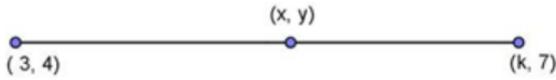**Solution**

COORDINATE GEOMETRY WS 6

Class 10 - Mathematics

Section A

1. -15

   Explanation:

   Since, (x, y) is the mid-point

   $x = \frac{3+k}{2}, y = \frac{4+7}{2} = \frac{11}{2}$

   Again,

   2x + 2y + 1 = 0

   $\Rightarrow 2 \times \frac{(3+k)}{2} + 2 \times \frac{11}{2} + 1 = 0$

   $\Rightarrow$ 3 + k + 11 + 1 = 0

   $\Rightarrow$ 3 + k + 12 = 0

   $\Rightarrow$ k + 15 = 0

   $\Rightarrow$ k = -15

2. 7

   Explanation:

   Mid-point of the line segment joining A(3, 4) and B(k, 6) = $\frac{3+k}{2}, \frac{4+6}{2}$

   $= \frac{3+k}{2}, 5$

   Then, $\frac{3+k}{2}, 5$ = (x, y)

   Therefore, $\frac{3+k}{2}$ = x and 5 = y

   Since x + y - 10 = 0, we have

   $\frac{3+k}{2}$ + 5 - 10 = 0

   i.e., 3 + k = 10

   Therefore, k = 7

3. -1

   Explanation:

   At mid-point of AB = $\left(\frac{\frac{x}{2} + x + 1}{2} \right) = 5$

   or, x = 6

   $\left(\frac{\frac{y+1}{2} + y - 3}{2} \right) = -2$

   or, y + 1 + 2y - 6 = -8

   y = -1

4. 29

   Explanation:

   Point C(-1, 2) divides internally the line segment A(2, 5) and B(x, y) in the ratio 3 : 4

   Then, by section formula

   $C = \left(\frac{3 \times x + 4 \times 2}{3+4}, \frac{3 \times y + 4 \times 5}{3+4} \right)$

   $\Rightarrow (-1, 2) = \left(\frac{3x+8}{7}, \frac{3y+20}{7} \right)$

   $\Rightarrow \frac{3x+8}{7} = -1$ and $\frac{3y+20}{7} = 2$
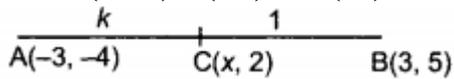
   $\Rightarrow$ 3x + 8 = -7 and 3y + 20 = 14

   $\Rightarrow$ 3x = -15 and 3y = -6

   $\Rightarrow$ x = -5 and y = -2

   $\therefore x^2 + y^2 = (-5)^2 + (-2)^2 = 25 + 4 = 29$

Section B

5. Given, A(-3 , -4) , B(3, 5) and C(x,2)

Let C divides AB in the ratio k: 1

By using section formula, we get

$(x,2) = (\frac{(k\times 3)+(1\times -3)}{k+1}, \frac{(k\times 5)+(1\times -4)}{k+1})$

∴ y coordinate of C = $\frac{(k\times 5)+(1\times -4)}{k+1}$

⇒ $2 = \frac{5k-4}{k+1}$

⇒ 2k + 2 = 5k - 4

⇒ k = 2

∴ C divides AB in the ratio 2 : 1

∴ x coordinates of C = $\frac{(2\times 3)+(1\times -3)}{2+1}$

⇒ x = 1

6. A (x_1, y_1), B(x_2, y_2) C(x_3, y_3) are the three vertices of parallelogram ABCD.

   Let D(a, b) be the fourth vertex.

   Let the diagonals AC and BD of parallelogram intersect at O.

   Now, diagonals of parallelogram bisect each other.

   ∴ O is the mid-point of AC.

   ⇒ Coordinates of $O = \left(\frac{x_1+x_3}{2}, \frac{y_1+y_3}{2}\right)$

   ∴ O is the also mid-point of BD.

   ⇒ Coordinates of $O = \left(\frac{x_2+a}{2}, \frac{y_2+b}{2}\right)$

   ⇒ $\frac{x_1+x_3}{2} = \frac{x_2+a}{2}$ and $\frac{y_1+y_3}{2} = \frac{y_2+b}{2}$

   ⇒ $x_1 + x_3 = x_2 + a$ and $y_1 + y_3 = y_2 + b$

   ⇒ $a = x_1 + x_3 - x_2$ and $b = y_1 + y_3 - y_2$

   Hence, coordinates of fourth vertex are $(x_1 + x_3 - x_2, y_1 + y_3 - y_2)$.

7. It can be observed that Niharika posted the green flag at $\frac{1}{4}$ th of the distance AD i.e., $\frac{1}{4} \times 100 = 25m$ from the starting point of 2nd line. Therefore, the coordinates of this point G is (2, 25)

   Similarly, Preet posted a red flag at $\frac{1}{5}$ th of the distance AD i.e., $\frac{1}{5} \times 100 = 20m$ from the starting point of 8th line. Therefore, the coordinates of this point R are (8, 20)

   Now we have the positions of posts by Preet and Niharika

   According to distance formula, the distance between points A(x_1, y_1) and B(x_2, y_2) is given by

   $D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

   Distance between these flags by using distance formula,

   $D = \sqrt{(8 - 2)^2 + (25 - 20)^2}$

   $= \sqrt{36 + 25}$m

   $= \sqrt{61}$m

   The point at which Rashmi should post her blue flag is the mid-point of the line joining these points. Let this point be A (*X,Y*)

   Now by midpoint formula,

   $(X,Y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

   $X = \left(\frac{2+8}{2}\right) = 5$

   $Y = \left(\frac{25+20}{2}\right) = 22.5$

   Hence, A (X,Y) = (5, 22.5)

   Therefore, Rashmi should post her blue flag at 22.5m on the 5th line.

8. Let the coordinates of A be (x, y) which lies on line joining P(6, -6) and Q (-4, -1)

   such that $\frac{PA}{PQ} = \frac{2}{5}$

   ⇒ $\frac{PA}{PQ-PA} = \frac{2}{5-2}$

$\Rightarrow \frac{PA}{AQ} = \frac{2}{3}$

$\Rightarrow PA : AQ = 2 : 3$

Now by section formula x and y becomes as shown below

Since, P(6, -6) and Q(-4, -1)

$\therefore x = \frac{mx_2 + nx_1}{m+n} = \frac{2(-4) + 3 \times 6}{2+3}$

$= \frac{-8+18}{5} = \frac{10}{5} = 2$

$y = \frac{my_2 + ny_1}{m+n} = \frac{2 \times (-1) + 3(-6)}{2+3}$

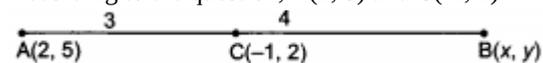$= \frac{-2-18}{5} = \frac{-20}{5} = -4$

Therefore Coordinates of A are (2, -4). As A lies on line segment joining the points P and Q so it must satisfy equation of line segment.

Therefore Substituting the value of x and y i.e; value of A (2,-4) in 3x + k(y + 1) =0

$\Rightarrow$ 3 × 2 + k(-4 + 1) = 0 $\Rightarrow$ 6 - 3k =0

$\Rightarrow 3k = 6 \Rightarrow k = \frac{6}{3} = 2$

9. According to the question, A(2, 5) and C(-1, 2).

point C divides the line segment AB in the ratio 3 : 4.

By using section formula,

$(x, y) = \frac{3 \times x + 4 \times 2}{3+4}, \frac{3 \times y + 4 \times 5}{3+4}$

Comparing x , we get

$\Rightarrow \frac{3 \times x + 4 \times 2}{3+4} = -1$

$\Rightarrow \frac{3x+8}{7} = -1$

$\Rightarrow$ 3x + 8 = -7

$\Rightarrow$ 3x = -15

$\Rightarrow$ x = -5

Comparing y , we get

$\Rightarrow \frac{3 \times y + 4 \times 5}{3+4} = 2$

$\Rightarrow \frac{3y+20}{7} = 2$

$\Rightarrow$ 3y + 20 = 14

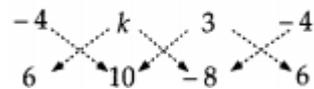$\Rightarrow$ 3y = 14 - 20

$\Rightarrow$ 3y = -6

$\Rightarrow$ y = -2

$\therefore$ Coordinates of B are (-5, -2).

10. If P (-4, 6) lies on the line segment joining A (k, 10) and B (3, -8), then P, A and B are collinear.

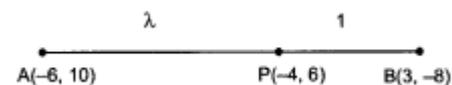$\therefore$ (-4 × 10 + k × -8 + 3 × 6) - (6k + 30 + -4 × -8) = 0

$\Rightarrow$ (-40 - 8k + 18) - (6k + 30 + 32) = 0

$\Rightarrow$ (-22 - 8k) - (6k + 62) = 0

$\Rightarrow$ -14k - 84 = 0

$\Rightarrow$ k = - 6

Suppose P divides AB in the ratio λ: 1. Then, the coordinates of P are $\left(\frac{3\lambda - 6}{\lambda+1}, \frac{-8\lambda+10}{\lambda+1} \right)$ .But, the coordinates of P are (-4, 6).

$\therefore \quad \frac{3\lambda-6}{\lambda+1} = -4$ and $\frac{-8\lambda+10}{\lambda+1} = 6$

$\Rightarrow \quad \lambda = \frac{2}{7}$

Hence, P divides AB in the ratio $\frac{2}{7}$: 1 or 2: 7.

11. Let P and Q be the point which trisect the line AB.

Therefore AP : PQ : QB = 1 : 1 : 1

So P divide AB in 1 : 2

Hence coordinates of P are

$P(\frac{1\times6+2\times3}{1+2}, \frac{1\times8+2\times(-1)}{1+2})$

$= P(\frac{12}{3}, \frac{6}{3})$

P(4, 2)

Similarly, Q divides AB in 2 : 1

so cordiantes of Q

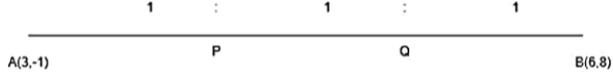$Q(\frac{2\times6+1\times3}{1+2}, \frac{2\times8+1\times(-1)}{1+2})$

$Q(\frac{15}{3}, \frac{15}{3})$

Q(5,5)

Hence points of trisection are P(4, 2) and Q(5, 5)

12. Given: The vertices of a $\triangle$ABC are A (5,5), B (1,5) and C (9,1)

We have,

$\frac{AP}{AB} = \frac{AQ}{AC} = \frac{3}{4}$

$\Rightarrow \quad \frac{AP}{AP+PB} = \frac{AQ}{AQ+QC} = \frac{3}{4}$

$\Rightarrow \frac{AP}{AP+PB} = \frac{3}{4}, \frac{AQ}{AQ+QC} = \frac{3}{4}$

$\Rightarrow$ 4AP = 3AP + 3PB and 4AQ = 3AQ + 3 QC

$\Rightarrow$AP = 3PB and AQ = 3QC

$\Rightarrow \quad \frac{AP}{PB} = \frac{3}{1}$ and $\frac{AQ}{QC} = \frac{3}{1}$

$\Rightarrow$P and Q divide AB and AC respectively in the same ratio 3 : 1

Thus, the coordinates of P and Q are

$\left(\frac{3\times1+1\times5}{3+1}, \frac{3\times5+1\times5}{3+1}\right)$=(2, 5) and $\left(\frac{3\times9+1\times5}{3+1}, \frac{3\times1+1\times5}{3+1}\right)$= (8, 2)

$\therefore \quad PQ = \sqrt{(2-8)^2 + (5-2)^2} = \sqrt{45} = 3\sqrt{5}$ units

13. Given points are A(3, -5) and B(-4, 8).

P divides AB in the ratio k:1

Using the section formula, we have:

Coordinate of point P are $\left\{ \left(\frac{-4k+3}{k+1}\right) \left(\frac{8k-5}{k+1}\right) \right\}$

Now it is given, that P lies on the line x + y =0

Therefore,

$\frac{-4k+3}{k+1} + \frac{8k-5}{k+1} = 0$

$\Rightarrow$ -4k + 3 + 8k - 5 =0

$\Rightarrow$ -4k + 3 + 8k - 5 = 0

$\Rightarrow$ 4k - 2 = 0

$\Rightarrow k = \frac{2}{4}$

$\Rightarrow k = \frac{1}{2}$

Thus, the value of k is 1/2.

14. Let A(-2, -3) and B(5, 6) be the given points.

i. Suppose x-axis divides AB in the ratio k:1 at point P

Then, the coordinates of the point of division are

$P\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1} \right]$

Since P lies on x-axis, and y-coordinates of every point on x-axis is zero.

$\therefore \quad \frac{6k-3}{k+1} = 0$

$\Rightarrow$ 6k - 3=0

$\Rightarrow$ 6k = 3

$\Rightarrow k = \frac{3}{6} \Rightarrow k = \frac{1}{2}$

Hence, the required ratio is 1:2.

Putting $K = \frac{1}{2}$ in the coordinates of P.

We find that its coordinates are $\left(\frac{1}{3}, 0\right)$.

ii. Suppose y-axis divides AB in the ratio k:1 at point Q.

Then, the coordinates of the point of division are

$Q\left[\frac{5k-2}{k+1}, \frac{6k-3}{k+1}\right]$

Since, Q lies on y-axis, and x-coordinates of every point on y-axis is zero.

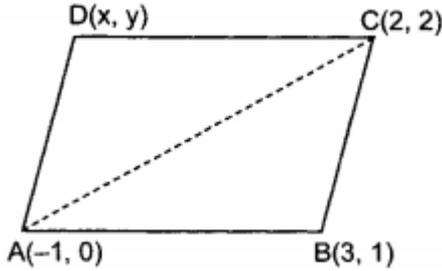$\therefore \quad \frac{5k-2}{k+1} = 0$

$\Rightarrow$ 5k - 2 = 0

$\Rightarrow \quad k = \frac{2}{5}$

Hence, the required ratio is $\frac{2}{5} : 1 = 2 : 5$

Putting $K = \frac{2}{5}$ in the coordinates of Q.

We find that the coordinates are $\left(0, \frac{-3}{7}\right)$

15.

Area of $\triangle$ABC

$= \frac{1}{2}$ [$x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)$]

$= \frac{1}{2}$ [-1(1 - 2) + 3(2 - 0) + 2(0 - 1)]

$= \frac{1}{2}$ [1 + 6 - 2] $= \frac{5}{2}$ sq. units

Area of ∥gm = 2 × area of $\triangle$ABC

$\Rightarrow$ Area of ∥gm = 2 × $\frac{5}{2}$ = 5 sq. units

Let coordinates of D are (x, y)

Mid point of AC $= \left(\frac{-1+2}{2}, \frac{0+2}{2}\right) = \left(\frac{1}{2}, 1\right)$

Mid-point of BD $= \left(\frac{3+x}{2}, \frac{1+y}{2}\right)$

$\therefore$ Diagonals of a ∥gm bisect each other

$\therefore$ Mid-point of BD = Mid-point of AC

$\Rightarrow \left(\frac{3+x}{2}, \frac{1+y}{2}\right) = \left(\frac{1}{2}, 1\right)$

$\Rightarrow \frac{3+x}{2} = \frac{1}{2}$ and $\frac{1+y}{2} = 1$

$\Rightarrow$ x = - 2

$\Rightarrow$ y = 1

Now AD $= \sqrt{(-1+2)^2 + (0+1)^2} = \sqrt{2}$

Also area of ∥gm = base × height

$\Rightarrow$ AD × height = 5

$\Rightarrow \sqrt{2}$ × height = 5

$\Rightarrow$ height $= \frac{5}{\sqrt{2}} = \frac{5}{2}\sqrt{2}$ units.

16. We know that any point on Y axis is in the form of (0, y)

let us suppose the required ratio is K : 1

by using section formula

cordinate of point P are $\left(\frac{Kx_2+x_1}{K+1}, \frac{Ky_2+y_1}{k+1}\right)$

$= \left(\frac{-2K+6}{K+1}, \frac{-7K-4}{k+1}\right)$

On comparing the X cordinates

$\frac{-2K+6}{K+1} = 0$

-2K = -6

K = 3

Hence the required ratio is 3 : 1

Now put the value of K in Y coordiante

$\frac{-7x3-4}{3+1}$

$= \frac{-25}{4}$

hence the required point is $(0, \frac{-25}{4})$

17.

$(5, 7) \quad \overline{\quad P \quad \quad Q \quad} \quad (8, 10)$

Let P(x, y) and Q(x_1, y_1) trisect AB.

P divides AB in the ratio 1 : 2

$\therefore \quad x = \frac{1(8)+2(5)}{3} = 6$

$y = \frac{1(10)+2(7)}{3} = 8$

Hence, P(6, 8)
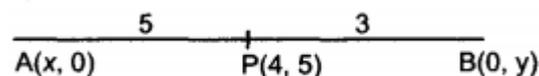
And Q is the mid point of PB.

$x_1 = \frac{6+8}{2} = 7$

$y_1 = \frac{8+10}{2} = 9$

Hence, Q(7, 9)

18. Let coordinates of A are (x, 0) and coordinates of B are (0, y)

```
        5              3
 ————————————————+——————————————
 A(x, 0)      P(4, 5)      B(0, y)
```

Using section formula, we get
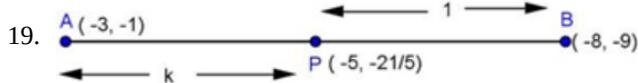
$4 = \frac{5 \times 0 + 3 \times x}{5+3}$

$\Rightarrow 32 = 3x$

$\Rightarrow x = \frac{32}{3}$

Similarly, $5 = \frac{5 \times y + 3 \times 0}{5+3}$

$\Rightarrow 40 = 5y$

$\Rightarrow y = 8$

$\therefore$ Coordinate of A are $\left(\frac{32}{3}, 0\right)$ and coordinates of B are (0, 8).

19.

A (−3, −1) $\quad \overleftarrow{\quad} 1 \overrightarrow{\quad}$ B (−8, −9)

P (−5, −21/5)

$\overleftarrow{\quad k \quad}$

Let the point P divide AB in the ratio K:1

Then, the coordinates of P are $\left[\frac{-8k-3}{k+1}, \frac{-9k-1}{k+1}\right]$

But the coordinates of P are given as $\left(-5, \frac{-21}{5}\right)$

$\therefore \frac{-8k-3}{k+1} = -5$

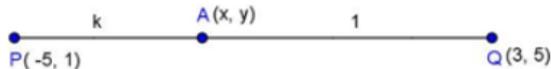$\Rightarrow$ -8k - 3 = -5k - 5

$\Rightarrow$ -8k + 5k = -5 + 3

$\Rightarrow$ -3k = -2

$\Rightarrow k = \frac{2}{3}$

Hence, the point P divides AB in the

ratio $\frac{2}{3} : 1 \Rightarrow 2 : 3$

20. Let A(x, y) divides the join of P(-5, 1) and Q(3, 5) in the ratio k : 1

```
        k      A(x, y)      1
 •————————————————•————————————————•
 P(−5, 1)                      Q(3, 5)
```

By using section formula, we get

$x = \frac{3k-5}{k+1}, y = \frac{5k+1}{k+1}$

Vertices of $\triangle$ABC are $A\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$, B(1, 5) and C(7, -2)

Area of $\triangle$ABC is

$= \frac{1}{2} \left| \frac{3k-5}{k+1}(5+2) + 1\left(-2 - \frac{5k+1}{k+1}\right) + 7\left(\frac{5k+1}{k+1} - 5\right) \right|$

$= \frac{1}{2} \left| \frac{3k-5}{k+1}(7) + 1\left(\frac{-2(k+1)-(5k+1)}{k+1}\right) + 7\left(\frac{5k+1-5(k+1)}{k+1}\right) \right|$

$= \frac{1}{2} \left| \frac{3k-5}{k+1}(7) + 1\left(\frac{-2(k+1)-(5k+1)}{k+1}\right) + 7\left(\frac{5k+1-5(k+1)}{k+1}\right) \right|$

$= \frac{1}{2} \left| \frac{3k-5}{k+1} \times 7 + \frac{-7k-3}{k+1} + \frac{-4 \times 7}{k+1} \right|$

$$= \frac{1}{2} \left| \frac{21k-35}{k+1} + \frac{-7k-3}{k+1} + \frac{-28}{k+1} \right|$$
$$= \frac{1}{2} \left| \frac{21k-35-7k-3-28}{k+1} \right|$$
$$= \frac{1}{2} \left| \frac{14k-66}{k+1} \right|$$
$$= \left| \frac{7k-33}{k+1} \right|$$

But area of $\triangle ABC$ = 2 sq. units

$\Rightarrow \frac{7k-33}{k+1} = \pm 2$

$\Rightarrow 7k - 33 = \pm 2(k + 1)$

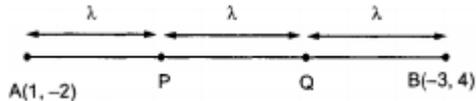$\Rightarrow 7k - 33 = 2k + 2$ or $7k - 33 = -2k - 2$

$\Rightarrow 5k = 35$ or $9k = 31$

$\Rightarrow$ k = 7 or $k = \frac{31}{9}$

21. Let A (1, -2) and B (-3,4) be the given points.

Let the points of trisection be P and Q. Then, AP = PQ = QB = X(say)

A(1, −2)   P   Q   B(−3, 4)

PB = PQ + QB = 2λ and AQ = AP + PQ = 2λ

$\Rightarrow$ AP : PB = λ : 2λ = 1 : 2 and AQ : QB = 2λ : λ = 2 : 1
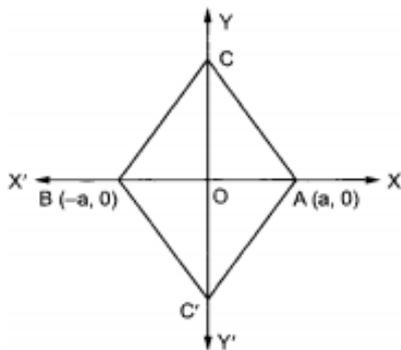
So, P divides AB internally in the ratio 1 : 2 while Q divides internally in the ratio 2: 1. Thus, the coordinates of P and Q are

$$P \left(\frac{1 \times -3 + 2 \times 1}{1+2}, \frac{1 \times 4 + 2 \times -2}{1+2} \right) = P \left(\frac{-1}{3}, 0 \right)$$
$$Q \left(\frac{2 \times -3 + 1 \times 1}{2+1}, \frac{2 \times 4 + 1 \times (-2)}{2+1} \right) = Q \left(\frac{-5}{3}, 2 \right)$$

Hence, the two points of trisection are (-1/3, 0) and (-5/3, 2)

22. Since the mid-point of AB is at the origin O and AB = 2a.

$\therefore$ OA = OB = a.

Therefore, the coordinates of A and B are (a, 0) and (- a, 0) respectively.

Since triangles ABC and ABC' are equilateral. Therefore, their third vertices C and C' lie on the perpendicular bisector of base AB.

Clearly, Y'OY is the perpendicular bisector of AB.

Thus, C and C' lie on Y-axis.

Consequently, their x-coordinates are equal to zero.

In $\triangle AOC$, we have $OA^2 + OC^2 = AC^2$ [Using Pythagoras theorem]

$\Rightarrow a^2 + OC^2 = (2a)^2$ [$\because$ AB = AC = BC and AB = 2a $\therefore$ AC = 2a]
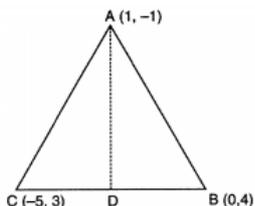
$\Rightarrow OC^2 = 4a^2 - a^2$

$\Rightarrow OC^2 = 3a^2$

$\Rightarrow OC = \sqrt{3}a$

Similarly, by applying Pythagoras theorem in $\triangle AOC'$, we have,

OC' = $\sqrt{3}$a

Therefore, the coordinates of C and C are (0, $\sqrt{3}$a) and (0, - $\sqrt{3}$a) respectively.

23.

Let the vertices of $\triangle ABC$ be A(1, - 1), B(0, 4) and C(-5, 3)

$\therefore$Using the distance formula,

AB = $\sqrt{(1-0)^2 + (-1-4)^2}$

$= \sqrt{1 + 5^2}$

$= \sqrt{1 + 25}$

$= \sqrt{26}$ units ....(i)

BC = $\sqrt{(-5-0)^2 + (3-4)^2}$

$= \sqrt{(-5)^2 + (-1)^2}$

$= \sqrt{25 + 1} = \sqrt{26}$ units ....(ii)

AC = $\sqrt{(-5-1)^2 + (3+1)^2}$

$= \sqrt{36 + 16}$

$= \sqrt{52} = 2\sqrt{13}$ units ....(iii)

from (i), (ii) and (iii)

$\Rightarrow$ AB = BC $\neq$ AC

$\therefore \triangle ABC$ is an isosceles triangle

Using mid-section formula, the coordinates of mid-point of BC are

$x = \frac{-5+0}{2} = -\frac{5}{2}$

$y = \frac{3+4}{2} = \frac{7}{2}$

$\Rightarrow$D(x, y) = $\left(-\frac{5}{2}, \frac{7}{2}\right)$

$\therefore$ AD is median

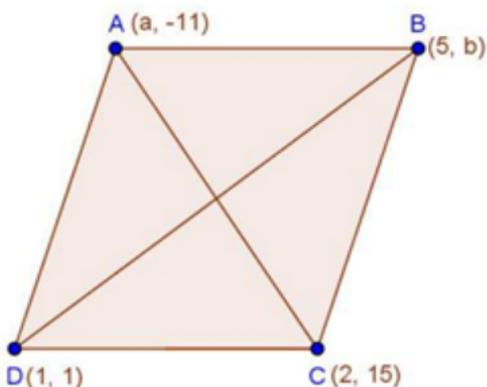$\therefore$ Length of median AD = $\sqrt{\left(\frac{-5}{2} - 1\right)^2 + \left(\frac{7}{2} + 1\right)^2}$

$= \sqrt{\left(\frac{-7}{2}\right)^2 + \left(\frac{9}{2}\right)^2}$

$= \sqrt{\left(\frac{49}{4}\right) + \left(\frac{81}{4}\right)}$

$= \sqrt{\frac{130}{4}} = \frac{\sqrt{130}}{2}$ units

$\therefore$ The length of median AD is $\frac{\sqrt{130}}{2}$ units.

24.

Let A(a, -11), B(5, b), C(2, 15) and D(1, 1) be the given points.

We know that diagonals of parallelogram bisect each other.

Therefore, Coordinates of mid-point of AC = Coordinates of mid-point of BD

$\left(\frac{a+2}{2}, \frac{15-11}{2}\right) = \left(\frac{5+1}{2}, \frac{b+1}{2}\right)$

$\Rightarrow \frac{a+2}{2} = 3$ and $\frac{b+1}{2} = 2$

$\Rightarrow$ a + 2 = 6 and b + 1 = 4

$\implies$ a=6-2 and b=4-1

$\implies$ a=4 and b=3

Hence value of a and b is equal to 4 and 3 respectively.

25. Coordinates of the midpoint P of A and B are

$$\left(\frac{-10+(-2)}{2}, \frac{4+0}{2}\right) = (-6, 2)$$

P lies on the line joining C and D.

Let P(-6, 2) divide C(-9, -4) and D(-4, y) in the ratio of r:1

Using the section formula for the x-coordinate we get

$-6 = \frac{-4r-9}{r+1} \Rightarrow$ -6r - 6 = -4r - 9

$\Rightarrow 2r = 3 \Rightarrow r = \frac{3}{2}$

Hence, P(-6, 2) divides C(-9, -4) and D(-4, y) in the ratio of 3:2

Using the section formula for y-coordinate we get

$-6 = \frac{-4r-9}{r+1} \Rightarrow -6r - 6 = -4r - 9$

$\Rightarrow 2r = 3 \Rightarrow r = \frac{3}{2}$

Hence, P(-6, 2) divides C(-9, -4) and D(-4, y) in the ratio of 3:2

Using the section formula for y-coordinate we get

$2 = \frac{3y-8}{3+2} \Rightarrow 10 = 3y - 8 \Rightarrow 3y = 18$

$\Rightarrow y = 6$

26. Here, G is the centroid of a triangle ABC.

Let A(b, c), B(0, 0) and C(a, 0) be the coordinates of $\triangle$ABC then coordinates of centroid are $G\left[\frac{a+b+0}{3}, \frac{c+0+0}{3}\right]$

To prove:-

$(AB)^2 + (BC)^2 + (CA)^2 = 3(GA^2 + GB^2 + GC^2)$

Consider : L.H.S.

$= (AB)^2 + (BC)^2 + (CA)^2$

$= b^2 + c^2 + a^2 + (a - b)^2 + c^2$

$= b^2 + c^2 + a^2 + a^2 + b^2 - 2ab + c^2$

$= 2a^2 + 2b^2 + 2c^2 - 2ab$

Consider : R.H.S.

$= 3(GA^2 + GB^2 + GC^2)$

$= 3\left[\left(\frac{a+b}{3} - b\right)^2 + \left(c - \frac{c}{3}\right)^2 + \left(\frac{a+b}{3}\right)^2 + \left(\frac{c}{3}\right)^2 + \left(\frac{a+b}{3} - a\right) + \left(\frac{c}{3}\right)^2\right]$

$= 3\left[\left(\frac{a-2b}{3}\right)^2 + \left(\frac{2c}{3}\right)^2 + \left(\frac{a+b}{3}\right)^2 + \left(\frac{c}{3}\right)^2 + \left(\frac{b-2a}{3}\right)^2 + \left(\frac{c}{3}\right)^2\right]$

$= 3\left[\frac{a^2+4b^2-4ab}{9} + \frac{4c^2}{9} + \frac{a^2+b^2+2ab}{9} + \frac{c^2}{9} + \frac{b^2+4a^2-4ab}{9} + \frac{c^2}{9}\right]$

$= 3\left[\frac{a^2+4b^2-4ab+4c^2+a^2+b^2+2ab+c^2+b^2+4a^2-4ab+c^2}{9}\right]$

$= 3\left[\frac{6a^2+6b^2+6c^2-6ab}{9}\right]$

$= 3 \times 3\left[\frac{2a^2+2b^2+2c^2-2ab}{9}\right]$

$= 2a^2 + 2b^2 + 2c^2 - 2ab$

L.H.S. = R.H.S.

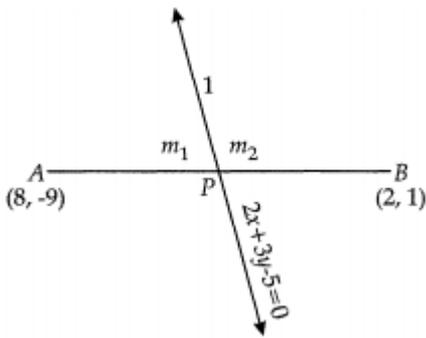Therefore, $(AB)^2 + (BC)^2 + (CA)^2 = 3(GA^2 + GB^2 + GC^2)$

27. We have to find the ratio in which the line 2x + 3y - 5 = 0 divides the line segment joining the points (8, - 9) and (2,1). Also we have to find the co-ordinates of the point of division.

P(x, y) divides AB in the ratio $m_1 : m_2$

A(8, - 9) : B(2,1)

$x = \frac{2m_1+8m_2}{m_1+m_2}$

$y = \frac{m_1-9m_2}{m_1+m_2}$

$$2\left(\frac{2m_1+8m_2}{m_1+m_2}\right)+3\left(\frac{m_1-9m_2}{m_1+m_2}\right)-5=0$$

$$\frac{2(2m_1+8m_2)+3(m_1-9m_2)-5(m_1+m_2)}{m_1+m_2}=0$$

$$\frac{2m_1-16m_2}{m_1+m_2}=0$$

$\therefore 2m_1 - 16m_2 = 0$

i.e, $m_1 : m_2 = 8 : 1$

$\therefore x=\left(\frac{2\times8+8\times1}{8+1}\right)=\frac{8}{3}$
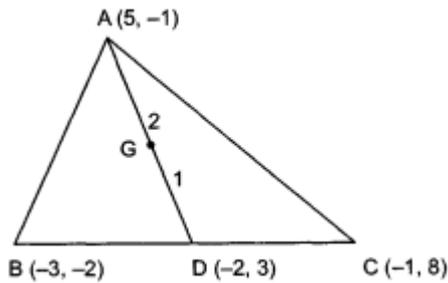
$y=\left(\frac{8\times1-9\times1}{8+1}\right)$

$=-\frac{1}{9}$

P(x, y) = $\left(\frac{8}{3}, -\frac{1}{9}\right)$

28. Let AD be the median through the vertex A of $\triangle$ABC.

Then, D is the mid-point of BC.

So, the coordinates of D are $\left(\frac{-3-1}{2}, \frac{-2+8}{2}\right)$ i.e, (-2, 3)

$\therefore\quad AD=\sqrt{(5+2)^2+(-1-3)^2}=\sqrt{49+16}=\sqrt{65}$ units

Let G be the centroid of $\triangle$ABC.

Then, G lies on median AD and divides it in the ratio 2:1.

So, coordinates of G are

$\left(\frac{2\times-2+1\times5}{2+1}, \frac{2\times3+1\times-1}{2+1}\right)=\left(\frac{-4+5}{3}, \frac{6-1}{3}\right)=\left(\frac{1}{3}, \frac{5}{3}\right)$

29. Let A $\to$ (1, 2), B $\to$ (4, y), C $\to$ (x, 6) and D $\to$ (3, 5).

We know that the diagonals of parallelogram bisect each other.

So, Coordinates of the mid-point of diagonal AC

= Coordinates of the mid-point of diagonal BD

$\Rightarrow \left(\frac{1+x}{2}, \frac{2+6}{2}\right)=\left(\frac{4+3}{2}, \frac{y+5}{2}\right)$

$\Rightarrow \left(\frac{1+x}{2}, 4\right)=\left(\frac{7}{2}, \frac{y+5}{2}\right)$

$\Rightarrow \frac{1+x}{2}=\frac{7}{2}$

$\Rightarrow 1 + x = 7$

$\Rightarrow x = 6$

and $4=\frac{y+5}{2}$

$\Rightarrow y + 5 = 8$

$\Rightarrow y = 3$

30.

We have P(p, -2) and $Q\left(\frac{5}{3}, q\right)$ are the points of trisection of the line segment joining A(3, -4) and B(1, 2)

We know AP : PB = 1 : 2

By section formula $\left[\frac{mx_2+nx_1}{m+n}\right], \left[\frac{my_2+ny_1}{m+n}\right]$ coordinates of P are

$\left(\frac{1\times1+2\times3}{1+2}, \frac{1\times2+2\times(-4)}{1+2}\right)$

$= \left(\frac{7}{3}, -2\right)$

Hence, $P = \frac{7}{3}$

Again we know that AQ : QB = 2 : 1
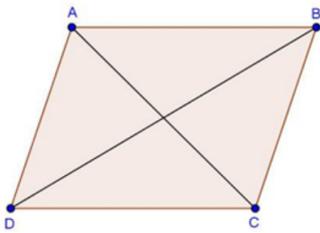
Therefore, Coordinates of Q are (using section formula)

$\left(\frac{2\times1+1\times3}{2+1}, \frac{2\times2+1\times(-4)}{2+1}\right)$

$= \left(\frac{5}{3}, 0\right)$

Hence, q = 0

Therefore, value of p and q is $\frac{7}{3}$ and 0 respectively.

31.

Let A(a + b, a - b), B(2a + b, 2a - b), C(a - b, a + b) and D(x, y) be the given points.

Since, the diagonals of a parallelogram bisect each other.

∴ Coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$\left(\frac{a+b+a-b}{2}, \frac{a-b+a+b}{2}\right) = \left(\frac{2a+b+x}{2}, \frac{2a-b+y}{2}\right)$

$\Rightarrow \quad (a, a) = \left(\frac{2a+b+x}{2}, \frac{2a-b+y}{2}\right)$

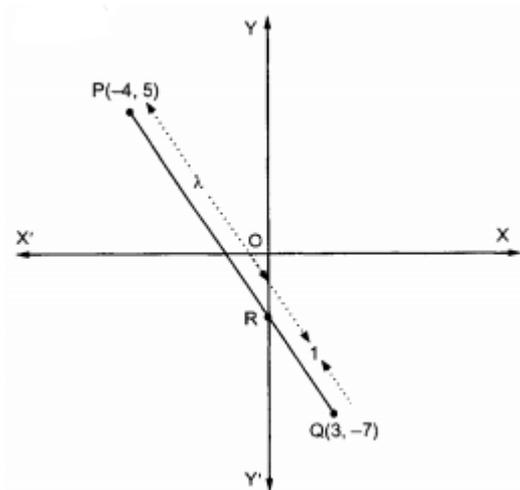$\Rightarrow \quad \frac{2a+b+x}{2} = a$ and $\frac{2a-b+y}{2} = a$

$\Rightarrow$ 2a + b + x = 2a $\Rightarrow$ 2a - b + y = 2a

$\Rightarrow$ x = -b $\Rightarrow$ y = b

Hence, the fourth vertex is (-b, b).

32. Suppose X-axis divides PQ in the ratio λ: 1. Then, the coordinates of the point of division are:

$R\left(\frac{3\lambda-4}{\lambda+1}, \frac{-7\lambda+5}{\lambda+1}\right)$ (by section formula)

Since R lies on y-axis, so the x-coordinate of every point on the y-axis will be zero.

By equating the x coordinate =0,

∴ $\frac{3\lambda-4}{\lambda+1} = 0 \Rightarrow 3\lambda - 4 = 0 \Rightarrow \lambda = \frac{4}{3}$

Hence, the required ratio is $\frac{4}{3}$: 1 i.e, 4: 3.

Putting $\lambda = 4/3$ in the coordinates of R, we find that its coordinates are $\left(0, \frac{-13}{7}\right)$

Hence, the coordinates of R are (0, -13/7)

33.

Consider QB.

The point S divides QB in the ratio 2:1

$6 = \frac{m_1 x_2 + m_2 x_1}{m_1 + m_2}$

$\Rightarrow 6 = \frac{2(7) + 1(x)}{2+1}$

$\Rightarrow 6 = \frac{x+14}{3}$

$\Rightarrow 18 = x + 14$

$\Rightarrow x = 4$

$y = \frac{m_1 y_2 + m_2 y_1}{m_1 + m_2}$

$\Rightarrow y = \frac{2(10) + 1(7)}{2+1}$

$\Rightarrow y = \frac{20+7}{3}$

$\Rightarrow y = 9$

Consider AS.

The point Q divides AS in the ratio 1:1.

$7 = \frac{y_1 + y_2}{2}$

$\Rightarrow 7 = \frac{p+9}{2}$

$\Rightarrow 14 = p + 9$

$\Rightarrow p = 5$

So, x = 4, y = 9, p = 5.

34. Given: points A(3, - 4) and B(1, 2) trisected at the points P and Q

Let the co-ordinates of P be (x, y)

Since AP = PQ = QB

$\therefore$AP: PB = 1: 2

Using intersection formula

$x = \frac{x_2(m_1) + x_1(m_2)}{m_1 + m_2}$, $y = \frac{y_2(m_1) + y_1(m_2)}{m_1 + m_2}$

$x = \frac{1 \times 1 + 2 \times 3}{1+2} = \frac{7}{3}$

$y = \frac{1 \times 2 + 2 \times -4}{1+2} = -2$

Hence point P is $\left(\frac{7}{3}, -2\right)$.

35. Let D, E, F be the mid-points of the sides BC, CA and AB respectively. Then, the coordinates of D, E and E are
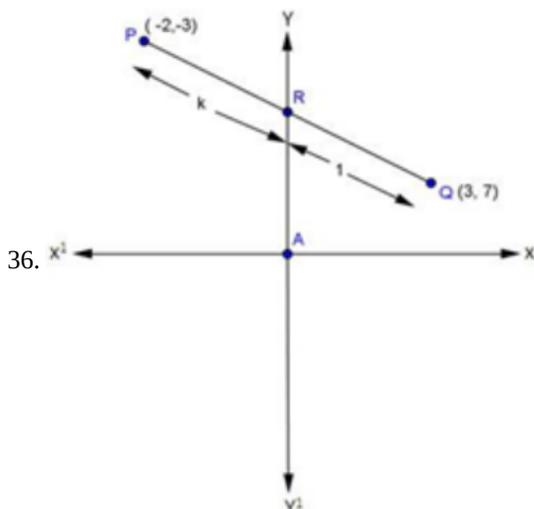
$D\left(\frac{5+3}{2}, \frac{3-1}{2}\right)$ = D(4, 1), $E\left(\frac{3+7}{2}, \frac{-1-3}{2}\right)$ = E(5, -2)

and, $F\left(\frac{7+5}{2}, \frac{-3+3}{2}\right)$ = F(6, 0)

$\therefore$AD = $\sqrt{(7-4)^2 + (-3-1)^2} = \sqrt{9+16}$ = 5 units

BE = $\sqrt{(5-5)^2 + (-2-3)^2} = \sqrt{0+25}$ = 5 units

and, CF = $\sqrt{(6-3)^2 + (0+1)^2} = \sqrt{9+1} = \sqrt{10}$ units

36.

Suppose y-axis divides PQ in the ratio K:1 at R.

Then, the coordinates of the point of division are:

$R\left[\frac{3k+(-2)\times 1}{k+1}, \frac{7k+(-3)\times 1}{k+1}\right]$

$= R\left[\frac{3k-2}{k+1}, \frac{7k-3}{k+1}\right]$

Since, R lies on y-axis and x-coordinate of every point on y-axis is zero

$\therefore \quad \frac{3k-2}{k+1} = 0$

$\Rightarrow$ 3k - 2 = 0

$\Rightarrow$ 3k = 2
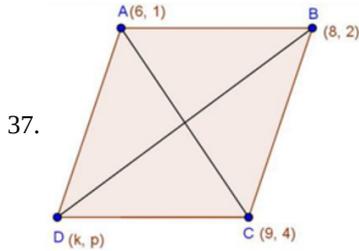
$\Rightarrow k = \frac{2}{3}$

Hence, the required ratio is $\frac{2}{3} : 1$

i.e., 2:3

Putting $k = \frac{2}{3}$ in the coordinates of R

We get, (0, 1)

37.

Let A(6, 1), B(8, 2), C(9, 4) and D(k, p) be the given points.

Since, ABCD is a parallelogram.

Coordinates of the mid-point of AC = Coordinates of the mid-point of BD

$\Rightarrow \quad \left(\frac{6+9}{2}, \frac{1+4}{2}\right) = \left(\frac{8+k}{2}, \frac{2+p}{2}\right)$

$\Rightarrow \quad \left(\frac{15}{2}, \frac{5}{2}\right) = \left(\frac{8+k}{2}, \frac{2+p}{2}\right)$

$\Rightarrow \quad \frac{8+k}{2} = \frac{15}{2}$ and $\frac{2+p}{2} = \frac{5}{2}$

$\Rightarrow$ 8 + k = 15 $\Rightarrow$ 2 + p = 5

$\Rightarrow$ k = 7 $\Rightarrow$ p = 3

38.

PQ is the line segment, A and B are the points of trisection of PQ.

We know that PA : QA = 1:2

So, the coordinates of A are

$\left(\frac{6\times 1+3\times 2}{2+1}, \frac{-6\times 1+3\times 2}{2+1}\right)$

$= \frac{12}{3}, 0$

$= (4, 0)$

Since, A lies on the line

2x + y + k = 0

$\Rightarrow 2 \times 4 + 0 + k = 0$

$\Rightarrow 8 + k = 0$

$\Rightarrow k = -8$

39. Let P(1, 1), Q(2, -3), R(3, 4) be the mid-points of sides AB, BC and CA respectively of triangle ABC. Let A(x_1, y_1), B(x_2, y_2) and

C(x_3, y_3) be the vertices of triangle ABC. Then,

P is the midpoint of BC

$\Rightarrow \frac{x_1+x_2}{2} = 1, \frac{y_1+y_2}{2} = 1$

$\Rightarrow x_1 + x_2 = 2$ and $y_1 + y_2 = 2$..........(i)

Q is the midpoint of BC

$\Rightarrow \frac{x_2+x_3}{2} = 2, \frac{y_2+y_3}{2} = -3$

$\Rightarrow x_2 + x_3 = 4$ and $y_2 + y_3 = -6$........(ii)

R is the midpoint of AC

$\Rightarrow \frac{x_1+x_3}{2} = 3, \frac{y_1+y_3}{2} = 4$

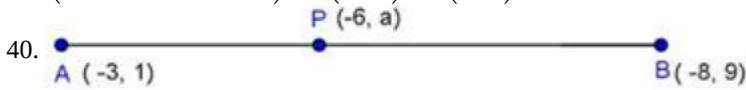$\Rightarrow x_1 + x_3 = 6$ and $y_1 + y_3 = 8$..........(iii)

From (i), (ii) and (iii), we get

$x_1 + x_2 + x_2 + x_3 + x_1 + x_3 = 2 + 4 + 6$

and, $y_1 + y_2 + y_2 + y_3 + y_3 + y_1 + y_3 = 2 - 6 + 8$

$\Rightarrow x_1 + x_2 + x_3 = 6$ and $y_1 + y_2 + y_3 = 2$.........(iv)

The coordinates of the centroid of $\triangle ABC$ are

$\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right) = \left(\frac{6}{3}, \frac{2}{3}\right) = \left(2, \frac{2}{3}\right)$ [Using (iv)]

40.

Let P(-6, a) divides the join of A(-3, 1) and B(-8, 9) in the ratio k:1

Then, the coordinates of P are

$\left(\frac{-8k-3}{k+1}, \frac{9k+1}{k+1}\right)$

But, $\frac{-8k-3}{k+1} = -6$

$\Rightarrow$ -8k - 3 = -6k - 6

$\Rightarrow$ -8k + 6k = -6 + 3

$\Rightarrow$ -2k = -3

$\Rightarrow \quad k = \frac{3}{2}$

Hence, P divides AB in the ratio 3:2

Again,

$\frac{9k+1}{k+1} = a$
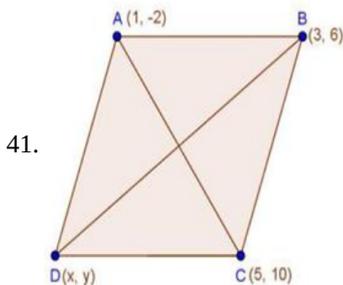
Substituting $k = \frac{3}{2}$

We get,

$\frac{9 \times \frac{3}{2} + 1}{\frac{3}{2} + 1} = a$

$\Rightarrow \frac{\frac{29}{2}}{\frac{5}{2}} = a$

$\Rightarrow \quad \frac{29}{5} = a$

$\therefore \quad a = \frac{29}{5}$

41.

Let A (1, -2), B(3, 6), C(5, 10) and D(x, y) be the given points taken in order.

Since, diagonals of parallelogram bisect each other.

Coordinates of mid-point of AC = Coordinates of mid-point of BD

$\left(\frac{5+1}{2}, \frac{10-2}{2}\right) = \left(\frac{x+3}{2}, \frac{y+6}{2}\right)$

$\Rightarrow \quad (3, 4) = \frac{x+3}{2}, \frac{y+6}{2}$

$\Rightarrow \quad \frac{x+3}{2} = 3 \quad$ and $\quad \frac{y+6}{2} = 4$
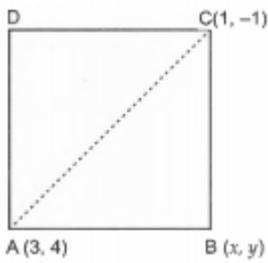
$\Rightarrow$ x + 3 = 6 $\Rightarrow$ y + 6 = 8

$\Rightarrow$ x = 3 $\Rightarrow$ y = 2

Hence, the fourth vertex is (3, 2)

42. Let ABCD be a square and let A (3,4) and C (1, -1) be the given angular points.

Let B(x, y) be the unknown vertex

Then, AB = BC

$\Rightarrow AB^2 = BC^2$

$\Rightarrow (x - 3)^2 + (y - 4)^2 = (x - 1)^2 + (y + 1)^2$

$\Rightarrow 4x + 10y - 23 = 0$

$\Rightarrow \quad x = \frac{23-10y}{4}$ .... (i)

In right-angled triangle ABC, we have

$AB^2 + BC^2 = AC^2$

$\Rightarrow (x - 3)^2 + (y - 4)^2 + (x - 1)^2 + (y + 1)^2 = (3 - 1)^2 + (4 + 1)^2$

$\Rightarrow x^2 + y^2 - 4x - 3y - 1 = 0$ ...(ii)

Substituting the value of x from (i) into (ii), we get

$\left(\frac{23-10y}{4}\right)^2 + y^2 - (23 - 10y) - 3y - 1 = 0$

$\Rightarrow 4y^2 - 12y + 5 = 0 \Rightarrow (2y - 1)(2y - 5) = 0 \Rightarrow y = \frac{1}{2}$ or $\frac{5}{2}$

Putting $y = \frac{1}{2}$ and $y = \frac{5}{2}$ respectively in (i), we get $x = \frac{9}{2}$ and $x = \frac{-1}{2}$ respectively.

Hence, the required vertices of the square are (9/2,1/2) and (-1 /2 ,5 /2).

43. Let ABCD be a parallelogram in which the co-ordinates of the vertices are A (3,-4); B (-1,-3) and C(-6,2). We have to find the co-ordinates of the forth vertex.

Let the forth vertex be D(x, y)

Since ABCD is a parallelogram, the diagonals bisect each other. Therefore the midpoint of the diagonals of the parallelogram will coincide.

Now to find the mid-point P(x, y) of two points $A(x_1, y_1)$ and $B(x_2, y_2)$.

we use section formula as,

$P(x,y) = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

The mid-point of the diagonals of the parallelogram will coincide.

So, coordinate of mid-point AC = Coordinate of mid-point of BD

Therefore,

$\left(\frac{x-1}{2}, \frac{y-3}{2}\right) = \left(\frac{3-6}{2}, \frac{2-4}{2}\right)$

$\left(\frac{x-1}{2}, \frac{y-3}{2}\right) = \left(-\frac{3}{2}, -1\right)$

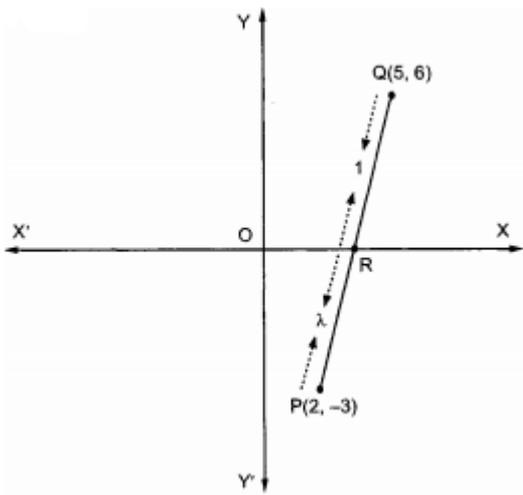Now equate the individual terms to get the unknown value. So,

x = -2

y = 1

So the forth vertex is D(-2, 1)

44. Let the required ratio be λ: 1. Then, the coordinates of the point of division are,

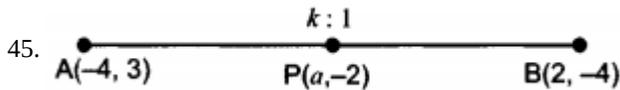$R\left(\frac{5\lambda+2}{\lambda+1}, \frac{6\lambda-3}{\lambda+1}\right)$

But, it is a point on x-axis on which y-coordinates of every point is zero.

$\therefore \quad \frac{6\lambda - 3}{\lambda + 1} = 0 \Rightarrow \lambda = \frac{1}{2}$

Thus, the required ratio is $\frac{1}{2} : 1$ or, $1 : 2$.

Putting $\lambda = 1/2$ in the coordinates of R, we find that its coordinates are (3, 0).

45.

Let P divides the joining AB in the ratio k : 1

$\therefore$ y coordinate of P = $\frac{k(-4) + 1 \times 3}{k+1}$

$\Rightarrow -2 = \frac{-4k + 3}{k+1}$

$\Rightarrow -2k - 2 = -4k + 3$

$\Rightarrow 2k = 5$

$\Rightarrow k = \frac{5}{2}$

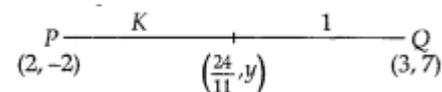$\therefore$ P divides the joining of A and B in the ratio of 5 : 2

Now,

x coordinate of P = $\frac{5 \times 2 + 2(-4)}{5+2}$

$\Rightarrow a = \frac{10 - 8}{7}$

$\Rightarrow a = \frac{2}{7}$

46. Here $\left(\frac{24}{11}, y\right)$ be any point on line formed by joining the points P and Q. Let it divide the line segment in the ratio of K:1.

Let PA : QA be K : 1

Where $\left(\frac{24}{11}, y\right)$ represent point A.

Using intersection formula,

$\frac{mx_2 + nx_1}{m+n} = x$

$\implies \frac{3k + 2}{k+1} = \frac{24}{11}$

$\implies (3k+2)11 = 24(k+1)$

$\implies 33k + 22 = 24k + 24$

$\implies 33k - 24k = 24 - 22$

$\implies 9k = 2$

$\implies k = \frac{2}{9}$
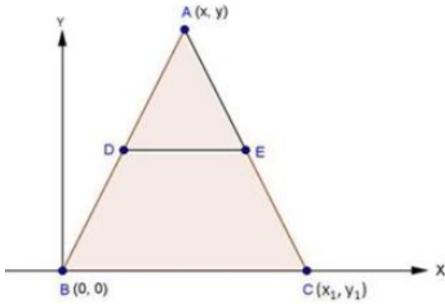
Thus the ratio in which line is divided is equal to k:1=2:9

Also $y = \frac{-18 + 14}{11} = -\frac{4}{11}$

Thus y= $-\frac{4}{11}$ which is coordinate of y.

Therefore the ratio in which the line is divided by point $\left(\frac{24}{11}, \frac{-4}{11}\right)$ is 2:9

47. Let ABC be a triangle such that BC is along x-axis.

Coordinates of A, B and C are (x, y), (0, 0) and (x_1, y_1)

D and E are the mid-points of AB and AC respectively.

Coordinates of D are $\left(\frac{x+0}{2}, \frac{y+0}{2} \right)$

$= \left(\frac{x}{2}, \frac{y}{2} \right)$

Coordinates of E are $\left(\frac{x+x_1}{2}, \frac{y+y_1}{2} \right)$

Length of BC $= \sqrt{x_1^2 + y_1^2}$

Length of $DE = \sqrt{\left(\frac{x+x_1}{2} - \frac{x}{2} \right)^2 + \left(\frac{y+y_1}{2} - \frac{y}{2} \right)^2}$

$= \sqrt{\left(\frac{x_1}{2} \right)^2 + \left(\frac{y_1}{2} \right)^2}$
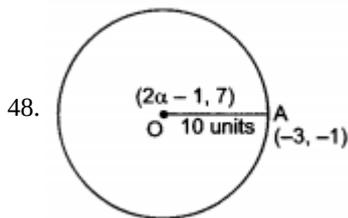
$= \sqrt{\frac{x_1^2}{4} + \frac{y_1^2}{4}}$

$= \sqrt{\frac{1}{4} \left(x_1^2 + y_1^2 \right)}$

$= \frac{1}{2} \sqrt{x_1^2 + y_1^2}$

$= \frac{1}{2} BC$

Hence proved that length of $DE = \frac{1}{2}$ of BC

48.

OA = 10 units

$\Rightarrow$ OA $= \sqrt{(2a - 1 + 3)^2 + (7 + 1)^2}$

$\Rightarrow$ 10 $= \sqrt{4a^2 + 4 + 8a + 64}$

Squaring 100 = 4a^2 + 8a + 68
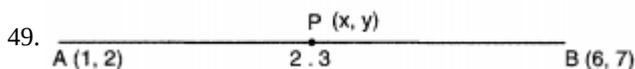
$\Rightarrow$ 4a^2 + 8a - 32 = 0

$\Rightarrow$ a^2 + 2a - 8 = 0

$\Rightarrow$ a^2 + 4a - 2a - 8 = 0

$\Rightarrow$ a(a + 4) - 2(a + 4) = 0

$\Rightarrow$ (a + 4)(a - 2) = 0

$\therefore$ a = - 4, a = 2.

49.

AP = $\frac{2}{5}$ AB

by Compodendo-Dividendo, $\frac{AP}{(AB - AP)} = \frac{2}{(5 - 2)}$

$\Rightarrow$ AP : PB = 2 : 3

$x = \frac{2 \times 6 + 3 \times 1}{2 + 3}$ and $y = \frac{2 \times 7 + 3 \times 2}{2 + 3}$

$\therefore \quad x = \frac{12 + 3}{5} = 3$, $y = \frac{14 + 6}{5} = 4$

P(x, y) = (3, 4)

50. Given the line segment joining the points (-4, 7) and (3, -7).

Let the y-axis cut the join of A(-4, 7) and B(3, -7) at the point P in the ratio k:1,

Then, by section formula, $\left[\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right]$

Coordinates of $P = \left(\frac{k\times 3+1\times(-4)}{k+1}, \frac{k\times(-7)+1\times 7}{k+1}\right)$

$= \left(\frac{3k-4}{k+1}, \frac{-7k+7}{k+1}\right)$

But P lies on y-axis. So, its abscissa is 0.

$\therefore \frac{3k-4}{k+1} = 0$
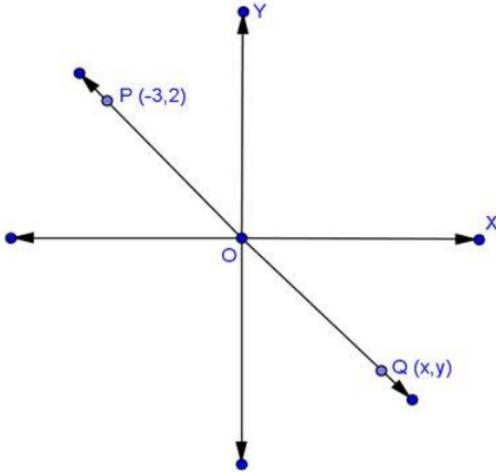
$\Rightarrow$ 3k - 4 =0

$\Rightarrow$ 3k = 4

$\Rightarrow k = \frac{4}{3}$

Therefore , the required ratio is 4:3.

51. Given coordinates of the point P are (-3, 2).

Let the coordinates of Q be (x, y)

Since Q lies on the line joining P and O (origin) and OP = OQ

By mid-point theorem

$\frac{(x-3)}{2} = 0$ and $\frac{(y+2)}{2} = 0$

$\implies$ x-3=0 and y+2=0

Therefore, x = 3, y = -2

Hence coordinates of point Q are (3, -2).

52. Given coordinates of point $P\left(\frac{3}{4}, \frac{5}{12}\right)$ and coordinates of the line segment joining the point $A\left(\frac{1}{2}, \frac{3}{2}\right)$ and B(2, -5).

Let the required ratio be k:1

Then, by section formula, $\left[\frac{mx_2+ny_1}{m+n}, \frac{my_2+nx_1}{m+n}\right]$

Coordinates of $P = \left(\frac{k\times 2+1\times\frac{1}{2}}{k+1}, \frac{k\times(-5)+1\times\frac{3}{2}}{k+1}\right)$

$= \left(\frac{2k+\frac{1}{2}}{k+1}, \frac{-5k+\frac{3}{2}}{k+1}\right)$

$= \left(\frac{4k+1}{2(k+1)}, \frac{-10k+3}{2(k+1)}\right)$

Given, coordinates of $P = \left(\frac{3}{4}, \frac{5}{12}\right)$

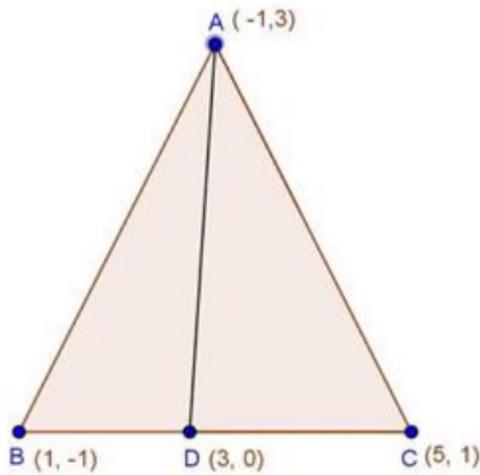$\therefore \frac{4k+1}{2(k+1)} = \frac{3}{4}$

$\Rightarrow$ 16k + 4 = 6k + 6

$\implies$ 16k-6k=6-4

$\Rightarrow$ 10k = 2

$\Rightarrow k = \frac{1}{5}$

So, the required ratio is 1:5.

53.

Let A(-1, 3), B(1, -1) and C(5, 1) be the vertices of triangle ABC and let AD be the median through A.

Since, AD is the median, D is the mid-point of BC

Coordinates of mid point are $\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$\therefore$ Coordinates of D are $\left(\frac{1+5}{2}, \frac{-1+1}{2}\right) = (3, 0)$

So, Length of median $AD = \sqrt{(3+1)^2 + (0-3)^2}$

$= \sqrt{(4)^2 + (-3)^2}$

$= \sqrt{16+9}$

$= \sqrt{25}$

=5 units

Hence, median AD is 5

Section C

54. We have, P(-3, 4), Q(3, 4) and R(-2, -1).

$\therefore$ Coordinates of centroid of $\triangle$PQR

$= \left(\frac{-3+3-2}{3}, \frac{4+4-1}{3}\right) = \left(\frac{-2}{3}, \frac{7}{3}\right)$

55. Coordinates of T $= \left(\frac{-2+3}{2}, \frac{-1+4}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$

56. Coordinates of U $= \left(\frac{-2-3}{2}, \frac{-1+4}{2}\right) = \left(\frac{-5}{2}, \frac{3}{2}\right)$

57. The centroid of the triangle formed by joining the mid-points of sides of a given triangle is the same as that of the given triangle.

So, centroid of $\triangle$STU $= \left(\frac{-2}{3}, \frac{7}{3}\right)$

58. Since, PQRS is a square

$\therefore$ PQ = QR = RS = PS

Length of PQ = 200 - (-200) = 400

$\therefore$ The coordinates of R = (200, 400)

and coordinates of S = (-200, 400)

59. Area of square PQRS = (side)^2

$= (PQ)^2$

$= (400)^2$

= 1,60,000 sq. units

60. By Pythagoras theorem

$(PR)^2 = (PQ)^2 + (QR)^2$

= 1,60,000 + 1,60,000

= 3,20,000

$\Rightarrow$ PR $= \sqrt{3,20,000}$

$= 400 \times \sqrt{2}$ units

61. Since, point S divides CA in the ratio K : 1

$\therefore \left(\frac{Kx_2+x_1}{K+1}, \frac{Ky_2+y_1}{K+1}\right) = (-200, 400)$

$\Rightarrow \left(\frac{K(200)+(-600)}{K+1}, \frac{K(800)+0}{K+1}\right) = (-200, 400)$

$\Rightarrow \left(\frac{200K-600}{K+1}, \frac{800K}{K+1} \right)$ = (-200, 400)

$\therefore \frac{800K}{K+1}$ = 400
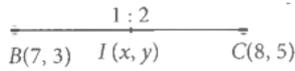
$\Rightarrow$ 800K = 400K + 400

$\Rightarrow$ 400K = 400

$\Rightarrow$ K = 1

62. The distance between A and C

$= \sqrt{(8-4)^2 + (5+3)^2} = \sqrt{4^2 + 8^2}$

$= \sqrt{16 + 64} = \sqrt{80} = 4\sqrt{5}$ units

63. Let the coordinates of I be (x, y)

Then, by section formula,

x = $\frac{1 \times 8 + 2 \times 7}{1+2} = \frac{8+14}{3} = \frac{22}{3}$

and y = $\frac{1 \times 5 + 2 \times 3}{1+2} = \frac{5+6}{3} = \frac{11}{3}$

Thus, the coordinates of I is $\left(\frac{22}{3}, \frac{11}{3} \right)$

64. The mid-point of A and C

$= \left(\frac{8+4}{2}, \frac{5-3}{2} \right)$ = (6, 1)

65. Let B divides the line segment joining A and C in the ratio k : 1. Then, the coordinates of B will be $\left(\frac{8k+4}{k+1}, \frac{5k-3}{k+1} \right)$.

Thus, we have $\left(\frac{8k+4}{k+1}, \frac{5k-3}{k+1} \right)$ = (7, 3)

$\Rightarrow \frac{8k+4}{k+1}$ = 7 and $\frac{5k-3}{k+1}$ = 3

Consider, $\frac{8k+4}{k+1}$ = 7 $\Rightarrow$ 8k + 4 = 7k + 7 $\Rightarrow$ k = 3

Hence, the required ratio is 3 : 1.

66. Point of intersection of diagonals is their midpoint

So, $\left[\frac{(1+7)}{2}, \frac{(1+5)}{2} \right]$

= (4, 3)

67. Length of diagonal AC

AC = $\sqrt{(7-1)(7-1) + (5-1)(5-1)}$

$= \sqrt{52}$ units

68. Area of campaign board

= 6 $\times$ 4

= 24 units square

69. Ratio of lengths = $\frac{AB}{AC}$

$= \frac{6}{\sqrt{52}}$

$= 6 : \sqrt{52}$