## Solution

## COORDINATE GEOMETRY WS 5

### Class 10 - Mathematics

### Section A

1.

**(b)** (-4, 6)

**Explanation:** Given: $(x_1, y_1) = (-6, 10)$, $(x_2, y_2) = (3, -8)$

and $m_1 : m_2 = 2 : 7$

$\therefore$ x = $\frac{m_1x_2+m_2x_1}{m_1+m_2}$

= $\frac{2\times3+7\times(-6)}{2+7}$ = $\frac{6-42}{9}$ = $\frac{-36}{9}$ -4

And y = $\frac{m_1y_2+m_2y_1}{m_1+m_2}$ = $\frac{2\times(-8)+7\times10}{2+7}$ = $\frac{-16+70}{9}$ = $\frac{54}{9}$ = 6

Therefore, the required coordinates are (-4, 6)

2. **(a)** (3, 6)

**Explanation:** Since, the point, where the perpendicular bisector of a line segment joining the points A(2, 5) and B(4, 7) cuts, is the mid-point of that line segment.

$\therefore$ Coordinates of Mid-point of line segment AB = $\left(\frac{2+4}{2}, \frac{5+7}{2}\right)$ = (3, 6)

3.

**(d)** 1 : 2

**Explanation:** Let the y-axis cut AB at p (0, y) in the ratio K : 1 Then

$P\left(\frac{8k-4}{k+1} \cdot \frac{3k+2}{k+1}\right) = P(0, y) = \frac{8k-4}{k+1} = 0$

= 8k - 4= 0 =k = $\frac{1}{2}$

required ratio =$\left(\frac{1}{2}; 1\right) = 1 : 2$

4.

**(c)** (3, 4)

**Explanation:** Since, the diagonals of rectangles are equal and bisect each other.

Let coordinate of vertex A = (x, y)

midpoint of AC = midpoint of BD

$(\frac{x+3}{2}, \frac{y}{2})$ = (0, 2)

on comparing; $\frac{x+3}{2}$ = 0

x = -3

and $\frac{y}{2}$ = 2

y = 4

$\therefore$ Coordinate of vertex A = (-3, 4)

5. **(a)** (0, 3)

**Explanation:** Given: $(x_1, y_1) = (3, -7)$, $(x_2, y_2) = (-8, 6)$ and $(x_3, y_3) = (5, 10)$

Coordinates of Centroid of triangle

x = $\frac{x_1+x_2+x_3}{3}$ and y = $\frac{y_1+y_2+y_3}{3}$

$\therefore$ x = $\frac{3-8+5}{3} = \frac{8-8}{3}$ = 0

and y = $\frac{-7+6+10}{3} = \frac{9}{3}$ = 3

Therefore, the coordinates of the centroid of the triangle are (0, 3)

6.

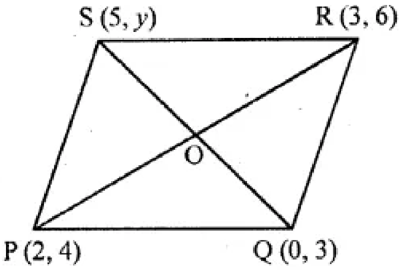
**(c)** $\left(\frac{7}{2}, \frac{9}{2}\right)$

**Explanation:** Midpoint of BC is D $\left(\frac{6+1}{2}, \frac{5+4}{2}\right) = D\left(\frac{7}{2}, \frac{9}{2}\right)$

7.

**(b)** 7

**Explanation:** P(2, 4), Q(0, 3), R(3, 6) and S(5, y) are the vertices of parallelogram PQRS




Join PR and QS which intersect at O

$\therefore$ O is mid-point of PR and QS

When O is mid point of PR then coordinates

of O will be = $\left(\frac{2+3}{2}, \frac{4+6}{2}\right)$

$= \left(\frac{5}{2}, 5\right)$

O is mid-point of QS

$\therefore 5 = \frac{y+3}{2} \Rightarrow y + 3 = 10$

$\Rightarrow$ y = 10 - 3 = 7

8. **(a)** 7

   **Explanation:** Given that R is the mid- point of the line segment AB.

   Th y-coordinate of R = $\frac{5+y}{2}$

   $\Rightarrow$ y = 7

9. **(a)** 2

   **Explanation:** O(k, -1) is the centroid of triangle whose vertices are

   A(3, -5), B(-7, 4), C(10, -k)

   $\therefore k = \frac{x_1+x_2+x_3}{3}$

   $\Rightarrow k = \frac{3-7+10}{3} = \frac{6}{3} = 2$

10. **(a)** 2

    **Explanation:** 2

11. 

    **(c)** 2, 2

    **Explanation:** Coordinates of mid-point of AB are

    $\left(\frac{2a-2}{2}, \frac{4+3b}{2}\right)$ i.e., $\left(a-1, \frac{4+3b}{2}\right)$

    But M(1, 2a + 1) is the mid-point of AB. [Given]

    $\therefore$ a - 1 = 1 $\Rightarrow$ a = 2 ...(i)

    and $\frac{4+3b}{2}$ = 2a + 1 $\Rightarrow$ 4 + 3b = 4a + 2

    $\Rightarrow$ 4 + 3b = 4(2) + 2 [Using (i)]

    $\Rightarrow$ 4 + 3b = 10 $\Rightarrow$ b = $\frac{10-4}{3}$ = $\frac{6}{3}$ = 2

    $\therefore$ a = 2, b = 2

12. 

    **(b)** $-y_1 : y_2$

    **Explanation:** Let a point A on x-axis divides the line segment joining the points $P(x_1, y_1)$ $Q(x_2, y_2)$ in the ratio $m_1 : m_2$ and let co-ordinates of A be (x, 0)

    $\therefore 0 = \frac{m_1y_2+m_2y_1}{m_1+m_2} \Rightarrow 0 = m_1y_2 + m_2y_1$

    $\Rightarrow m_1y_2 = -m_2y_1 \Rightarrow \frac{m_1}{m_2} = \frac{-y_1}{y_2}$

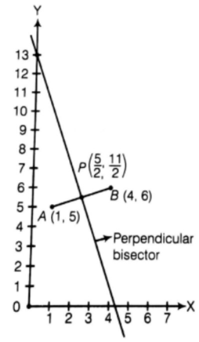
    $\therefore$ Ratio is $-y_1 : y_2$

13. 

    **(c)** (0, 13)

    **Explanation:**

First, we have to plot the points of the line segment on the paper and join them.




As we know that the perpendicular bisector of line segment AB, perpendicular at AB and passes through the mid-point of AB.

Let P be the mid-point of AB

Now find the mid-point,

Mid-point of AB = $\frac{1+4}{2}, \frac{5+6}{2}$

∵ Mid-point of line segment passes through the points ($x_1$, $y_1$) and ($x_2$, $y_2$)

= $\left[\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right]$

$\Rightarrow P = \frac{5}{2}, \frac{11}{2}$

Find the slope of the bisector:

Slop of the given line = $\frac{(y_1-y_2)}{(x_1-x_2)}$

Slope = $\frac{5-6}{1-4} = \frac{1}{3}$

Slope of given line multiplied by slope of bisector = - 1

Slope of bisector = $\frac{-1}{\frac{1}{3}} = \frac{-3}{1}$

= - 3

Now, we find the bisector's formula by using the point slope form;

Which is;

$-3 = \frac{\frac{11}{2}-y}{\frac{3}{2}-x} = \frac{5.5-y}{2.5-x}$

- 3(2.5 – x) = 5.5 - y

- 7.5 + 3x = 5.5 - y3x + y - 13 = 0

Transform the formula into slope - intercept form

3x + y - 13 = 0y = - 3x + 13

because, slope - intercept form is y = mx + c,

Where, m is the slope and c is the y - intercept

Thus, perpendicular bisector cuts the y - axis at (0, 13)

So, the required point is (0, 13).

14.

**(b)** -12

**Explanation:**

Given, P is the mid - point of the line segment joining the points Q and R

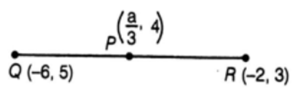
Where;

P = $\left(\frac{a}{3}, 4\right)$

Q = ( - 6, 5)

R = ( - 2, 3)

Shown in the figure given below;




∴ Mid - point of QR = P$\left(\frac{-6-2}{2}, \frac{5+3}{2}\right)$ = (-4, 4)

P = ( - 4, 4)

Since, midpoint of line segment having points ($x_1$, $y_1$) and ($x_2$, $y_2$);

= $\left(\frac{(x_1+x_2)}{2}, \frac{(y_1+y_2)}{2}\right)$

But given coordinates of mid - point P is $\left(\frac{a}{3}, 4\right)$;

$\therefore$ $\left(\frac{a}{3}, 4\right)$ = ( - 4, 4)

On comparing the coordinates, we get

$\frac{a}{3}$ = - 4

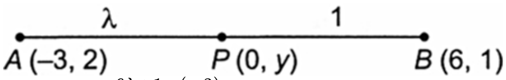
$\therefore$ a = - 12

Hence, the required value of a = - 12

15.

**(c)** 1 : 2

**Explanation:** We know, co-ordinates on Y-axis is (0, y). Now, let P(0, y) divides the line segment joining the points A(-3, 2) and B(6, 1) in $\lambda$ : 1.




Then, 0 = $\frac{6\lambda+1\times(-3)}{\lambda+1} \Rightarrow \lambda = \frac{1}{2}$

$\therefore$ Required ratio = 1 : 2

16.

**(b)** (6, -12)

**Explanation:** If (a, b) and (c, d) be the coordinates of any two points, then the coordinates of the mid-point joining those points be $\left(\frac{(a+c)}{2}, \frac{(b+d)}{2}\right)$.

The line segment is formed by points are (0, 0) and (x, y), whose mid-point is (3, -6).

Then,

$\frac{(0+x)}{2}$ = 3 and $\frac{(0+y)}{2}$ = -6

or, $\frac{x}{2}$ = 3 or, $\frac{y}{2}$ = -6

or, x = 6 or, y = -12

Therefore the required point is (6, -12).

17.

**(d)** $\left(\frac{20}{3}, \frac{5}{3}\right)$

**Explanation:** AD is the angle bisector of $\angle$BAC.

So, by the angle bisector theorem in $\triangle$ABC, we have

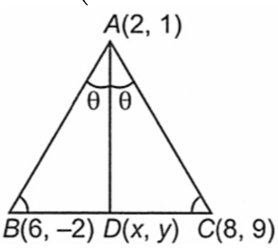
$\frac{AB}{AC} = \frac{BD}{DC}$ ...(i)

Now, AB = 5 and AC = 10

$\therefore$ $\frac{1}{2} = \frac{BD}{DC}$ [using (i)]

Thus, D divides BC in the ratio 1 : 2.

$\therefore$ D = $\left(\frac{2\times6+1\times8}{2+1}, \frac{-2\times2+9\times1}{2+1}\right) = \left(\frac{20}{3}, \frac{5}{3}\right)$




18.

**(b)** (-1, -8)

**Explanation:** (-1, -8)

19. **(a)** 2 : 3

**Explanation:** Given: (x, y) = (1, 3), ($x_1$, $y_1$) = (-6, 10), ($x_2$, $y_2$) = (3, -8)

Let $m_1$ : $m_2$ = k : 1

$\therefore$ x = $\frac{m_1x_2+m_2x_1}{m_1+m_2}$

$1 = \frac{k \times 4 + 1 \times (1)}{k+1}$

k + 1 = 4k - 1

$\Rightarrow$ k = $\frac{2}{3}$

Therefore, the required ratio is 2 : 3

20.

**(d)** (-1, 2)

**Explanation:** Let the coordinates of centre O be (x, y).

The endpoints of a diameter of the circle are A(- 4, - 3) and B(2, 7).

Since centre is the midpoint of diameter.

$\therefore$ x = $\frac{x_1+x_2}{2}$ = $\frac{-4+2}{2} = \frac{-2}{2}$ = -1 and

y = $\frac{y_1+y_2}{2}$ = $\frac{-3+7}{2} = \frac{4}{2}$ = 2

Therefore, the coordinates of the centre O is (-1, 2)

21.

**(d)** IV

**Explanation:** The point p is given by P $\left(\frac{2\times5+3\times2}{2+3}, \frac{2\times2-3\times5}{2+3}\right) = P\left(3, \frac{-11}{5}\right)$

so, p lies in IV quadrant.

$$\begin{array}{c|c} (-,+) & (+\infty) \\ & \text{if} \\ \hline & \\ & \text{if} \\ (-,-) & (+,-) \end{array}$$

22.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:** Centroid: x = $\frac{1+2-3}{3}$ = 0, y = $\frac{1+2-3}{3}$ = 0

Using formula, x = $\frac{x_1+x_2+x_3}{3}$ and y = $\frac{y_1+y_2+y_3}{3}$

23.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:** Distance of point (h, k) from its image under x-axis is 2k units and distance of point (h, k) under y-axis is 2h units.

24. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** Both A and R are true and R is the correct explanation of A.

25.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:** We know that the mid-point of the line segment joining the points

$\boldsymbol{P}(\boldsymbol{x}_1, \boldsymbol{y}_1)$ and $\boldsymbol{Q}(\boldsymbol{x}_2, \boldsymbol{y}_2)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So, the Reason is correct.

Given, the points A(4, 3) and B(x, 5) lie on a circle with center O(2, 3).

Then OA = OB $\Rightarrow$ $(OA)^2 = (OB)^2$

$\Rightarrow (4-2)^2 + (3-3)^2 = (x-2)^2 + (5-3)^2$

$\Rightarrow (2)^2+(0)^2 = (x-2)^2 + (2)^2 \Rightarrow 4 = (x-2)^2 + 4 \Rightarrow (x-2)^2 = 0$

$\Rightarrow$ x - 2 = 0 $\Rightarrow$ x = 2

So Assertion is correct.

The correct option is Both A and R are true but R is not the correct explanation of A.

26.

**(d)** A is false but R is true.

**Explanation:** Let joining of (1, 1) and (5, 5) meet x-axis in k : 1. Now, x = $\frac{5k+1}{k+1}$ and y = $\frac{5k+1}{k+1}$

But for x-axis y = 0. So, $\frac{5k+1}{k+1} = 0 \Rightarrow k = -\frac{1}{5}$

$\Rightarrow$ 1 : 5 externally

27. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** We know that the coordinates of the point P(x, y) which divides the line segment A($x_1$, $y_1$) and B($x_2$, $y_2$) in the ratio $m_1 : m_2$ is $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$ So, Reason is correct.

Here, x-coordinate = $\frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{(1\times-1)+(2\times1)}{1+2}$ = $\frac{1}{3}$

and y-coordinate = $\frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{(1\times1)+(2\times2)}{1+2}$ = $\frac{1+4}{3}$ = $\frac{5}{3}$

So, Assertion is correct.

28.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:** Using section formula, we have

-1 = $\frac{k\times6+1\times(-3)}{k+1}$

-k - 1 = 6k - 3

7k = 2

k = $\frac{2}{7}$

Ratio is 2 : 7 internally.

Also, if ar(△ABC) = 0

A, B and C all these points are collinear.

29. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** We know that the mid-point of the line segment joining the points $P\left(x_1, y_1\right)$ and $Q\left(x_2, y_2\right)$ is $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

So, Reason is correct.

Since, C(y, −1) is the mid-point of P(4, x) and Q(−2, 4).

We have, $\frac{4-2}{2}$ = y ⇒ y = 1

and $\frac{x+4}{2}$ = -1 ⇒ X + 4 = -2

⇒ X = -6

So, Assertion is correct

Correct option is Both A and R are true and R is the correct explanation of A.

30.

**(d)** A is false but R is true.

**Explanation:** We know that the coordinates of the point P(x, y) which divides the line segment joining the points A($x_1$, $y_1$) and B($x_2$, $y_2$) in the ratio $m_1 : m_2$ is $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

So, Reason is correct.

Let the ratio is k:1.Here, $x_1$ = 1, $y_1$ = 2, $x_2$ = −2, $y_2$ = 1, $m_1$ = k,$m_2$ = 1

Now x-coordinate = $\frac{m_1x_2+m_2x_1}{m_1+m_2} = \frac{(k\times-2)+(1\times1)}{k+1}$ = $\frac{-2k+1}{k+1}$

and y-coordinate = $\frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{(k\times1)+(1\times2)}{k+1}$ = $\frac{k+2}{k+1}$

Now, −6k + 3 + 4k + 8 = 7k + 7 ⇒ 7k + 2k = 11 − 7 ⇒ 9k = 4 ⇒ k = $\frac{4}{9}$

So, the Assertion is not correct

31. **(a)** Both A and R are true and R is the correct explanation of A.

**Explanation:** We know that the coordinates of the point P(x, y) which divides the line segment joining the points A($x_1$ , $y_1$) and B($x_2$, $y_2$) in the ratio $m_1 : m_2$ is $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

So, Reason is correct.

Here, $x_1$ = -5, $y_1$ = 11, $x_2$ = 4, $y_2$ = -7, $m_1$ = 7, $m_2$ = 2

Now, x-coordinate = $\frac{m_1x_2+m_2\bar{x}_1}{m_1+m_2} = \frac{(7\times4)+(2\times-5)}{7+2}$ = $\frac{28-10}{9} = \frac{18}{9}$ = 2

and y-coordinate = $\frac{m_1y_2+m_2y_1}{m_1+m_2} = \frac{(7\times-7)+(2\times11)}{7+2}$ = $\frac{-49+22}{9}$ = $\frac{-27}{9}$ = -3

So, Assertion is also correct

Correct option is both A and R are true and R is the correct explanation of A.

32.

**(b)** Both A and R are true but R is not the correct explanation of A.

**Explanation:** We know that the coordinates of the point P(x, y) which divides the line segment Joining the points A($x_1$, $y_1$)

So Reason is correct.

Let P(x, 0) be a point on X-axis such that, AP = BP

$\Rightarrow (x + 2)^2 + (0 - 3)^2 = (x - 5)^2 + (0 + 4)^2$

$\Rightarrow x^2 + 4x + 4 + 9 = x^2 - 10x + 25 + 16 \Rightarrow 14x = 28 \Rightarrow x = 2$

Hence, required point = (2, 0)

So Assertion is correct.

33. Fill in the blanks:

(i) 1. (0, 0)

34.

**(c)** -4

**Explanation:** Q (3, y) divides AB in the ratio 2 : 1

so Q is $\left(\frac{2\times1+1\times7}{2+1}, \frac{2\times(-5)+1\times(-2)}{2+1}\right)$, is

hence, = -4

35.

**(b)** centroid

**Explanation:** The point where three medians of a triangle meet is called the centroid of the triangle. It is the centre of gravity of the triangle. It divides the median in the ratio 2 :1

36. Diagonals of a parallelogram bisect each other.

$\therefore$Mid-points of AC and BD are same

or, $\left(3, \frac{-1+y}{2}\right) = (3, 2)$

$\frac{-1+y}{2} = 2$

y = 5.

37. Mid point of AC = $\left(\frac{3+1}{2}, \frac{8+2}{2}\right)$ = (2, 5)

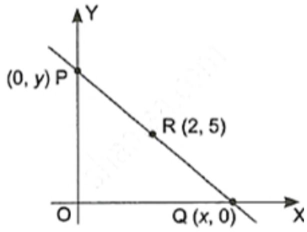
Mid point of BD = $\left(\frac{5-1}{2}, \frac{4+6}{2}\right)$ = (2, 5)

$\Rightarrow$ Mid point of AC = Mid point of BD

Hence, ABCD is a parallelogram.

38. Let P point be (0, y)

and Q point be (x, 0)




Since, on y-axis, x = 0 and x-axis, y = 0

Let coordinates of point P be (0, y)

and coordinates of point Q be (x, 0)

Since, mid-point $(x_1, y_1)$ and $(x_2, y_2)$ is

$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\therefore 2 = \frac{0+x}{2} \Rightarrow x = 4$

and $5 = \frac{y+0}{2} \Rightarrow y = 10$

$\therefore$ coordinates of P = (0, 10)

and coordinates of Q = (4, 0)

39. Let the coordinates of other end be (x, y)

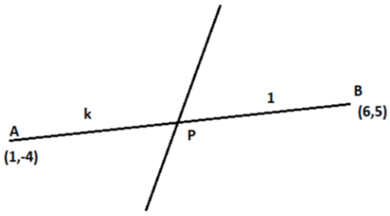
$\left(\frac{x+4}{2}, \frac{y+0}{2}\right)$ = (4, 4)

$\therefore$ x = 4, y = 8

So, coordinates of other end are (4, 8)

40. (2, p) is the mid point of the line segment joining the points A(6, -5), B (-2, 11)

$\therefore p = \frac{-5+11}{2} = \frac{6}{2} = 3$

41. 


Let the ratio be k:1

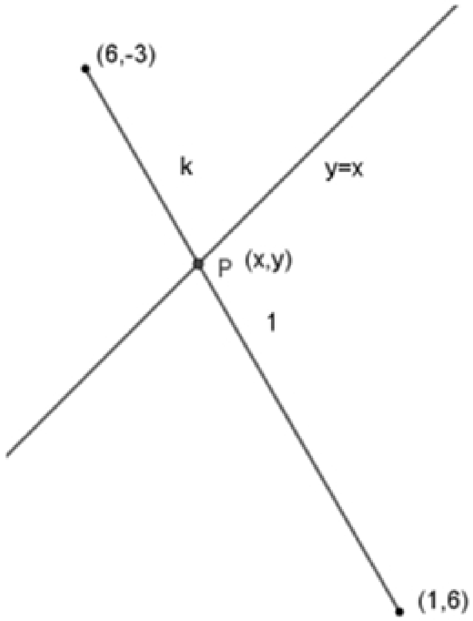
Point P is $\left(\frac{6k+1}{k+1}, \frac{5k-4}{k+1}\right)$

P lies on x-axis so y = 0

$\frac{5k-4}{k+1}$ = 0

$\Rightarrow$ k = $\frac{4}{5}$

$\therefore$ P divides AB in the ratio 4:5

42. 


x = $\frac{k+6}{k+1}$

y = $\frac{6k-3}{k+1}$

P(x, y) lies on y = x

$\Rightarrow$ k + 6 = 6k - 3

$\Rightarrow$ k = $\frac{9}{5}$

Ratio is 9 : 5

43. Given, P(4, - 2) is mid-point of A(5k, 3) and B(-k, -7)

Using Mid Point Formula,

$\therefore$ $\left(\frac{5k-k}{2}, \frac{3-7}{2}\right)$ = (4, -2)

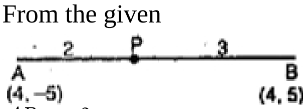
$\Rightarrow \frac{5k-k}{2} = 4$

$\Rightarrow$ 4k = 8

$\Rightarrow$ k = 2

Hence k = 2

44. From the given




$\frac{AP}{AB} = \frac{2}{5}$

$\Rightarrow \frac{AP}{AB} = \frac{5}{2}$

$\frac{AB}{AP}$ - 1 = $\frac{5}{2}$ - 1

$\Rightarrow \frac{\mathrm{PB}}{\mathrm{AP}} = \frac{3}{2}$

$\therefore$ corrdinates of P

$= \left(\frac{mx_2+nx_1}{m+x}, \frac{my_2+xy_1}{m+x}\right)$

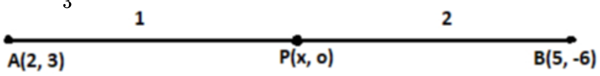
$= \left(\frac{2\times4+3\times4}{2+3}, \frac{2\times5+3\times(-5)}{2+3}\right)$
$= \left(\frac{8+12}{5}, \frac{10-15}{5}\right)$
= (4, -1)

45. y = $\frac{3(2)+3(1)}{1+2}$
$\Rightarrow$ y = 3
Coordinate of P(0, 3)

46. False, since, Distance between PC
$= \sqrt{(-2-3)^2 + (4-5)^2}$
$= \sqrt{25+1} = \sqrt{26}$ =5.09,is less than its radius, i.e, 6cm.
So, the above given statement is false

47. The co-ordinates of the mid-point of the line segment joining the points (6, 8) and (2, 4) are $\left(\frac{6+2}{2}, \frac{8+4}{2}\right)$ i.e. (4, 6)
$\therefore$ Required distance $= \sqrt{(1-4)^2 + (2-6)^2} = \sqrt{9+16} = 5 units.$

48. Vertices of $\triangle PQR$ are P(a, b), Q(b, c) and R(c, a) and its centroid = O(0, 0)
$\therefore \frac{x_1+x_2+x_3}{3} = 0$
$\Rightarrow \frac{a+b+c}{3} = 0 \Rightarrow$ a + b + c = 0

49. 

Since, x-axis dividing the line joingin AB
$\therefore$ y coordinat of point must be 0.
x = $\frac{1\times5+2\times2}{1+2}$
x = $\frac{5+4}{3}$
x = $\frac{9}{3}$
x = 3
$\therefore$ P = (3, 0)

50. Let the coordinates of the required point be (x, y), then
$x = \frac{mx_2+nx_1}{m+n}$ and $y = \frac{my_2+ny_1}{m+n}$
Now, $x = \frac{1\times3+5\times2}{1+5} = \frac{3+10}{6} = \frac{13}{6}$
and $y = \frac{1\times4+5\times3}{1+5} = \frac{4+15}{6} = \frac{19}{6}$
Hence coordinates of the required point will be $\left(\frac{13}{6}, \frac{19}{6}\right)$

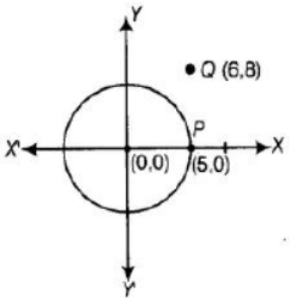
**Section B**

51. State True or False:

(i) **(a)** True
**Explanation:** True

(ii) **(a)** True
**Explanation:** True

(iii) **(a)** True
**Explanation:**
True
First, we draw a circle and a point from the given information




Now, distance between origin i.e., O(0, 0) and P(5, 0)
OP = $\sqrt{(5-0)^2 + (0-0)^2}$

[∵ Distance between two points $(x_1, y_1)$ and $(x_2, y_2)$, d = $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$ ]

= $\sqrt{5^2+0^2}=5$

Radius of circle and distance between origin O(0, 0) and Q (6, 8), OQ = $\sqrt{(6-0)^2+(8-0)^2}$

= $\sqrt{6^2+8^2}=\sqrt{36+64}=\sqrt{100}=10$

We know that, if the distance of any point from the centre is less than/equal to/ more than the radius, then the point is inside/on/outside the circle, respectively.

Here, we see that, OQ > OP

Hence, it is true that point Q(6,8), lies outside the circle.

(iv) **(a)** True

**Explanation:** True

Let P (5, - 3) be the point which divides the line segment joining the points A (7, - 2) and B (1, - 5) in the ratio k : 1 internally.

By using the section formula,

The coordinate of point P = $\left(\frac{\mathrm{k}(1)+(1)(7)}{\mathrm{k}+1}, \frac{\mathrm{k}(-5)+1(-2)}{\mathrm{k}+1}\right)$

P = $\left(\frac{k+7}{k+1}, \frac{-5k-2}{k+1}\right)$

Given coordinate of P = (5, - 3)

So,

(5, -3) = $\left(\frac{k+7}{k+1}, \frac{-5k-2}{k+1}\right)$

$\Rightarrow \frac{\mathrm{k}+7}{\mathrm{k}+1}=5$

⇒ k + 7 = 5k + 5

⇒ - 4k = - 2

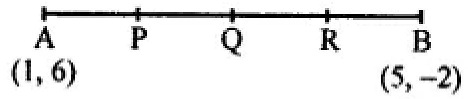
∴ k = $\frac{1}{2}$

So the point P divides the line segment AB in ratio 1 : 2

Hence, Point P is one of the two points of trisection of the line segment AB.

**Section C**

52. Points P, Q and R in order divide a line segment joining the points A(1, 6) and B(5, -2) in 4 equal parts.




P divides AB in the ratio of 1:3 Let coordinates of P be (x, y), then

$x=\frac{mx_2+nx_1}{m+n}=\frac{1\times5+3\times1}{1+3}$

$=\frac{5+3}{4}=\frac{8}{4}=2$

$y=\frac{my_2+ny_1}{m+n}=\frac{1\times(-2)+3\times6}{1+3}$

$=\frac{-2+18}{4}=\frac{16}{4}=4$

∴ Coordinates of P are (2, 4)

Similarly,

Q divides AB in 2:2 or 1:1 and Q is midpoint of AB.

∴ Coordinates of Q will be $\left(\frac{1+5}{2}, \frac{6-2}{2}\right)$

or $\left(\frac{6}{2}, \frac{4}{2}\right)$ or (3, 2)

and R divides AB in the ratio of 3:1

Coordinates of R will be

$\left(\frac{3\times5+1\times1}{3+1}, \frac{3\times(-2)+1\times6}{3+1}\right)$

or $\left(\frac{15+1}{4}, \frac{-6+6}{4}\right)$ or $\left(\frac{16}{4}, \frac{0}{4}\right)$ or $(4, 0)$

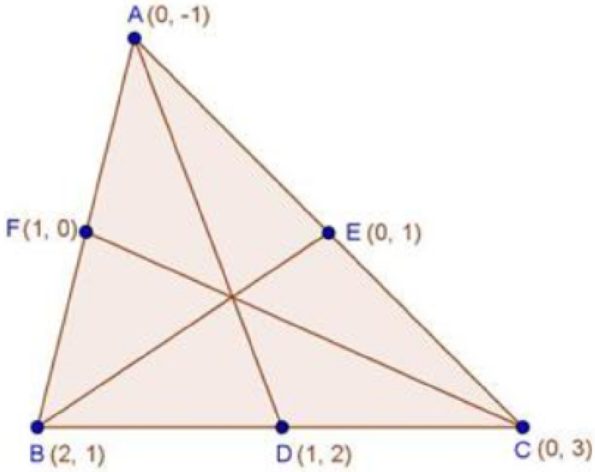
53. Let the point $P\left(\frac{1}{2}, y\right)$ divides the line segment joining the points A(3, -5) and B(-7, 9) in the ratio k:1.

Then, by section formula,

Coordinates of $P=\left(\frac{k\times(-7)+1\times3}{k+1}, \frac{k\times9+1\times(-5)}{k+1}\right)$

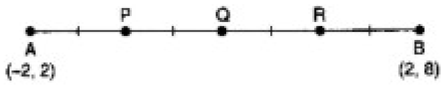
$=\left(\frac{-7k+3}{k+1}, \frac{9k-5}{k+1}\right)$

Given, coordinates of $P = \left(\frac{1}{2}, y\right)$
$\therefore \frac{-7k+3}{k+1} = \frac{1}{2}$
$\Rightarrow$ -14k + 6 = k + 1
$\Rightarrow$15k = 5
$\Rightarrow k = \frac{1}{3}$
So, the required ratio is 1:3.
Also, $\frac{9k-5}{k+1} = y$
$\Rightarrow \frac{9\times\frac{1}{3}-5}{\frac{1}{3}+1} = y$
$\Rightarrow \frac{3-5}{4/3} = y$
$\Rightarrow y = \frac{-6}{4} = \frac{-3}{2}$

54. 


Let A(0, -1), B(2, 1) and C(0, 3) be the given points.
Let AD, BE and CF be the medians
Coordinates of D are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right) = (1, 2)$
Coordinates of E are $\left(\frac{0}{2}, \frac{3-1}{2}\right) = (0, 1)$
Coordinates of F are $\left(\frac{2+0}{2}, \frac{1-1}{2}\right) = (1, 0)$
Length of median $AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{10}\, units$
Length of median $BE = \sqrt{(2-0)^2 + (1-1)^2} = 2 units$
Length of median $CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{10}\, units$

55. Let P ($x_1$, $y_1$) Q($x_2$, $y_2$) and R($x_3$, $y_3$) be the points which divide the line segment AB into four equal parts.




Then, P divides AB in the ratio 1 : 3 internally.
$x = \frac{mx_2+nx_1}{m+n}$
$\therefore x_1 = \frac{(1)(2)+(3)(-2)}{1+3}$
$= \frac{2-6}{4} = -\frac{4}{4} = -1$
$y = \frac{my_2+ny_1}{m+n}$
$y_1 = \frac{(1)(8)+(3)(2)}{1+3}$
$= \frac{8+6}{4} = \frac{14}{4} = \frac{7}{2}$
So, P $\rightarrow \left(-1, \frac{7}{2}\right)$
Also, Q divides AB in the ratio 1 : 1 i.e.
Q is the mid point of AB
$x_2 = \frac{-2+2}{2} = 0$
$y_2 = \frac{2+8}{2} = \frac{10}{2} = 5$
So, $Q \rightarrow (0, 5)$
and, R divides AB in the ratio 3 : 1

$\therefore x_2 = \frac{(3)(2)+(1)(-2)}{3+1}$
$= \frac{6-2}{4} = \frac{4}{4} = 1$
$y_3 = \frac{(3)(8)+(1)(2)}{3+1}$
$= \frac{24+2}{4} = \frac{26}{4} = \frac{13}{2}$
So, $\mathrm{R} \rightarrow \left(1, \frac{13}{2}\right)$

56. Let the vertex of triangles are A($x_1$, $y_1$), B($x_2$, $y_2$) and C($x_3$, $y_3$)

and L(3,4),M(4,6) and N(5,7) are the midpoints of side AB, BC and AC respective.
Since, L, M and N are mid-point than they divide the sides AB, BC and AC in the ratio of 1:1
Midpoint formula
$x = \frac{x_1+x_2}{2}$
$y = \frac{y_1+y_2}{2}$
So,
$\frac{x_1+x_2}{2} = 3$
$x_1 + x_2 = 6$ ..... (1)
$\frac{y_1+y_2}{2} = 4$
$y_1 + y_2 = 8$ ..... (2)

by the same way
$x_2 + x_3 = 8$ .... (3)

$y_2 + y_3 = 12$ ... (4)

$x_1 + x_3 = 10$ .... (5)

$y_1 + y_3 = 14$ ... (6)

Subtract eq (3) and (5)
$x_2 - x_1 = 2$ .... (7)

subtract eq. (4) and (6)
$y_2 - y_1 = -2$ ... (8)

on adding eq. (1) and (7)
$x_1 + x_2 = 6$

$-x_1 + x_2 = -2$

$2x_2 = 4$

$x_2 = 2$

Therefore $x_1 = 4$ and $x_3 = 6$

on adding eq. (2) and (8), we get
$y_1 + y_2 = 8$

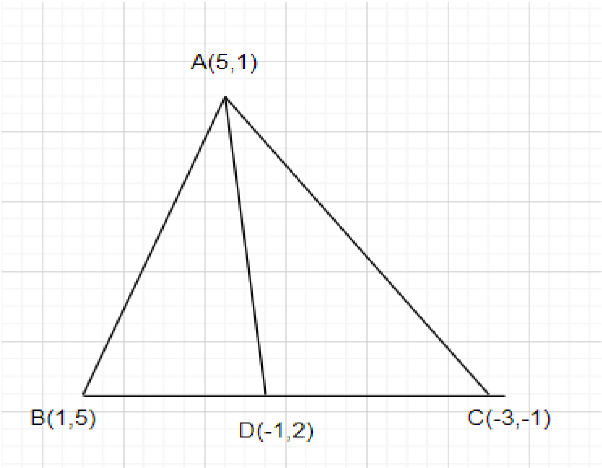
$-y_1 + y_2 = -2$

$2y_2 = 6$

$y_2 = 3$

Therefore, $y_1 = 5$ and $y_3 = 9$

So A(4, 5), B(2, 3) and C(6, 9).

57. 


Let A(5, 1), B(1, 5) and C(-3, -1) be vertices of $\Delta ABC$

Let AD be the median. Since, D is the midpoint of BC, So,

Coordinates of D are $\left(\frac{1-3}{2}, \frac{5-1}{2}\right) = (-1, 2)$

Length of median $AD = \sqrt{(5+1)^2 + (1-2)^2} = \sqrt{37}\, units$

58. Let A($x_1$, $y_1$), B($x_2$, $y_2$) and C($x_3$, $y_3$) be the vertices of a triangle D (10, 5) and F(6, 6) are mid-points of sides BC, CA and AB respectively.

Therefore, $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$ = (6, 6)

$\Rightarrow x_1 + x_2 = 12$ ......(i)

and $y_1 + y_2 = 12$ .....(ii)

$\left(\frac{x_2+x_3}{2}, \frac{y_2+y_3}{2}\right)$ = (10, 5)

$x_2 + x_3 = 20$ .......(iii)

and $y_2 + y_2 = 10$ .......(iv)

and $\left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$ = (8, 4)

$\Rightarrow x_1 + x_3 = 16$ ...(v)

and $y_1 + y_3 = 8$ ...(vi)

Adding (i), (iii) and (v), we get

$2 (x_1 + x_2 + x_3) = 48$

$\Rightarrow x_1 + x_2 + x_3 = 24$........(vii)

From (i), (iii) and (v) and (vii), we get

$x_1 = 4$, $x_2 = 8$, $x_3 = 12$ .....(viii)

Adding (ii), (iv) and (vi), we get

$2 (y_1 + y_2 + y_3) = 30$

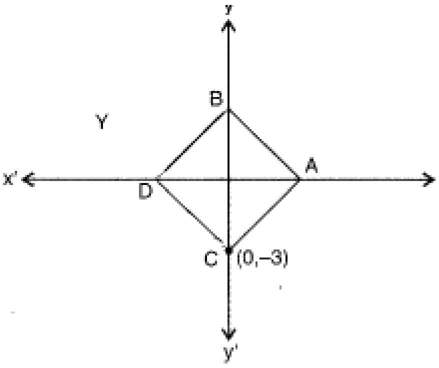
$y_1 + y_2 + y_3 = 15$ ......(ix)

From (ii), (iv), (vi) and (ix), we get

$y_1 = 5$, $y_2 = 7$, y3 = 3 .......(x)

From (viii) and (x), we get

Coordinates of vertices are A (4, 5), B (8, 7) and C (12, 3).

59. 


Co-ordinates of point B are (0,3)

$\therefore$BC = 6 unit

Let the co-ordinates of point A be (x, 0)

or, AB = $\sqrt{x^2+9}$

$\because$ AB = BC

$\therefore$ $x^2$ + 9 = 36

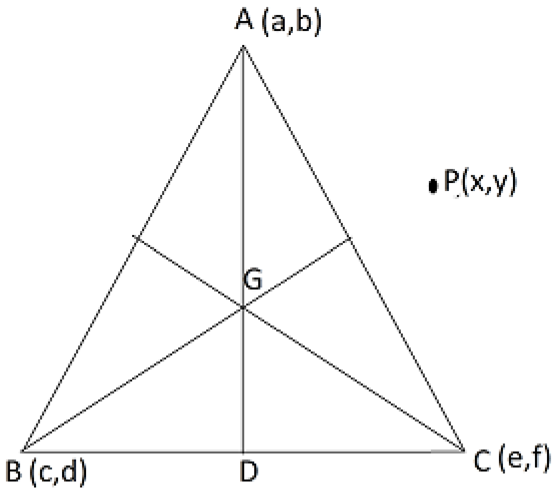
or, $x^2$ = 27 or, x = $\pm 3\sqrt{3}$

Co-ordinates of point A = ($3\sqrt{3}$ ,0)

Since ABCD is a rhombus

or, AB = AC = CD = DB

$\therefore$ Co-ordinate of point D = ($-3\sqrt{3}$ ,0)

60. 


A(a, b), B(c, d) and C(e, f) are the co-ordinates of triangle ABC.

G is the intersection of the median

D is the mid-point of BC.

Let G(0,0)

Coordinates of D = $\left(\frac{c+e}{2}, \frac{d+f}{2}\right)$

The centroid of triangle, G = $\left(\frac{a+c+e}{3}, \frac{b+d+f}{3}\right)$

$\therefore \frac{a+c+e}{3} = 0$ and $\frac{b+d+f}{3} = 0$

$\Rightarrow$ e = -(a+c) and f = -(b+d)

To prove: $PA^2 + PB^2 + PC^2 = GA^2 + GB^2 + GC^2 + 3GP^2$

$\Rightarrow\{(x-a)^2 + (y-b)^2\} + \{(x-c)^2 + (y-d)^2\} + \{(x+a+ c)^2 + (y+b+d)^2\} = \{(0-a)^2 + (0-b)^2\} + \{(0-c)^2 + (0-d)^2\} + \{(a+c)^2 + (b+d)^2\} + 3\{(x-0)^2 + (y-0)^2\}$

From L.H.S.

$x^2 + a^2 - 2ax + y^2 + b^2 - 2by + x^2 + c^2 - 2cx + y^2 + d^2 - 2dy + x^2 + a^2 + c^2 + 2ax + 2ac + 2cx + y^2 + b^2 + d^2 + 2by + 2bd + 2yd$

$\Rightarrow 3(x^2+y^2) + 2(a^2 + b^2 + c^2 + d^2 + ca + bd)$

From R.H.S.

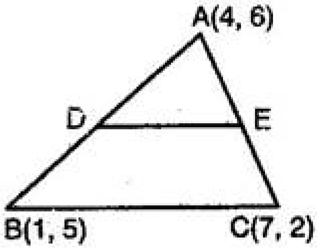
$\Rightarrow a^2 + b^2 + c^2 + d^2 + a^2 + c^2 + 2ac + b^2 + d^2 + 2bd + 3(x^2 + y^2)$

$\Rightarrow 3(x^2+y^2) + 2(a^2 + b^2 + c^2 + d^2 + ac + bd)$

L.H.S. = R.H.S

Hence proved.

61. Since, $\frac{AD}{AB} = \frac{AE}{AC} = \frac{1}{4}$




$\therefore$ DE || BC [By Thales theorem]

$\therefore \Delta\text{ADE} \sim \Delta\text{ABC}$

$\therefore \frac{\text{Area }(\Delta\text{ADC})}{\text{Area }(\Delta\text{ABC})} = \frac{\text{AD}^2}{\text{AB}^2}$

$= \left(\frac{\text{AD}}{\text{AB}}\right)^2 = \left(\frac{1}{4}\right)^2 = \frac{1}{16}$ ……….(i)

Now, Area ($\Delta$ABC) = $\frac{1}{2}[4(5-2)+1(2-6)+7(6-5)]$

$= \frac{1}{2}[12-4+7] = \frac{15}{2}$ sq. units……….(ii)

From eq. (i) and (ii),

Area ($\Delta$ADE) = $\frac{1}{16} \times$ Area ($\Delta$ABC) = $\frac{1}{16} \times \frac{15}{2} = \frac{15}{32}$ sq. units

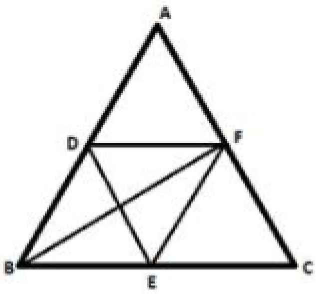
$\Delta$ Area ($\Delta$ADE): Area ($\Delta$ABC) = 1 : 16.

62. Let the vertices of the triangle are A($x_1$, $y_1$), B($x_2$, $y_2$), C($x_3$, $y_3$)

D,E AND F are the mid points of sides AB, BC AND AC

Given, D(1,2),E(0,-1)and F(2,-1).

Draw DE, DF, FE and BF




As D and F are mid points of AB and AC

$\therefore$ DF || BE

E and F are mid points of BC and AC

$\therefore$ EF || BD

Hence, DBEF is a parallelogram

We know that, the diagonals of a parallelogram bisect each other.

That means, both diagonals have same mid - point.

Midpoint BF = Midpoint of DE

$\Rightarrow \left(\frac{x_2-2}{2}, \frac{y_2+1}{2}\right) = \left(\frac{1+0}{2}, \frac{2-1}{2}\right)$

On comparing both sides, we get

$\frac{x_2-2}{2} = \frac{1}{2}$ and $\frac{y_2+1}{2} = \frac{1}{2}$

$\Rightarrow x_2 - 2 = 1, y_2 + 1 = 1$

$\therefore x_2 = 3, y_2 = 0$

D is the midpoint of AB

D = $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

(1, 2) = $\left(\frac{x_1+3}{2}, \frac{y_1+0}{2}\right)$

$\Rightarrow x_1 = -1$ and $y_1 = 4$

Now, F is the midpoint of AC

$F = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

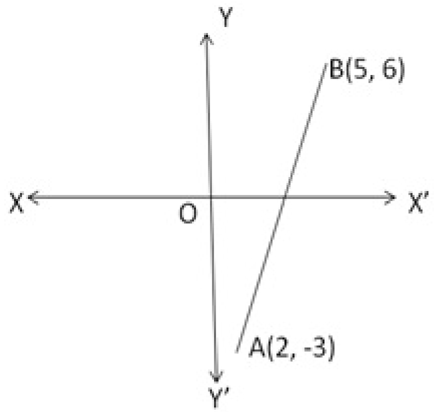
$(2, -1) = \left(\frac{-1+x_3}{2}, \frac{4+y_3}{2}\right)$

$\Rightarrow x_3 = 5$ and $y_3 = -6$

The vertices of the triangle are (1, 2), (3, 0) and (5, - 6)

63. Let the x-axis cut the join of A(2, -3) and B(5, 6) in the ratio k:1 at the point P

Then, by the section formula, the coordinates of P are $\left(\frac{5k+2}{k+1}, \frac{6k-3}{k+1}\right)$




But P lies on the x-axis so, its ordinate must be 0

$\therefore \frac{6k-3}{k+1} = 0$

$\Rightarrow 6k - 3 = 0, k = \frac{1}{2}$

So the required ratio is 1:2

Thus the x-axis divides AB in the ratio 1:2

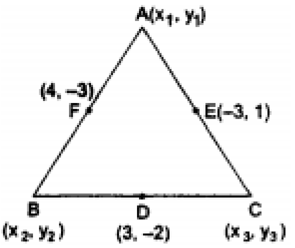
Putting $k = \frac{1}{2}$ in $\frac{5k+2}{k+1}$ , we get the point P as

$P\left(\frac{5\times\frac{1}{2}+2}{\frac{1}{2}+1}, 0\right)$ or $P(3, 0)$

Thus, P is (3, 0) and k= 1:2

64. According to the question, D(3, -2), E(-3, 1) and F(4, -3) are the midpoints of sides BC, CA and AB respectively.

Let A($x_1$, $y_1$), B($x_2$, $y_2$) and C($x_3$, $y_3$) be the vertices of triangle.




D is the midpoint of BC

$\Rightarrow \frac{x_2+x_3}{2} = 3, \frac{y_2+y_3}{2} = -2$

$\Rightarrow \begin{cases} x_2 + x_3 = 6 \ldots (i) \\ y_2 + y_3 = -4 \ldots (ii) \end{cases}$

E is the midpoint of CA

$\Rightarrow \frac{x_1+x_3}{2} = -3, \frac{y_1+y_3}{2} = 1$

$\Rightarrow \begin{cases} x_1 + x_3 = -6 \ldots (iii) \\ y_1 + y_3 = 2 \ldots (iv) \end{cases}$

F is the midpoint of AB

$\Rightarrow \frac{x_1+x_2}{2} = 4, \frac{y_1+y_2}{2} = -3$

$\Rightarrow \begin{cases} x_1 + x_2 = 8 \ldots (v) \\ y_1 + y_2 = -6 \ldots (vi) \end{cases}$

Adding (i), (iii) and (v). we get

$2(x_1 + x_2 + x_3) = 8$

$\Rightarrow (x_1 + x_2 + x_3) = 4$ ...(vii)

Using (i), (iii) and (v) with (vii), we get

$x_1 = -2$, $x_2 = 10$ and $x_3 = -4$ ...(viii)

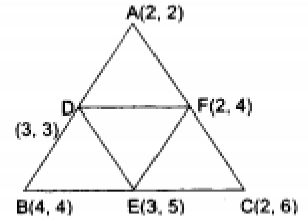
Adding (ii), (iv) and (vi) with (viii), we get

$y_1 = 0$, $y_2 = -6$ and $y_3 = 2$

Hence, the vertices of ΔABC are $A(-2,0), B(10,-6)\ and\ C(-4,2)$.

65. Let A(2, 2), B(4, 4) and C(2, 6) be the vertices of the given ΔABC. Let D, E and F be the midpoints of AB, BC and CA respectively.

Then, the coordinates of D, E and F are




$D\left(\frac{2+4}{2}, \frac{2+4}{2}\right), E\left(\frac{4+2}{2}, \frac{4+6}{2}\right)$ and $F\left(\frac{2+2}{2}, \frac{2+6}{2}\right)$

i.e., D(3, 3), E(3, 5) and F(2, 4).

For $\Delta DEF$, we have

($x_1 = 3$, $y_1 = 3$), ($x_2 = 3$, $y_2 = 5$) and ($x_3 = 2$, $y_3 = 4$)

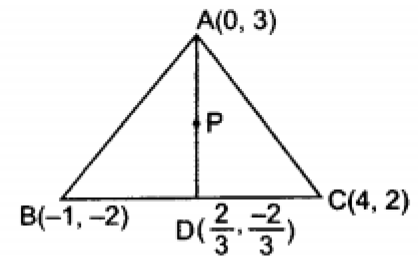
$\therefore \quad \operatorname{art}(\Delta DEF) = \frac{1}{2}|x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)|$

$= \frac{1}{2}|3 \cdot (5-4) + 3 \cdot (4-3) + 2 \cdot (3-5)|$

$= \frac{1}{2}|(3 \times 1) + (3 \times 1) + 2 \times (-2)|$

$= \frac{1}{2}|3 + 3 - 4| = \left(\frac{1}{2} \times 2\right) = 1 sq.unit.$

Hence, the area of ΔDEF is 1 sq unit.

66. 


∵ BD : CD = 1 : 2

∴ Coordinate of D are

$\left(\frac{1\times4+2\times-1}{1+2}, \frac{1\times2+2\times-2}{1+2}\right)$, i.e, $\left(\frac{2}{3}, \frac{-2}{3}\right)$

AD = $\sqrt{\left(\frac{2}{3}-0\right)^2 + \left(\frac{-2}{3}-3\right)^2}$

$= \sqrt{\frac{4}{9} + \frac{121}{9}} = \sqrt{\frac{125}{9}} = \frac{5\sqrt{5}}{3}$ units

DP = AD - AP = $\frac{5\sqrt{5}}{3} - \frac{2\sqrt{5}}{3} = \frac{3\sqrt{5}}{3} = \sqrt{5}$ units

$\therefore \frac{\text{AP}}{\text{DP}} = \frac{\frac{2\sqrt{5}}{3}}{\sqrt{5}} = \frac{2}{3}$

⇒ P divides AD in the ratio 2 : 3.

∴ x-coordinates of P is

x = $\frac{2\times\frac{2}{3}+3\times0}{2+3} = \frac{4}{15}$

Similarly, y-coordinates of P is

y = $\frac{2\times\frac{-2}{3}+3\times3}{2+3} = \frac{23}{15}$

∴ Coordinates of P are $\left(\frac{4}{15}, \frac{23}{15}\right)$.

67. Let Q(x, 0) be a point on x–axis which lies on the perpendicular bisector of AB.

Therefore, QA = QB

⇒ $QA^2 = QB^2$

⇒ $(-5 - x)^2 + (-2 - 0)^2 = (4 - x)^2 + (-2 - 0)^2$

⇒ $(x + 5)^2 + (-2)^2 = (4 - x)^2 + (-2)^2$

$\Rightarrow x^2 + 25 + 10x + 4 = 16 + x^2 - 8x + 4$

$\Rightarrow 10x + 8x = 16 - 25$

$\Rightarrow 18x = -9$

$\Rightarrow x = \frac{-9}{18} = \frac{-1}{2}$

Hence, the point Q is $\left(\frac{-1}{2}, 0\right)$.

Now, $QA^2 = \left[-5 + \frac{1}{2}\right]^2 + [-2 - 0]^2$

$= \left(\frac{-9}{2}\right)^2 + \frac{4}{1}$

$\Rightarrow QA^2 = \frac{81}{4} + \frac{4}{1} = \frac{81+16}{4} = \frac{97}{4}$

$\Rightarrow QA = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2}$ units

Now, $QB^2 = \left(4 + \frac{1}{2}\right)^2 + (-2 - 0)^2 = \left(\frac{9}{2}\right)^2 + (-2)^2$

$\Rightarrow QB^2 = \frac{81}{4} + \frac{4}{1} = \frac{81+16}{4} = \frac{97}{4}$

$\Rightarrow QB = \sqrt{\frac{97}{4}} = \frac{\sqrt{97}}{2}$ units

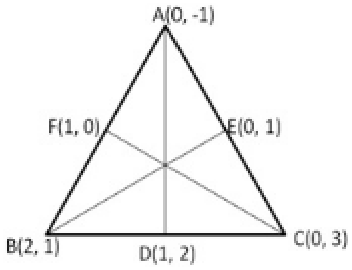
and $AB = \sqrt{(4+5)^2 + [-2-(-2)]^2} = \sqrt{(9)^2}$ = 9 units

$\Rightarrow$ AB = 9 units

As QA = QB

So, Δ QAB is an isosceles Δ.

68. Let D, E, F be the midpoint of the side BC, CA and AB respectively in $\Delta$ABC




Then, by the midpoint formula, we have

$D\left(\frac{2+0}{2}, \frac{1+3}{2}\right), E\left(\frac{0+0}{2}, \frac{3-1}{2}\right) F\left(\frac{0+2}{2}, \frac{-1+1}{2}\right)$

i.e., D(1, 2), E(0, 1), F(1, 0)

Hence the lengths of medians AD, BE and CF are given by

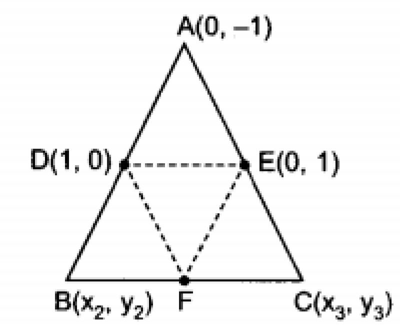
$AD = \sqrt{(1-0)^2 + (2+1)^2} = \sqrt{1+9} = \sqrt{10}\ units$

$BE = \sqrt{(0-2)^2 + (1-1)^2} = \sqrt{4+0} = \sqrt{4} = 2\ units$

$CF = \sqrt{(1-0)^2 + (0-3)^2} = \sqrt{1+9} = \sqrt{10}\ units$

Hence, AD = $\sqrt{10}$, BE= 2, CF = $\sqrt{10}$

69. According to the question, A(0, - 1),D(1, 0) and E(0,1).




Let coordinates of B and C are $(x_2, y_2)$ and $(x_3, y_3)$ respectively.

D is mid-point of AB,

$\therefore 1 = \frac{0+x_2}{2}$

$\Rightarrow x_2 = 2$

and $0 = \frac{-1+y_2}{2}$

$\Rightarrow y_2 = 1$

$\therefore$ Coordinates of B are (2, 1)

E is mid-point of AC

$\therefore 0 = \frac{0+x_3}{2}$

$\Rightarrow x_3 = 0$

and $1 = \frac{-1+y_3}{2}$

$\Rightarrow y_3 = 3$

$\therefore$ Coordinates of C are (0, 3)

Area of $\triangle$ABC

$= \frac{1}{2}\ [0(1-3)+2(3+1)+0(-1+-1)]$

$= \frac{1}{2}\ \times 8$ = 4 sq. units

F is mid-point of BC

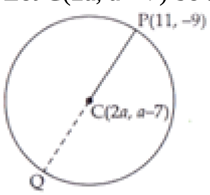
$\therefore$ Coordiantes F are $\left(\frac{2+0}{2}, \frac{1+3}{2}\right)$, i.w, (1, 2).

Area $\triangle$DEF

$= \frac{1}{2}\ [1(2-1)+(1-0)+0(0-2)]$

$= \frac{1}{2}\ [1 + 1]$ = 1 sq. units

70. Let C(2a, a – 7) be the centre of the circle and it passes through the point P(11, –9).




$\therefore PQ = 10\sqrt{2}$

$\Rightarrow CP = 5\sqrt{2}$

$\Rightarrow CP^2 = (5\sqrt{2})^2 = 50$

$\Rightarrow (2a - 11)^2 + (a - 7 + 9)^2 = 50$

$\Rightarrow (2a)^2 + (11)^2 - 2(2a)(11) + (a + 2)^2 = 50$

$\Rightarrow 4a^2 + 121 - 44a + (a)^2 + (2)^2 + 2(a)(2) = 50$

$\Rightarrow 5a^2 - 40a + 125 = 50$

$\Rightarrow a^2 - 8a + 25 = 10$

$\Rightarrow a^2 - 8a + 25 - 10 = 0$

$\Rightarrow a^2 - 8a + 15 = 0$

$\Rightarrow a^2 - 5a - 3a + 15 = 0$

$\Rightarrow a(a - 5) - 3(a - 5) = 0$

$\Rightarrow (a - 5)(a - 3) = 0$

$\Rightarrow a - 5 = 0$ or $a - 3 = 0$

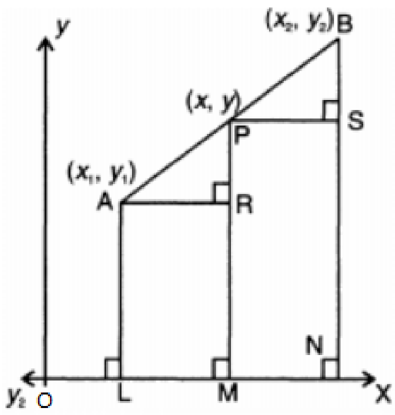
$\Rightarrow a = 5$ or $a = 3$

Hence, the required values of a are 5 and 3.

71. i. Derivation of Section Formula:

Let $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points.

Let P(x, y) be a point on line AB, such that P divides it in the ratio $m_1 : m_2$



Let AB be a line segment joining the points A($x_1$, $y_1$), B($x_2$, $y_2$).

Let P have coordinates (x, y).

Draw AL, PM, BN $\perp$ to x-axis.

It is clear from fig., that

AR = LM = (distance between origin and point M) - (distance between origin and point L).

$\therefore$ AR = LM = OM - OL = x - $x_1$

Similarly, PR = PM - RM = y - $y_1$

And, PS = (Distance between origin and point N) - (Distance between point M and origin) = ON - OM = $x_2$- x

Similarly, BS = BN - SN = $y_2$ - y

$\Delta APR \sim \Delta PBS$ [AAA]

$\frac{AR}{PS} = \frac{PR}{BS} = \frac{AP}{PB}$

Now, $\frac{AR}{PS} = \frac{AP}{PB}$

$\Rightarrow \frac{x-x_1}{x_2-x} = \frac{m_1}{m_2}$

$\Rightarrow m_2(x-x_1) = m_1(x_2-x)$

$\Rightarrow m_2x - m_2x_1 = m_1x_2 - m_1x$

$\therefore x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

Similarly, $\frac{PR}{BS} = \frac{AP}{PB}$

$\Rightarrow \frac{y-y_1}{y_2-y} = \frac{m_1}{m_2}$

$\therefore y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

$\therefore$ Coordinates of P are $\left(\frac{m_1x_2+m_2x_1}{m_1+m_2}, \frac{m_1y_2+m_2y_1}{m_1+m_2}\right)$

ii. Let (- 4, 6) divides the line segment joining the point. A(-6, 10) and B(3, - 8) in k : 1

So, $x_1$ = -6, $y_1$ = 10, $x_2$ = 3, $y_2$ = -8, x = -4, y = 6, $m_1$ = k, $m_2$ = 1

Using section formula,

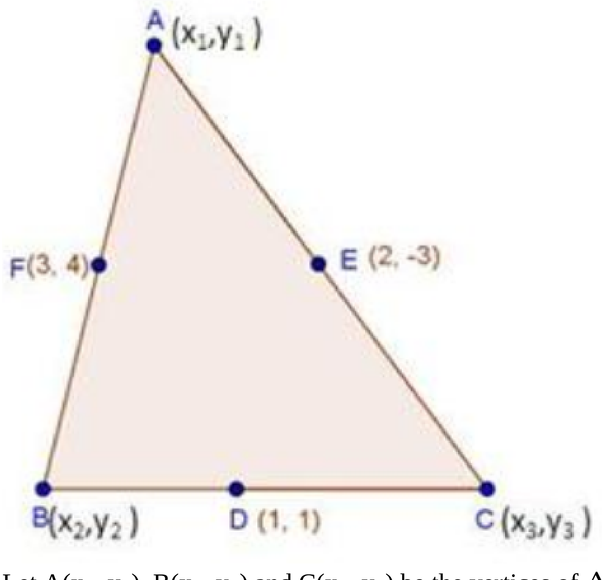
$-4 = \frac{k(3)+1(-6)}{k+1}$

$\Rightarrow -4k - 3k = -6 + 4$

$\Rightarrow -7k = -2$

$\Rightarrow k = \frac{2}{7}$

Therefore, the ratio = 2 : 7

72. 


Let A($x_1$, $y_1$), B($x_2$, $y_2$) and C($x_3$, $y_3$) be the vertices of $\Delta ABC$.

Let D(1, 1), E(2, -3) and F(3, 4) be the mid-points of sides BC, CA and AB respectively.

Since, D is the mid-point of BC

$\therefore \frac{x_2+x_3}{2} = 1$ and $\frac{y_2+y_3}{2} = 1$

$\Rightarrow x_2 + x_3 = 2$ and $y_2 + y_3 = 2$ ...(i)

Similarly E and F are the mid-points of CA and AB respectively.

$\therefore \quad \frac{x_1+x_3}{2}=2 \text{ and } \frac{y_1+y_3}{2}=-3$

$\Rightarrow x_1 + x_3 = 4$ and $y_1 + y_3 = -6$ ...(ii)

and, $\frac{x_1+x_2}{2}=3 \text{ and } \frac{y_1+y_2}{2}=4$

$\Rightarrow x_1 + x_2 = 6$ and $y_1 + y_2 = 8$ ...(iii)

From (i), (ii) and (iii) we get

$x_2 + x_3 + x_1 + x_3 + x_1 + x_2 = 2 + 4 + 6$

and, $y_2 + y_3 + y_1 + y_3 + y_1 + y_2 = 2 +(-6) + 8$

$\Rightarrow 2(x_1 + x_2 + x_3) = 12$ and $2(y_1 + y_2 + y_3) = 4$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$ ...(iv)

From (i) and (iv) we get

$x_1 + 2 = 6$ and $y_1 + 2 = 2$

$\Rightarrow x_1 = 6 - 2 \Rightarrow y_1= 2 - 2$

$\Rightarrow x_1 = 4 \Rightarrow y_1 = 0$

So the coordinates of A are (4, 0)

From (ii) and (iv) we get

$x_2 + 4 = 6$ and $y_2 + (-6) = 2$

$\Rightarrow x_2 = 2 \Rightarrow y_2 - 6 = 2$

$\Rightarrow y_2 = 8$

So the coordinates of B are (2, 8)

From (iii) and (iv) we get

$6 + x_3 = 6$ and $8 + y_3 = 2$

$\Rightarrow x_3 = 6 - 6 \Rightarrow y_3 = 2 - 8$

$\Rightarrow x_3 = 0 \Rightarrow y_3 = -6$

So the coordinates of c are (0, -6)

Hence, the vertices of triangle ABC are:

A(4, 0), B(2, 8) and C(0, -6)