Solution

COORDINATE GEOMETRY WS 4

Class 10 - Mathematics

1. 1

   Explanation:

   It is given that A(0, 2) is equidistant from the points B(3, p) and C(p, 5).

   ∴ AB = AC

   $\Rightarrow \sqrt{(3-0)^2 + (p-2)^2} = \sqrt{(p-0)^2 + (5-2)^2}$ (Distance formula)

   Squaring on both sides, we get

   $9 + p^2 - 4p + 4 = p^2 + 9$

   $\Rightarrow -4p + 4 = 0$

   $\Rightarrow p = 1$

   Thus, the values of p is 1.

2. 39

   Explanation:

   Required distance

   $= \sqrt{(36-0)^2 + (15-0)^2}$

   $= \sqrt{(36)^2 + (15)^2}$

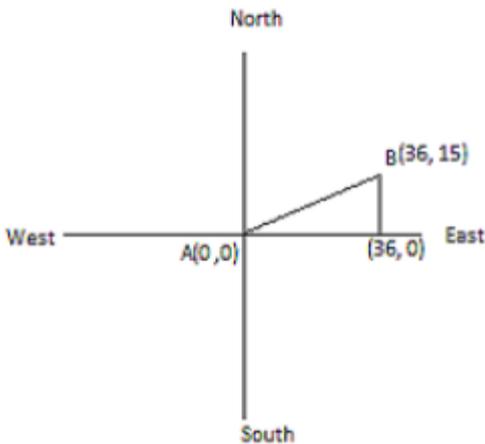 $= \sqrt{1296 + 225} = \sqrt{1521}$

   = 39

3. 39

   Explanation:

   Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

   d = $\sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39$

   Town B is located at 36 km east and 15 km north of town A. So, the location of town A and B can be shown as:

   Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

   To find the distance between them, we use Distance formula:

   d = $\sqrt{(36-0)^2 + (15-0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521}$ = 39 km

4. -2

   Explanation:

   Let the required point on y-axis be P(0, y) which is equidistant from A(5, -2) and B(-3, 2)

   According to question, PA = PB

   $\Rightarrow PA^2 = PB^2$

   $\Rightarrow (5 - 0)^2 + (-2 - y)^2 = (-3 - 0)^2 + (2 - y)^2$ [By using distance formula]

   $\Rightarrow 25 + 4 + y^2 + 4y = 9 + 4 + y^2 - 4y$

   $\Rightarrow 29 - 13 = -8y$

   $\Rightarrow 16 = -8y$
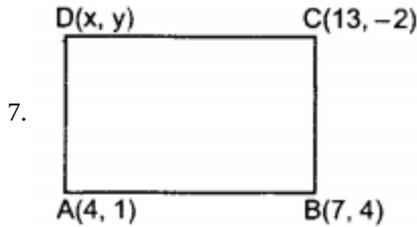
$\Rightarrow$ y = -2

∴ Required point is (0, -2)

5. 13

Explanation:

The given point is A(5, -12) and let O(0, 0) be the origin

Then, $AO = \sqrt{(5-0)^2 + (-12-0)^2}$

$= \sqrt{5^2 + (-12)^2} = \sqrt{25 + 144} = \sqrt{169}$

= 13 units

6. (a) - (ii), (b) - (iv), (c) - (i), (d) - (iii)

7.

ABCD is a rectangle, and we know that diagonal of the rectangle are equal and bisect each other.

Let O be the mid point where diagonals intersects.

$\Rightarrow$ Mid point of AC = Mid point of BD

∴ $\left(\frac{13+4}{2}, \frac{1-2}{2}\right) = \left(\frac{x+7}{2}, \frac{y+4}{2}\right)$

∴ $\left(\frac{17}{2}, \frac{-1}{2}\right) = \left(\frac{x+7}{2}, \frac{y+4}{2}\right)$

$\Rightarrow \frac{x+7}{2} = \frac{17}{2}, \frac{y+4}{2} = -\frac{1}{2}$

$\Rightarrow$ x+7 = 17, y+4 = -1

$\Rightarrow$ x = 17-7 , y = -1 - 4

$\Rightarrow$ x = 10 and y = - 5.

Hence, fourth vertex is D(10, - 5).

8. Let P(x, 0) be the point on the x - axis. Then, as per the question, we have

AP = 10

$\Rightarrow \sqrt{(x-11)^2 + (0+8)^2} = 10$

Squaring both sides,

$\Rightarrow$ (x - 11)^2 + 8^2 = 100

$\Rightarrow$ (x - 11)^2 = 100 - 64 = 36

$\Rightarrow$ x - 11 = ± 6

$\Rightarrow$ x = 11± 6

$\Rightarrow$ x = 11 - 6, 11 + 6

$\Rightarrow$ x = 5, 17

Hence, the points on the x - axis are (5, 0) and (17, 0).

9. PA = PB (Given)

∴ PA^2 = PB^2

$\Rightarrow$ (5 - x)^2 + (1 - y)^2 = (1 - x)^2 + (5 - y)^2

$\Rightarrow$ 25 + x^2 - 10x + 1 + y^2 - 2y = 1 + x^2 - 2x + 25 + y^2 - 10y

$\Rightarrow$ -8x = -10y + 2y

$\Rightarrow$ -8x = -8y

$\Rightarrow$ x = y

10. $P(\sqrt{2}, \sqrt{2}), Q(-\sqrt{2}, -\sqrt{2})$ and $R(-\sqrt{6}, \sqrt{6})$ are the vertices of ΔPQR.

Now,

$PQ = \sqrt{(-\sqrt{2}-\sqrt{2})^2 + (-\sqrt{2}-\sqrt{2})^2} = \sqrt{(-2\sqrt{2})^2 + (-2\sqrt{2})^2} = = \sqrt{8+8} = \sqrt{16} = 4\ units$

$QR = \sqrt{(-\sqrt{6}+\sqrt{2})^2 + (\sqrt{6}+\sqrt{2})^2} = \sqrt{6+2-2\sqrt{12}+6+2+2\sqrt{12}} = \sqrt{16} = 4\ units$

$PR = \sqrt{(-\sqrt{6}-\sqrt{2})^2 + (\sqrt{6}-\sqrt{2})^2} = \sqrt{6+2+2\sqrt{12}+6+2-2\sqrt{12}} = \sqrt{16} = 4\ units$

Since PQ = QR = PR, ΔPQR is an equilateral triangle.
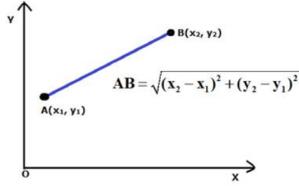
11. According to the question,

Distance between AB = 9

Coordinates of A (- 3, - 14)

Coordinates of B (a, - 5)

Distance between two points (x_1, y_1) and (x_2, y_2);

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

So,

AB = $\sqrt{(a + 3)^2 + (-5 + 14)^2}$ = 9

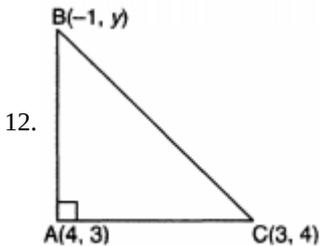AB = $\sqrt{(a + 3)^2 + (9)^2}$ = 9

On squaring both the sides,

We get;

$(a + 3)^2 + 81 = 81$

$(a + 3)^2 = 0$

a = - 3

Hence, the value of a is - 3.

12.

If A(4, 3), B (-1, y), and C(3,4) are the vertices of a right triangle ABC, right angled at A, then, we have to find the value of y.

By Pythagoras theorm,

$AB^2 + AC^2 = BC^2$

or, $(4 + 1)^2 + (3 - y)^2 + (3 - 4)^2 + (4 - 3)^2 = (3 + 1)^2 + (4 - y)^2$

or, $(5)^2 + (3 - y)^2 + (-1)^2 + (1)^2 = (4)^2 + (4 - y)^2$

or, $25 + 9 - 6y + y^2 + 1 + 1 = 16 + 16 - 8y + y^2$

or, $36 + 2y - 32 = 0$

or, y = - 2

13. We have to find a point on the y-axis which is equidistant from the points A(6, 5) and B (- 4, 3).

We know that a point on y-axis is of the form (0, y). So, let the required point be P (0, y).

Then,

PA = PB

$\Rightarrow \sqrt{(0 - 6)^2 + (y - 5)^2} = \sqrt{(0 + 4)^2 + (y - 3)^2}$

$\Rightarrow 36 + (y - 5)^2 = 16 + (y - 3)^2$

$\Rightarrow 36 + y^2 - 10y + 25 = 16 + y^2 - 6y + 9$

$\Rightarrow 4y = 36$

$\Rightarrow y = 9$

So, the required point is (0, 9).

14. We have,

PQ = 10

$\Rightarrow PQ^2 = 100$

$\Rightarrow (9 - x)^2 + (10 - 4)^2 = 100$

$\Rightarrow (9 - x)^2 + 6^2 = 100$

$\Rightarrow (9 - x)^2 + 36 = 100$

$\Rightarrow (9 - x)^2 = 64$

$\Rightarrow 9 - x = \pm 8$

$\Rightarrow 9 - x = 8$ or $9 - x = -8$

$\Rightarrow x = 1$ or $x = 17$

Hence, the required values of x are 1 and 17.

15. The point P on x-axis will have its ordinate = 0

Let the coordinates of point P be (x, 0)

Let the given points be A(–2, 5) and B(2, –3)

Then PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (x + 2)^2 + (0 - 5)^2 = (x - 2)^2 + (0 + 3)^2$

$\Rightarrow x^2 + 4 + 4x + 5^2 = x^2 + 4 - 4x + 3^2$

$\Rightarrow x^2 + 4x + 4 + 25 = x^2 + 4 - 4x + 9$

$\Rightarrow 4x + 4x = 9 - 25$

$\Rightarrow 8x = -16$

$\Rightarrow x = \frac{-16}{8} = -2$

$\therefore$ The point equidistant from given points is (–2, 0)

16. Distance between the given points

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(0 - 5\sin 60°)^2 + (5\sin 30° - 0)^2}$

$= \sqrt{\left(-5 \times \frac{\sqrt{3}}{2}\right)^2 + \left[5\left(\frac{1}{2}\right)\right]^2}$

$= \sqrt{\left(-5 \times \frac{\sqrt{3}}{2}\right)^2 + \left[5\left(\frac{1}{2}\right)\right]^2} \left\{ \because \sin 60° = \frac{\sqrt{3}}{2}, \ \sin 30° = \frac{1}{2} \right\}$

$= \sqrt{\frac{25 \times 3}{4} + \frac{25 \times 1}{4}} = \sqrt{\frac{75}{4} + \frac{25}{4}}$

$= \sqrt{\frac{100}{4}} = \sqrt{25} = 5 \text{ units}$

17. Distance between (x, 0) and (0, 3) = 5

$\Rightarrow \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = 5$

$\Rightarrow \sqrt{(0 - x)^2 + (3 - 0)^2} = 5$

$\Rightarrow \sqrt{x^2 + 9} = 5$

Squaring,

$x^2 + 9 = 25 \Rightarrow x^2 = 25 - 9 = 16$

$\Rightarrow x^2 - 16 = 0 \Rightarrow (x + 4)(x - 4) = 0$

Either x + 4 = 0, then x = -4

or x - 4 = 0, then x = 4

Hence x = 4, -4

18. (4, 5), (7, 6), (4, 3), (1, 2)

Let A → (4, 5), B → (7, 6), C → (4, 3) and D → (1, 2)

Then, $AB = \sqrt{(7 - 4)^2 + (6 - 5)^2}$

$= \sqrt{(3)^2 + (1)^2} = \sqrt{9 + 1} = \sqrt{10}$

$BC = \sqrt{(4 - 7)^2 + (3 - 6)^2}$

$\sqrt{(-3)^2 + (-3)^2} = \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$CD = \sqrt{(1 - 4)^2 + (2 - 3)^2}$

$= \sqrt{(-3)^2 + (-1)^2} = \sqrt{9 + 1} = \sqrt{10}$

$DA = \sqrt{(4 - 1)^2 + (5 - 2)^2}$

$= \sqrt{9 + 9} = \sqrt{18} = 3\sqrt{2}$

$AC = \sqrt{(4 - 4)^2 + (3 - 5)^2} = 2$

$BD = \sqrt{(1-7)^2 + (2-6)^2}$

$= \sqrt{36 + 16} = \sqrt{52}$

We see that

AB = CD, opposite sides are equal

BC = DA

and AC $\neq$ BD ..... Diagonals are unequal

Hence, the quadrilateral ABCD is a parallelogram.

19. The given points are A (7,13) and B (10, 9)

    Then, $x_1 = 7$, $y_1 = 13$ and $x_2 = 10$, $y_2 = 9$.

    Therefore, by the distance formula, we have,

    $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

    $= \sqrt{(10-7)^2 + (9-13)^2} = \sqrt{3^2 + (-4)^2}$

    $= \sqrt{9 + 16} = \sqrt{25} = 5\ units.$

20. Let A(-2, 3), B(8, 3) and C(6, 7) be the vertices of a given triangle. Then,

    $(AB)^2 = (8+2)^2 + (3-3)^2 = (10)^2 + 0^2 = 100$

    $(BC)^2 = (8-6)^2 + (3-7)^2 = 2^2 + 4^2 = 4 + 16 = 20$

    $(AC)^2 = (-2-6)^2 + (3-7)^2 = 8^2 + 4^2 = 64 + 16 = 80$

    Clearly, $(AB)^2 = (AC)^2 + (BC)^2$ [pythagoras theorem]

    Therefore, A(-2,3), B(8,3) and C(6,7) are the vertices of right-angled triangle.

21. Mid-point of AC = $\left(\frac{1}{2}, 1\right)$

    Mid-point of BD = $\left(\frac{1}{2}, 1\right)$

    Since Mid-point of AC = BD, therefore ABCD is a parallelogram.

    Now AC = $\sqrt{9+4} = \sqrt{13}$

    and BD = $\sqrt{25+0} = \sqrt{25} = 5$

    $\therefore$ AC $\neq$ BD therefore ABCD is not a rectangle.

22. In an equilateral triangle, the height 'h' is given by

    $h = \frac{\sqrt{3}(\text{ Side of the equilateral triangle })}{2}$

    Here it is given that 'PQ' forms the base of two equilateral triangle whose side measure '2a' units.

    The height of these two equilateral triangles has got to be

    $h = \frac{\sqrt{3}(\text{ Side of the equilateral triangle })}{2}$

    $= \frac{\sqrt{3}(2a)}{2}$

    $h = a\sqrt{3}$

    In an equilateral triangle, the height drawn from one vertex meets the midpoint of the side opposite this vertex. So here we have 'PQ' being the base lying along the y-axis with its midpoint at the origin, that is at (0, 0).

    So the vertices 'R' and 'R'' will lie perpendicular to the y-axis on either side of the origin at a distance of '$a\sqrt{3}$'. Hence the coordinates of R and R' are

    $R(a\sqrt{3}, 0)$ and $R'(-a\sqrt{3}, 0)$

23. Let P(x, y) be equidistant from the points A(7, 1) and B(3, 5)

    AP = BP (Given)

    $\Rightarrow AP^2 = BP^2$

    $\Rightarrow (x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$

    $\Rightarrow x^2 + 49 - 14x + y^2 + 1 - 2y = x^2 + 9 - 6x + y^2 + 25 - 10y$

    $\Rightarrow 49 - 14x + 1 - 2y = 9 - 6x + 25 - 10y$

    $\Rightarrow -14x + 6x - 2y + 10y = 34 - 50$

    $\Rightarrow -8x + 8y = -16$

    $\Rightarrow x - y = 2$

24. It is given that P (x,y) is equidistant from A(3, 6) and B(-3, 4).

    Using Distance formula, we can write

    PA = PB

$$\sqrt{(x-3)^2 + (y-6)^2} = \sqrt{[x-(-3)]^2 + (y-4)^2}$$
$$\Rightarrow \sqrt{x^2 + 9 - 6x + y^2 + 36 - 12y} = \sqrt{x^2 + 9 + 6x + y^2 + 16 - 8y}$$

Squaring both sides, we get

$\Rightarrow x^2 + 9 - 6x + y^2 + 36 - 12y = x^2 + 9 + 6x + y^2 + 16 - 8y$

$\Rightarrow -6x - 12y + 45 = 6x - 8y + 25$

$\Rightarrow 12x + 4y = 20$

$\Rightarrow 3x + y = 5$

25. Let us denote the points A(20, 3), B(19, 8) and C(2, –9) are all equidistant from the point (7, 3) by P.

Then,

$PA = \sqrt{(7-20)^2 + (3-3)^2} = \sqrt{169} = 13\ units$

$PB = \sqrt{(7-19)^2 + (3-8)^2} = \sqrt{144+25} = \sqrt{169} = 13\ units$

$PC = \sqrt{(7-2)^2 + (3+9)^2} = \sqrt{25+144} = \sqrt{169} = 13\ units$

Since PA = PB = PC

$\therefore$ (20, 3), (19, 8) and (2 – 9) are all equidistant from the point (7, 3)

26. As given Q(0, 1) is equidistant from P(5, -3) and R(x, 6)

$\Rightarrow$ PQ = QR

$\Rightarrow \sqrt{(0-5)^2 + (1-(-3))^2} = \sqrt{(x-0)^2 + (1-6)^2}$ [By using distance formula $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$]

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow 41 = x^2 + 25$

$\Rightarrow x^2 = 16$

$\Rightarrow x = 4$

27. PQ = RQ

$\Rightarrow PQ^2 = RQ^2$

$\Rightarrow (0 - 5)^2 + [1 - (-3)]^2 = (0 - x)^2 + (1 - 6)^2$

$\Rightarrow 25 + 16 = x^2 + 25$

$\Rightarrow x^2 = 16$

$\Rightarrow x = \pm 4$

$\therefore$ R $\rightarrow (\pm 4, 6)$

$QR = \sqrt{(0 \pm 4)^2 + (1-6)^2} = \sqrt{41}$

$PR = \sqrt{(\pm 4 - 5)^2 + \{6 - (-3)\}^2}$

$= \sqrt{(4-5)^2 + 81}$ or $\sqrt{(-4-5)^2 + 81}$

$= \sqrt{82}$ or $9\sqrt{2}$.

28. Let ABC be the triangle P(5, -2), Q(6, 4), R(7, -2)

By distance formula,

$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ = \sqrt{(6-5)^2 + (4+2)^2} = \sqrt{1+36}$

$PQ = \sqrt{37}$

$QR = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{1+36}$

$QR = \sqrt{37}$

$PR = \sqrt{(7-5)^2 + (-2+2)^2} = \sqrt{4} = 2$

$\because$ PQ = QR = $\sqrt{37}$

$\therefore \triangle$PQR is an isosceles triangle.

29. We have,

AB = 10

$\Rightarrow AB^2 = 100$

$\Rightarrow (9 - x)^2 + (8 - 2)^2 = 100$

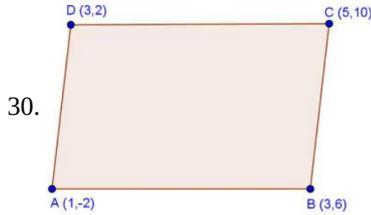$\Rightarrow$ (9 - x)^2 + 6^2 =100

$\Rightarrow$ (9 - x)^2 + 36 = 100

$\Rightarrow$ (9 - x)^2 =64

$\Rightarrow$ 9 - x = $\pm$8

$\Rightarrow$ 9 - x = 8 or 9 - x = -8

$\Rightarrow$ x = 1 or x = 17

Hence, the required values of x are 1 and 17.

30.

Let A(1, -2), B(3, 6), C(5, 10), D(3, 2) be the given points

$AB = \sqrt{(3-1)^2 + (6+2)^2}$

$\Rightarrow \quad AB = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow AB = \sqrt{4 + 64}$

$\Rightarrow \quad AB = \sqrt{68}$

$CD = \sqrt{(5-3)^2 + (10-2)^2}$

$\Rightarrow \quad CD = \sqrt{(2)^2 + (8)^2}$

$\Rightarrow \quad CD = \sqrt{4 + 64}$

$\Rightarrow \quad CD = \sqrt{68}$

$AD = \sqrt{(3-1)^2 + (2+2)^2}$

$\Rightarrow \quad AD = \sqrt{(2)^2 + (4)^2}$

$\Rightarrow \quad AD = \sqrt{4 + 16}$

$\Rightarrow \quad AD = \sqrt{20}$

$BC = \sqrt{(5-3)^2 + (10-6)^2}$

$\Rightarrow \quad BC = \sqrt{(2)^2 + (4)^2}$

$\Rightarrow \quad BC = \sqrt{4 + 16}$

$\Rightarrow \quad BC = \sqrt{20}$

$\therefore$ AB = CD and AD = BC

Since opposite sides of a parallelogram are equal

Hence, ABCD is a parallelogram.

31. Let the given points be A(5, -2) and B(-3, 2) and let the required point be P(x, 0). Then,

PA = PB $\Rightarrow$ PA^2 = PB^2

$\Rightarrow$ (x - 5)^2 + (0 + 2)^2= (x + 3)^2 + (0 - 2)^2

$\Rightarrow$ (x - 5)^2 + 4 = (x + 3)^2 + 4

$\Rightarrow$ (x - 5)^2 - (x + 3)^2 =0

$\Rightarrow$ (x^2 -10x + 25) - (x^2 + 6x + 9)=0

$\Rightarrow$ -16x + 16 =0

$\Rightarrow$ 16x = 16 $\Rightarrow$ x = 1.

Hence, the required point is P(1, 0).

32. Given: A(-2,2), B(5,2) and C(k,8) are the vertices of a right angle triangle ABC with $\angle B = 90°$.

Using distance formula : $D = \sqrt{(c-a)^2 + (d-b)^2}$,

Where D is distance between (a, b) and (c, d).

$AB = \sqrt{(5-(-2))^2 + (2-2)^2} = \sqrt{(7)^2 + 0^2}$ = 7 units

$AC = \sqrt{(k-(-2))^2 + (8-2)^2} = \sqrt{(k+2)^2 + 6^2} = \sqrt{(k+2)^2 + 36}$ units

$BC = \sqrt{((k-5)^2 + (8-2)^2} = \sqrt{(k-5)^2 + 6^2} = \sqrt{(k-5)^2 + 36}$ units

Using Pythagoras theorem,

AC^2 = AB^2 + BC^2 [AC is opposite to angle B is the Hypotenuse of triangle]

$\sqrt{((k+2)^2 + 36)}^2 = 7^2 + \sqrt{((k-5)^2 + 36)}^2$

$(k + 2)^2 + 36 = 49 + (k - 5)^2 + 36$

$k^2 + 2^2 + 2(k)(2) + 36 = 49 + k^2 + 5^2 - 2(k)(5) + 36$

$k^2 + 4 + 4k + 36 = 49 + k^2 + 25 - 10k + 36$
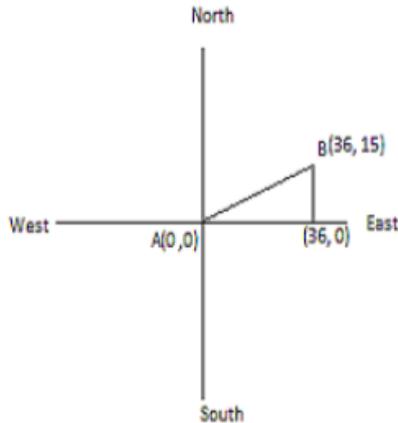
$4k + 40 = 110 - 10k$

$4k + 10k = 110 - 40$

$14k = 70$

k = 5 [Divide both sides by 14]

33. Applying Distance Formula to find distance between points (0, 0) and (36, 15), we get

$$d = \sqrt{(36 - 0)^2 + (15 - 0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39$$

Town B is located at 36 km east and 15 km north of town A. So, the location of town A and B can be shown as:

Clearly, the coordinates of point A are (0, 0) and coordinates of point B are (36, 15).

To find the distance between them, we use Distance formula:

$$d = \sqrt{(36 - 0)^2 + (15 - 0)^2} = \sqrt{(36)^2 + (15)^2} = \sqrt{1296 + 225} = \sqrt{1521} = 39 \text{ km}$$

34. Let P(6, -6) be the given point and O(0, 0) be the origin.

Then, $OP = \sqrt{(6 - 0)^2 + (-6 - 0)^2} = \sqrt{6^2 + (-6)^2}$

$= \sqrt{36 + 36} = \sqrt{72} = \sqrt{36 \times 2} = 6\sqrt{2} \, units.$

35. The given points are A(-6, -4) and B(9, -12)

Then, $(x_1 = -6, y_1 = -4)$ and $(x_2 = 9, y_2 = -12)$

$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(6 + 9)^2 + (-12 + 4)^2} = \sqrt{(15)^2 + (-8)^2}$

$= \sqrt{225 + 64} = \sqrt{289} = 17 \, units$

36. It is being given that the two vertices of $\triangle ABC$ are A{6, 4} and B (-2, 2). Let its third vertex be C(x, y).

Then, the coordinates of the centroid of $\triangle ABC$ are

$G\left(\frac{6+(-2)+x}{3}, \frac{4+2+y}{3}\right)$ i.e., $G\left(\frac{4+x}{3}, \frac{6+y}{3}\right)$

But, it is given that the centroid of $\triangle ABC$ is G(3, 4).

$\therefore \quad \frac{4+x}{3} = 3$ and $\frac{6+y}{3} = 4$

$\Rightarrow 4 + x = 9$ and $6 + y = 12$

$\Rightarrow x = 5$ and $y = 6$

Hence, the coordinates of the third vertex is C(5, 6)

37. A = (x, y) = (0,0),

B = (x_1, y_1) = (1,0) and C = (x_2, y_2) = (0,1)

The perimeter is sum of length of three sides, so first find the length of three sides and add them.

First side= AB= $\sqrt{(x_1 - x)^2 + (y_1 - y)^2} = \sqrt{(1 - 0)^2 + (0 - 0)^2} = 1$

Second side = BC = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(0 - 1)^2 + (1 - 0)^2} = \sqrt{2}$

Third side = AC = $\sqrt{(x_2 - x)^2 + (y_2 - y)^2} = \sqrt{(0 - 0)^2 + (1 - 0)^2} = 1$

Adding lengths of three sides $= 1 + 1 + \sqrt{2} = (2 + \sqrt{2}) \, units$

38. P(6, -1), Q(1, 3) and R(x, 8) are the given points.

PQ = QR

$PQ^2 = QR^2$

$\Rightarrow (6 - 1)^2 + (-1 - 3)^2 = (x - 1)^2 + (8 - 3)^2$

$\Rightarrow (5)^2 + (-4)^2 = x^2 + 1 - 2x + (5)^2$

$\Rightarrow 25 + 16 = x^2 + 1 - 2x + 25$

$\Rightarrow 41 = x^2 - 2x + 26$

$\Rightarrow x^2 - 2x + 26 - 41 = 0$

$\Rightarrow x^2 - 2x - 15 = 0$

$\Rightarrow x^2 - 5x + 3x - 15 = 0$

$\Rightarrow x(x - 5) + 3(x - 5) = 0$

$\Rightarrow (x + 3)(x - 5) = 0$

$\Rightarrow x = -3$ or $x = 5$

39. Let A(6, -1) and B(2,3) be the given point and P(x,y) be the required point.

Since point P is equidistant from points A and B, therefore,

$PA = PB \Rightarrow (PA)^2 = (PB)^2$

$\Rightarrow (x - 6)^2 + (y + 1)^2 = (2 - x)^2 + (3 - y)^2$

$\Rightarrow 36 + x^2 - 12x + y^2 + 1 + 2y = 4 + x^2 - 4x + 9 + y^2 - 6y$

$\Rightarrow -12x + 4x + 2y + 6y = 4 + 9 - 1 - 36$

$\Rightarrow -8x + 8y = -24$

$-8(x - y) = -24$

$\Rightarrow x - y = 3$

Hence, x - y = 3

40. Let us apply the distance formula to find the distances PQ, QR and PR, where

P↔(3, 2),

Q↔(–2, –3) and

R↔(2, 3)

are the given points. We have

$PQ = \sqrt{(3 + 2)^2 + (2 + 3)^2} = \sqrt{5^2 + 5^2} = \sqrt{50} = 7.07 (\text{ approx. })$

$QR = \sqrt{(-2 - 2)^2 + (-3 - 3)^2} = \sqrt{(-4)^2 + (-6)^2} = \sqrt{52} = 7.21 (\text{ approx. })$

$PR = \sqrt{(3 - 2)^2 + (2 - 3)^2} = \sqrt{1^2 + (-1)^2} = \sqrt{2} = 1.41 (\text{ approx. })$

Since the sum of any two of these distances is greater than the third distance, therefore, the points P, Q and R form a triangle.

Also, $PQ^2 + PR^2 = QR^2$, by the converse of Pythagoras theorem, we have $\angle$ P = 90°. Therefore, PQR is a right triangle.

41. Using Distance formula, we have

$10 = \sqrt{(2 - 10)^2 + (-3 - y)^2}$

$\Rightarrow 10 = \sqrt{(-8)^2 + 9 + y^2 + 6y}$

$\Rightarrow 10 = \sqrt{64 + 9 + y^2 + 6y}$

Squaring both sides, we get

$100 = 73 + y^2 + 6y$

$\Rightarrow y^2 + 6y - 27 = 0$

Solving this Quadratic equation by factorization, we can write

$\Rightarrow y^2 + 9y - 3y - 27 = 0$

$\Rightarrow y(y + 9) - 3(y + 9) = 0$

$\Rightarrow (y + 9)(y - 3) = 0$

$\Rightarrow y = 3, -9$

42. i. A(2, 2), B(5, 4), C(7, 7), D(4, 5)

ii. $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(5 - 2)^2 + (4 - 2)^2}$

$= \sqrt{3^2 + 2^2}$

$= \sqrt{9 + 4}$

$= \sqrt{13}$ units

CD $= \sqrt{(7 - 4)^2 + (7 - 5)^2}$

$= \sqrt{9^2 + 4^2}$

$= \sqrt{13}$

$\therefore$ AB = CD

AD $= \sqrt{(n - 2)^2 + (5 - 2)^2}$

$= \sqrt{2^2 + 3^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

BC $= \sqrt{(7 - 5)^2 + (7 - 4)^2}$

$= \sqrt{2^2 + 3^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

midpoint of diagonal AC = $\left(\frac{2+7}{2}, \frac{2+7}{2}\right)$

$= \left(\frac{9}{2}, \frac{9}{2}\right)$

midpoint of diagonal BD = $\left(\frac{5+4}{2}, \frac{4+5}{2}\right)$

$= \left(\frac{9}{2}, \frac{9}{2}\right)$

As opposite side of quadrilateral are equal and midpoint of its diagonal are same means diagonal bisect each other this is sufficient to say ABCD is a ‖ gm.

43. We have

AB = 10 $\Rightarrow$ AB^2 = 100

$\Rightarrow$ (11 - 3)^2 + (y + 1)^2 = 100

$\Rightarrow 8^2$ + (y + 1)^2 = 100

$\Rightarrow$ (y + 1)^2 = 100 - 64 = 36 = 6^2

$\Rightarrow$ y + 1 = ± 6

$\Rightarrow$ y + 1 = 6 or y + 1 = -6

$\Rightarrow$ y = 5 or y = -7

Hence, the required values of y are 5 and -7.

44. The given points are A(x, -1) and B(5, 3)

Also given, AB = 5

$\Rightarrow$ AB^2 = 25

$\Rightarrow$ (5 - x)^2 + (3 + 1)^2 = 25 [using distance formula]

$\Rightarrow$ 25 + x^2 - 10x + 16 = 25

$\Rightarrow$ x^2 - 10x + 16 = 0

$\Rightarrow$ x^2 - 8x - 2x + 16 = 0

$\Rightarrow$ x (x - 8) -2(x - 8) = 0

$\Rightarrow$ (x - 8) (x - 2) = 0

$\Rightarrow$ x - 8 = 0 or x - 2 = 0

$\Rightarrow$ x = 8 or x = 2

Hence, possible values of x are 8 & 2.

45. Slope of line joining (7,1) and (3,5) is given by,

$m = \frac{5-1}{3-7} = \frac{4}{-4}$ = -1

Therefore, slope of perpendicular bisector to line joining (7,1) and (3,5) = $-\frac{1}{m} = -\frac{1}{-1}$ = 1

Mid-point of the line joining (7,1) and (3,5) is $\left(\frac{7+3}{2}, \frac{1+5}{2}\right)$ = (5,3)

Therefore, perpendicular bisector passes through (5,3) and has slope 1, therefore it's equation is,
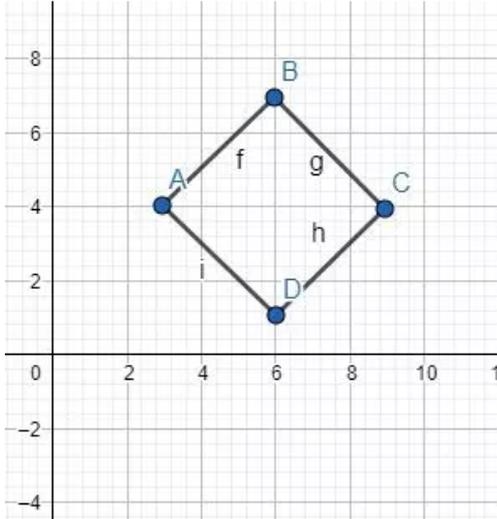
y - 3 = 1(x-5)

y - 3 = x - 5

x - y - 2 = 0

46. The given points are P(a sin α, a cos α) and Q(a cos α, - a sin α)

$(x_1 = a\sin\alpha, y_1 = a\cos\alpha)$ and $(x_2 = a\cos\alpha, y_2 = -a\sin\alpha)$

Therefore,by using distance formula,we have,

$$PQ = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$= \sqrt{(a\cos\alpha - a\sin\alpha)^2 + (-a\sin\alpha - a\cos\alpha)^2}$$
$$= \sqrt{a^2\cos^2\alpha + a^2\sin^2\alpha - 2a^2\cos\alpha\sin\alpha + a^2\cos^2\alpha + a^2\sin^2\alpha + 2a^2\cos\alpha\sin\alpha}$$
$$= \sqrt{a^2\cos^2\alpha + a^2\sin^2\alpha + a^2\cos^2\alpha + a^2\sin^2\alpha}$$
$$= \sqrt{a^2\left(\cos^2\alpha + \sin^2\alpha\right) + a^2\left(\cos^2\alpha + \sin^2\alpha\right)}$$
$$= \sqrt{a^2 + a^2} = \sqrt{2a^2} = \sqrt{2}a\ units$$

47. It can be seen that A (3, 4), B (6, 7), C (9, 4), and D (6, 1) are the positions of 4 friends

Distance between two points A(x_1, y_1) and B(x_2, y_2) is given by

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Hence,

AB = [(3- 6)^2 + (4 - 7)^2]$^{1/2}$
$= \sqrt{9 + 9} = \sqrt{18}$
$= 3\sqrt{2}$

BC = [(6 - 9)^2 + (7- 4)^2]$^{1/2}$
$= \sqrt{9 + 9} = \sqrt{18}$
$= 3\sqrt{2}$

CD = [(9- 6)^2 + (4- 1)^2]$^{1/2}$
$= \sqrt{9 + 9} = \sqrt{18}$
$= 3\sqrt{2}$

AD = [(3 - 6)^2 + (4 - 1)^2]$^{1/2}$
$= \sqrt{9 + 9} = \sqrt{18}$
$= 3\sqrt{2}$

Diagonal AC = [(3 - 9)^2 + (4 - 4)^2]$^{1/2}$
$= \sqrt{36 + 0} = 6$

Diagonal BD = [(6- 6)^2 + (7- 1)^2]$^{1/2}$
$= \sqrt{36 + 0} = 6$

It can be seen that all sides of quadrilateral ABCD are of the same length and diagonals are of the same length

Therefore, ABCD is a square and hence, Champa was correct.

48. Let A $\rightarrow$ (1, 5)

B $\rightarrow$ (2, 3)

C $\rightarrow$ (-2, -11)

Then $AB = \sqrt{(2 - 1)^2 + (3 - 5)^2} = \sqrt{1 + 4} = \sqrt{5}$

$BC = \sqrt{(-2 - 2)^2 + (-11 - 3)^2} = \sqrt{(-4)^2 + (-14)^2}$

$$= \sqrt{16 + 196} = \sqrt{212}$$

$$CA = \sqrt{[1 - (-2)]^2 + [5 - (-11)]^2} = \sqrt{(3)^2 + (16)^2}$$

$$= \sqrt{9 + 256} = \sqrt{265}$$

We see that

AB + BC $\neq$ CA

BC + CA $\neq$ AB

and CA + AB $\neq$ BC

Hence, the given points are not collinear.

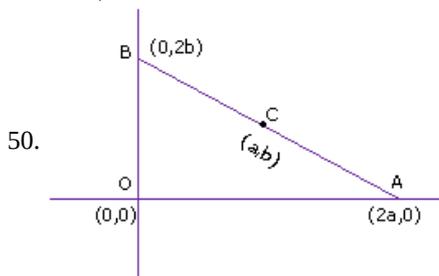49. (a, b), (-a, -b)

Required distance

$$= \sqrt{(-a - a)^2 + (-b - b)^2}$$

$$= \sqrt{(-2a)^2 + (-2b)^2}$$

$$= \sqrt{4a^2 + 4b^2}$$

$$= \sqrt{4(a^2 + b^2)}$$

$$= 2\sqrt{a^2 + b^2}$$

50.

Let A(2a, 0), B(0, 2b) and O(0, 0) are the vertices of right-angled triangle

Coordinate of $C\left(\frac{2a+0}{2}, \frac{0+2b}{2}\right)$

i.e. (a, b)

$$OC = \sqrt{a^2 + b^2}$$
$$AC = \sqrt{a^2 + b^2}$$
$$BC = \sqrt{a^2 + b^2}$$

Hence, C is Equidistant from the vertices.

51. Let vertices of a triangle are A(0,0),B(5,5) and C(-5,5)

Then, AB = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ = $\sqrt{(5 - 0)^2 + (5 - 0)^2} = \sqrt{(5)^2 + (5)^2} = 5\sqrt{2}$ units

BC = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ = $\sqrt{(-5 - 5)^2 + (5 - 5)^2} = \sqrt{(-10)^2 + (0)^2} = 10$ units
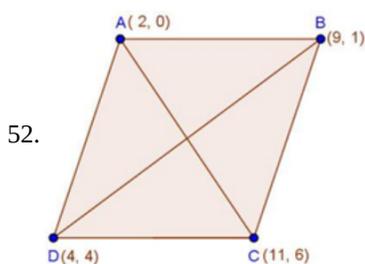
and CA = $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$ = $\sqrt{(-5 - 0)^2 + (5 - 0)^2} = \sqrt{(-5)^2 + (5)^2} = 5\sqrt{2}$ units

Here, AB = CA

and AB^2 + CA^2 = 50 + 50 = 100 = BC^2

So, $\angle A = 90^o$

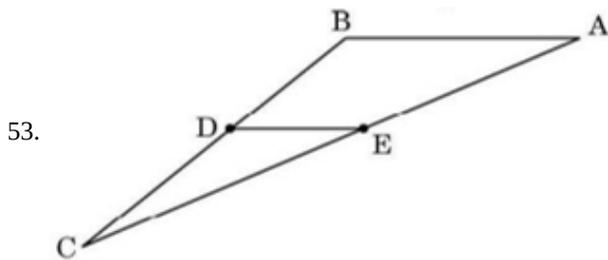Hence, $\triangle ABC$ is an isosceles right-angled triangle, right angled at A.

52.

Let A(2, 0), B(9, 1), C(11, 6) and D(4, 4) be the given points.

Coordinates of mid-point of AC are $\left(\frac{11+2}{2}, \frac{6+0}{2}\right) = \left(\frac{13}{2}, 3\right)$

Coordinates of mid-point of BD are $\left(\frac{9+4}{2}, \frac{1+4}{2}\right) = \left(\frac{13}{2}, \frac{5}{2}\right)$

Since, coordinates of mid-point of AC $\neq$ coordinates of mid-point of BD.

So, ABCD is not a parallelogram, Hence, it is not a rhombus.

53.

Point D is $\left(-2, \frac{-9}{2}\right)$

Point E is $\left(-1, \frac{-9}{2}\right)$

$\therefore$ DE $= \sqrt{1^2 + 0^2} = 1$ and AB $= \sqrt{2^2 + 0^2} = 2$

$\therefore$ DE $= \frac{1}{2}$ AB

54. The given point is C(-4, -6) and let O(0,0) be the origin

Then, $CO = \sqrt{(-4-0)^2 + (-6-0)^2}$

$= \sqrt{16 + 36} = \sqrt{52} = 2\sqrt{13} \, units$

55. According to the question, A(7, -3), B(5, 3) and C(3, -1).

AD and BE are medians of ΔABC.

D is the mid-point of BC and

E is the mid-point of AC

$\therefore$ Coordinates of $D = \left(\frac{5+3}{2}, \frac{3-1}{2}\right) = \left(\frac{8}{2}, \frac{2}{2}\right) = (4, 1)$

Coordinates of $E = \left(\frac{7+3}{2}, \frac{-3-1}{2}\right) = \left(\frac{10}{2}, \frac{-4}{2}\right) = (5, -2)$

Now, median $AD = \sqrt{(4-7)^2 + (1+3)^2} = \sqrt{(-3)^2 + (4)^2} = \sqrt{9 + 16} = \sqrt{25} = 5 \, units$

And, median $BE = \sqrt{(5-5)^2 + (-2-3)^2} = \sqrt{0 + (-5)^2} = \sqrt{25} = 5 \, units$

56. Required distance

$= \sqrt{(36-0)^2 + (15-0)^2}$

$= \sqrt{(36)^2 + (15)^2}$

$= \sqrt{1296 + 225} = \sqrt{1521}$

$= 39$

57. We have, AP = AQ

$AP^2 = AQ^2$

$(8 - 3)^2 + (-3 - y)^2 = (7 - 3)^2 + (6 - y)^2$

$25 + 9 + y^2 + 6y = 16 + 36 + y^2 - 12y$

$34 = 52 - 18y$

$18y = 52 - 34 = 18$

Thus y = 1

AQ $= \sqrt{(7-3)^2 + (6-y)^2} = \sqrt{16 + (6-1)^2} = \sqrt{16 + 25} = \sqrt{41}$ unit

58. Let A $\to$ (5, -2), B $\to$ (6, 4) and C $\to$ (7, -2)

Then,

$AB = \sqrt{(6-5)^2 + (4-(-2))^2} = \sqrt{(1)^2 + (6)^2}$

$= \sqrt{1 + 36} = \sqrt{37}$

$BC = \sqrt{(7-6)^2 + (-2-4)^2} = \sqrt{(1)^2 + (-6)^2}$

$= \sqrt{1 + 36} = \sqrt{37}$

CA $= \sqrt{(7-5)^2 + (-2-(-2))^2} = \sqrt{(2)^2 + (0)^2}$

We see that AB = BC $\neq$ CA

So, the A, B and C are vertices of an isosceles triangle.

59. The given points are A(7, -4) and B(-5, 1)

Then, $(x_1 = 7, y_1 = -4)$ and $(x_2 = -5, y_2 = 1)$

Therefore, by distance formula, we have,

$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(-5-7)^2 + (1+4)^2} = \sqrt{(-12)^2 + (5)^2}$

$= \sqrt{144 + 25} = \sqrt{169} = 13 \; units$

60. Distance between (3, 0) and (0, y) is 5 units

$\therefore \sqrt{(0-3)^2 + (y-0)^2} = 5$

$\sqrt{9 + y^2} = 5$

$9 + y^2 = 25 \Rightarrow y^2 = 25 - 9 = 16 = (\pm 4)^2$

$\therefore y = \pm 4$

But y is positive

$\therefore$ y = 4

61. The vertices of $\triangle$OAB are O(0, 0), A(a, 0), and B(0, b)

Now length of OA

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$= \sqrt{(a-0)^2 + (0-0)^2}$

$= \sqrt{a^2 + 0^2} = \sqrt{a^2} = a$

$OB = \sqrt{(0-0)^2 + (b-0)^2}$

$= \sqrt{0^2 + b^2} = \sqrt{b^2} = b$

$AB = \sqrt{(0-a)^2 + (b-0)^2}$

$= \sqrt{a^2 + b^2}$

$\therefore$ Area of $\triangle$ABC

$= \frac{1}{2} \times$ OA $\times$ OB

$= \frac{1}{2}$ ab sq. units

62. Using distance formula, we obtain

$AB = \sqrt{\left(at_2^2 - at_1^2\right)^2 + (2at_2 - 2at_1)^2}$

$\Rightarrow AB = \sqrt{a^2 (t_2 - t_1)^2 (t_2 + t_1)^2 + 4a^2 (t_2 - t_1)^2}$

$\Rightarrow AB = a(t_2 - t_1) \sqrt{(t_2 + t_1)^2 + 4}$

63. Let the given points (1, 2), (5, 3) and (18, 6) be denoted by A, B, and C respectively.

Now $AB = \sqrt{(5-1)^2 + (3-2)^2} = \sqrt{16 + 1} = \sqrt{17}$

$BC = \sqrt{(18-5)^2 + (6-3)^2} = \sqrt{169 + 9} = \sqrt{178}$

$AC = \sqrt{(18-1)^2 (6-2)^2} = \sqrt{289 + 16} = \sqrt{305}$

Here, we see that AB + BC $\neq$ AC, BC + AC $\neq$ AB and AB + AC $\neq$ BC. Hence, the points A, B and C are not collinear.

64. Let AOB be a right-angled triangle with base OA taken along x-axis and the perpendicular OB taken along y-axis. Let OA = a and OB = b.

Let D be the mid-point of the hypotenuse AB. Then, the coordinates of A, B and D are respectively (a, 0), (0, b) and (a/2, b/2).
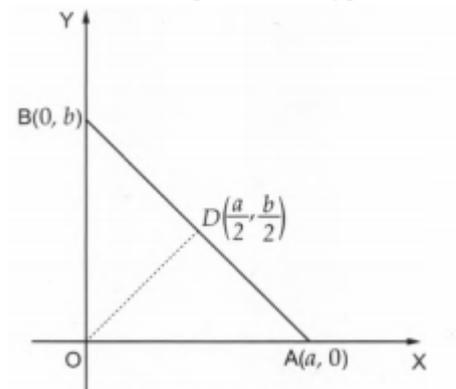
Now, $DO = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \frac{1}{2}\sqrt{a^2 + b^2}$

$DA = \sqrt{\left(\frac{a}{2} - a\right)^2 + \left(\frac{b}{2} - 0\right)^2} = \frac{1}{2}\sqrt{a^2 + b^2}$

and $DB = \sqrt{\left(\frac{a}{2} - 0\right)^2 + \left(\frac{b}{2} - b\right)^2} = \frac{1}{2}\sqrt{a^2 + b^2}$

Hence, DA = DB = DC i.e., D is equidistant from the vertices of triangle ABC.

65. i. A(3, 1)

   B(6, 4)

   C(8, 6)

   AB = $\sqrt{(6-3)^2 + (4-1)^2}$

   = $\sqrt{3^2 + 3^2}$

   = $\sqrt{9+9}$

   = $3\sqrt{2}$

   BC = $\sqrt{(8-6)^2 + (6-4)^2}$

   = $\sqrt{2^2 + 2^2}$

   = $\sqrt{4+4}$

   = $2\sqrt{2}$

   AC = $\sqrt{(8-3)^2 + (6-1)^2}$

   = $\sqrt{5^2 + 5^2}$

   = $\sqrt{25+25}$

   = $5\sqrt{2}$

   ∵ AB + BC = $3\sqrt{2}$ + $2\sqrt{2}$ = $5\sqrt{2}$ and AC = $5\sqrt{2}$

   ∵ AB + BC = AC = $5\sqrt{2}$

   hence, these three girls are sitting in a line

   ii.

A(3, 1) B(6, 4) C(8, 6)

   Let point B divide the live joining AC in K : 1 ration
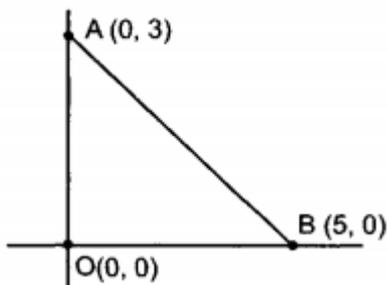
   $\frac{8k+3}{k+1} = 6$

   8k + 3 = 6k + 6

   8k - 6k = 6 - 3

   2k = 3

   k = $\frac{3}{2}$

   ∴ Point B divide AC in 3 : 2 ratio

66.

   AB = $\sqrt{(5-0)^2 + (0-3)^2} = \sqrt{25+9} = \sqrt{34}$

   Hence, the length of diagonals is $\sqrt{34}$ units.

67. It is given that A(0, 2) is equidistant from the points B(3, p) and C(p, 5).

   ∴ AB = AC

   $\Rightarrow \sqrt{(3-0)^2 + (p-2)^2} = \sqrt{(p-0)^2 + (5-2)^2}$ (Distance formula)

   Squaring on both sides, we get

   9 + p^2 - 4p + 4 = p^2 + 9

   $\Rightarrow$ -4p + 4 = 0

   $\Rightarrow$ p = 1

   Thus, the values of p is 1.

68. Let A(7, 10), B(-2, 5) and C(3, -4) be the vertices of given isosceles triangle.

   Then, $AB = \sqrt{(7+2)^2 + (10-5)^2} = \sqrt{106}$

   $AC = \sqrt{(7-3)^2 + (10+4)^2} = \sqrt{212}$

   $BC = \sqrt{(3+2)^2 + (-4-5)^2} = \sqrt{106}$

   Thus, AB = AC

   Also AB^2 + BC^2 = 106 + 106 = 212 = AC^2 (pythagoras theorem)

   Hence, given vertices are coordinates of an isosceles right triangle.

69. We know that a point on the x-axis is of the form (x, 0). So, let the point P(x, 0) be equidistant from A(2, –5) and B(–2, 9). Then

PA = PB

$\Rightarrow PA^2 = PB^2$

$\Rightarrow (2 - x)^2 + (-5 - 0)^2 = (-2 - x)^2 + (9 - 0)^2$

$\Rightarrow 4 + x^2 - 4x + 25 = 4 + x^2 + 4x + 81$

$\Rightarrow -4x + 25 = 4x + 81$

$\Rightarrow 8x = -56$

$\Rightarrow x = \frac{-56}{8} = -7$

Hence, the required point is (-7, 0)

Check:

$PA = \sqrt{\{2 - (-7)\}^2 + (-5 - 0)^2}$

$= \sqrt{81 + 25} = \sqrt{106}$

$PB = \sqrt{\{-2 - (-7)\}^2 + (9 - 0)^2}$

$= \sqrt{25 + 81} = \sqrt{106}$

∵ PA = PB

∴ Our solution is checked.

70. Given that points A(-1, y) and B(5, 7) lie on a circle with centre O(2, -3y).

Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Now,

OA = OB (radii of a circle)

$\Rightarrow OA^2 = OB^2$

$\Rightarrow (-1 - 2)^2 + [y - (-3y)]^2 = (5 - 2)^2 + [7 - (-3y)]^2$ [using distance formula]

$\Rightarrow (-3)^2 + (y+3y)^2 = (3)^2 + (7+3y)^2$

$\Rightarrow 9 + (4y)^2 = 9 + (49 + 9y^2 + 42y)$

$\Rightarrow 16y^2 = 49 + 9y^2 + 42y$

$\Rightarrow 7y^2 - 42y - 49 = 0$

$\Rightarrow 7(y^2 - 6y - 7) = 0$

$\Rightarrow y^2 - 6y - 7 = 0$

$\Rightarrow y^2 - 7y + y - 7 = 0$

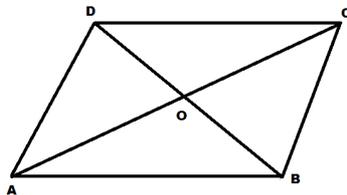$\Rightarrow y(y-7) + 1(y-7) = 0$

$\Rightarrow (y-7)(y+1)=0$

$\Rightarrow y - 7 = 0$ or $y + 1 = 0$

$\Rightarrow y = 7$ or $y = -1$

When y = 7, Coordinate of A(-1,7) and O(2,-21).

When y = -1, Coordinate of A(-1,-1) and O(2,3).

71.

We know that all the sides of a rhombus are equal and diagonals are not equal.

i.e. In rhombus ABCD, AB = BC = CD = DA and $AC \neq BD$

Distance between two points $= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Here A(-3, 2), B(-5, -5), C(2, - 3) and D(4, 4)

AB $= \sqrt{\{-5 - (-3)\}^2 + (-5 - 2)^2} = \sqrt{4 + 49} = \sqrt{53}$

BC $= \sqrt{\{2 - (-5)\}^2 + \{-3 - (-5)\}^2} = \sqrt{49 + 4} = \sqrt{53}$

CD $= \sqrt{(4 - 2)^2 + \{4 - (-3)\}^2} = \sqrt{4 + 49} = \sqrt{53}$

AD $= \sqrt{\{4 - (-3)\}^2 + (4 - 2)^2} = \sqrt{49 + 4} = \sqrt{53}$

Thus, AB = CD = BC = AD i.e. all sides are equal.

Now,

$AC = \sqrt{(2+3)^2 + (-3-2)^2}$

$= \sqrt{5^2 + 5^2} = = \sqrt{25+25} = \sqrt{50} = 5\sqrt{2}$

$BD = \sqrt{(4+5)^2 + (4+5)^2} = \sqrt{9^2 + 9^2} = \sqrt{81+81} = 9\sqrt{2}$

Thus, $AC \neq BD$ i.e diagonals are not equal.

$\therefore$ ABCD is a rhombus.

72. i. A(2, 2)

B(5, 4)

C(7, 6)

ii. $AB = \sqrt{(5-2)^2 + (2-2)^2}$

$= \sqrt{9+4}$

$= \sqrt{13}$

$BC = \sqrt{(7-5)^2 + (6-4)^2}$

$= \sqrt{4+4}$

$= 2\sqrt{2}$

$AC = \sqrt{(7-2)^2 + (6-2)^2}$

$= \sqrt{25+16}$

$= \sqrt{41}$

$\because AB + BC = \sqrt{13} + 2\sqrt{2}$

$AC = \sqrt{41}$

$\therefore AB + BC \neq AC$

$\therefore$ A, B, C are not collinear

73. Let the given points (–2, 1), (2 –2) and (5, 2) be denoted by A, B and C respectively.

$AB = \sqrt{(2+2)^2 + (-2-1)^2} = \sqrt{16+9} = \sqrt{25} = 5$

$BC = \sqrt{(5-2)^2 + (2+2)^2} = \sqrt{9+16} = \sqrt{25} = 5$

$AC = \sqrt{(5+2)^2 + (2-1)^2} = \sqrt{49+1} = \sqrt{50} = 5\sqrt{2}$

Since $AB^2 + BC^2 = 25 + 25 = 50 = AC^2$,

$\therefore \triangle ABC$ is right angled at B.

Hence, the given points are the vertices of a right triangle.

74. Since (2, 1) and (1, -2) are equidistant from (x, y), therefore,

Distance of (2, 1) from (x, y) = Distance of (1, -2) from (x, y)

$\sqrt{(x-2)^2 + (y-1)^2} = \sqrt{(x-1)^2 + (y+2)^2}$

$(x - 2)^2 + (y - 1)^2 = (x - 1)^2 + (y + 2)^2$

$x^2 + 4 - 4x + y^2 + 1 - 2y = x^2 + 1 - 2x + y^2 + 4 + 4y$

$4 - 4x + 1 - 2y = 1 - 2x + 4 + 4y$

$2x + 6y = 0$

$x + 3y = 0$

75. Here, A(–3, 2), B(p, q) and C (–1, 4) are the vertices of $\triangle$ABC such that AB = BC

i.e, $AB^2 = BC^2$

$\therefore (-3 - p)^2 + (2 - q)^2 = (p + 1)^2 + (q - 4)^2$

$\Rightarrow 9 + 6p + p^2 + 4 + q^2 - 4q = p^2 + 2p + 1 + q^2 - 8q + 16$

$\Rightarrow p^2 + q^2 + 6p - 4q + 13 = p^2 + q^2 + 2p - 8q + 17$

$\Rightarrow 6p - 4q + 13 = 2p - 8q + 17$

$\Rightarrow 6p - 2p - 4q + 8q = 17 - 13$

$\Rightarrow 4p + 4q = 4$

$\Rightarrow p + q = 1$