Solution

INTRODUCTION TO TRIGONOMETRY WS 1

Class 10 - Mathematics

Section A

1.

(b) $\frac{7}{8}$

Explanation: $\tan^2\theta = \frac{8}{7} \Rightarrow \tan\theta = \frac{\sqrt{8}}{\sqrt{7}} = \frac{2\sqrt{2}}{\sqrt{7}} \quad = \frac{\text{Perpendicular}}{\text{Base}}$

By Pythagoras Theorem

(Hyp.)^2 = (Base)^2 + (Perp.)^2

$= (\sqrt{7})^2 + (2\sqrt{2})^2 = 7 + 8 = 15$

$\therefore \text{Hyp.} = \sqrt{15}$

$\therefore \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{2\sqrt{2}}{\sqrt{15}}$

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{\sqrt{7}}{\sqrt{15}}$

Now, $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

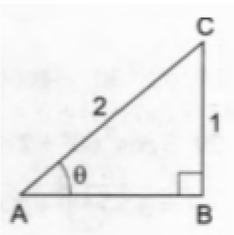
$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{1-\left(\frac{2\sqrt{2}}{\sqrt{15}}\right)^2}{1-\left(\frac{\sqrt{7}}{\sqrt{15}}\right)^2}$

$= \frac{1-\frac{8}{15}}{1-\frac{7}{15}} = \frac{\frac{15-8}{15}}{\frac{15-7}{15}} = \frac{\frac{7}{15}}{\frac{8}{15}}$

$= \frac{7}{15} \times \frac{15}{8} = \frac{7}{8}$

2.

(d) $\sqrt{3}$

Explanation: 

$\sin\theta = \frac{1}{2} = \frac{BC}{AC}$

$\therefore$ AB^2 = AC^2 - BC^2 = 4 - 1 = 3

$\therefore \quad AB = \sqrt{3}$

$\cot\theta = \frac{AB}{BC} = \frac{\sqrt{3}}{1} = \sqrt{3}$

3.

(d) $\sqrt{3}$

Explanation: Given: $\sin\theta = \frac{\sqrt{3}}{2}$ and cosec $\theta = \frac{2}{\sqrt{3}}$

cosec^2 θ - cot^2 θ = 1

$\Rightarrow$ cot^2 θ = cosec$^2\theta$ - 1

$\Rightarrow$ cot^2 θ = $\frac{4}{3}$ - 1 [Given]

$\Rightarrow$ cot θ = $\frac{1}{\sqrt{3}}$

$\therefore$ cosec θ + cot θ = $\frac{2}{\sqrt{3}} + \frac{1}{\sqrt{3}}$

= $\frac{3}{\sqrt{3}}$

= $\frac{\sqrt{3}\times\sqrt{3}}{\sqrt{3}}$

= $\sqrt{3}$

4. **(a)** $\frac{\sqrt{b^2-a^2}}{b}$

Explanation: Given: $\sin\theta = \frac{a}{b}$

we know that cosθ = $\sqrt{1 - sin^2\theta}$

[∵, $\sin^2\theta + \cos^2\theta = 1$]

or, cosθ = $\sqrt{1 - a^2/b^2}$

or, cosθ = $\frac{\sqrt{b^2-a^2}}{b}$

5.

(d) 0

Explanation: $5\tan\theta - 4 = 0 \Rightarrow 5\tan\theta = 4$

$\Rightarrow \tan\theta = \frac{4}{5}$

Now, $\frac{5\sin\theta - 4\cos\theta}{5\sin\theta + 4\cos\theta} = \frac{5\frac{\sin\theta}{\cos\theta} - 4\frac{\cos\theta}{\cos\theta}}{5\frac{\sin\theta}{\cos\theta} + 4\frac{\cos\theta}{\cos\theta}}$ (Dividing by cos θ)

= $\frac{5\tan\theta - 4}{5\tan\theta + 4}$

= $\frac{5\times\frac{4}{5} - 4}{5\times\frac{4}{5} + 4} = \frac{4-4}{4+4} = \frac{0}{8} = 0$

6.

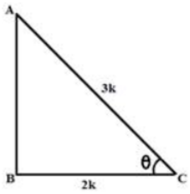
(c) 0

Explanation: Given,

cos θ = $\frac{2}{3}$ = $\frac{b}{h}$ = k

2sec^2 θ + 2 tan^2 θ - 7

b = 2k, h = 3k



In △ABC,

$h^2 = p^2 + b^2$

$\Rightarrow (3k)^2 = p^2 + (2k)^2$

$\Rightarrow 9k^2 = p^2 + 4k^2$

$\Rightarrow p^2 = 9k^2 - 4k^2$

$\Rightarrow p^2 = 5k^2$

$\Rightarrow p = \sqrt{5}k$

Then,

sec θ = $\frac{3k}{2k} = \frac{3}{2}$ and tan θ = $\frac{\sqrt{5}k}{2k} = \frac{\sqrt{5}}{2}$

⇒ 2 sec^2 θ + 2 tan^2 θ - 7

$\Rightarrow 2\left(\frac{3}{2}\right)^2 + 2\left(\frac{\sqrt{5}}{2}\right)^2 - 7$

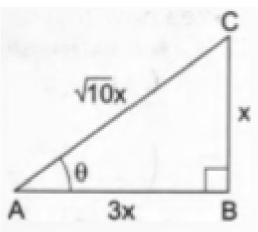
$\Rightarrow 2 \times \frac{9}{4} + 2 \times \frac{5}{4} - 7$

$\Rightarrow \frac{9}{2} + \frac{5}{2} - 7$

$\Rightarrow \frac{9+5-14}{2} = 0$

7.

(c) $\frac{\sqrt{10}}{3}$

Explanation: 

$cosec\theta = \frac{AC}{BC} = \frac{\sqrt{10}}{1} = \frac{\sqrt{10}x}{x} \Rightarrow AC = \sqrt{10x}$ and BC = x.

$\therefore AB^2 = AC^2 - BC^2 = (\sqrt{10}x)^2 - (x^2) = 10x^2 - x^2 = 9x^2$

$\Rightarrow AB = \sqrt{9x^2} = 3x$

$\therefore \sec\theta = \frac{AC}{AB} = \frac{\sqrt{10}x}{3x} = \frac{\sqrt{10}}{3}$

8.

(c) $\frac{3}{4}$

Explanation: $\tan\theta = \frac{1}{\sqrt{7}} = \frac{\text{Perpendicular}}{\text{Base}}$

By Pythagoras Theorem,

$(\text{Hyp.})^2 = (\text{Base})^2 + (\text{Perp.})^2$

$= (1)^2 + (\sqrt{7})^2 = 1 + 7 = 8$

$\therefore$ Hyp. $= \sqrt{8} = 2\sqrt{2}$

Now, $\text{cosec}\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{2\sqrt{2}}{1}$

$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{2\sqrt{2}}{\sqrt{7}}$

Now, $\frac{cosec^2\theta - \sec^2\theta}{cosec^2\theta + \sec^2\theta} = \frac{\left(\frac{2\sqrt{2}}{1}\right)^2 - \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(\frac{2\sqrt{2}}{1}\right)^2 + \left(\frac{2\sqrt{2}}{\sqrt{7}}\right)^2}$

$= \frac{8 - \frac{8}{7}}{8 + \frac{8}{7}}$

$= \frac{\frac{56-8}{7}}{\frac{56+8}{7}} = \frac{\frac{48}{7}}{\frac{64}{7}}$

$= \frac{48}{7} \times \frac{7}{64} = \frac{3}{4}$

9.

(d) $\frac{a^2+b^2}{a^2-b^2}$

Explanation: $\tan\theta = \frac{a}{b}$

$\frac{a\sin\theta + b\cos\theta}{a\sin\theta - b\cos\theta} = \frac{a\frac{\sin\theta}{\cos\theta} + b\frac{\cos\theta}{\cos\theta}}{a\frac{\sin\theta}{\cos\theta} - b\frac{\cos\theta}{\cos\theta}}$ (Dividing by $\cos\theta$)

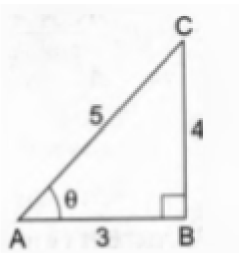
$= \frac{a\tan\theta + b}{a\tan\theta - b} = \frac{a \times \frac{a}{b} + b}{a \times \frac{a}{b} - b}$

$= \frac{\frac{a^2}{b} + b}{\frac{a^2}{b} - b} = \frac{\frac{a^2+b^2}{b}}{\frac{a^2-b^2}{b}}$

$= \frac{a^2+b^2}{b} \times \frac{b}{a^2-b^2}$

$= \frac{a^2+b^2}{a^2-b^2}$

10. **(a)** $\frac{7}{5}$

Explanation: 

$\tan\theta = \frac{4}{3} = \frac{BC}{AB}$

$\therefore AC^2 = AB^2 + BC^2 = (3)^2 + (4)^2 = 25$

$\Rightarrow \quad AC = \sqrt{25} = 5$

$\therefore (\sin\theta + \cos\theta) = \left(\frac{4}{5} + \frac{3}{5}\right) = \frac{7}{5}$

11. **(a)** $\frac{1}{2}$

Explanation: We know that $\sec^2 A - \tan^2 A = 1$.

$\therefore (2x)^2 - \left(\frac{2}{x}\right)^2 = 1 \Rightarrow 4x^2 - \frac{4}{x^2} = 1 \Rightarrow 4\left(x^2 - \frac{1}{x^2}\right) = 1$

$\Rightarrow \left(x^2 - \frac{1}{x^2}\right) = \frac{1}{4} \Rightarrow 2\left(x^2 - \frac{1}{x^2}\right) = 2 \times \frac{1}{4} = \frac{1}{2}$

12.

(b) $-\frac{17}{7}$

Explanation: tanθ = $\frac{5}{12}$ = $\frac{P}{b}$

h = $\sqrt{5^2 + 12^2}$

= 13

Now, sinθ = $\frac{5}{13}$

cosθ = $\frac{12}{13}$

Now, $\frac{sin\theta + cos\theta}{sin\theta - cos\theta}$ = $\frac{\frac{5}{13} + \frac{12}{13}}{\frac{5}{13} - \frac{12}{13}}$

= $-\frac{17}{7}$

13.

(d) $\frac{3}{160}$

Explanation: $\cos\theta = \frac{3}{5} = \frac{\text{Base}}{\text{Hypotenuse}}$

By Pythagoras Theorem, $(\text{Hypotenuse})^2 = (\text{Base})^2 + (\text{Alt.})^2$

$\Rightarrow (5)^2 = (3)^2 + (alt.)^2$

$\Rightarrow 25 = 9 + (alt)^2 \Rightarrow (alt)^2 = 25 - 9 = 16 = (4)^2$

Alt. = 4

Now, $\sin\theta = \frac{\text{Alt.}}{\text{Hypotenuse}} = \frac{4}{5}$

and $\tan\theta = \frac{\text{Alt.}}{\text{Base}} = \frac{4}{3}$

$\therefore \frac{\sin\theta\tan\theta - 1}{2\tan^2\theta} = \frac{\frac{4}{5} \times \frac{4}{3} - 1}{2 \times \left(\frac{4}{3}\right)^2} = \frac{\frac{16}{15} - 1}{2 \times \frac{16}{9}}$

$= \frac{\frac{1}{15}}{\frac{32}{9}} = \frac{1}{15} \times \frac{9}{32} = \frac{3}{160}$

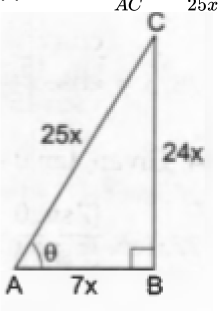
14.

(b) $\frac{24}{25}$

Explanation: $\sec\theta = \frac{AC}{AB} = \frac{25}{7} = \frac{25x}{7x} \Rightarrow$ AC = 25x and AB = 7x

$\therefore BC^2 = AC^2 - AB^2 = 625x^2 - 49x^2 = 576x^2 \Rightarrow BC = 24x$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{24x}{25x} = \frac{24}{25}$



15.

(b) $\frac{3}{4}$

Explanation: Given: cos A = $\frac{4}{5}$ …(i)

we know that tan A = $\frac{\sin A}{\cos A}$

Also we know that, sin A = $\sqrt{(1 - \cos^2 A)}$ …(ii)

Thus,

Substituting eq. (i) in eq. (ii), we get

Sin A = $\sqrt{1 - \frac{16}{25}}$

$= \sqrt{(9/25)} = \frac{3}{5}$

Therefore, $\tan A = \frac{3}{5} \times \frac{5}{4} = \frac{3}{4}$

16. **(a)** $\frac{7}{25}$

Explanation: tan $\theta = \frac{3}{4} = \frac{\text{Perpendicular}}{\text{Base}}$

By Pythagoras Theorem,

$(\text{Hyp.})^2 = (\text{Base})^2 + (\text{Perp.})^2$

= $(4)^2 + (3)^2 = 16 + 9 = 25$

$\therefore Hyp. = \sqrt{25} = 5$

Now, sin $\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{5}$

and cos $\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{4}{5}$

$\cos^2\theta - \sin^2\theta = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

= $\frac{16}{25} - \frac{9}{25} = \frac{16-9}{25} = \frac{7}{25}$

17.

(c) $\frac{7}{17}$

Explanation: 8 tan x = 15 ⇒ $\tan x = \frac{15}{8} = \frac{\text{Perpendicular}}{\text{Base}}$

By Pythagoras Theorem,

$(\text{Hyp.})^2 = (\text{Base}\}^2 + (\text{Perp.})^2$

= $(8)^2 + (15)^2$

= 64 + 225 = 289 = $(17)^2$

∴ Hyp. = 17 units

∴ sin x = $\frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15}{17}$

cos x = $\frac{\text{Base}}{\text{Hypotenuse}} = \frac{8}{17}$

sin x - cos x = $\frac{15}{17} - \frac{8}{17} = \frac{15-8}{17}$

$= \frac{7}{17}$

18.

(c) $\frac{16}{29}$

Explanation: 5 cotθ = 3 ⇒ cotθ = $\frac{3}{5}$

Now, $\frac{5\sin\theta - 3\cos\theta}{4\sin\theta + 3\cos\theta}$

= $\frac{5-3\cot\theta}{4+3\cot\theta}$ = $\frac{5-3\left(\frac{3}{5}\right)}{4+3\left(\frac{3}{5}\right)}$ = $\frac{25-9}{20+9}$ = $\frac{16}{29}$

19.

(c) 9

Explanation: Given, $\cot\theta = \frac{4}{3}$

$\therefore \frac{(5\sin\theta + 3\cos\theta)}{(5\sin\theta - 3\,cos\,\theta)} = \frac{5+3\,cot\,\theta}{5-3\,cot\,\theta}$ [dividing num. and denom. by sin θ]

= $\frac{\left(5+3\times\frac{4}{3}\right)}{\left(5-3\times\frac{4}{3}\right)} = \frac{(5+4)}{(5-4)} = \frac{9}{1} = 9$

20.

(d) $\frac{1}{3}$

Explanation: cosec^2 θ- cot$^{2\theta}$ =1

$\implies (3x)^2 - \left(\frac{3}{x}\right)^2 = 1 \Rightarrow 9x^2 - \frac{9}{x^2} = 1 \Rightarrow 9\left(x^2 - \frac{1}{x^2}\right) = 1$

$\Rightarrow \left(x^2 - \frac{1}{x^2}\right) = \frac{1}{9}$

$\Rightarrow 3\left(x^2 - \frac{1}{x^2}\right) = 3 \times \frac{1}{9} = \frac{1}{3}$

21.

(c) Only (i)

Explanation:

i. We know, 3sinθ - cos$\theta)^4$ = 3((sinθ - cos$\theta)^2)^2$

= 3(1^2 + 4 sin$^2\theta$ cos^2 θ - 4 sinθcosθ) ...(i)

and, 6(sinθ + cos$\theta)^2$ = 6 + 12 sinθcosθ ...(ii)

Also, 4($\sin^6\theta + \cos^6\theta$) = 4($(\sin^2\theta)^3 + (\cos^2\theta)^3$)

= 4(1) - 12$\sin^2\theta\cos^2\theta$ ...(iii)

Adding (i), (ii) and (iii), we get

3 + 12$\sin^2\theta\cos^2\theta$ - 12$\sin\theta\cos\theta$ + 6 + 12$\sin\theta\cos\theta$ + 4 - 12$\sin^2\theta\cos^2\theta$

= 3 + 6 + 4 = 13, which is independent of θ.

ii. We have,

$\operatorname{cosec}\theta - \sin\theta = \frac{1}{\sin\theta} - \sin\theta = \frac{1-\sin^2\theta}{\sin\theta}$

$\Rightarrow a^3 = \frac{\cos^2\theta}{\sin\theta} \Rightarrow a^2 = \left(\frac{\cos^2\theta}{\sin\theta}\right)^{\frac{2}{3}}$

Similarly, $\sec\theta - \cos\theta = \frac{1}{\cos\theta} - \cos\theta = \frac{1-\cos^2\theta}{\cos\theta}$

$\Rightarrow b^3 = \frac{\sin^2\theta}{\cos\theta} \Rightarrow b^2 = \left(\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}}$

$\therefore a^2b^2(a^2 + b^2) = \left(\frac{\cos^2\theta}{\sin\theta} \times \frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}} \left(\left(\frac{\cos^2\theta}{\sin\theta}\right)^{\frac{2}{3}} + \left(\frac{\sin^2\theta}{\cos\theta}\right)^{\frac{2}{3}}\right)$

$= (\sin\theta\cdot\cos\theta)^{\frac{2}{3}} \left(\frac{\cos^2\theta+\sin^2\theta}{(\sin\theta\cdot\cos\theta)^{\frac{2}{3}}}\right) = 1$

22.

(b) $\frac{271}{979}$

Explanation: $3\cos\theta = 5\sin\theta \Rightarrow \frac{\sin\theta}{\cos\theta} = \frac{3}{5}$

$\Rightarrow \tan\theta = \frac{3}{5} = \frac{\text{Perpendicular}}{\text{Base}}$

By Pythagoras Theorem,

$(\text{Hyp.})^2 = (\text{Base})^2 + (\text{Perp})^2$

$= (5) + (3)^2$

$= 25 + 9 = 34$

$\therefore \text{Hyp} = \sqrt{34}$

Now $\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{3}{\sqrt{34}}$

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{5}{\sqrt{34}}$

$\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{34}}{5}$

Now, $\frac{5\sin\theta - 2\sec^2\theta + 2\cos\theta}{5\sin\theta + 2\sec^3\theta - 2\cos\theta}$

$= \frac{3\cos\theta + 2\cos\theta - \frac{2}{\cos^3\theta}}{3\cos\theta - 2\cos\theta + \frac{2}{\cos^3\theta}}$

$= \frac{\frac{2}{\cos^3\theta}}{\cos\theta + \frac{2}{\cos^3\theta}} \quad (3\cos\theta = 5\sin\theta)$

$= \frac{\frac{5\cos^4\theta - 2}{\cos^3}}{\frac{\cos^4\theta + 2}{\cos^3\theta}} = \frac{5\cos^4\theta - 2}{\cos^3\theta} \times \frac{\cos^3\theta}{\cos^4\theta + 2}$

$= \frac{5\cos^4\theta - 2}{\cos^4\theta + 2}$

$= \frac{5\left(\frac{5}{\sqrt{34}}\right)^4 - 2}{\left(\frac{5}{\sqrt{34}}\right)^4 + 2} = \frac{\frac{5\times625}{1156} - 2}{\frac{625}{1156} + 2^2}$

$= \frac{\frac{3125-2372}{1156}}{\frac{625+2312}{1156}}$

$= \frac{813}{1156} \times \frac{1156}{2937} = \frac{271}{979}$

23.

(b) $\frac{5}{8}$

Explanation: $\frac{5}{8}$

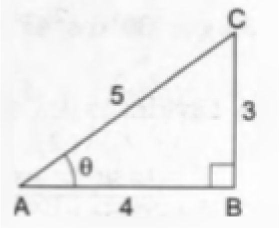
24. **(a)** $\frac{3}{4}$

Explanation: $\cos\theta = \frac{4}{5} = \frac{AB}{AC}$

$\therefore \quad BC^2 = AC^2 - AB^2 = 25 - 16 = 9$

$\Rightarrow$ BC = 3

$\therefore \tan\theta = \frac{BC}{AB} = \frac{3}{4}$



25.

(b) $\frac{1}{7}$

Explanation: Given, $\tan\theta = \frac{4}{7}$

$\therefore \quad \frac{(7\sin\theta - 3\cos\theta)}{(7\sin\theta + 3\cos\theta)} = \frac{(7\tan\theta - 3)}{(7\tan\theta + 3)}$ [Dividing numerator and denom. by cosθ]

$= \frac{\left(7\times\frac{4}{7}-3\right)}{\left(7\times\frac{4}{7}+3\right)} = \frac{(4-3)}{(4+3)} = \frac{1}{7}$

26. **(a)** 3

Explanation: 2tan A = 3

tan A = $\frac{3}{2}$ =$\frac{P}{b}$

h = $\sqrt{P^2 + b^2}$

= $\sqrt{3^2 + 2^2}$

= $\sqrt{13}$

Now, Sin A = $\frac{P}{h}$ = $\frac{3}{\sqrt{13}}$

Cos A = $\frac{b}{h}$ = $\frac{2}{\sqrt{13}}$

$\frac{4Sin\ A + 3Cos\ A}{4Sin\ A - 3cos\ A} = \frac{4(\frac{3}{\sqrt{13}}) + 3(\frac{2}{\sqrt{13}})}{4(\frac{3}{\sqrt{13}}) - 3(\frac{2}{\sqrt{13}})}$

= $\frac{\frac{12}{\sqrt{13}} + \frac{6}{\sqrt{13}}}{\frac{12}{\sqrt{13}} - \frac{6}{\sqrt{13}}}$

= $\frac{18}{6}$ = 3

27. **(a)** $\sqrt{3}$

Explanation: Given: $\sin A = \frac{1}{2}$ ...(i)

And we know that, $\cot A = \frac{1}{\tan A} = \frac{\cos A}{\sin A}$ ...(ii)

We need to find the value of cos A.

$\cos A = \sqrt{1 - \sin^2 A}$ ...(iii)

Substituting eq. (i) in eq. (iii), we get

cos A = $\sqrt{\left(1 - \frac{1}{4}\right)}$

$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$

Substituting values of sin A and cos A in eq. (ii), we get

$\cot A = \frac{\sqrt{3}}{2} \times 2 = \sqrt{3}$

28.

(d) $\frac{1}{7}$

Explanation: 16 cot x = 12 $\Rightarrow \cot x = \frac{12}{16} = \frac{3}{4}$

Now, $\frac{\sin x - \cos x}{\sin x + \cos x} = \frac{\frac{\sin x}{\sin x} - \frac{\cos x}{\sin x}}{\frac{\sin x}{\sin x} + \frac{\cos x}{\sin x}}$

$= \frac{1 - \cot x}{1 + \cot x}$ (Dividing by sin x)

$= \frac{1-\frac{3}{4}}{1+\frac{3}{4}} = \frac{\frac{1}{4}}{\frac{7}{4}}$
$= \frac{1}{4} \times \frac{4}{7} = \frac{1}{7}$

29.
(b) Both A and R are true but R is not the correct explanation of A.

Explanation: For, θ = 45^o, we have tan 45^o = 1 and cot 45^o = 1, so tan^{2}45 + cot^{2}45 = 1 + 1 = 2

30.
(d) A is false but R is true.
Explanation: sin θ cannot be equal to $\frac{1}{\mathrm{Tan}\,\theta}$. But tan$\theta$ = $\frac{Sin\,\theta}{\mathrm{Cos}\,\theta}$

31.
(d) A is false but R is true.
Explanation: sin θ and cosec θ are reciprocal of each other so sin θ $\times$ cosec θ = 1
sin θ $\times$ cosec $\theta \neq$ cot θ

32. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: tan30^o = $\frac{AB}{BC} = \frac{AB}{20}$
AB = $\frac{1}{\sqrt{3}} \times 20 = \frac{20}{1.73}$ = 11.56 m

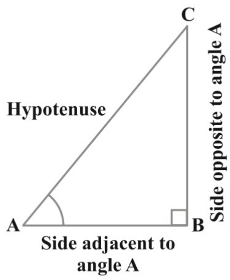
33. **(a)** Both A and R are true and R is the correct explanation of A.
Explanation: sinθ = $\frac{P}{H} = \frac{4}{3}$
Here, perpendicular is greater than the hypotenuse which is not possible in any right triangle.

34.
(c) $\frac{4}{5}$
Explanation: We know that the sum of all the angles on one side of a straight line is 180°.
These angles are said to be in linear pairs.
Therefore, using the figure, we get
$\theta + \phi + 90^\circ = 180^\circ$
Therefore, $\theta = 90^\circ - \phi$ ...(a)
Using trigonometric ratio in $\triangle ABC$, we get
$\sin\theta = \frac{4}{5}$ ...(b)
Using equation (a) in equation (b), we get
$\sin(90^\circ - \phi) = \frac{4}{5}$
We know that for any angle theta,
$\sin(90^\circ - \theta) = \cos\theta$.
Therefore, we get
$\cos\phi = \frac{4}{5}$
Therefore, the correct option is option is $\frac{4}{5}$

35.
(b) $\frac{12}{5}$
Explanation: $BA = \sqrt{(AD)^2 + (BD)^2}$
$= \sqrt{(4)^2 + (3)^2}$
$= \sqrt{25} = 5$
$\therefore AC = \sqrt{(BC)^2 + (AB)^2}$
$= \sqrt{(12)^2 + (5)^2}$
$= \sqrt{144 + 25}$
$= \sqrt{169} = 13$
Hence,
$\cos\theta = 12/13$
$and \sin\theta = 5/13$
$\Rightarrow \cot\theta = 12/5$

36.
(d) $\frac{\sin A}{\cos A} = \tan A$

Explanation: In right Ld triangle ABC rt Ld at B



Sin A $= \frac{BC}{AC}$

Cos A $= \frac{AB}{AC}$

Then $\frac{SinA}{CosA} = \frac{\frac{BC}{AC}}{\frac{AB}{AC}}$

$\frac{BC}{AC} \times \frac{AC}{AB} = \frac{BC}{AB}$

$L.H.S = \frac{\sin A}{\cos A} = \frac{BC}{AB}$

$R.H.S = tanA$

= $\frac{sinA}{cosA} = \frac{BC}{AB}$ = $\frac{BC}{AB}$

L.H.S = R.H.S

Therefore $\frac{sinA}{cosA}$ = tan A is true.

37.

(c) cos A sec A = 1

Explanation: cosA secA

= cosA $\times$ $\frac{1}{cosA}$

= 1 $\times$1 = 1

Therefore cos A sec A = 1 is true.

38. sec $\theta = \frac{25}{7} = \frac{h}{b}$

p = $\sqrt{h^2 - b^2}$

p = $\sqrt{27^2 - 7^2}$

p = $\sqrt{625 - 49}$

p = 24

cot $\theta = \frac{b}{p}$

cot $\theta = \frac{7}{24}$

39. $\frac{5\tan\theta - 3}{4\tan\theta + 3}$ = 0

40. tan θ = $\frac{4}{5}$

LHS = $\frac{5\tan\theta - 3}{5\tan\theta + 3}$

$\Rightarrow \frac{5\times\frac{4}{5}-3}{5\times\frac{4}{5}+3} = \frac{1}{7}$

41. cos (A + B) = $\frac{1}{2}$

A + B = 60^o ...(i)

sin (A - B) = $\frac{1}{2}$

A - B = 30^o ...(ii)

Solving (i) & (ii)

A = 45^o

B = 15^o

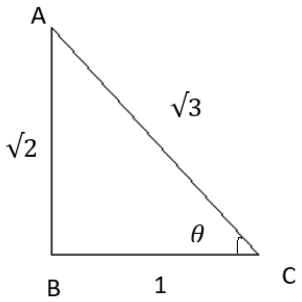
42. If $\sqrt{3}$ tan θ = 3 sin θ

$\Rightarrow \sqrt{3}\frac{sin\ \theta}{cos\ \theta}$ = 3 sin θ

$\Rightarrow$ Cos θ = $\frac{\sqrt{3}}{3}$

$\Rightarrow$ Cos θ = $\frac{1}{\sqrt{3}}$

$\Rightarrow \sin\theta = \frac{\sqrt{2}}{3}$



43. cosec $\theta = \frac{5}{4} = \frac{h}{p}$

b = $\sqrt{h^2 - p^2}$

b = $\sqrt{25 - 16}$

b = 3

cot $\theta = \frac{b}{p} = \frac{3}{4}$

44. In ΔABC, we have,

A + B + C = 180°

$\Rightarrow$ B + C =180° - A

Dividing 2 on both sides, we get

$\Rightarrow \quad \frac{B+C}{2} = 90^\circ - \frac{A}{2}$

Applying Tan on both sides

$\Rightarrow \tan\left(\frac{B+C}{2}\right) = \tan\left(90^\circ - \frac{A}{2}\right)$

$\Rightarrow \tan\left(\frac{B+C}{2}\right) = \cot\frac{A}{2}$

45. tan $\theta = \frac{\sqrt{3}}{1} = \frac{p}{b}$

h = $\sqrt{p^2 + b^2}$

h = $\sqrt{(\sqrt{3})^2 + 1^2}$

h = 2

sec $\theta = \frac{h}{b} = \frac{1}{2}$

$\left(\frac{2\sec\theta}{1+\tan^2\theta}\right) = \left(\frac{2\sec\theta}{\sec^2\theta}\right)$

= $\frac{2}{\sec\theta}$

= $\frac{2}{\left(\frac{1}{2}\right)}$

= 4

46. 3 sinA = 1

$\sin A = \frac{1}{3}$

P = 1x, H = 3x

By using Pythagoras theorem,

$H^2 = P^2 + B^2$

$9x^2 = x^2 + B^2$

$B^2 = 8x^2$

B = $2\sqrt{2}x$

Now, sec A = $\frac{H}{B}$

$secA = \frac{3x}{2\sqrt{2}x} = \frac{3}{2\sqrt{2}}$

$secA = \frac{3\sqrt{2}}{4}$

Section B

47. Fill in the blanks:

(i) 1. hypotenuse

(ii) 1. trigonometric ratios

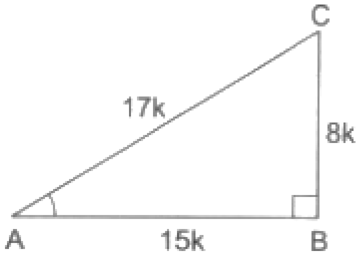
(iii) 1. 1

(iv) 1. 1

Section C

48. Let us draw a △ABC in which ∠B = 90°

Then, $\sin A = \frac{BC}{AC} = \frac{8}{17}$



Let BC = 8k and AC = 17k, where k is positive.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$\Rightarrow AB^2 = (17k)^2 - (8k)^2 = 289k^2 - 64k^2 = 225k^2$

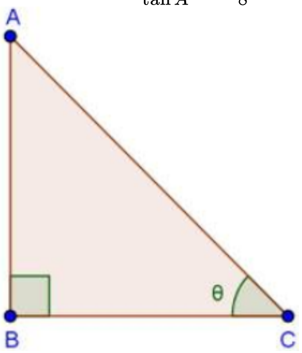
$\Rightarrow AB = \sqrt{225k^2} = 15k$

$\therefore \quad \sin A = \frac{BC}{AC} = \frac{8k}{17k} = \frac{8}{17}; \cos A = \frac{AB}{AC} = \frac{15k}{17k} = \frac{15}{17}$

$\tan A = \frac{\sin A}{\cos A} = \left(\frac{8}{17} \times \frac{17}{15}\right) = \frac{8}{15}$

$cosec A = \frac{1}{\sin A} = \frac{17}{8}; \sec A = \frac{1}{\cos A} = \frac{17}{15}$

and $\cot A = \frac{1}{\tan A} = \frac{15}{8}$

49. 

Let us draw a right angled ΔABC ,right angled at B .

Let $\angle C = \theta$

Hence, Given $\cos\theta = \frac{3}{5} = \frac{BC}{AC}$

Let BC = 3K and, AC = 5K , where K is positive integer.

In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

or, $AB^2 + (3K)^2 = (5K)^2$

or, $AB^2 + 9K^2 = 25K^2$

or, $AB^2 = 16K^2$

$\therefore AB = \sqrt{16K^2} = 4K$

$\therefore \sin\theta = \frac{AB}{AC} = \frac{4K}{5K} = \frac{4}{5}$

$\tan\theta = \frac{AB}{BC} = \frac{4K}{3K} = \frac{4}{3}$

Now,

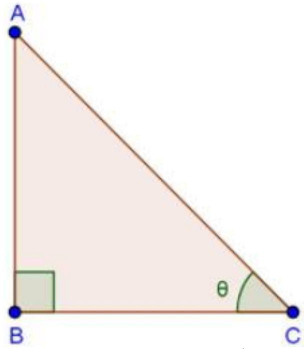
$\frac{\sin\theta - \frac{1}{\tan\theta}}{2\tan\theta}$

$= \frac{\frac{4}{5} - \frac{3}{4}}{2 \times \frac{4}{3}}$

$= \frac{\frac{16-15}{20}}{\frac{8}{3}}$

$= \frac{\frac{1}{20}}{\frac{8}{3}}$

$= \frac{1}{20} \times \frac{3}{8} = \frac{3}{160}$

50. 

Given, $\sec\theta = \frac{13}{5} = \frac{AC}{BC}$

Let AC = 13K

and BC = 5K

In △ABC, By Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$AB^2 + (5K)^2 = (13K)^2$

$AB^2 + 25K^2 = 169K^2$

$AB^2 = 169K - 25K^2 = 144K^2$

$AB = \sqrt{144K^2} = 12K$

$\therefore \sin\theta = \frac{AB}{AC} = \frac{12K}{13K} = \frac{12}{13}$

$\cos\theta = \frac{BC}{AC} = \frac{5K}{13K} = \frac{5}{13}$

LHS = $\frac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

$= \frac{2\times\frac{12}{13} - 3\times\frac{5}{13}}{4\times\frac{12}{13} + 9\times\frac{5}{13}}$

$= \frac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} + \frac{45}{13}}$

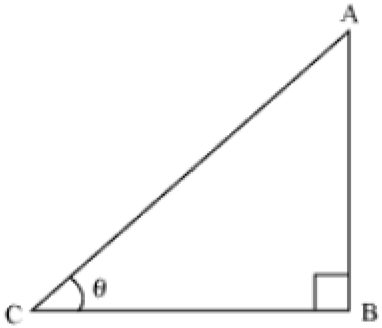
$= \frac{\frac{9}{13}}{\frac{3}{13}}$

$= \frac{9}{13} \times \frac{13}{3}$

= 3

= R.H.S.

51. Let us consider a right angled ΔABC right angled at point B.



Let $\angle C = \theta$.

Given,

$\cot\theta = \frac{7}{8} = \frac{Side\ adjacent\ to\ \angle\theta}{Side\ opposite\ to\ \angle\theta} = \frac{BC}{AB}$

If BC is 7K, AB will be 8K , where K is a positive integer.

Now applying Pythagoras theorem in ΔABC

$AC^2 = AB^2 + BC^2$

Or, $AC^2 = 64K^2 + 49K^2$

Or, $AC^2 = 113K^2$

$\therefore AC = \sqrt{113}K$

Now,

$\sin\theta = \frac{Side\ opposite\ to\ \angle\theta}{hypotenuse} = \frac{AB}{AC}$

$= \frac{8K}{\sqrt{113}K} = \frac{8}{\sqrt{113}}$

And,

$\cos\theta = \frac{Side\ adjacent\ to\ \angle\theta}{hypotenuse} = \frac{BC}{AC}$

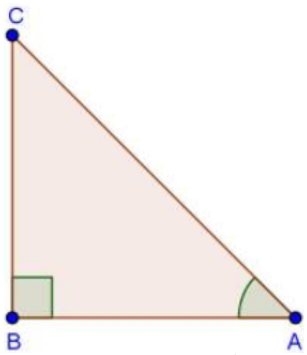
$= \frac{7K}{\sqrt{113}K} = \frac{7}{\sqrt{113}}$

i. $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)} = \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)}$

$= \frac{1-\left(\frac{8}{\sqrt{113}}\right)^2}{1-\left(\frac{7}{\sqrt{113}}\right)^2} = \frac{1-\frac{64}{113}}{1-\frac{49}{113}}$

$= \frac{\frac{49}{113}}{\frac{64}{113}} = \frac{49}{64}$

ii. $\cot^2\theta = (\cot\theta)^2 = \left(\frac{7}{8}\right)^2 = \frac{49}{64}$

52. 

Given $\sec A = \frac{5}{4} = \frac{AC}{AB}$

Let AC = 5K

and, AB = 4K

In ΔABC, by Pythagoras theorem

$BC^2 + AB^2 = AC^2$

$BC^2 + (4K)^2 = (5K)^2$

$BC^2 + 16K^2 = 25K^2$

$BC^2 = 25K^2 - 16K^2$

$BC^2 = 9K^2$

$BC = \sqrt{9K^2} = 3K$

$\therefore \sin A = \frac{BC}{AC} = \frac{3K}{5K} = \frac{3}{5}$

$\cos A = \frac{AB}{AC} = \frac{4K}{5K} = \frac{4}{5}$

$\tan A = \frac{BC}{AB} = \frac{3K}{4K} = \frac{3}{4}$

LHS

$= \frac{3\sin A - 4\sin^3 A}{4\cos^3 A - 3\cos A}$

$= \frac{3\times\frac{3}{5} - 4\times\left(\frac{3}{5}\right)^3}{4\left(\frac{4}{5}\right)^3 - 3\left(\frac{4}{5}\right)}$

$= \frac{\frac{9}{5} - \frac{108}{125}}{\frac{256}{125} - \frac{12}{5}}$

$= \frac{\frac{225-108}{125}}{\frac{256-300}{125}}$

$= \frac{\frac{117}{125}}{\frac{-44}{125}}$

$= \frac{117}{125} \times \frac{125}{-44} = \frac{-117}{44}$

RHS

$= \frac{3\tan A - \tan^3 A}{1-3\tan^2 A}$

$= \frac{3 \times \frac{3}{4} - \left(\frac{3}{4}\right)^3}{1 - 3\left(\frac{3}{4}\right)^2}$

$= \frac{\frac{9}{4} - \frac{27}{64}}{1 - \frac{27}{16}}$

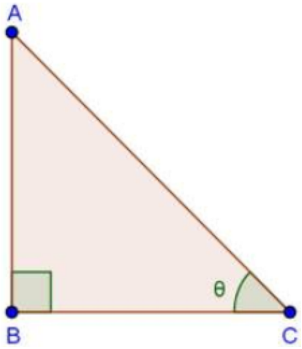
$= \frac{\frac{144-27}{64}}{\frac{16-27}{16}}$

$= \frac{\frac{117}{64}}{\frac{-11}{16}}$

$= \frac{117}{64} \times \frac{-16}{11}$

$= \frac{-117}{44}$

Hence proved

53. 

Given $\cos\theta = \frac{5}{13} = \frac{BC}{AC}$

Let BC = 5K

and, AC = 13K

In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$AB^2 + (5K)^2 = (13K)^2$

$AB^2 + 25K^2 = 169K^2$

$AB^2 = 169K^2 - 25K^2$

$AB^2 = 144K^2$

$AB = \sqrt{144K^2} = 12K$

$\therefore \sin\theta = \frac{AB}{AC} = \frac{12K}{13K} = \frac{12}{13}$

$\tan\theta = \frac{AB}{BC} = \frac{12K}{5} = \frac{12}{5}$

Now, $\frac{\sin^2\theta - \cos^2\theta}{2\sin\theta \times \cos\theta} \times \frac{1}{\tan^2\theta}$

$= \frac{\left(\frac{12}{13}\right)^2 - \left(\frac{5}{13}\right)^2}{2 \times \frac{12}{13} \times \frac{5}{13}} \times \frac{1}{\left(\frac{12}{5}\right)^2}$

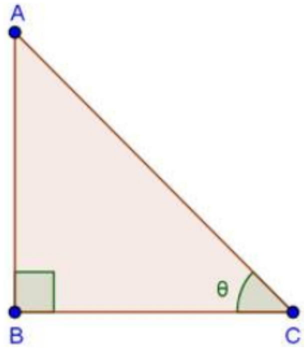
$= \frac{\frac{144}{169} - \frac{25}{169}}{\frac{120}{169}} \times \frac{1}{\frac{144}{25}}$

$= \frac{\frac{144-25}{169}}{\frac{120}{169}} \times \frac{25}{144}$

$= \frac{\frac{119}{169}}{\frac{120}{169}} \times \frac{25}{144}$

$= \frac{119}{169} \times \frac{169}{120} \times \frac{25}{144}$

$= \frac{595}{3456}$

54. 

Given $\cot\theta = \frac{1}{\sqrt{3}} = \frac{BC}{AB}$

Let BC = 1K

and, $AB = \sqrt{3}K$

In ΔABC, By Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$\Rightarrow \left(\sqrt{3K}\right)^2 + (1K)^2 = AC^2$

$\Rightarrow 3K^2 + K^2 = AC^2$

$\Rightarrow AC^2 = 4K^2$

$\Rightarrow AC = \sqrt{4K^2} = 2K$

$\therefore \cos\theta = \frac{BC}{AC} = \frac{1K}{2K} = \frac{1}{2}$

$\sin\theta = \frac{AB}{AC} = \frac{\sqrt{3}K}{2K} = \frac{\sqrt{3}}{2}$

LHS $= \frac{1-\cos^2\theta}{2-\sin^2\theta}$

$= \frac{1-\left(\frac{1}{2}\right)^2}{2-\left(\frac{\sqrt{3}}{2}\right)^2}$

$= \frac{1-\frac{1}{4}}{2-\frac{3}{4}}$

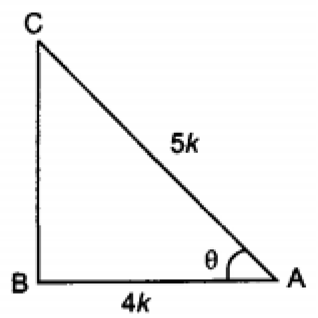
$= \frac{\frac{4-1}{4}}{\frac{8-3}{4}}$

$= \frac{\frac{3}{4}}{\frac{5}{4}}$

$= \frac{3}{4} \times \frac{4}{5} = \frac{3}{5}$

= RHS.

Hence Proved

55. 

If ABC is a triangle, right-angled at B and $\angle A = \theta$, then

sec θ = $\frac{AC}{AB} = \frac{5}{4}$

$\Rightarrow$ AC = 5k and AB = 4k

Since, $AC^2 = AB^2 + BC^2$

$\Rightarrow (5k)^2 = (4k)^2 + BC^2$

$\Rightarrow BC^2 = 25k^2 - 16k^2 = 9k^2$

$\Rightarrow$ BC = $\sqrt{9k^2}$ = 3k

Now, $\frac{\tan\theta}{1+\tan^2\theta} = \frac{\frac{\mathrm{BC}}{\mathrm{AB}}}{1+\left(\frac{\mathrm{BC}}{\mathrm{AB}}\right)^2} = \frac{\frac{3k}{4k}}{1+\left(\frac{3k}{4k}\right)^2} = \frac{\frac{3}{4}}{\left(1+\frac{9}{16}\right)}$

$= \frac{3}{4} \div \frac{25}{16} = \frac{3}{4} \times \frac{16}{25} = \frac{12}{25}$

Also, $\frac{\sin\theta}{\sec\theta} = \frac{\frac{BC}{AC}}{\frac{AC}{AB}} = \frac{\frac{3k}{5k}}{\frac{5k}{4k}}$

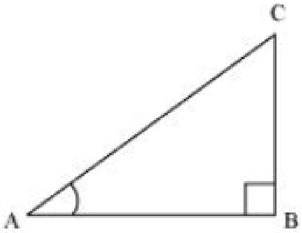
$= \frac{3}{5} \times \frac{4}{5} = \frac{12}{25}$

Hence, verified

56. Give that 3cot A = 4

Or cot $A = \frac{4}{3}$

Consider a right angle triangle ΔABC right angled at point B.



cot $A = \frac{Side\ adjacent\ to\ \angle A}{Side\ opposite\ to\ \angle A}$

$\frac{AB}{BC} = \frac{4}{3}$

If AB is 4K, BC will be 3K. where K is a positive integer

Now in ΔABC

$(AC)^2 = (AB)^2 + (BC)^2$

$= (4K)^2 + (3K)^2$

$= 16\ K^2 + 9K^2$

$= 25K^2$

AC = 5K

$\cos A = \frac{Side\ adjacent\ to\ \angle A}{hypotenuse} = \frac{AB}{AC}$

$= \frac{4K}{5K} = \frac{4}{5}$

$\sin A = \frac{Side\ opposite\ to\ \angle A}{hypotenuse} = \frac{BC}{AC}$

$= \frac{3K}{5K} = \frac{3}{5}$

$\tan A = \frac{Side\ opposite\ to\ \angle A}{Side\ adjacent\ to\ angle\ A} = \frac{BC}{AB}$

$= \frac{3K}{4K} = \frac{3}{4}$

$\frac{1-\tan^2 A}{1+\tan^2 A} = \frac{1-\left(\frac{3}{4}\right)^2}{1+\left(\frac{3}{4}\right)^2} = \frac{1-\frac{9}{16}}{1+\frac{9}{16}}$

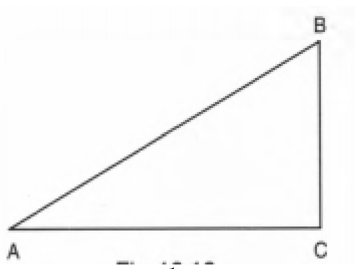
$= \frac{\frac{7}{16}}{\frac{25}{16}} = \frac{7}{25}$

$\cos^2 A - \sin^2 A = \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2$

$= \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Hence $\frac{1-\tan^2 A}{1+\tan^2 A} = \cos^2 A - \sin^2 A$

57. Let us draw a $\triangle$ ABC, right angled at C in which tan A = $\frac{1}{\sqrt{3}}$



Now, tan $A = \frac{1}{\sqrt{3}}$

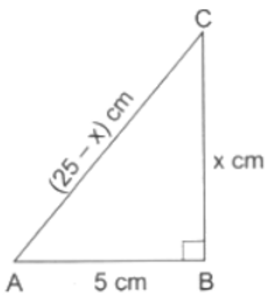
Let, $\frac{BC}{AC} = \frac{1}{\sqrt{3}}$ $\left[\because \tan A = \frac{BC}{AC}\right]$

$\Rightarrow BC = x$ and $AC = \sqrt{3}x$ ....(i)

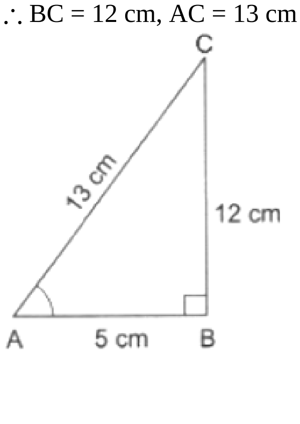
By Pythagoras theorem, we have

$AB^2 = AC^2 + BC^2$
$\Rightarrow AB^2 = (\sqrt{3}x)^2 + x^2$
$\Rightarrow AB^2 = 3x^2 + x^2$
$\Rightarrow AB^2 = 4x^2$
$\Rightarrow AB = 2x$
To find the ratios of $\angle A$, we have
Base =AC = $\sqrt{3x}$, Perpendicular = BC = x and Hypotenuse = AB = 2x
$\therefore \sin A = \frac{BC}{AB} = \frac{x}{2x} = \frac{1}{2}$ and $\cos A = \frac{AC}{AB} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$
When we consider the ratios of $\angle B$, we have
Base = B C = x ,Perpendicular = AC = $\sqrt{3x}$ and, Hypotenuse = AB = 2x
$\therefore \cos B = \frac{BC}{AB} = \frac{x}{2x} = \frac{1}{2}$ and, $\sin B = \frac{AC}{AB} = \frac{\sqrt{3}x}{2x} = \frac{\sqrt{3}}{2}$
$\therefore \sin A \cos B + \cos A \sin B = \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = 1$

58. In right $\triangle$AMB,
tan B = $\frac{3}{4}$
$\Rightarrow \frac{AM}{BM} = \frac{3}{4}$
$\Rightarrow$ 4AM = 3BM $\Rightarrow$BM = $\frac{4}{3}$AM ...(i)
In right $\triangle$AMC,
tan C = $\frac{\mathrm{AM}}{\mathrm{MC}}$
$\Rightarrow \frac{5}{12} = \frac{\mathrm{AM}}{\mathrm{MC}}$
$\Rightarrow$ MC = $\frac{12}{5}$ AM ...(ii)
Now, BM + MC = BC
$\frac{4}{3}$AM + $\frac{12}{5}$AM = 56
AM $\left(\frac{4}{3} + \frac{12}{5}\right)$ = 56
AM $\left(\frac{20+36}{15}\right)$ = 56
$\Rightarrow$ AM = $\frac{56 \times 15}{56}$
= 15 cm

59. 

Let BC = x cm. Then, AC = (25 - x) cm.
By Pythagoras' theorem, we have
$AB^2 + BC^2 = AC^2$
$\Rightarrow 5^2 + x^2 = (25 - x)^2$
$\Rightarrow 25 + x^2 = 625 - 50x + x^2$
$\Rightarrow$ 50x = 600
$\Rightarrow$ x = 12
$\therefore$ BC = 12 cm, AC = 13 cm and AB = 5 cm.



For T-ratios of ∠A, we have

$\sin A = \frac{BC}{AC} = \frac{12}{13}$ and $\cos A = \frac{AB}{AC} = \frac{5}{13}$

For T-ratios of ∠C, we have

base, BC = 12

perpendicular, AB = 5 cm

and hypotenuse, AC = 13 cm.

$\therefore cosec C = \frac{AC}{AB} = \frac{13}{5}$ and $\sec C = \frac{AC}{BC} = \frac{13}{12}$

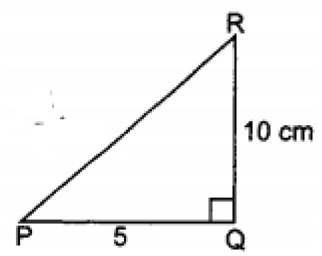
60. LHS = $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\text{cosec}^3\theta}$

= $\frac{\left(1+\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}}$

= $\frac{\frac{\left(\cos\theta\sin\theta+\cos^2\theta+\sin^2\theta\right)(\sin\theta-\cos\theta)}{\cos\theta\sin\theta}}{\frac{\sin^3\theta-\cos^3\theta}{\cos^3\theta\sin^3\theta}}$

= $\frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta} \times \frac{\cos^3\theta\sin^3\theta}{\sin^3\theta-\cos^3\theta}$

= $\cos^2\theta\sin^2\theta$ = RHS

61. 

In right △RQP

$PR^2 = PQ^2 + QR^2$

$= 5^2 + 10^2 = 125$

⇒ PR = $\sqrt{125} = 5\sqrt{5}$ cm

$\sin P = \frac{RQ}{PR} = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$;

$\cos P = \frac{PQ}{PR} = \frac{5}{5\sqrt{5}} = \frac{1}{\sqrt{5}}$

$\cos R = \frac{RQ}{PR} = \frac{10}{5\sqrt{5}} = \frac{2}{\sqrt{5}}$;

$\tan R = \frac{PQ}{RQ} = \frac{5}{10} = \frac{1}{2}$

i. $\sin P = \left(\frac{2}{\sqrt{5}}\right)$

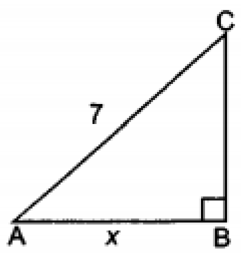
ii. $\cos^2 R = \left(\frac{2}{\sqrt{5}}\right)^2 = \frac{4}{5}$

$\tan R = \frac{1}{2}$

iii. $\sin P \times \cos P = \frac{2}{\sqrt{5}} \times \frac{1}{\sqrt{5}} = \frac{1}{2}$

iv. $\sin^2 P - \cos^2 P = \left(\frac{2}{\sqrt{5}}\right)^2 - \left(\frac{1}{\sqrt{5}}\right)^2$

$= \frac{4}{5} - \frac{1}{5} = \frac{3}{5}$

62. 

In right △ABC,

$AC^2 = AB^2 + BC^2$

⇒ $49 = x^2 + BC^2$

⇒ $BC^2 = 49 - x^2$

⇒ BC = $\sqrt{49 - x^2}$

Also, tan C = $\frac{\mathrm{AB}}{\mathrm{BC}}=\frac{x}{\sqrt{49-x^2}}$;

cot A $=\frac{AB}{BC}=\frac{x}{\sqrt{49-x^2}}$

cos A $=\frac{\mathrm{AB}}{\mathrm{AC}}=\frac{x}{7}$

sin C $=\frac{AB}{AC}=\frac{x}{7}$

cos B = 0 (given)

Now, $\sqrt{7-x}$ tan C + $\sqrt{7+x}$ cot A - 14 cos A + 21 sin C + $\sqrt{49+x^2}$ cos B

By putting the values in given eq, we get,

$=\sqrt{7-x}\times\frac{x}{\sqrt{49-x^2}}+\sqrt{7+x}\times\frac{x}{\sqrt{49-x^2}}-14\times\frac{x}{7}+21\times\frac{x}{7}+\sqrt{49+x^2}\times 0$

$=\sqrt{7-x}\times\frac{x}{\sqrt{(7-x)(7+x)}}+\sqrt{7+x}\times\frac{x}{\sqrt{(7-x)(7+x)}}$ - 2x + 3x + 0

$=\frac{x}{\sqrt{7+x}}+\frac{x}{\sqrt{7-x}}+x$

$=\frac{x\sqrt{7-x}+x\sqrt{7+x}+x\sqrt{7+x}\sqrt{7-x}}{\sqrt{7+x}\sqrt{7-x}}$

$=\frac{x\sqrt{7-x}+x\sqrt{7+x}+x\sqrt{49-x^2}}{\sqrt{49+x^2}}$

63. In $\triangle PQR, \quad \therefore \angle Q=90^{\circ}$

$\therefore$ PR^2 = PQ^2 + QR^2 .... By Pythagoras theorem

$\Rightarrow$ (13)^2 = (12)^2 + QR^2

$\Rightarrow$ 169 = 144 + QR^2

$\Rightarrow$ QR^2 = 169 - 144 $\Rightarrow$ QR^2 = 25

$\Rightarrow QR=\sqrt{25}=5\mathrm{cm}$

$\therefore \tan\mathrm{P}-\cot\mathrm{R}=\frac{\mathrm{QR}}{\mathrm{PQ}}-\frac{\mathrm{QR}}{\mathrm{PQ}}=0$

Section D

64. State True or False:

(i) **(b)** False
Explanation: False

(ii) **(b)** False
Explanation: False

(iii) **(b)** False
Explanation: False

(iv) **(a)** True
Explanation: True

(v) **(b)** False
Explanation: False

(vi) **(b)** False
Explanation: False,
$\sin\theta=\frac{4}{3}$
In a right-angled triangle, hypotenuse is always greater than the remaining two sides. Therefore, such value of sin θ is not possible
Hence, the given statement is false

(vii) **(a)** True
Explanation: True

(viii) **(a)** True
Explanation: True

(ix) **(b)** False
Explanation: False