = $\sqrt{25}$
   = 5 unit
3. Given that,
   ⇒ A(2, 3), B(-5, 6), C(6, 7) and D(p, 4)
   We know that, diagonals of a parallelogram bisect each other
   So, midpoint of line segment joining points A and C is same as midpoint of line segment joining points B and D
   ⇒ $\left[\frac{2+6}{7}, \frac{3+7}{2}\right] = \left[\frac{-5+\mathrm{p}}{2}, \frac{6+4}{2}\right]$
   ⇒ (4, 5) = $\left[\frac{p-5}{2}, 5\right]$
   On comparing,
   ⇒ $\frac{\mathrm{p}-5}{2}$ = 4
   ⇒ p - 5 = 8
   ⇒ p = 13
4. True.
   By using the distance formula
   Distance between centre (origin) and Q(6, 8) = $\sqrt{(6-0)^2 + (8-0)^2} = \sqrt{36 + 64} = 10$, which is greater than its radius, i.e, 5.
   Distance from origin to point P is 5.
5. Let the point P(0, 2) is equidistant from A(3, k) and B(k, 5)
   PA = PB
   $PA^2 = PB^2$
   $(3 - 0)^2 + (k - 2)^2 = (k - 0)^2 + (5 - 2)^2$
   $\Rightarrow 9 + k^2 + 4 - 4k = k^2 + 9$
   $\Rightarrow 9 + k^2 + 4 - 4k - k^2 - 9 = 0$
   ⇒4 - 4k = 0
   ⇒-4k = -4
   ⇒k = 1
6. Here, A→(0,4),B→(0,0),C→(3,0)
   AB = $\sqrt{(0-0)^2 + (0-4)^2} = \sqrt{16} = 4$
   BC = $\sqrt{(3-0)^2 + (0-0)^2} = \sqrt{9}$ = 3
   $CA = \sqrt{(0-3)^2 + (4-0)^2}$
   $= \sqrt{9 + 16} = \sqrt{25}$ =5
   Therefore, Perimeter of triangle = 4 + 3 + 5 = 12
7. Let the point on y-axis be P(0, y).Then,
   PA = PB
   $PA^2 = PB^2$
   or, $(0 - 5)^2 + (y - 3)^2 = (0 - 1)^2 + (y + 5)^2$
   or, $5^2 + y^2 - 6y + 9 = 1 + y^2 + 10y + 25$

or, 16y = 8

$y = \frac{1}{2}$

Hence, point on y-axis is $\left(0, \frac{1}{2}\right)$

8. Centre = $\left(\frac{3+1}{2}, \frac{-10+4}{2}\right)$ = (2, -3)

   Radius = $\sqrt{(2-1)^2 + (-3-4)^2}$ = $\sqrt{50}$ = $5\sqrt{2}$ units.

9. Let, the point on X-axis be (x,0).

   Now, by using distance formula,

   $\sqrt{(x_2 \ - \ x_1)^2 + \ (y_2 - \ y_2)^2}$

   $\Rightarrow \sqrt{(x-7)^2 + (0+4)^2} = 2\sqrt{5}$

   Squaring both sides,

   $\Rightarrow (x-7)^2 + 4^2 = (2\sqrt{5})^2$

   $\Rightarrow x^2 - 14x + 49 + 16 = 20$

   $\Rightarrow x^2 - 14x + 45 = 0$

   ⇒ (x - 9) (x - 5) = 0

   $\Rightarrow x = 9 \ or \ x = 5$

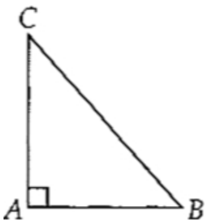
   Hence, two points exists (9,0) and (5,0)

10. A(1, -3), B(4, 1)

    $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(4-1)^2 + [1-(-3)]^2}$

    $= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5units$

11. Given A(3, 0), B(6, 4) and C(-1, 3)

    $AB^2 = (3 - 6)^2 + (0 - 4)^2 = 9 + 16 = 25$

    $BC^2 = (6 + 1)^2 + (4 - 3)^2 = 49 + 1 = 50$

    $CA^2 = (-1 - 3)^2 + (3 - 0)^2 = 16 + 9 = 25$

    $AB^2 + CA^2 = BC^2$

    Since, pythagoras theorem is verified, therefore triangle is right-angled triangle.

12. Distance between the points A(10 cosθ, 0) and B(0, 10 sinθ)

    $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

    $= \sqrt{(0 - 10\cos\theta)^2 + (10\sin\theta - 0)^2}$

    $= \sqrt{100\cos^2\theta + 100\sin^2\theta}$

    $= \sqrt{100\left(\sin^2\theta + \cos^2\theta\right)}$

    $= \sqrt{100 \times 1} = \sqrt{100} \ \left\{\because \sin^2\theta + \cos^2\theta = 1\right\}$

    = 10

13. Let the vertices of the triangle be A(-2, 0), B(2, 3) and C(1, -3).

    We compute the length of the sides as follows:

    AB = $\sqrt{(2-(-2))^2 + (3-0)^2}$ = $\sqrt{4^2 + 3^2} = \sqrt{16+9}$ = 5

    BC = $\sqrt{(1-2)^2 + (-3-3)^2}$ = $\sqrt{(-1)^2 + (-6)^2} = \sqrt{1+36}$ = $\sqrt{37}$

    AC = $\sqrt{(1-(-2))^2 + (-3-0)^2}$ = $\sqrt{3^2 + (-3)^2} = \sqrt{9+9}$ = $\sqrt{18}$

    Since none of the lengths of the sides are equal, therefore, it is a scalene triangle.

14. Point P(x, 0) is equidistant from point A(-2, 0) and B(6, 0) i.e. AP = BP

    $\Rightarrow AP^2 = BP^2$

    $\Rightarrow (-2-x)^2 + (0-0)^2 = (6-x)^2 + (0-0)^2$ (by distance formula)

    $\Rightarrow [(-1)(2+x)]^2 = (6-x)^2$

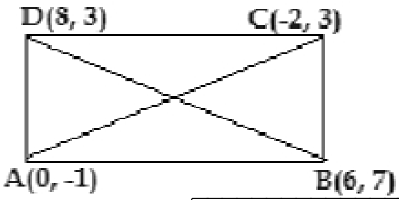
    $\Rightarrow 4 + 4x + x^2 = 36 - 12x + x^2$

    $\Rightarrow 4 + 4x = 36 - 12x$

$\Rightarrow 36-12x-4-4x=0$
$\Rightarrow 32-16x=0$
$\Rightarrow 32=16x$
$\Rightarrow x=2$
Hence, point P(2, 0) is equidistant from point A(-2, 0) and B(6, 0).

15. Let the coordinates of A be(a, b) and B be (c, d),
The coordinates of origin be O(0, 0)
$\therefore \angle AOB = 90^\circ$ (given)
$\Rightarrow$ $AB^2 = AO^2 + BO^2$ (By using pythagoras theorem)
By using distance formula,
$\therefore (c - a)^2 + (d - b)^2 = a^2 + b^2 + c^2 + d^2$
$c^2+a^2-2ac+d^2+b^2-2bd=a^2+b^2+c^2+d^2$
ac+bd=0
Hence proved

16. Distance of the point (2, 4) from x-axis is 4 units. There is no point on x-axis which is at a distance of 2 units from the given point.

17. 

length of AB = $\sqrt{(6-0)^2+(7+1)^2}$
= $\sqrt{36+64}$
= $\sqrt{100}$ = 10 units.
length of BC = $\sqrt{(6+2)^2+(7-3)^2}$
= $\sqrt{8^2+4^2}$
= $\sqrt{64+16}$
= $\sqrt{80}$ units.
= 2 $\times$ $2\sqrt{5}$ units.
= $4\sqrt{5}$ units.
length of CD = $\sqrt{(-2+8)^2+(3-3)^2}$
= $\sqrt{100}$
= 10 units.
length of DA = $\sqrt{(0-8)^2+(-1-3)^2}$
= $\sqrt{64+16}$
= $4\sqrt{5}$ units.
$\Rightarrow$ AB = CD and AD = CB.
Now diagonal BD = $\sqrt{(6-8)^2+(7-3)^2}$
= $\sqrt{4+16}$ = $\sqrt{20}$ = $2\sqrt{5}$ units.
diagonal AC
= $\sqrt{(0+2)^2+(-1-3)^2}$
= $\sqrt{4+16}$.
= $\sqrt{20}$.
$\Rightarrow$ AC = BD.
quad. with opposite sides equal and diagonal also equal is Rectangle.
$\therefore$ ABCD is Rectangle.

18. Given $A\left(\frac{-7}{3},5\right)$ and $B\left(\frac{2}{3},5\right)$
Using distance formula,
$AB=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$
$AB=\sqrt{\left(\frac{2}{3}+\frac{7}{3}\right)^2+(5-5)^2}=\sqrt{9+0}$
AB = 3 units

19. Here the point is given to be R(-4,0). Comparing this with the standard form of (x, y)
We have
x = -4
y = 0
Here we see that y=0, x $\neq$ 0.
Hence the given point lies on the x-axis.
20. We can find distance between point A and B by using distance formula
Distance AB
$= \sqrt{(2a+\sqrt{3}a-2a)^2+(5a-6a)^2}$
$= \sqrt{3a^2+a^2} = \sqrt{4a^2}$
= 2a
So distance AB= 2a
21. Reflection of a point through x-axis is obtained by changing the sign of the ordinate and reflection through y-axis is obtained by changing the sign of the ordinate.
So, the reflection of (3, 5) through x- axis is (3, -5) and reflection of P through y-axis is (-3, 5).
22. AB = 5
Applying distance formula,
$\Rightarrow \sqrt{(x-0)^2+(-4-0)^2}$ = 5
Squaring both side,
x^2 + 16 = 25
x^2 = 9
x = $\pm$ 3
23. Required distance $= \sqrt{[(a+b)-(a-b)]^2+[(b+c)-(c-b)]^2}$
$= \sqrt{(2b)^2+(2b)^2} = \sqrt{2(2b)^2} = 2b\sqrt{2}$
24. We can find distance AB y using the distance formula,
AB = $\sqrt{(8+1)^2+(-2+1)^2} = \sqrt{82}$
25. Applying Distance Formula to find distance between points (2, 3) and (4,1), we get
d = $\sqrt{(4-2)^2+(1-3)^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{4+4} = \sqrt{8} = 2\sqrt{2}\ units$
26. Here the point is given to be S(0, 5). Comparing this with the standard form of (x, y)
we have
x = 0
y = 5
Here we see that x = 0, y $\neq$ 0.
Hence the given point lies on the y - axis.
27. A(a, 0), B(0, a)
$AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(0-a)^2+(a-0)^2}$
$= \sqrt{(a^2+a^2)} = \sqrt{2a^2} = \sqrt{2}a\ units$
28. Distance of the point, from the centre
a = $\sqrt{(5-3)^2+(8-4)^2}$
= $\sqrt{4+16} = \sqrt{20}$
= $2\sqrt{5}$
Since, $2\sqrt{5}$ is less than radius=7
$\therefore$ The point lies inside the circle.

Section B

29. The distance d between two point (x_1, y_1) and (x_2, y_2) is given by the formula
$d = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$
Here it is given that one end of a line segment has co-ordinates (2, -3). The abscissa of the other end of the line segment is given to be 10. Let the ordinate of this point be 'y'.
So, the co-ordinates of the other end of the line segment is (10, y).
The distance between these two points is given to be 10 units.
Substituting these values in the formula for distance between two points we have,

$d = \sqrt{(2-10)^2 + (-3-y)^2}$
$10 = \sqrt{(-8)^2 + (-3-y)^2}$
Squaring on both sides of the equation we have,

$100 = (-8)^2 + (-3 - y)^2$

$100 = 64 + 9 + y^2 + 6y$

$27 = y^2 + 6y$

We have a quadratic equation for 'y'. Solving for the roots of this equation we have,

$y^2 + 6y - 27 = 0$

$y^2 + 9y - 3y - 27 = 0$

$y(y + 9) - 3(y + 9) = 0$

$(y + 9)(y - 3) = 0$

The roots of the above equation are '-9' and '3'

Thus the ordinates of the order end of the line segment could be -9 or 3.

30. $(2a - 11)^2 + (a - 7 + 9)^2 = (5\sqrt{2})^2$

$\Rightarrow 5a^2 - 40a + 75 = 0$

$\Rightarrow (a - 5)(5a - 15) = 0$

a = 5, a = 3

31. i. The location of doorknob and medicine bottle is (-3, -1) and (2, -1) respectively.

So, distance = $\sqrt{[2 - (-3)]^2 + [-1 - (-1)]^2}$ = 5

Now, since each unit is equal to 5 meters, so actual distance = 5×5 = 25 m

ii. The location of medicine bottle and pottery jug is (2, -1) and (2, 4) respectively.

So, distance = $\sqrt{[2 - 2]^2 + [4 - (-1)]^2}$ = 5

Now, since each unit is equal to 5 meters, so actual distance = 5×5 = 25 m

iii. The location of doorknob and pottery jug is (-3, -1) and (2, 4) respectively.

So, distance = $\sqrt{[2 - (-3)]^2 + [4 - (-1)]^2}$ = 7.07

Now, since each unit is equal to 5 meters, so actual distance = 7.07×5 = 35.35 m

32. Let A(1, 7), B(4, 2), C(-1, -1) and D(-4, 4) be the given points. One way of showing that ABCD is a square is to use the property that all its sides should be equal both its diagonals should also be equal. Now,

$AB = \sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$
$BC = \sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$
$CD = \sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$
$DA = \sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$
$AC = \sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$
$BD = \sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

33. Given: P(2, 2) is equidistant from the points A(-2, k) and B(-2k, -3),

We have, AP = BP

$AP^2 = BP^2$

$(2 + 2)^2 + (2 - k)^2 = (2 + 2k)^2 + (2 + 3)^2$

$16 + 4 + k^2 - 4k = 4 + 4k^2 + 8k + 25$

$20 + k^2 - 4k = 29 + 4k^2 + 8k$

$3k^2 + 12k + 9 = 0$

$k^2 + 4k + 3 = 0$

$k^2 + 3k + k + 3 = 0$

$k(k + 3) + 1(k + 3) = 0$

$(k + 1)(k + 3) = 0$

k = -1, -3

For k = -1, we have,

AP = $\sqrt{(2+2)^2+(2-k)^2} = \sqrt{16+(2+1)^2} = \sqrt{25} = 5$

For k = -3, we have,

AP = $\sqrt{(2+2)^2+(2+3)^2} = \sqrt{16+25} = \sqrt{41}$

34. Required distance $= \sqrt{(asin\alpha + acos\alpha)^2 + (-bcos\alpha - b\sin\alpha)^2}$

$= \sqrt{a^2sin^2\alpha + a^2cos^2\alpha + 2a^2sin\alpha cos\alpha + b^2cos^2\alpha + b^2sin\alpha + 2b^2sin\alpha cos\alpha}$

$= \sqrt{a^2(sin^2\alpha + cos^2\alpha) + b^2(sin^2\alpha + cos^2\alpha) + (2a^2+2b^2)sin\alpha cos\alpha}$

$= \sqrt{a^2+b^2+2(a^2+b^2)sin\alpha cos\alpha}$

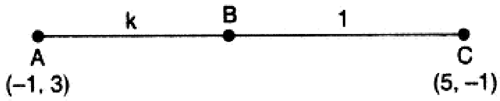
$= \sqrt{(a^2+b^2)[1+2sin\alpha cos\alpha]}$

$= \sqrt{(a^2+b^2)[sin^2\alpha + cos^2\alpha + 2sin\alpha cos\alpha]}$

$= \sqrt{(a^2+b^2)[sin\alpha + cos\alpha]^2}$

$= (sin\alpha + cos\alpha)\sqrt{a^2+b^2}$

35. Let A → (–1, 3) B → (2, p) and C → (5, –1)



If the points A, B and C are collinear then let B divide AC in the ratio K : 1 internally.

Then, $B \rightarrow \left\{\frac{(K)(5)+(1)(-1)}{K+1}, \frac{(K)(-1)+(1)(3)}{K+1}\right\}$

$\Rightarrow$ B $\rightarrow \left(\frac{5K-1}{K+1}, \frac{-K+3}{K+1}\right)$

But, B is given to be (2,p)

$\therefore \frac{5K-1}{K+1} = 2$

$\Rightarrow$ 5K - 1 = 2(K + 1)

$\Rightarrow$ 5K - 1 = 2K + 2

$\Rightarrow$5K - 2K = 2 + 1

$\Rightarrow$3K = 3

$\Rightarrow K = \frac{3}{3} = 1$ and $\frac{-K+3}{K+1} = p$

$\Rightarrow \frac{-1+3}{1+1} = p$

$\Rightarrow$ p = 1

Hence, the required value of p is 1.

36. A(2, 3) and B(-4, 1) are the given points.

Let C (0, y) be the points on y-axis

$AC = \sqrt{(0-2)^2+(y-3)^2}$

$\Rightarrow \quad AC = \sqrt{4+y^2+9-6y}$

$\Rightarrow \quad AC = \sqrt{y^2-6y+13}$

$BC = \sqrt{(0+4)^2+(y-1)^2}$

$\Rightarrow \quad BC = \sqrt{16+y^2+1-2y}$

$\Rightarrow \quad BC = \sqrt{y^2-2y+17}$

Since AC = BC

$AC^2 = BC^2$

$y^2 - 6y + 13 = y^2 - 2y + 17$

$\Rightarrow$ -6y + 2y = 17 - 13

$\Rightarrow$ -4y = 4

$\Rightarrow$ y = -1

$\therefore$ The point on y-axis is (0, -1)

37. In equilateral $\triangle$ABC , coordinates of points A and B are (2,0) and (5,0) respectively.we have to find the co-ordinates of the other two vertices.

Let co-ordinates of C be (x, y)

Since $AC^2 = BC^2$ (sides of equilateral triangle)

$(x - 2)^2 + (y - 0)^2 = (x - 5)^2 + (y - 0)^2$

or, $x^2 + 4 - 4x + y^2 = x^2 + 25 - 10x + y^2$

or, 6x = 21

x = $\frac{7}{2}$

And $(x - 2)^2 + (y - 0)^2 = 9$

or, $\left(\frac{7}{2} - 2\right)^2 + y^2 = 9$

or, $\frac{9}{4} + y^2 = 9$

or, $y^2 = \frac{27}{4} = \frac{3\sqrt{3}}{2}$

Hence, C = $\left(\frac{7}{2}, \frac{3\sqrt{3}}{2}\right)$

38. Coordinates of points on a circle are A(2, 1), B(5, -8) and C(2, -9).

Let the coordinates of the centre of the circle be O(x, y).

OA = OB

$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-5)^2 + (y+8)^2}$

$(x - 2)^2 + (y - 1)^2 = (x - 5)^2 + (y + 8)^2$

$x^2 + 4 - 4x + y^2 + 1 - 2y = x^2 + 25 - 10x + y^2 + 64 + 16y$

6x - 18y - 84 = 0

x - 3y - 14 = 0 .... (i)

Similarly, OC = OB

$\sqrt{(x-2)^2 + (y+9)^2} = \sqrt{(x-5)^2 + (y+8)^2}$

$(x - 2)^2 + (y + 9)^2 = (x - 5)^2 + (y + 8)^2$

$x^2 + 4 - 4x + y^2 + 81 + 18y = x^2 + 25 - 10x + y^2 + 64 + 16y$

6x + 2y - 4 = 0

3x + y - 2 = 0 .... (ii)

By solving (i) and (ii), we get,

x = 2 and y = -4

So, the coordinates of the centre of circle are (2, -4).

39. By using the distance formula lets determine the length of all three sides.

∵ Distance between the points (x_1, y_1) and (x_2, y_2);

d = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

So,

AB = $\sqrt{(-4+5)^2 + (-2-6)^2}$

= $\sqrt{(1)^2 + (-8)^2}$

= $\sqrt{1 + 64} = \sqrt{65}$

BC = $\sqrt{(7+4)^2 + (5+2)^2}$

= $\sqrt{(11)^2 + (7)^2}$

= $\sqrt{121 + 49}$

= $\sqrt{170}$

CA = $\sqrt{(-5-7)^2 + (6-5)^2}$

= $\sqrt{(-12)^2 + (1)^2}$

= $\sqrt{144 + 1}$

= $\sqrt{145}$

By calculating the distance between two points, now we can find out the type of triangle.

As we can see;

AB $\neq$ BC $\neq$ CA

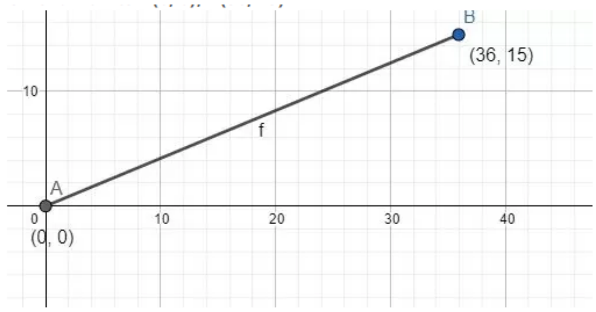
$\triangle$ABC also not satisfies the condition Pythagoras i.e.,

$(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Perpendicular})^2$

Hence, the triangle is scalene because all of its sides are not equal i.e., different from each other.

40. Distance between two points

Given: Points A(0, 0), B(36, 15)



For two points A(x_1, y_1) and B(x_2, y_2),

Distance is given by $f = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Distance between (0, 0) and (36, 15) is:

$f = \sqrt{(36 - 0)^2 + (15 - 0)^2}$

$f = \sqrt{1296 + 225}$

$f = \sqrt{1521}$ = 39

Hence Distance between points A and B is 39 units

Yes, we can find the distance between the given towns A and B. Let us take town A at origin point (0, 0)

Hence, town B will be at point (36, 15) with respect to town A

And, as calculated above, the distance between town A and B will be 39 km

41. The point is on x-axis

Its ordinates of the point P is (x, 0)

P is equidistant from A(-3, 4) and B(2, 5)

Now $PA = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(x + 3)^2 + (0 - 4)^2} = \sqrt{(x + 3)^2 + 16}$

and $PA^2 = (x + 3)^2 + 16$

Similarly $PB^2 = [\sqrt{(x - 2)^2 + (0 - 5)^2}]^2$

$= (x - 2)^2 + 25$

$\because PA = PB \Rightarrow PA^2 = PB^2$

$\therefore (x + 3)^2 + 16 = (x - 2)^2 + 25$

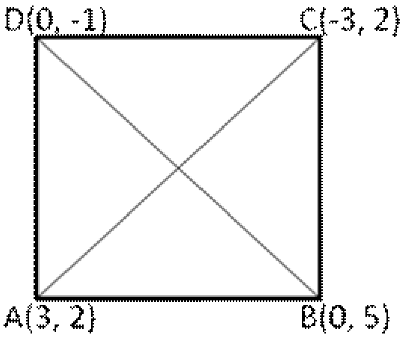
$x^2 + 6x + 9 + 16 = x^2 - 4x + 4 + 25$

$\Rightarrow x^2 + 6x - x^2 + 4x = 25 + 4 - 9 - 16$

$\Rightarrow 10x = 4 \Rightarrow x = \frac{4}{10} = \frac{2}{5}$

$\therefore$ co-ordinates of point P will be $\left(\frac{2}{5}, 0\right)$

42. Let the angular points of quadrilateral ABCD are A(3, 2), B(0, 5), C(-3, 2) and D(0, -1)



By using distance formula,

$AB = \sqrt{(0 - 3)^2 + (5 - 2)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$BC = \sqrt{(-3 - 0)^2 + (2 - 5)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(0 + 3)^2 + (-1 - 2)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$DA = \sqrt{(3 - 0)^2 + (2 + 1)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$\therefore$ AB = BC = CD = DA = $3\sqrt{2}$ units

$Diag.\ AC = \sqrt{(-3 - 3)^2 + (2 - 2)^2} = 6$ units

$Diag.\ BD = \sqrt{(0 - 0)^2 + (-1 - 5)^2} = 6$ units

$\therefore$ Diag. AC = Diag. BD

Since all the sides of quad. ABCD are equal and the diagonals are also equal, hence, the quad. ABCD is a square.

43. Distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by PQ = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$\therefore$ Distance between $(0, a\cos 55°)$ and $(a\cos 35°, 0)$

$= \sqrt{(a\cos 35° - 0)^2 + (0 - a\cos 55°)^2}$

$= \sqrt{(a\cos 35°)^2 + (-a\cos 55°)^2}$

$= \sqrt{a^2\cos^2 35° + a^2\cos^2 55°}$

$= \sqrt{a^2(\cos^2 35° + \cos^2 55°)}$

$= a\sqrt{\cos^2(90° - 55°) + \cos^2 55°}$

$= a\sqrt{\sin^2 55° + \cos^2 55°}$

$= a\sqrt{1}$

$= a$ units.

44. Let the coordinates of third vertex be (x,y)

Each length of equilateral triangle = $\sqrt{(0-0)^2+(3+3)^2} = \sqrt{6^2} = 6$

Since the triangle is equilateral, therefore length of each side = 6.

Thus, Distance between (x, y) and (0, 3) = 6

$\sqrt{(x-0)^2+(y-3)^2} = 6$

$(x - 0)^2 + (y - 3)^2 = 36$

$x^2 + (y - 3)^2 = 36$ .... (i)

Also, Distance between (x, y) and (0, -3) = 6

$\sqrt{(x-0)^2+(y+3)^2} = 6$

$x^2 + (y + 3)^2 = 36$ .... (ii)

From (i) and (ii), we get,

$x^2 + (y - 3)^2 = x^2 + (y + 3)^2$

$\Rightarrow$ 12y = 0

$\Rightarrow$ y = 0

From (i), $x^2 + (0 - 3)^2 = 36$

$x^2 = 36 - 9 = 27$

x = $\pm 3\sqrt{3}$

Thus, the third vertex is $(\pm 3\sqrt{3}, 0)$

45. Let A(1, 2), B(4, 3), C(6, 6) and D(3, 5) be the angular point of a quadrilateral ABCD.

Now,

$AB = \sqrt{(4-1)^2+(3-2)^2} = \sqrt{(3)^2+(1)^2} = \sqrt{9+1} = \sqrt{10}\, units$

$BC = \sqrt{(6-4)^2+(6-3)^2} = \sqrt{(2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13}\, units$

$CD = \sqrt{(3-6)^2+(5-6)^2} = \sqrt{(-3)^2+(-1)^2} = \sqrt{9+1} = \sqrt{10}\, units$

$DA = \sqrt{(3-1)^2+(5-2)^2} = \sqrt{(2)^2+(3)^2} = \sqrt{4+9} = \sqrt{13}\, units$

$\therefore$ AB = CD and BC = DA

Thus, ABCD can be either a parallelogram or a rectangle.

$Diagonal\ AC = \sqrt{(6-1)^2+(6-2)^2} = \sqrt{(5)^2+(4)^2} = \sqrt{25+16} = \sqrt{41}\, units$

$Diagonal\ BD = \sqrt{(3-4)^2+(5-3)^2} = \sqrt{(-1)^2+(2)^2} = \sqrt{1+4} = \sqrt{5}\, units$

$\therefore$ Diagonal AC $\neq$ Diagonal BD

Since the opposite sides are equal and the diagonals are not equal, hence ABCD is a parallelogram and it is not a rectangle.

46. i. It can be observed that Niharika posted the green flag at 5th position flower of the distance AD i.e., 5×5 = 25m from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25).
Similarly, Preet posted a red flag at the distance of 4th flower position of AD i.e., 4×5 = 20m from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20).

ii. According to distance formula,
Distance between these flags by using the distance formula, D
$= [(8 - 2)^2 + (25 - 20)^2]^{1/2} = (36 + 25)^{1/2} = \sqrt{61}m$

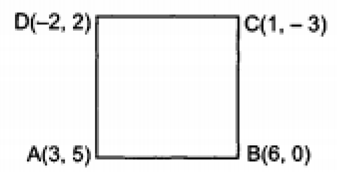
iii. The point at which Rashmi should post her blue flat is the mid-point of the line joining these points. Let this point be A(x, y)
Now by midpoint formula,
$(x, y) = \frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}$

$x = \frac{2+8}{2} = 5$

$y = \frac{25+20}{2} = 22.5$

Hence, A(x, y) = (5, 22.5)

Therefore, Rashmi should post her blue flag at 22.5 m on 5th line.

47. 

AB = $\sqrt{(6-3)^2+(0-5)^2}$

$= \sqrt{9+25} = \sqrt{34}$

BC = $\sqrt{(6-1)^2+(0+3)^2}$

$= \sqrt{25+9} = \sqrt{34}$

CD = $\sqrt{(1+2)^2+(-3-2)^2}$

$= \sqrt{9+25} = \sqrt{34}$

DA = $\sqrt{(-2-3)^2+(2-5)^2}$

$= \sqrt{25+9} = \sqrt{34}$

AC = $\sqrt{(1-3)^2+(-3-5)^2}$

$= \sqrt{4+64} = \sqrt{68}$

BD = $\sqrt{(6+2)^2+(0-2)^2}$

$= \sqrt{64+4} = \sqrt{68}$

AB = BC = CD = DA = $\sqrt{3}4$

Diagonal AC = diagonal BD =$\sqrt{6}8$

Hence A, B, C and D are vertices of a square.

48. Let A(2, 3), B(-4, -6) and C(1, 3/2) be the given points.

$AB = \sqrt{(-4-2)^2+(-6-3)^2}$

$\Rightarrow \quad AB = \sqrt{(-6)^2+(-9)^2}$

$\Rightarrow \quad AB = \sqrt{36+81}$

$\Rightarrow \quad AB = \sqrt{117}$

$BC = \sqrt{(1+4)^2+\left(\frac{3}{2}+6\right)^2}$

$\Rightarrow \quad BC = \sqrt{(5)^2+\left(\frac{15}{2}\right)^2}$

$\Rightarrow \quad BC = \sqrt{25+\frac{225}{4}}$

$\Rightarrow \quad BC = \sqrt{\frac{325}{4}}$

$\Rightarrow \quad BC = \sqrt{81.25}$

$AC = \sqrt{(2-1)^2+\left(3-\frac{3}{2}\right)^2}$

$\Rightarrow \quad AC = \sqrt{(1)^2+\left(\frac{3}{2}\right)^2}$

$\Rightarrow \quad AC = \sqrt{1+\frac{9}{4}}$

$\Rightarrow \quad AC = \sqrt{\frac{13}{4}}$

$\Rightarrow \quad AC = \sqrt{3.25}$

We know that for a triangle sum of two sides is greater than the third side.

Here Ac + BC is not greater than AB.

$\therefore$ ABC is not a triangle.

49. Let A(2a, 4a), B(2a, 6a) and C(2a + $\sqrt{3}a$, 5a) be the given point:

$AB = \sqrt{(2a-2a)^2+(6a-4a)^2}$

$\Rightarrow \quad AB = \sqrt{(0)^2+(2a)^2}$

$\Rightarrow \quad AB = \sqrt{4a^2}$

$\Rightarrow$AB = 2a

$BC = \sqrt{(2a+\sqrt{3a}-2a)^2+(5a-6a)^2}$

$\Rightarrow \quad BC = \sqrt{(\sqrt{3}a)^2 + (-a)^2}$

$\Rightarrow \quad BC = \sqrt{3a^2 + a^2}$

$\Rightarrow \quad BC = \sqrt{4a^2}$

$\Rightarrow$BC = 2a

$AC = \sqrt{(2a + \sqrt{3}a - 2a)^2 + (5a - 4a)^2}$

$\Rightarrow \quad AC = \sqrt{(\sqrt{3}a)^2 + (a)^2}$

$\Rightarrow \quad AC = \sqrt{3a^2 + a^2}$

$\Rightarrow \quad AC = \sqrt{4a^2}$

$\Rightarrow$AC = 2a

Since, AB = BC = AC

$\therefore$ ABC is an equilateral triangle.

50. We have P(x, y), Q(-3, 0) and R(3, 0)

$PQ = \sqrt{(x+3)^2 + (y-0)^2}$

$\Rightarrow \quad 4 = \sqrt{x^2 + 9 + 6x + y^2}$

Squaring both sides

$\Rightarrow \quad (4)^2 = \left(\sqrt{x^2 + 9 + 6x + y^2}\right)^2$

$\Rightarrow$ 16 = x^2 + 9 + 6x + y^2

$\Rightarrow$$x^2$ + y^2 = 16 - 9 - 6x

$\Rightarrow$$x^2$ + y^2 = 7 - 6x ...(i)

$PR = \sqrt{(x-3)^2 + (y-0)^2}$

$\Rightarrow 4 = \sqrt{x^2 + 9 - 6x + y^2}$

Squaring both sides

$(4)^2 = \left(\sqrt{x^2 + 9 - 6x + y^2}\right)^2$

$\Rightarrow$16 = x^2 + 9 - 6x + y^2

$\Rightarrow$$x^2$ + y^2 - 16 - 9 + 6x ...(ii)

Equating (i) and (ii)

7 - 6x = 7 + 6x

$\Rightarrow$7 - 7 = 6x + 6x

$\Rightarrow$0 = 12x

$\Rightarrow$x = 0

Substituting the values of x = 0 in (ii)

x^2 + y^2 = 7 + 6x

0 + y^2 = 7 + 6 $\times$0

y^2 = 7

y = $\pm\sqrt{7}$

51. Diameter of a circle = $10\sqrt{2}$ units

$\Rightarrow$ Radius of a circle = $5\sqrt{2}$ units

Let the centre of a circle be O(2a, a - 7) which passes through the point P(11, -9).

$\Rightarrow$OP is the radius of the circle.

$\Rightarrow OP = 5\sqrt{2}$ units

$\Rightarrow OP^2 = (5\sqrt{2})^2$

$\Rightarrow$ $(11 - 2a)^2$ + $(-9 - a + 7)^2$ = 50

$\Rightarrow$ 121 + $4a^2$ - 44a + $(-2 - a)^2$ = 50

$\Rightarrow$ 121 + $4a^2$ - 44a + 4 + a^2 + 4a = 50

$\Rightarrow$ $5a^2$ - 40a + 125 = 50

$\Rightarrow$ $5a^2$ - 40a + 75 = 0

$\Rightarrow$ a^2 - 8a + 15 = 0

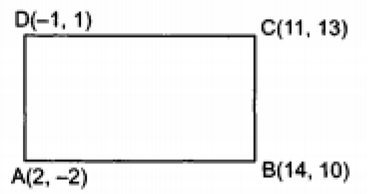
$\Rightarrow$ a^2 - 5a - 3a + 15 = 0

$\Rightarrow$ a(a - 5) -3(a - 5) =0

⇒ (a - 5)(a - 3) = 0
⇒ a - 5 = 0 or a - 3 = 0
⇒ a = 5 or a = 3

52. i. The perpendiculars from P, Q, R and S intersect the x-axis at 3, 10, 10 and 3 respectively.
Also, the perpendiculars from P, Q, R and S intersect the y-axis at 6, 6, 2, 2 respectively.
Hence coordinates of P, Q, R and S are: P(3, 6), Q(10, 6), R(10, 2) and S(3, 2).
ii. Let M be the mid-point of QS.
So using mid-point formula, Coordinates of M are $\left(\frac{3+10}{2}, \frac{2+6}{2}\right) = \left(\frac{13}{2}, 4\right)$
iii. Now, PQ $= \sqrt{(10-3)^2+(6-6)^2} = \sqrt{49} = 7\text{ m}$
PS $= \sqrt{(3-3)^2+(2-6)^2} = \sqrt{16} = 4\text{ m}$
Hence, area of rectangle PQRS = PQ × PS = 7 × 4 = 28 m^2

53. According to the question, A (2,-2), B(14,10), C (11, 13) and D(-1, 1)

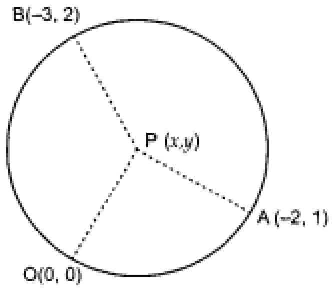


AB = $\sqrt{(14-2)^2+(10+2)^2} = 12\sqrt{2}$ units
BC = $\sqrt{(11-14)^2+(13-10)^2} = 3\sqrt{2}$ units
CD = $\sqrt{(-1-11)^2+(1-13)^2} = 12\sqrt{2}$ units
AD = $\sqrt{(-1-2)^2+(1+2)^2} = 3\sqrt{2}$ units
⇒ AB = CD and BC = AD
∴ ABCD is a parallelogram.
Now, AC = $\sqrt{(11-2)^2+(13+2)^2} = \sqrt{306}$
⇒ AC^2 = 306 units, AB^2 = 288 units.
BC^2 = 18 units
$AB^2 + BC^2$ = 306 units.
⇒ $AC^2 = AB^2 + BC^2$
⇒ $\angle ABC = 90^\circ$
⇒ ABCD is a rectangle

54. Two vertices of an isosceles triangle are A(2, 0) and B(2, 5), Let C(X, y) be the third vertex.
$AB = \sqrt{(2-2)^2+(5-0)^2} = \sqrt{(0)^2+(5)^2} = \sqrt{25} = 5$
$BC = \sqrt{(x-2)^2+(y-5)^2} = \sqrt{x^2+4-4x+y^2+25-10y} = \sqrt{x^2-4x+y^2-10y+29}$
$AC = \sqrt{(x-2)^2+(y-0)^2} = \sqrt{x^2+4-4x+y^2}$
Also we are given that
AC = BC = 3
⇒ $AC^2 = BC^2$ = 9
⇒ $x^2 + 4 - 4x - y^2 = x^2 - 4x + y^2 - 10y + 29$
⇒ 10y = 25
⇒ $y = \frac{25}{10} = \frac{5}{2}$ =2.5
AC^2 = 9
$x^2 + 4 - 4x + y^2 = 9$
$x^2 + 4 - 4x + (2.5)^2 = 9$
$x^2 + 4 - 4x + 6.25 = 9$
$x^2 - 4x + 1.25 = 0$
D = $(-4)^2 - 4\times1\times1.25$
D = 16 - 5
D = 11
$x = \frac{-(-4)+\sqrt{11}}{2\times1} = \frac{4+3.31}{2} = \frac{7.31}{2} = 3.65$

or, $x=\frac{-(-4)-\sqrt{11}}{2}=\frac{4-\sqrt{11}}{2}=\frac{4-3.31}{2}=0.35$

$\therefore$ The third vertex is (3.65, 2.5) or (0.35, 2.5)

55. 

Let P (x, y) be the centre of the circle passing through the points O (0, 0), A (-2, 1) and B (-3, 2). Then,

$OP=AP=BP$

Now, OP = AP

$\Rightarrow OP^2 = AP^2$

$\Rightarrow x^2 + y^2 = (x + 2)^2 + (y - 1)^2$

$\Rightarrow x^2 + y^2 = x^2 + y^2 + 4x - 2y + 5$

$\Rightarrow 4x - 2y + 5 = 0$ ...(i)

and, OP = BP

$\Rightarrow OP^2 = BP^2$

$\Rightarrow x^2 + y^2 = (x + 3)^2 + (y - 2)^2$

$\Rightarrow x^2 + y^2 = x^2 + y^2 + 6x - 4y + 13$

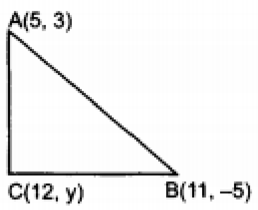
$\Rightarrow 6x - 4y +13 = 0$.........(ii)

On solving equations (i) and (ii), we get $x=\frac{3}{2}$ and $y=\frac{11}{2}$

Thus, the coordinates of the centre are $\left(\frac{3}{2},\frac{11}{2}\right)$

Therefore, Radius = OP = $\sqrt{x^2+y^2}=\sqrt{\frac{9}{4}+\frac{121}{4}}=\frac{1}{2}\sqrt{130}$ sq. units.

56. Given, A(5, 3), B(11, -5) and C(12, y)



AB = $\sqrt{(11-5)^2+(-5-3)^2}$

$=\sqrt{100}$ = 10

AC = $\sqrt{(12-5)^2+(y-3)^2}$

$=\sqrt{49+(y-3)^2}$

BC $=\sqrt{(12-11)^2+(y+5)^2}$

$=\sqrt{1+(y+5)^2}$

Using Pythagoras theorem, we get

$AB^2 = AC^2 + BC^2$

$(10)^2 = 49 + (y - 3)^2 + 1 + (y + 5)^2$

$100 = 50 + y^2 + 9 - 6y + y^2 + 25 + 10y$

$\Rightarrow 2y^2 + 4y - 16 = 0$

$\Rightarrow y^2 + 2y - 8 = 0$

$\Rightarrow (y+4)(y-2)=0$

$\Rightarrow y=-4, y=2$

57. Given: A(3, -1), B(5, -1) and C(3, -3)

$AB=\sqrt{(5-3)^2+(-1+1)^2}=\sqrt{2^2+0^2}=2$

$BC=\sqrt{(5-3)^2+(-1+3)^2}=\sqrt{2^2+2^2}=2\sqrt{2}$

$AC=\sqrt{(3-3)^2+(-1+3)^2}=\sqrt{2^2}=2$

Clearly, AB = AC

Therefore, $\triangle$ABC is an isosceles triangle.

Now, AB^2 = 4, BC^2 = 8 and AC^2 = 4

Therefore, $BC^2 = AB^2 + AC^2$

Therefore, $\triangle$ABC is right angled also.

Hence, $\triangle$ABC is right isosceles triangle.

58. According to question,

Let A(-1, -1), B(2, 3) and C(8, 11) be the given points.

Then,

$AB = \sqrt{(2+1)^2 + (3+1)^2} = \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5\ units$

$BC = \sqrt{(8-2)^2 + (11-3)^2} = \sqrt{(6)^2 + (8)^2} = \sqrt{36+64} = \sqrt{100} = 10\ units$

$AC = \sqrt{(8+1)^2 + (11+1)^2} = \sqrt{(9)^2 + (12)^2} = \sqrt{81+144} = \sqrt{225} = 15\ units$

We know that three points A,B,C are collinear if they lie on the same straight line.That is three points A, B and C are collinear if AB + BC = AC.

Now, AB + BC = 15 units

And AC = 15 units.

$\Rightarrow$ AB + BC = AC. Hence A,B and C are collinear.

59. Let A(1, 7), B(4, 2), C(-1, -1) and D(-4, 4) be the given points. One way of showing that ABCD is a square is to use the property that all its sides should be equal both its diagonals should also be equal. Now,

AB = $\sqrt{(1-4)^2 + (7-2)^2} = \sqrt{9+25} = \sqrt{34}$

BC = $\sqrt{(4+1)^2 + (2+1)^2} = \sqrt{25+9} = \sqrt{34}$

CD = $\sqrt{(-1+4)^2 + (-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

DA = $\sqrt{(1+4)^2 + (7-4)^2} = \sqrt{25+9} = \sqrt{34}$

AC = $\sqrt{(1+1)^2 + (7+1)^2} = \sqrt{4+64} = \sqrt{68}$

BD = $\sqrt{(4+4)^2 + (2-4)^2} = \sqrt{64+4} = \sqrt{68}$

Since, AB = BC = CD = DA and AC = BD, all the four sides of the quadrilateral ABCD are equal and its diagonals AC and BD are also equal. Therefore, ABCD is a square.

60. i. The positions of the students are (3, 5), B(7, 9), C(11, 5) and D(7, 1).

To find the distance between them, we use distance formula.

So,

$AB = \sqrt{(7-3)^2 + (9-5)^2} = \sqrt{(4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$

$BC = \sqrt{(11-7)^2 + (5-9)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

CD = $\sqrt{(7-11)^2 + (1-5)^2} = \sqrt{(4)^2 + (-4)^2} = \sqrt{16+16} = \sqrt{32}$

And $DA = \sqrt{(3-7)^2 + (5-1)^2} = \sqrt{(-4)^2 + (4)^2} = \sqrt{16+16} = \sqrt{32}$

ii. We see that, AB = BC = CD = DA i.e., all sides are equal.

Now, we find the length of both diagonals;

$AC = \sqrt{(11-3)^2 + (5-5)^2} = \sqrt{(8)^2 + 0} = 8$

and $BD = \sqrt{(7-7)^2 + (1-9)^2} = \sqrt{0 + (-8)^2} = 8$

Here, AC = BD

Since AB = BC = CD = DA and AC = BD, so we can say that ABCD is a square.

Thus, it is possible to place Jaspal in the drill in such a way that he is equidistant from each of the four students A, B, C and D.

iii. As we also know that diagonals of a square bisect each other, so, let P be a position of Jaspal in which he is equidistant from each of the four students A, B, C and D.

Coordinates of P = Mid - point of AC

$= \left(\frac{3+11}{2}, \frac{5+5}{2}\right) = \left(\frac{14}{2}, \frac{10}{2}\right) = (7, 5)$

Hence, the required position of Jaspal is (7, 5).

61. i. Given: A(3, 4), B(6, 7), C(9, 4), D(6, 1)

Using distance formula,

AB = $\sqrt{(6-3)^2 + (7-4)^2} = 3\sqrt{2}$ units

BC = $\sqrt{(9-6)^2 + (4-7)^2} = 3\sqrt{2}$ units

CD = $\sqrt{(6-9)^2 + (1-4)^2} = 3\sqrt{2}$ units

DA = $\sqrt{(3-6)^2 + (4-1)^2} = 3\sqrt{2}$ units

AC = $\sqrt{(9-3)^2+(4-4)^2} = 6 \text{ units}$

BD = $\sqrt{(6-6)^2+(1-7)^2} = 6 \text{ units}$

As sides AB = BC = CD = DA, and diagonals AC and BD are equal, so ABCD is a square.

Now as diagonals of a square bisect each other, so midpoint of the diagonal gives the position of Anjali to sit in the middle of the four students.

Here diagonal is AC or BD.

So, mid-point of AC = $\left(\frac{3+9}{2}, \frac{4+4}{2}\right)$ = (6, 4)

So, position of Anjali is (6, 4).

ii. Position of Sita is at point A i.e. (3, 4) and Position of Anita is at point D i.e. (6, 1).

So, distance between Sita and Anita, AD =$\sqrt{(6-3)^2+(1-4)^2} = 3\sqrt{2} \text{ units}$

iii. Now, Gita is at position B and as BA and BC are equal and equidistant from point B.

So, we can say Sita and Rita are the two students who are equidistant from Gita.

62. (-3, 5), (3, 1), (0, 3), (-1, -4)

Let A $\rightarrow$ (-3, 5), B $\rightarrow$ (3, 1), C $\rightarrow$ (0, 3) and D $\rightarrow$ (-1, -4)

Then, $AB = \sqrt{(3-(-3))^2+(1-5)^2} = \sqrt{(6)^2+(-4)^2}$

$= \sqrt{36+16} = \sqrt{52} = 2\sqrt{13}$

$BC = \sqrt{(0-3)^2+(3-1)^2} = \sqrt{9+4} = \sqrt{13}$

$CD = \sqrt{(-1-0)^2+(-4-3)^2} = \sqrt{1+49} = \sqrt{50}$

$DA = \sqrt{[(-3)-(-1)]^2+[5-(-4)]^2} = \sqrt{4+81} = \sqrt{85}$

$AC = \sqrt{[0-(-3)]^2+(3-5)^2} = \sqrt{13}$

$BD = \sqrt{(-1-3)^2+(-4-1)^2} = \sqrt{41}$

We see that BC + AC = AB

Hence, the points A, B and C are collinear.

So, ABCD is not a quadrilateral.

63. Let point P(x, 0) be a point on x–axis, and A be the point (7, –4).

So, AP = $2\sqrt{5}$ [Given]

⇒ $AP^2 = 4 \times 5 = 20$

⇒ $(x-7)^2 + [0-(-4)]^2 = 20$

⇒ $x^2 + 49 - 14x + 16 = 20$

⇒ $x^2 - 14x - 20 + 65 = 0$

⇒ $x^2 - 14x + 45 = 0$

⇒ $x^2 - 9x - 5x + 45 = 0$

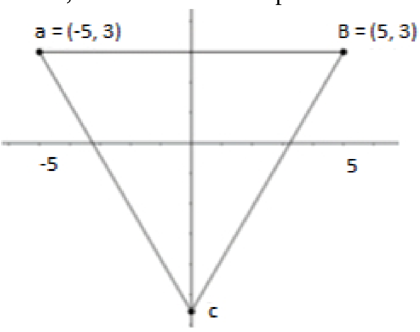
⇒ x(x – 9) –5(x – 9) = 0

⇒ (x – 9)(x – 5) = 0

⇒ x – 9 = 0 or x – 5 = 0

⇒ x = 9 or x = 5

Hence, there are two such points on x–axis whose distance from (7, –4) is $2\sqrt{5}$. Hence, required points are (9, 0), (5, 0).

64. 

Let the third vertex be (x, y)

A(-5, 3) B(5, 3) C(x, y)

AB = 10 = AC

$AC^2 = 100$

$(-5 - x)^2 + (3 - y)^2 = (5 - x)^2 + (3 - y)$

20x = 0

x = 0

$(3 - y)^2 = 75$

3 - y = $\pm 5\sqrt{3}$

y = 3 - $5\sqrt{3}$

y = -5.5

The coordinates of the third vertex are (0, -5.5)

65. i. The positions of the four friends are A = (3,4), B = (6,7) and C = (9,4), D = (6,1)

ii. $\Rightarrow AB = \sqrt{(6-3)^2 + (7-4)^2} = \sqrt{(3)^2 + (3)^2} = 3\sqrt{2}$

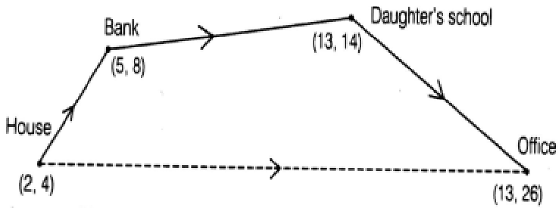
$\Rightarrow BC = \sqrt{(9-6)^2 + (4-7)^2} = \sqrt{(3)^2 + (-3)^2} = 3\sqrt{2}$

$\Rightarrow \text{CD} = \sqrt{(6-9)^2 + (1-4)^2} = \sqrt{(-3)^2 + (-3)^2} = 3\sqrt{2}$

$\Rightarrow AD = \sqrt{(6-3)^2 + (1-4)^2} = \sqrt{(3)^2 + (-3)^2} = 3\sqrt{2}$

iii. As all the sides are equal the ABCD is square. Jarina is correct.

66.



i. Distance between house and bank $= \sqrt{(5-2)^2 + (8-4)^2}$

$= \sqrt{(3)^2 + (4)^2} = \sqrt{9+16} = \sqrt{25} = 5$

ii. Distance between bank and daughter's school $= \sqrt{(13-5)^2 + (14-8)^2}$

$= \sqrt{(8)^2 + (6)^2} = \sqrt{64+36} = \sqrt{100} = 10$

iii. Distance between daughter's school and office $= \sqrt{(13-13)^2 + (26-14)^2}$

$= \sqrt{0 + (12)^2} = 12$

Total distance (House + Bank + School + Office) travelled = 5 + 10 + 12 = 27 km

Distance between house to office $= \sqrt{(13-2)^2 + (26-4)^2} = \sqrt{(11)^2 + (22)^2}$

$= \sqrt{121 + 484} = \sqrt{605}$ = 24.59 = 24.6 km

So, Extra distance travelled by Ayush in reaching his office = 27 - 24.6 = 2.4 km

67. PQ = QR $\Rightarrow$ $PQ^2 = QR^2$

$(5 - 0)^2 + (-3 - 1)^2 = (x - 0)^2 + (6 - 1)^2$

$x^2 = 16$

x = 4, -4

$\Rightarrow$ R(4, 6) or (-4, 6)

If R(4, 6), PR = $\sqrt{(5-4)^2 + (-3-6)^2}$ = $\sqrt{82}$

If R(-4, 6), PR = $\sqrt{(5+4)^2 + (-3-6)^2}$ = $\sqrt{162}$ or $9\sqrt{2}$

68. Distance between the points P(x, 2) and Q(3, - 6) is 10.

Using distance formula PQ = 10

$\Rightarrow \sqrt{(x-3)^2 + \{2-(-6)\}^2}$ = 10

$\Rightarrow \sqrt{x^2 + 3^2 - 2 \times 3 \times x + (2+6)^2}$ = 10

$\Rightarrow \sqrt{x^2 + 9 - 6x + 8^2}$ = 10

$\Rightarrow \sqrt{x^2 + 9 - 6x + 64}$ = 10

$\Rightarrow \sqrt{x^2 - 6x + 73}$ = 10

Squaring both sides, we get

$x^2 - 6x + 73 = 100$

$\Rightarrow x^2 - 6x + 73 - 100 = 0$

$\Rightarrow x^2 - 6x - 27 = 0$

$\Rightarrow x^2 - 9x + 3x - 27 = 0$

$\Rightarrow x(x - 9) + 3(x - 9) = 0$

$\Rightarrow (x - 9)(x + 3) = 0$

$\Rightarrow$ either x-9 = 0 or x + 3 = 0

$\Rightarrow$ x = 9 or x = - 3

Ignoring x = - 3 as it is given that x is a positive integer.

Thus, only solution is x = 9.

69. Distance between P(2, -3) and Q(x, 5) = 10

$\Rightarrow \sqrt{\left(x_2-x_1\right)^2+\left(y_2-y_1\right)^2}$

$\Rightarrow \sqrt{(x-2)^2+(5+3)^2}=10$

squaring,

$\Rightarrow (x - 2)^2 + (8)^2 = (10)^2$

$\Rightarrow x^2 - 4x + 4 + 64 = 100$

$\Rightarrow x^2 - 4x + 68 - 100 = 0$

$\Rightarrow x^2 - 4x - 32 = 0$

$\Rightarrow x^2 - 8x + 4x - 32 = 0$

$\Rightarrow$ x(x - 8) + 4(x - 8) = 0

$\Rightarrow$ (x - 8) (x + 4) = 0

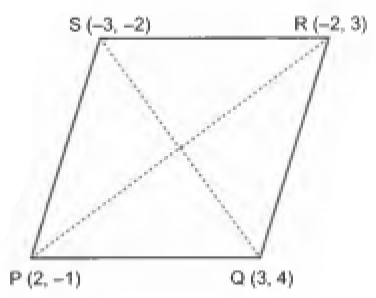
Either x - 8 = 0, then x = 8

or x + 4 = 0, then x = -6

$\therefore$ x = 8, -4

70. The given points are P (2, -1), Q (3,4), R(-2,3) and S (-3, -2).

We have,



$PQ=\sqrt{(3-2)^2+(4+1)^2}=\sqrt{1^2+5^2}=\sqrt{26}$ units

$QR=\sqrt{(-2-3)^2+(3-4)^2}=\sqrt{25+1}=\sqrt{26}$ units

$RS=\sqrt{(-3+2)^2+(-2-3)^2}=\sqrt{1+25}=\sqrt{26}$ units

$SP=\sqrt{(-3-2)^2+(-2+1)^2}=\sqrt{26}$units

and $QS=\sqrt{(-3-3)^2+(-2-4)^2}=\sqrt{36+36}=6\sqrt{2}$ units

$\therefore$ PQ = QR = RS = SP = $\sqrt{26}$ units

$PR=\sqrt{(-2-2)^2+(3+1)^2}=\sqrt{16+16}=\sqrt{32}$

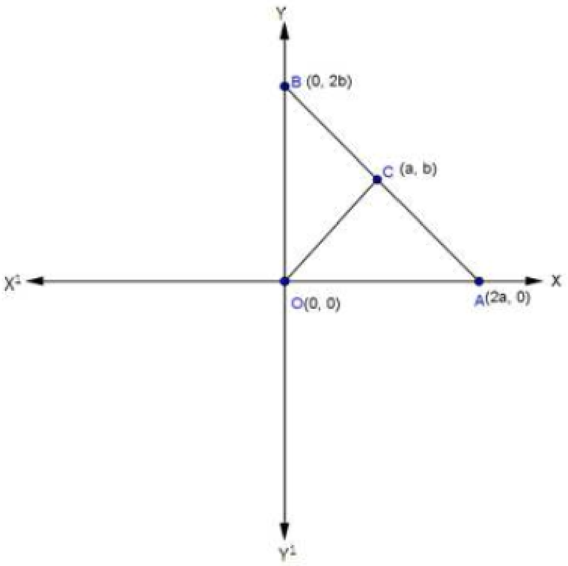
Therefore, $PR \neq QS$

This means that PQRS is a quadrilateral whose sides are equal but diagonals are not equal.

Thus, PQRS is a rhombus but not a square.

Now, Area of rhombus PQRS $=\frac{1}{2} \times$ (Product of lengths of diagonals)

$=\frac{1}{2} \times(PR \times QS)=\left(\frac{1}{2} \times 4\sqrt{2} \times 6\sqrt{2}\right)$ sq. units = 24 sq. units

71. 

Given a right triangle BOA with vertices B(0, 2b), O(0, 0) and A(2a, 0)

Since, C is the mid-point of AB

$\therefore$ Coordinates of C are $\left(\frac{2a+0}{2}, \frac{0+2b}{2}\right)$

= (a, b)

Now, $CO = \sqrt{(a-0)^2 + (b-0)^2} = \sqrt{a^2 + b^2}$

$CA = \sqrt{(2a-a)^2 + (0-b)^2} = \sqrt{a^2 + b^2}$

$CB = \sqrt{(a-0)^2 + (b-2b)^2} = \sqrt{a^2 + b^2}$

Since, CO = CA = CB.

$\therefore$ C is equidistant from O, A and B.

72. Let P(x, y) be equidistant from the points A(-5, 4) and B(-1, 6).

Now,

AP = BP

$\Rightarrow AP^2 = BP^2$

$\Rightarrow (x + 5)^2 + (y - 4)^2 = (x + 1)^2 + (y - 6)^2$

$\Rightarrow x^2 + 25 + 10x + y^2 + 16 - 8y = x^2 + 1 + 2x + y^2 + 36 - 12y$

$\Rightarrow$ 10x + 41 - 8y = 2x + 37 - 12y

$\Rightarrow$ 8x + 4y + 4 = 0

$\Rightarrow$ 2x + y + 1 = 0

Thus, all the points which lie on line 2x + y + 1 =0 are equidistant from A and B.

73. The distances of P (x, y) from A (5,1) and B (-1,5) are equal, then we have to prove that 3x = 2y.

PA = PB

$\therefore PA^2 = PB^2$

By distance formula,

$(5 - x)^2 + (1 - y)^2 = (-1 - x)^2 + (5 - y)^2$

$\Rightarrow 25 - 10x + x^2 + 1 - 2y + y^2 = 1 + 2x + x^2 + 25 - 10y + y^2$

$\Rightarrow$ -10x - 2y = 2x - 10y

$\Rightarrow$ 8y = 12x

$\Rightarrow$4(2y)= 4(3x)

$\Rightarrow$3x = 2y

Hence Proved.

74. Let Home represented by point H(4, 5), Library by point L(-1, 3), Skate Park by point P(3, 0) and School by S(4, 2).

i. Distance between Home and School, HS = $\sqrt{(4-4)^2 + (2-5)^2} = 3\ metres$

ii. Now, HL = $\sqrt{(-1-4)^2 + (3-5)^2} = \sqrt{25+4} = \sqrt{29}$

LS = $\sqrt{[4-(-1)]^2 + (2-3)^2} = \sqrt{25+1} = \sqrt{26}$

Thus, HL + LS = $\sqrt{29}+\sqrt{26}=10.48\ metres$

So, extra distance covered by Ramesh is = HL + LS - HS = 10.48 - 3 = 7.48 metres

iii. Now, HP = $\sqrt{(3-4)^2+(0-5)^2}=\sqrt{1+25}=\sqrt{26}$

PS = $\sqrt{[4-3]^2+(2-0)^2}=\sqrt{1+4}=\sqrt{5}$

Thus, HP + PS = $\sqrt{26}+\sqrt{5}=7.33\ metres$

So, extra distance covered by Ramesh is = HP + PS - HS = 7.33 - 3 = 4.33 metres

75. The distance d between two points (x_1, y_1) and (x_2, y_2) is given by the formula

$d=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

In a right angled triangle the angle opposite the hypotenuse subtends an angle of 90^o.

Here let the given points be A(0,100), B(10,0). Let the origin be denoted by O(0,0).

Let us find the distance between all the pairs of points

$AB=\sqrt{(0-10)^2+(100-0)^2}$

$=\sqrt{(-10)^2+(100)^2}$

$=\sqrt{(-10)^2+(100)^2}$

$=\sqrt{100+10000}$

$AB=\sqrt{10100}$

$AO=\sqrt{(0-0)^2+(100-0)^2}$

$=\sqrt{(0)^2+(100)^2}$

$AO=\sqrt{10000}$

$BO=\sqrt{(10-0)^2+(0-0)^2}$

$=\sqrt{(10)^2+(0)^2}$

$BO=\sqrt{100}$

Here we can see that $AB^2 = AO^2 + BO^2$.

So, $\triangle$AOB is a right angled triangle with 'AB' being the hypotenuse. So the angle opposite it has to be 90^o. This angle is nothing but the angle subtended by the line segment 'AB' at the origin.

Hence the angle subtended at the origin by the given line segment is 90^o.