Solution

INTRODUCTION TO TRIGONOMETRY WS 2

Class 10 - Mathematics

Section A

1. Given: $\tan\alpha = \dfrac{5}{12}$ …(1)

   By definition,
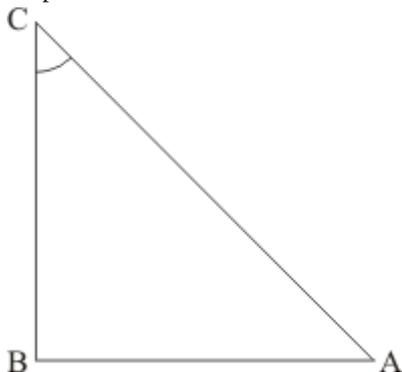
   $\tan\alpha = \dfrac{\text{Perpendicular}}{\text{Base}}$ ... (2)

   By Comparing (1) and (2)

   We get,

   Base = 12k and

   Perpendicular = 5k

   Therefore,

   By Pythagoras theorem,

   $AC^2 = AB^2 + BC^2$

   Now we substitute the value of base side (AB) and the perpendicular side (BC) and get hypotenuse (AC)

   $AC^2 = (12k)^2 + (5k)^2$

   $AC^2 = (144 + 25)k^2$

   $AC^2 = 144 + 25$

   $AC^2 = 169k^2$

   $AC = 13k$

   Hence, Hypotenuse = 13k

   Now, $\sin\alpha = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

   Therefore,

   $\sin\alpha = \dfrac{5}{13}$

   Next, $cosec\alpha = \dfrac{1}{\sin \alpha}$

   Therefore,

   $cosec\alpha = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

   $cosec\alpha = \dfrac{13}{5}$

   Next, $\cos\alpha = \dfrac{\text{Base}}{\text{Hypotenuse}}$

   Therefore,

   $\cos\alpha = \dfrac{12}{13}$

   Next, $\sec\alpha = \dfrac{1}{\cos \alpha}$

   Therefore,

   $\sec\alpha = \dfrac{\text{Hypotenuse}}{\text{Base}}$

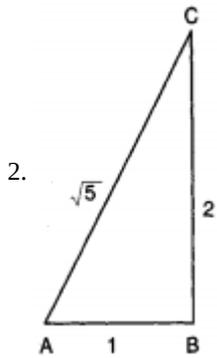 $\sec\alpha = \dfrac{13}{12}$

Now, $\cot\alpha = \dfrac{1}{\tan\alpha}$

Therefore,

$\cot\alpha = \dfrac{\text{Base}}{\text{Perpendicular}}$

$\cot\alpha = \dfrac{12}{5}$

2.

According to the question,

$\tan A = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{2}{1}$

So, we draw a right triangle ABC, right angled at **B** such that

Perpendicular $= BC = 2\,unit$ and Base $= AB = 1\,unit$

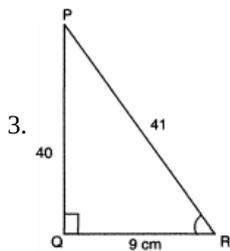Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad AC^2 = 2^2 + 1^2 = 5$

$\Rightarrow \quad AC = \sqrt{5}$

$\therefore \sec A = \dfrac{AC}{AB} = \sqrt{5},\ \tan A = \dfrac{BC}{AB} = \dfrac{2}{1} = 2,\ \sin A = \dfrac{BC}{AC} = \dfrac{2}{\sqrt{5}}$

and, $cosec\,A = \dfrac{AC}{BC} = \dfrac{\sqrt{5}}{2}$

Now, $\sec A \sin A + \tan^2 A - cosec A = \sqrt{5} \times \dfrac{2}{\sqrt{5}} + (2)^2 - \left(\dfrac{\sqrt{5}}{2}\right) = 2 + 4 - \dfrac{\sqrt{5}}{2} = 6 - \dfrac{\sqrt{5}}{2} = \dfrac{12-\sqrt{5}}{2}$

3.

$P^2 + B^2 = H^2 \text{(By Pythagoras theorem)}$

$PQ^2 + QR^2 = PR^2$

$PQ^2 + 9^2 = PR^2$

$PQ^2 + 81 = PR^2$

$PQ^2 + 81 = (PQ + 1)^2 \ (\because PR - PQ = 1)$

$PQ^2 + 81 = PQ^2 + 1 + 2PQ$

$PQ^2 - PQ^2 + 81 - 1 = 2PQ$
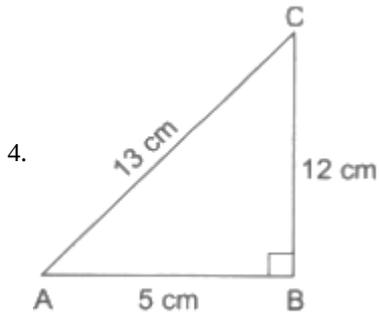
$80 = 2PQ$

or, $PQ = 40$

$PR - PQ = 1 \text{ (Given)}$

or, $PR = 1 + 40$

or, $PR = 41$

Now, $\sin R = \dfrac{P}{H} = \dfrac{PQ}{PR} = \dfrac{40}{41}$

$\cos R = \dfrac{B}{H} = \dfrac{9}{41}$

$\therefore \ \sin R + \cos R = \dfrac{40}{41} + \dfrac{9}{41} = \dfrac{49}{41}$

4.

In $\triangle ABC$, $\angle B = 90°$, AB = 5 cm and BC = 12 cm.

By Pythagoras' theorem, we have

$AC^2 = (AB^2 + BC^2) = \{(5)^2 + (12)^2\} \ cm^2$

$= (25 + 144) \ cm^2 = 169 \ cm^2$.

$\therefore AC = \sqrt{169 cm^2} = 13 cm$.

For T-ratios of $\angle A$, we have

base = AB = 5 cm,
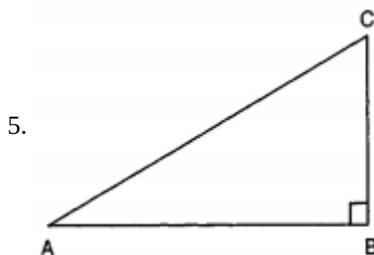
perpendicular = BC = 12 cm

and hypotenuse = AC = 13 cm.

$\therefore \sin A = \dfrac{BC}{AC} = \dfrac{12}{13}$ and $\sec A = \dfrac{AC}{AB} = \dfrac{13}{5}$

For T-ratios of $\angle C$, we have

base = BC = 12 cm,

perpendicular = AB = 5 cm and hypotenuse = AC = 13 cm.

$\therefore \sin C = \dfrac{AB}{AC} = \dfrac{5}{13}$ and $\sec C = \dfrac{AC}{BC} = \dfrac{13}{12}$

5.

we have,

$\tan A = \dfrac{1}{\sqrt{3}} = \tan 30°$

$\therefore A = 30°$

In $\triangle ABC$, we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 30° + 90° + \angle C = 180°$

$\Rightarrow 120° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 120° = 60°$

So,

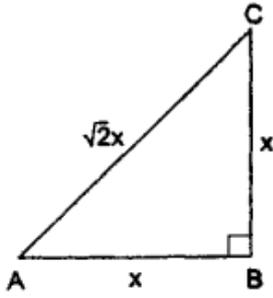$\sin A . \cos C + \cos A . \sin C$

$= \sin 30° . \cos 60° + \cos 30° . \sin 60°$

$= \dfrac{1}{2} \cdot \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2} = 1$

and, $\cos A . \cos C - \sin A . \sin C$

$= \cos 30° . \cos 60° - \sin 30° . \sin 60°$

$= \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{3}}{2} = 0$

6. Let us draw a triangle ABC in which ∠B = 90° and AB: AC = 1: $\sqrt{2}$.

Let AB = x.Then, AC = $\sqrt{2}$x.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

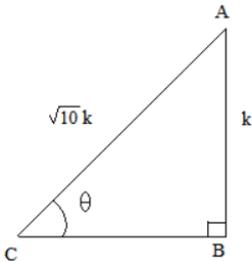$\Rightarrow BC^2 = (\sqrt{2}x)^2 - (x)^2 = 2x^2 - x^2$

$= x^2$

$\Rightarrow BC = x$

$\therefore \tan A = \frac{BC}{AB} = \frac{x}{x} = 1$

So, the given expression = $\left(\frac{2\tan A}{1+\tan^2 A}\right) = \left(\frac{2\times 1}{1+1}\right) = \frac{2}{2} = 1$

7. Let us first draw a right $\triangle$ABC, right angled at B and ∠$C = \theta$.

Now, we know that $cosec\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{AC}{AB} = \frac{\sqrt{10}}{1}$.

So, if AC = ($\sqrt{10}$)k, then AB = k, where k is a positive number.

Now, by using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2 = 10k^2 - k^2$

$\Rightarrow BC^2 = 9k^2$

$\Rightarrow BC = 3k$

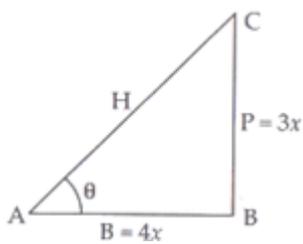Now, finding the other T-ratios using their definitions, we get:

$\tan\theta = \frac{AB}{BC} = \frac{k}{3k} = \frac{1}{3}$

$\cos\theta = \frac{BC}{AC} = \frac{3k}{\sqrt{10k}} = \frac{3}{\sqrt{10}}$

$\therefore \sin\theta = \frac{1}{\cos ec\theta} = \frac{1}{\sqrt{10}}, \cot\theta = \frac{1}{\tan\theta} = 3$ and $\sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{10}}{3}$

8. Given, tan A = $\frac{3}{4}$

$\Rightarrow$ tan A = $\frac{P}{B} = \frac{3x}{4x}$ [from figure]

$H^2 = P^2 + B^2$ [By Pythagoras theorem]

$= (3x)^2 + (4x)^2$

$= 9x^2 + 16x^2$

$\Rightarrow H^2 = 25x^2$

or, $H^2 = (5x)^2$

$\Rightarrow H = 5x$

Therefore,

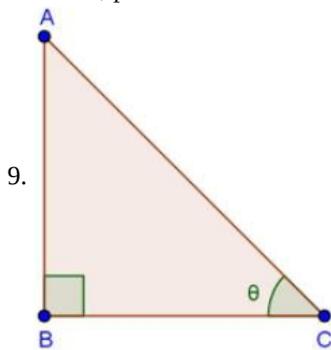$\sin A = \dfrac{P}{H} = \dfrac{3x}{5x} = \dfrac{3}{5}$

and $\cos A = \dfrac{B}{H} = \dfrac{4x}{5x} = \dfrac{4}{5}$

Now, LHS = sin A cos A

$= \dfrac{3}{5} \times \dfrac{4}{5} = \dfrac{12}{25} = $ RHS

Hence, proved.

9.

Given $\sec\theta = \dfrac{5}{4} = \dfrac{AC}{BC}$

Let AC = 5K & BC = 4K , where K is a positive integer.

In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

or, $AB^2 + (4K)^2 = (5K)^2$

or, $AB^2 + 16K^2 = 25K^2$

or, $AB^2 = 25K^2 - 16K^2$

or, $AB^2 = 9K^2$

$\therefore AB = \sqrt{9K^2} = 3K$

$\therefore \sin\theta = \dfrac{AB}{AC} = \dfrac{3K}{5K} = \dfrac{3}{5}$

$\cos\theta = \dfrac{BC}{AC} = \dfrac{4K}{5K} = \dfrac{4}{5}$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{3K}{4K} = \dfrac{3}{4}$

$\cot\theta = \dfrac{BC}{AB} = \dfrac{4K}{3K} = \dfrac{4}{3}$

Now $\dfrac{\sin\theta - 2\cos\theta}{\tan\theta - \cot\theta} = \dfrac{\dfrac{3}{5} - 2 \times \dfrac{4}{5}}{\dfrac{3}{4} - \dfrac{4}{3}}$

$$= \frac{\frac{3}{5} - \frac{8}{5}}{\frac{3}{4} - \frac{4}{3}}$$

$$= \frac{\frac{3-8}{5}}{\frac{9-16}{12}}$$

$$= \frac{\frac{-5}{5}}{\frac{-7}{12}} = \frac{-5}{5} \times \frac{-12}{7} = \frac{12}{7}$$

10. Here, $\sin \theta - \cos \theta = \frac{1}{2}$

Squaring both sides, we get

$(\sin \theta - \cos \theta)^2 = \left(\frac{1}{2} \right)^2$

$\Rightarrow \sin^2 \theta + \cos^2 \theta - 2 \sin \theta . \cos \theta = \frac{1}{4}$

$1 - 2 \sin \theta . \cos \theta = \frac{1}{4}$ ($\therefore \sin^2 \theta . \cos^2 \theta = 1$)

$\Rightarrow 1 - \frac{1}{4} = 2 \sin \theta . \cos \theta$
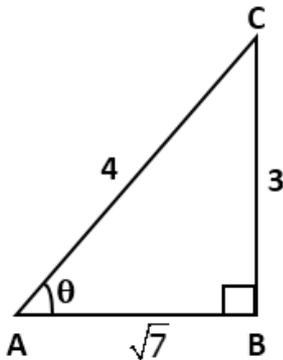
$\Rightarrow 2 \sin \theta . \cos \theta = \frac{3}{4}$ ...(i)

Now $(\sin \theta + \cos \theta)^2 = \sin^2 \theta + \cos^2 \theta + 2 \sin \theta \cos \theta$

$= 1 + \frac{3}{4}$ (using (i)

$\Rightarrow (\sin \theta + \cos \theta) = \sqrt{\frac{7}{4}}$

$\Rightarrow \frac{1}{\sin \theta + \cos \theta} = \frac{2}{\sqrt{7}} = \frac{2\sqrt{7}}{7}$

11. Let us draw a triangle ABC in which ∠B = 90°.

Let ∠A = θ°.

Given,

$\sin\theta = \frac{3}{4}$

$\Rightarrow \mathrm{cosec}\theta = \frac{4}{3}$......(1)

But ,$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3}{4}$

Let BC = 3 and AC = 5,

By Pythagoras' theorem in Δ ABC , we have :-

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$= 4^2 - 3^2 = 16 - 9 = 7$

$\Rightarrow AB = \sqrt{7}$

Now,

$\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{\sqrt{7}}{3}$ ....(2)

$\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{AC}{AB} = \dfrac{4}{\sqrt{7}}$ ....(3)

LHS

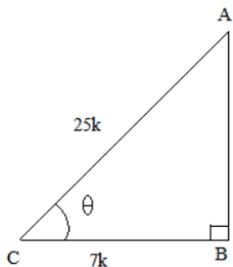$= \sqrt{\dfrac{\cos ec^2\theta - \cot^2\theta}{\sec^2\theta - 1}}$

$= \sqrt{\dfrac{(4/3)^2 - (\sqrt{7}/3)^2}{(4/\sqrt{7})^2 - 1}}$ [from (2) & (3)]

$= \sqrt{\dfrac{16/9 - 7/9}{12}}$

$= \sqrt{\dfrac{1}{9/7}}$

$= \sqrt{\dfrac{7}{9}}$

$= \dfrac{\sqrt{7}}{3}$

= RHS

Hence, Proved.

12. Let us first draw a right $\triangle ABC$, right angled at B and $\angle C = \theta$.

Now, we know that $\cos\theta = \dfrac{\text{Base}}{\text{hypotenuse}} = \dfrac{BC}{AC} = \dfrac{7}{25}$
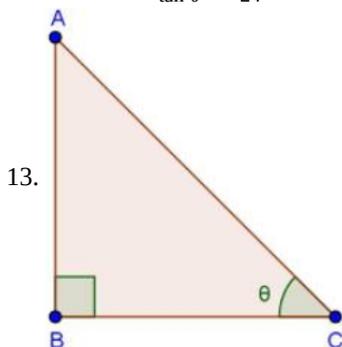

So, if BC = 7k, then AC = 25k, where k is a positive number.

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2 = (25k)^2 - (7k)^2$.

$\Rightarrow AB^2 = 625k^2 - 49k^2 = 576k^2$

$\Rightarrow AB = 24k$

Now, finding the other trigonometric ratios using their definitions, we get:

$\sin\theta = \dfrac{AB}{AC} = \dfrac{24k}{25k} = \dfrac{24}{25}$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{24k}{7k} = \dfrac{24}{7}$

$\therefore \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{7}{24}, cosec\theta = \dfrac{1}{\sin\theta} = \dfrac{25}{24}$ and $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{25}{7}$

13.

Given $\tan\theta = \dfrac{24}{7} = \dfrac{AB}{BC}$ (θ is $\angle C$, see figure)

Let AB = 24K and BC = 7K, where K is positive integer.

In ΔABC, by Pythagoras theorem

$AC^2 = AB^2 + BC^2$

Or, $AC^2 = (24K)^2 + (7K)^2$

Or, $AC^2 = 576K^2 + 49K^2$

Or, $AC^2 = 625K^2$

$\therefore AC = \sqrt{625K^2} = 25K$

Now,

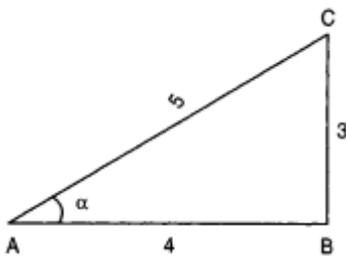$\sin\theta = \dfrac{AB}{AC} = \dfrac{24K}{25K} = \dfrac{24}{25}$

$\cos\theta = \dfrac{BC}{AC} = \dfrac{7K}{25K} = \dfrac{7}{25}$

$\therefore \sin\theta + \cos\theta$

$= \dfrac{24}{25} + \dfrac{7}{25}$

$= \dfrac{24+7}{25} = \dfrac{31}{25}$

14.

We have,

$\sec\alpha = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{5}{4}$

So, Let us draw a right triangle ABC, $\angle B = 90°$ such that

hypotenuse = AC = 5 units, Base = AB = 4 units, and $\angle BAC = \alpha$.

Applying Pythagoras theorem in ΔABC, we get

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 5^2 = 4^2 + BC^2$
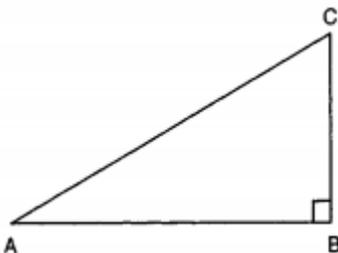
$\Rightarrow \quad BC^2 = 5^2 - 4^2 = 9$

$\Rightarrow \quad BC = \sqrt{9} = 3$

$\therefore \quad \tan\alpha = \dfrac{BC}{AB} = \dfrac{3}{4}$

Now, we have, $\dfrac{1-\tan\alpha}{1+\tan\alpha} = \dfrac{1-\frac{3}{4}}{1+\frac{3}{4}} = \dfrac{\frac{1}{4}}{\frac{7}{4}} = \dfrac{1}{7}$

therefore, $\dfrac{1-\tan\alpha}{1+\tan\alpha} = \dfrac{1}{7}$

15.

we have,

$\tan A = \dfrac{1}{\sqrt{3}} = tan30°$

$\therefore A = 30°$

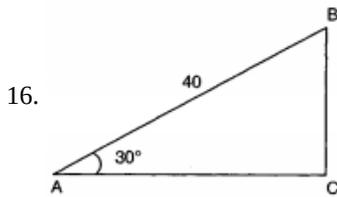In $\triangle ABC$, we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 30° + 90° + \angle C = 180°$

$\Rightarrow 120° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 120° = 60°$

So,

$\sin A . \cos C + \cos A . \sin C$

$= \sin 30°. \cos 60° + \cos 30°. \sin 60°$

$= \dfrac{1}{2} \cdot \dfrac{1}{2} + \dfrac{\sqrt{3}}{2} \cdot \dfrac{\sqrt{3}}{2} = 1$

16.

We know that

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 30° + \angle B + 90° = 180°$

$\Rightarrow \quad \angle B = 180° - 120° = 60°$

Now, $\cos A = \dfrac{AC}{AB}$

$\Rightarrow \quad \cos 30° = \dfrac{AC}{40}$

$\Rightarrow \dfrac{\sqrt{3}}{2} = \dfrac{AC}{40}$

$\Rightarrow \quad AC = \dfrac{\sqrt{3}}{2} \times 40$

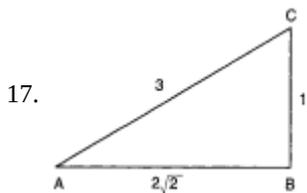$\Rightarrow AC = 20\sqrt{3}$ units

and, $\sin A = \dfrac{BC}{AB}$

$\Rightarrow \quad \sin 30° = \dfrac{BC}{40}$

$\Rightarrow \quad \dfrac{1}{2} = \dfrac{BC}{40}$

$\Rightarrow BC = 40 \times \dfrac{1}{2} = 20$

Hence, $\angle B = 60°, AC = 20\sqrt{3}$ units

and BC = 20 units

17.

According to the question,

$\sin A = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{1}{3}$

So, we draw a right triangle, right angled at B such that Perpendicular $= BC = 1$ *unit*,

Hypotenuse $= AC = 3$ *units*.

Using Pythagoras theorem,

$\Rightarrow AC^2 = AB^2 + BC^2$

$\Rightarrow \quad 3^2 = AB^2 + 1^2$

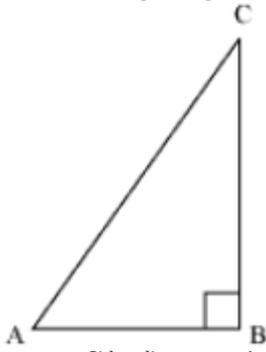$\Rightarrow \quad 9 - 1 = AB^2$

$\Rightarrow \quad AB = \sqrt{8} = 2\sqrt{2}$

$\cos A = \dfrac{AB}{AC} = \dfrac{2\sqrt{2}}{3}, cosec\,A = \dfrac{AC}{BC} = \dfrac{3}{1} = 3$

$\tan A = \dfrac{BC}{AB} = \dfrac{1}{2\sqrt{2}}$ and $\sec A = \dfrac{AC}{AB} = \dfrac{3}{2\sqrt{2}}$

$$\therefore \quad \cos A \operatorname{cosec} A + \tan A \sec A = \frac{2\sqrt{2}}{3} \times 3 + \frac{1}{2\sqrt{2}} \times \frac{3}{2\sqrt{2}}$$

$$\Rightarrow \quad \cos A \operatorname{cosec} A + \tan A \sec A = 2\sqrt{2} + \frac{3}{8} = \frac{16\sqrt{2}+3}{8}$$

18. Consider a right angled triangle, right angled at B

$$\cot A = \frac{\textit{Side adjacent to } \angle A}{\textit{Side opposite to } \angle A}$$

$$= \frac{AB}{BC}$$

Now given that:-

$$\cot A = \frac{8}{15}$$

$$\Rightarrow \frac{AB}{BC} = \frac{8}{15} \text{....... (1)}$$

Let AB be 8K so BC will be 15K where K is a positive integer.

Now applying pythagoras theorem in ΔABC

$$AC^2 = AB^2 + BC^2$$

$$\text{or, } AC^2 = (8K)^2 + (15K)^2$$

$$\text{or, } AC^2 = 64K^2 + 225K^2$$

$$\text{or, } AC^2 = 289 \ K^2$$

or, AC = 17K [taking square root both sides]

$$\cos A = \frac{\textit{Side adjacent to } \angle A}{\textit{hypotenuse}} = \frac{AB}{AC}$$

$$= \frac{8K}{17K} = \frac{8}{17}$$

$$\tan A = \frac{\textit{side opposite to } \angle A}{\textit{Side adjacent to } \angle A}$$

$$= \frac{BC}{AB} = \frac{15}{8}$$

19. Given: $\operatorname{cosec}\theta = \sqrt{10}$

$$\operatorname{cosec}\theta = \frac{\sqrt{10}}{1} \ \ldots\ldots \ (1)$$

By definition,

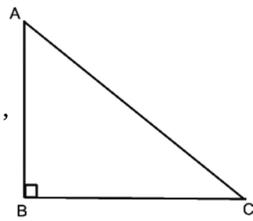$$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$$

$$\operatorname{cosec}\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} \ \ldots\ldots (2)$$

Without loss of generality, Comparing (1) and (2) gives

We get,

Perpendicular side = 1 and

Hypotenuse = $\sqrt{10}$

Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Now we substitute the value of perpendicular side (*BC*) and hypotenuse (*AC*) and get the base side (*AB*)

$(\sqrt{10})^2 = AB^2 + 1^2$

$AB^2 = (\sqrt{10})^2 - 1^2$

$AB^2 = 10 - 1$

$AB^2 = 9$

$AB = \sqrt{9}$

$AB = 3$

Hence, Base side = 3

Now, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

$\sin\theta = \dfrac{1}{\sqrt{10}}$

Now, $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

$\cos\theta = \dfrac{3}{\sqrt{10}}$

Next, $\sec\theta = \dfrac{1}{\cos\theta}$

Therefore,

$\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$

$\sec\theta = \dfrac{\sqrt{10}}{3}$

Next, $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$

Therefore,

$\tan\theta = \dfrac{1}{3}$
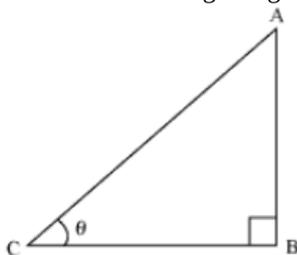
Finally, $\cot\theta = \dfrac{1}{\tan\theta}$

Therefore,

$\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$

$\cot\theta = \dfrac{3}{1}$

$\cot\theta = 3$

20. Let us consider a right angled ΔABC right angled at point B.

Let $\angle C = \theta$

Given,

$\cot\theta = \dfrac{7}{8} = \dfrac{\textit{Side adjacent to} \angle\theta}{\textit{Side opposite to} \angle\theta} = \dfrac{BC}{AB}$

If BC is 7K then AB will be 8K, where K is a positive integer.

Now applying Pythagoras theorem in ΔABC

$AC^2 = AB^2 + BC^2$

Or, $AC^2 = 64K^2 + 49K^2$

Or, $AC^2 = 113K^2$

$\therefore AC = \sqrt{113}K$

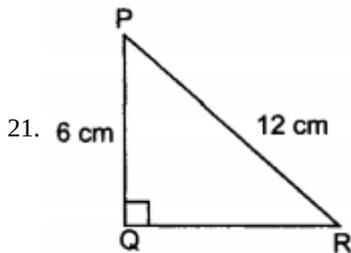Now,

$\sin\theta = \dfrac{\textit{Side opposite to} \angle\theta}{\textit{hypotenuse}} = \dfrac{AB}{AC}$

$= \dfrac{8K}{\sqrt{113}K} = \dfrac{8}{\sqrt{113}}$

And,

$\cos\theta = \dfrac{\textit{Side adjacent to} \angle\theta}{\textit{hypotenuse}} = \dfrac{BC}{AC}$

$= \dfrac{7K}{\sqrt{113}K} = \dfrac{7}{\sqrt{113}}$

Now $\cot^2\theta = \left(\dfrac{\cos\theta}{\sin\theta}\right)^2 = \left(\dfrac{7}{\sqrt{113}} \times \dfrac{\sqrt{113}}{8}\right)^2 = \left(\dfrac{7}{8}\right)^2 = \dfrac{49}{64}$

21.

In $\triangle PQR$

$\sin(\angle PRQ) = \dfrac{PQ}{PR}$

$\Rightarrow \sin(\angle PRQ) = \dfrac{6}{12} = \dfrac{1}{2}$

Also, $\sin 30° = \dfrac{1}{2}$

$\Rightarrow \angle PRQ = 30°$

In right $\triangle PQR$, $\angle Q + \angle R + \angle P = 180°$

$90° + 30° + \angle P = 180°$

$\Rightarrow \angle P = 60°$

$\therefore$ **$\angle QPR = 60°, \angle PRQ = 30°$**

22. We have,

AB = a

$\Rightarrow$ AD + DB = a

$\Rightarrow$ AD + AD = a

$\Rightarrow$ 2 AD = a

$\Rightarrow AD = \dfrac{a}{2}$

Thus, $AD = DB = \dfrac{a}{2}$

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow b^2 = a^2 + BC^2$

$\Rightarrow BC^2 = b^2 - a^2$

Thus, in $\triangle BCD$, we have

Base $= BC = \sqrt{b^2 - a^2}$ and Perpendicular $= BD = \dfrac{a}{2}$

Applying Pythagoras theorem in $\triangle BCD$, we have

$\Rightarrow BC^2 + BD^2 = CD^2$

$\Rightarrow \quad (\sqrt{b^2 - a^2})^2 + \left(\dfrac{a}{2}\right)^2 = CD^2$

$\Rightarrow \quad CD^2 = b^2 - a^2 + \dfrac{a^2}{4}$

$\Rightarrow \quad CD^2 = \dfrac{4b^2 - 4a^2 + a^2}{4}$

$\Rightarrow \quad CD^2 = \dfrac{4b^2 - 3a^2}{4}$
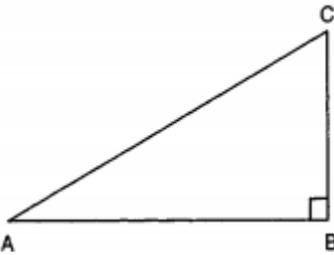
$\Rightarrow \quad CD = \dfrac{\sqrt{4b^2 - 3a^2}}{2}$

Now,

i. $\sin\theta = \dfrac{BD}{CD} \Rightarrow \sin\theta = \dfrac{a/2}{\frac{\sqrt{4b^2 - 3a^2}}{2}} = \dfrac{a}{\sqrt{4b^2 - 3a^2}}$

ii. $\cos\theta = \dfrac{BC}{CD} \Rightarrow \cos\theta = \dfrac{\sqrt{b^2 - a^2}}{\frac{\sqrt{4b^2 - 3a^2}}{2}} = \dfrac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}$

iii. $\tan\theta = \dfrac{BD}{BC} \Rightarrow \tan\theta = \dfrac{a/2}{\sqrt{b^2 - a^2}} = \dfrac{a}{2\sqrt{b^2 - a^2}}$

iv. $\sin^2\theta + \cos^2\theta = \left(\dfrac{a}{\sqrt{4b^2 - 3a^2}}\right)^2 + \left(\dfrac{2\sqrt{b^2 - a^2}}{\sqrt{4b^2 - 3a^2}}\right)^2 = \dfrac{a^2}{4b^2 - 3a^2} + \dfrac{4\left(b^2 - a^2\right)}{4b^2 - 3a^2} = \dfrac{4b^2 - 3a^2}{4b^2 - 3a^2} = 1$

23.

we have,

$\tan A = \dfrac{1}{\sqrt{3}} = \tan 30°$

$\therefore A = 30°$

In $\triangle ABC$, we have

$\angle A + \angle B + \angle C = 180°$

$\Rightarrow 30° + 90° + \angle C = 180°$

$\Rightarrow 120° + \angle C = 180°$

$\Rightarrow \angle C = 180° - 120° = 60°$

So,

cos A.cos C - sin A.sin C

$= \cos 30°. \cos 60° - \sin 30°. \sin 60°$

$= \dfrac{\sqrt{3}}{2} \cdot \dfrac{1}{2} - \dfrac{1}{2} \cdot \dfrac{\sqrt{3}}{2} = 0$

24.
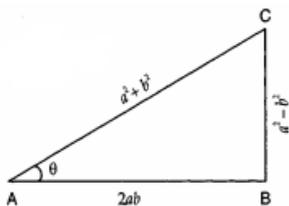
We have,

$\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{a^2 - b^2}{a^2 + b^2}$

So, Let us draw a right triangle ABC in which $\angle B$ is right angle, we have

Perpendicular = BC = a² - b² Hypotenuse = AC = a² + b² and, $\angle BAC = \theta$

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad \left(a^2 + b^2\right)^2 = AB^2 + \left(a^2 - b^2\right)^2$

$\Rightarrow \quad AB^2 = \left(a^2 + b^2\right)^2 - \left(a^2 - b^2\right)^2$

$\Rightarrow \quad AB^2 = \left(a^4 + b^4 + 2a^2b^2\right) - \left(a^4 + b^4 - 2a^2b^2\right)$

$\Rightarrow \quad AB^2 = 4a^2b^2 = (2ab)^2$

$\Rightarrow \quad AB = 2ab$

Now, Let$\angle BAC = \theta$

We have

Base = AB = 2ab, Perpendicular = BC = $a^2 - b^2$, Hypotenuse = AC = a² + b²

Therefore, $\quad \cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}} = \dfrac{2ab}{a^2+b^2}, \tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{a^2-b^2}{2ab}$

$\quad cosec\ \theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{a^2+b^2}{a^2-b^2}, \quad \sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}} = \dfrac{a^2+b^2}{2ab}$

and, $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{2ab}{a^2-b^2}$

25. In ΔACB, we have

$AB^2 = AC^2 + BC^2$

$\Rightarrow \quad AC = \sqrt{AB^2 - BC^2} = \sqrt{29^2 - 21^2} = \sqrt{(29+21)(29-21)} = \sqrt{400} = 20$ units

$\therefore \quad \sin\theta = \dfrac{AC}{AB} = \dfrac{20}{29}$ and $\cos\theta = \dfrac{BC}{AB} = \dfrac{21}{29}$

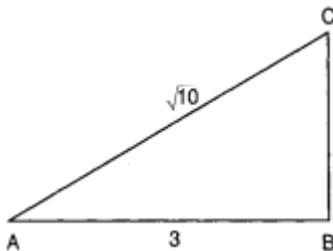i. Using the values of $\sin\theta$ and $\cos\theta$, we get

$\cos^2\theta + \sin^2\theta = \left(\dfrac{21}{29}\right)^2 + \left(\dfrac{20}{29}\right)^2$

$= \dfrac{441 + 400}{841} = 1$

ii. Using the values of $\sin\theta$ and $\cos\theta$, we obtain

$\cos^2\theta - \sin^2\theta = \left(\dfrac{21}{29}\right)^2 - \left(\dfrac{20}{29}\right)^2 = \dfrac{21^2 - 20^2}{29^2} = \dfrac{(21+20)\,(21-20)}{841} = \dfrac{41}{841}$

26.

We have,

$cosec\ A = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}} = \dfrac{\sqrt{10}}{1}$

So, we draw a right triangle ABC, right-angled at B such that

Perpendicular = BC = 1 unit and, Hypotenuse = $AC = \sqrt{10}$.

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow \quad (\sqrt{10})^2 = AB^2 + 1^2$

$\Rightarrow \quad AB^2 = 10 - 1 = 9$

$\Rightarrow \quad AB = \sqrt{9} = 3$

When we consider the trigonometric ratios of $\angle A$, we have

Base = AB = 3 units, Perpendicular = BC = 1 units and, Hypotenuse = $AC = \sqrt{10}$ units

$$\therefore \quad \sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{1}{\sqrt{10}}, \cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{3}{\sqrt{10}}$$

$$\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{1}{3}, \quad \sec A = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{\sqrt{10}}{3}.$$

and, $\cot A = \dfrac{\text{Base}}{\text{Perpendicular}} = \dfrac{3}{1} = 3$

27. We have

$cosec\ \theta = \dfrac{13}{12}$

$\sin\theta = \dfrac{1}{cosec\ \theta} = \dfrac{12}{13}$

$\sin^2\theta = \left(\dfrac{12}{13}\right)^2 = \dfrac{144}{169}$

We know that,

$\cos^2\theta = 1 - \sin^2\theta$

$\cos^2\theta = 1 - \dfrac{144}{169}$

$\cos^2\theta = \dfrac{25}{169}$

$\cos\theta = \dfrac{5}{13}$

Now, $\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta}$

$= \dfrac{2 \times \frac{12}{13} - 3 \times \frac{5}{13}}{4 \times \frac{12}{13} - 9 \times \frac{5}{13}}$

$= \dfrac{\frac{24}{13} - \frac{15}{13}}{\frac{48}{13} - \frac{45}{13}}$

$= \dfrac{\frac{9}{13}}{\frac{3}{13}} = \dfrac{9}{3} = 3$

Hence $\dfrac{2\sin\theta - 3\cos\theta}{4\sin\theta - 9\cos\theta} = 3$

28. Given, $\tan\theta = \dfrac{2mn}{m^2 - n^2}$ ......(1)

We know that,

$\sec^2\theta = 1 + \tan^2\theta$

$= 1 + \dfrac{(2mn)^2}{\left(m^2 - n^2\right)^2}$ [from (1)]

$= \dfrac{\left(m^2 - n^2\right)^2 + 4m^2n^2}{\left(m^2 - n^2\right)^2}$

$\Rightarrow \sec^2\theta = \dfrac{\left(m^2 + n^2\right)^2}{\left(m^2 - n^2\right)^2}$

$\Rightarrow \sec\theta = \sqrt{\dfrac{\left(m^2 + n^2\right)^2}{\left(m^2 - n^2\right)^2}}$

$\Rightarrow \sec\theta = \dfrac{m^2 + n^2}{m^2 - n^2}$ ....(2)

Now,

$\cos\theta = \dfrac{1}{\sec\theta} = \dfrac{m^2 - n^2}{m^2 + n^2}$ [from (2)]

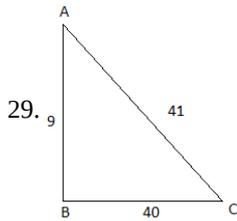$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{m^2 - n^2}{2mn}$ [from (1)]

Also, $\tan \theta = \dfrac{\sin \theta}{\cos \theta}$

$\Rightarrow \tan \theta \cdot \cos \theta = \sin \theta$

$\Rightarrow \dfrac{2mn}{m^2-n^2} \times \dfrac{m^2-n^2}{m^2+n^2} = \sin \theta$

$\Rightarrow \sin \theta = \dfrac{2mn}{m^2+n^2}$

$\&\ , \operatorname{cosec} \theta = \dfrac{1}{\sin \theta} = \dfrac{m^2+n^2}{2mn}$

29.

Let us draw a $\triangle$ABC in which $\angle$B = 90°.

Then, $\cos A = \dfrac{AB}{AC} = \dfrac{9}{41}$

Let AB = 9k and AC = 41k, where k is positive.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

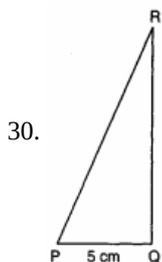$\Rightarrow BC^2 = (41k)^2 - (9k)^2 = 1681k^2 - 81k^2 = 1600k^2$

$\Rightarrow BC = \sqrt{1600k^2} = 40k$

$\sin A = \dfrac{BC}{AC} = \dfrac{40k}{41k} = \dfrac{40}{41}; \cos A = \dfrac{9}{41}$ (given);

$\tan A = \dfrac{\sin A}{\cos A} = \left(\dfrac{40}{41} \times \dfrac{41}{9}\right) = \dfrac{40}{9};$

$\csc A = \dfrac{1}{\sin A} = \dfrac{41}{40}; \sec A = \dfrac{1}{\cos A} = \dfrac{41}{9}$

and $\cot A = \dfrac{1}{\tan A} = \dfrac{9}{40}.$

30.

In $\triangle PQR$, by Pythagoras theorem

$PR^2 = PQ^2 + QR^2$

$\Rightarrow (25 - QR)^2 = 5^2 + QR^2 [\because PR + QR = 25 \text{ cm} \Rightarrow PR = 25 - QR]$

625 - 50QR + QR^2 = 25 + QR^2

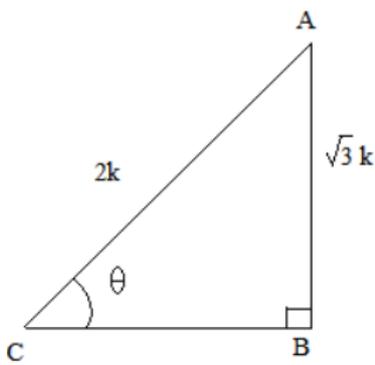$\Rightarrow 600 - 50QR = 0$

$\Rightarrow QR = \dfrac{600}{50} = 12$ cm

Now, PR + QR = 25 cm

$\Rightarrow PR = 25 - QR = 25 - 12 = 13$ cm

Hence, $\sin P = \dfrac{QR}{PR} = \dfrac{12}{13}, \cos P = \dfrac{PQ}{PR} = \dfrac{5}{13}$ and, $\tan P = \dfrac{QR}{PQ} = \dfrac{12}{5}$

31. Let us first draw a right $\triangle$ABC, right angled at B and $\angle C = \theta$.

Now, we know that $\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}} = \dfrac{AB}{AC} = \dfrac{\sqrt{3}}{2}.$

So, if $AB = \sqrt{3}k$, then AC = 2k, where k is a positive number.

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2 = (2k)^2 - (\sqrt{3}k)^2$

$\Rightarrow BC^2 = 4k^2 - 3k^2 = k^2$

$\Rightarrow BC = k$

Now, finding the other T-ratios using their definitions, we get:

$\cos\theta = \dfrac{BC}{AC} = \dfrac{k}{2k} = \dfrac{1}{2}$

$\tan\theta = \dfrac{AB}{BC} = \dfrac{\sqrt{3}k}{k} = \sqrt{3}$

$\therefore \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{1}{\sqrt{3}}, \csc\theta = \dfrac{1}{\sin\theta} = \dfrac{2}{\sqrt{3}}$ and $\sec\theta = \dfrac{1}{\cos\theta} = 2$

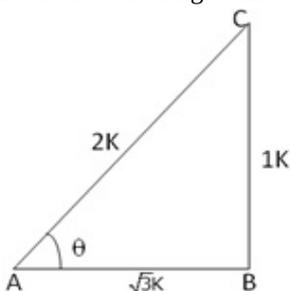32. We have,

$5\tan\theta = 4 \Rightarrow \tan\theta = \dfrac{4}{5}$

Now, L.H.S $= \dfrac{5\sin\theta - 3\cos\theta}{5\sin\theta + 2\cos\theta} = \dfrac{\frac{5\sin\theta - 3\cos\theta}{\cos\theta}}{\frac{5\sin\theta + 2\cos\theta}{\cos\theta}}$ [Dividing Numbers and Denominator by $\cos\theta$]

$= \dfrac{\frac{5\sin\theta}{\cos\theta} - \frac{3\cos\theta}{\cos\theta}}{\frac{5\sin\theta}{\cos\theta} + \frac{2\cos\theta}{\cos\theta}} = \dfrac{5\tan\theta - 3}{5\tan\theta + 2} = \dfrac{5 \times \frac{4}{5} - 3}{5 \times \frac{4}{5} + 2}$ $\left[\because \tan\theta = \dfrac{4}{5} \right]$

$= \dfrac{4 - 3}{4 + 2} = \dfrac{1}{6}$ = R.H.S

Hence proved.

33. Let us draw a triangle ABC in which ∠B = 90°.

Let ∠A = $\theta°$.

Then, $\csc\theta = \dfrac{AC}{BC} = \dfrac{2}{1}$

Let BC = 1k and AC = 2k, where k is positive.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$= (2k)^2 - (1k)^2 = 4k^2 - 1k^2 = 3k^2$

$\Rightarrow BC = \sqrt{3}k$

$\sin\theta = \dfrac{BC}{AC} = \dfrac{1k}{2k} = \dfrac{1}{2}$

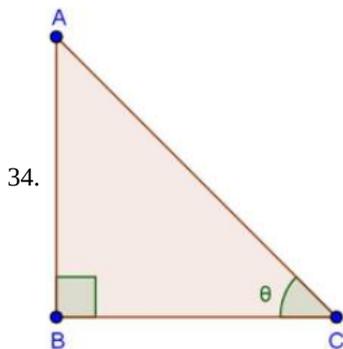$$\cos\theta = \frac{AB}{AC} = \frac{\sqrt{3}k}{2k} = \frac{\sqrt{3}}{2}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta} = \left(\frac{\sqrt{3}}{2} \times \frac{2}{1}\right) = \sqrt{3}$$

$$\Rightarrow \left[\cot\theta + \frac{\sin\theta}{1+\cos\theta}\right] = \left[\sqrt{3} + \frac{\frac{1}{2}}{1+\frac{\sqrt{3}}{2}}\right]$$

$$= \left(\sqrt{3} + \frac{1}{2+\sqrt{3}}\right) = \left(\frac{2\sqrt{3}+3+1}{2+\sqrt{3}}\right)$$

$$= \left(\frac{2\sqrt{3}+4}{2+\sqrt{3}}\right) = 2\left(\frac{\sqrt{3}+2}{2+\sqrt{3}}\right) = 2$$

Hence, $\left[\cot\theta + \frac{\sin\theta}{1+\cos\theta}\right] = 2$

34.

Given $\tan\theta = \frac{12}{13} = \frac{AB}{BC}$ (θ is $\angle C$, see figure)

Let AB = 12K and, BC = 13K ,where K is positive integer.

In ΔABC, by Pythagoras theorem

$AB^2 + BC^2 = AC^2$

Or, $(12K)^2 + (13K)^2 = AC^2$

Or, $144K^2 + 169K^2 = AC^2$

Or, $313K^2 = AC^2$

$\Rightarrow AC = \sqrt{313K^2} = \sqrt{313}K$

Now,

$\sin\theta = \frac{AB}{AC} = \frac{12K}{\sqrt{313}K} = \frac{12}{\sqrt{313}}$

$\cos\theta = \frac{BC}{AC} = \frac{13K}{\sqrt{313}K} = \frac{13}{\sqrt{313}}$

Now,

$$\frac{2\sin\theta\cos\theta}{\cos^2\theta - \sin^2\theta} = \frac{2 \times \frac{12}{\sqrt{313}} \times \frac{13}{\sqrt{313}}}{\left(\frac{13}{\sqrt{313}}\right)^2 - \left(\frac{12}{\sqrt{313}}\right)^2}$$

$$= \frac{\frac{312}{313}}{\frac{169}{313} - \frac{144}{313}}$$

$$= \frac{\frac{312}{313}}{\frac{169-144}{313}}$$

$$= \frac{\frac{312}{313}}{\frac{25}{313}}$$

$$= \frac{312}{313} \times \frac{313}{25}$$

$$= \frac{312}{25}.$$

35. In $\triangle$ABD,

Using Pythagoras theorem, we get

AB $= \sqrt{AD^2 - BD^2}$

$= \sqrt{10^2 - 8^2}$

$= \sqrt{100 - 64}$

$= \sqrt{36}$

$= 6$ cm

Again,

In $\triangle$ABC,

Using Pythagoras therem, we get

AC $= \sqrt{AB^2 + BC^2}$

$= \sqrt{6^2 + 4^2}$

$= \sqrt{36 + 16}$

$= \sqrt{52}$

$= 2\sqrt{13}$cm

Now,

i. $\sin\theta = \frac{BC}{AC}$

$= \frac{4}{2\sqrt{13}}$

$= \frac{2}{\sqrt{13}}$

$= \frac{2\sqrt{13}}{13}$

ii. $\cos\theta = \frac{AB}{AC}$

$= \frac{6}{2\sqrt{13}}$

$= \frac{3}{\sqrt{13}}$

$= \frac{3\sqrt{13}}{13}$
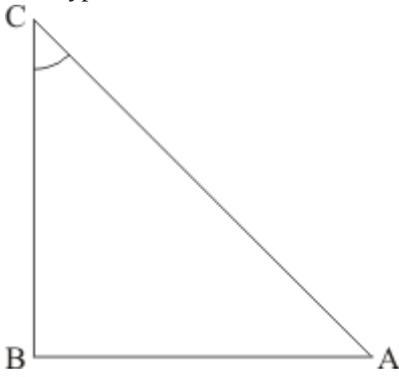
36. Given: $\sec\theta = \frac{13}{5}$ …(1)

By definition,

$\sec\theta = \frac{1}{\cos\theta}$

$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}}$ …(2)

By Comparing (1) and (2), without loss of generality we can take

Base $= 5$

and Hypotenuse = 13

Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Next we substitute the value of base side (AB) and hypotenuse (AC) and get the perpendicular side (BC)

$13^2 = 5^2 + BC^2$

$BC^2 = 13^2 - 5^2$

$BC^2 = 169 - 25$

$BC^2 = 144$

$BC = \sqrt{144}$

BC = 12

Hence, Perpendicular side = 12

Next, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

$\sin\theta = \dfrac{12}{13}$

Next, $cosec\theta = \dfrac{1}{\sin\theta}$

Therefore,

$cosec\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

$cosec\theta = \dfrac{13}{12}$

Next, $\cos\theta = \dfrac{1}{\sec\theta}$

Therefore,

$\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$

$\cos\theta = \dfrac{5}{13}$

Next, $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$

Therefore,

$\tan\theta = \dfrac{12}{5}$
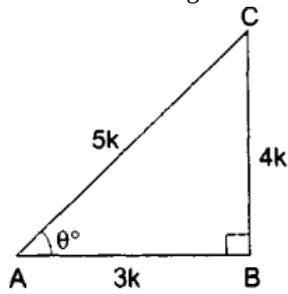
Finally, $\cot\theta = \dfrac{1}{\tan\theta}$

Therefore,

$\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$

$\cot\theta = \dfrac{5}{12}$

37. Let us draw a triangle ABC in which ∠B = 90°.

Let ∠A = $\theta°$.

Then, $\cos\theta = \dfrac{AB}{AC} = \dfrac{3}{5}$

Let AB = 3k and AC = 5k, where k is positive.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

$= (5k)^2 - (3k)^2 = 25k^2 - 9k^2 = 16k^2$

$\Rightarrow BC = \sqrt{16k^2} = 4k$

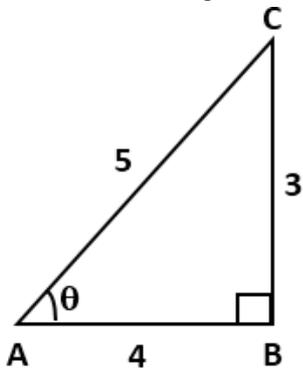$\therefore \sec\theta = \dfrac{1}{\cos\theta} = \dfrac{5}{3}, \tan\theta = \dfrac{BC}{AB} = \dfrac{4k}{3k} = \dfrac{4}{3}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{3}{4}$, and $\mathrm{cosec}\theta = \dfrac{AC}{BC} = \dfrac{5k}{4k} = \dfrac{5}{4}$

$\therefore \left(\dfrac{5\mathrm{cosec}\theta - 4\tan\theta}{\sec\theta + \cot\theta}\right) = \dfrac{\left(5 \times \frac{5}{4} - 4 \times \frac{4}{3}\right)}{\left(\frac{5}{3} + \frac{3}{4}\right)} = \dfrac{\left(\frac{25}{4} - \frac{16}{3}\right)}{\left(\frac{5}{3} + \frac{3}{4}\right)}$

$= \dfrac{\frac{75-64}{12}}{\frac{20+9}{12}} = \left(\dfrac{11}{12} \times \dfrac{12}{29}\right) = \dfrac{11}{29}$

38. Let us draw a triangle ABC in which ∠B = 90°.

Let ∠A = $\theta°$.

$\sec\theta = \dfrac{5}{4} \Rightarrow \cos\theta = \dfrac{4}{5}$

Let AB = 4 and AC = 5,

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow BC^2 = AC^2 - AB^2$

$= 5^2 - 4^2 = 25 - 16 = 9$
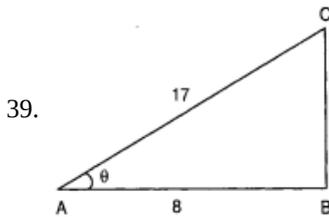
$\Rightarrow BC = 3$

Now,

$\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{3}{5}$

$\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}} = \dfrac{BC}{AB} = \dfrac{3}{4}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{4}{3}$

$$\text{LHS} = \frac{(\sin\theta - 2\cos\theta)}{(\tan\theta - \cot\theta)}$$

$$= \frac{\frac{3}{5} - 2 \times \frac{4}{5}}{\frac{3}{4} - \frac{4}{3}}$$

$$= \frac{\frac{3-8}{5}}{\frac{9-16}{12}}$$

$$= \frac{-5/5}{-7/12}$$

$$= \frac{-1}{-7/12}$$

$$= \frac{12}{7}$$

= RHS

39.

We have,

$$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{8}{17}$$

So, Let ABC is a right angle triangle in which $\angle B = 90°$

Base = AB = 8 units, Hypotenuse = AC = 17 units and, $\angle BAC = \theta$

By Pythagoras theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow 17^2 = 8^2 + BC^2$

$\Rightarrow BC^2 = 17^2 - 8^2$

$\Rightarrow BC^2 = 289 - 64 = 225$

$\Rightarrow BC = \sqrt{225} = 15$

When we consider the trigonometric-ratios of $\angle BAC = \theta$ we have

Base = AB = 8 units, perpendicular = BC = 15 units, and Hypotenuse = AC =17 units

$$\therefore \sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{15}{17}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{15}{8}$$

$$cosec\,\theta = \frac{\text{Hypotenuse}}{\text{Perpendicular}} = \frac{17}{15}$$

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{17}{8}$$

$$\text{and,}\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{8}{15}$$

40. Let us consider two right triangles LMN and PQR

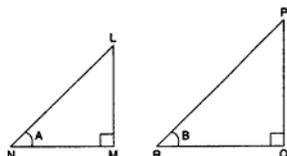Such that $\angle LNM = \angle A$ and $\angle PRO = \angle B$

cos A = cos B ...... Given

$$\Rightarrow \frac{NM}{NL} = \frac{RQ}{RP}$$

$$\Rightarrow \frac{NM}{RQ} = \frac{NL}{RP} = k(\text{say}) \text{ .... (1)}$$

where k is a positive number

$\Rightarrow$ NM = kRQ

NL = K RP

Now, using Pythagoras theorem,

$$ML = \sqrt{NL^2 - NM^2} = \sqrt{(kRP)^2 - (kRQ)^2}$$

$$= \sqrt{k^2 RP^2 - k^2 RQ^2} = k\sqrt{RP^2 - RQ^2}$$

and $QP = \sqrt{RP^2 - RQ^2}$

So, $\dfrac{ML}{QP} = \dfrac{k\sqrt{RP^2 - RQ^2}}{RP^2 - RQ^2} = k$

From (1) and (2), we have

$\dfrac{NM}{RQ} = \dfrac{NL}{RP} = \dfrac{ML}{QP}$

∴ $\Delta LMN \sim \Delta PQR$ ... SSS similarly criterion

∴ ∠LNM = ∠PRQ ∵ Corresponding angles of two similar triangles are equal.

⇒ ∠A = ∠B

41. Given: $\sin\theta = \dfrac{\sqrt{3}}{2}$ …(1)

By definition,
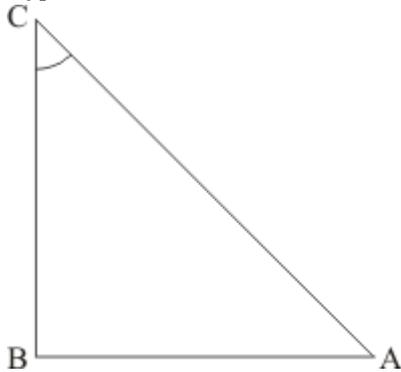
$\sin\theta = \dfrac{\text{Perpendiular}}{\text{Hypotenuse}}$ ... (2)

By Comparing (1) and (2)

We get,

Perpendicular side = $\sqrt{3}k$ and

Hypotenuse = 2k

Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Now we substitute the value of perpendicular side (*BC*) and hypotenuse(*AC*) and get the base side (*AB*)

$2^2 = AB^2 + (\sqrt{3})^2$

$AB^2 = (2k)^2 - (\sqrt{3}k)^2$

$AB^2 = (4 - 3)k^2$

$AB^2 = 1k^2$

$AB = \sqrt{1}k$

$AB = 1k$

Hence, Base = 1k

Now, $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

$\cos\theta = \dfrac{1}{2}$

Next, $cosec\theta = \dfrac{1}{\sin\theta}$

Therefore,

$cosec\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

$cosec\theta = \dfrac{2}{\sqrt{3}}$

Next, $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$

Therefore,

$\sec\theta = \dfrac{2}{1}$

Next, $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$
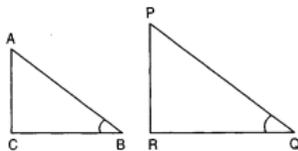
Therefore,

$\tan\theta = \dfrac{\sqrt{3}}{1}$

Finally, $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$

Therefore,

$\cot\theta = \dfrac{1}{\sqrt{3}}$

42. Consider two right triangles ABC and PQR in which $\angle B$ and $\angle Q$ are the right angles.

We have,

In $\triangle ABC$

$\sin B = \dfrac{AC}{AB}$

and, In $\triangle PQR$

$\sin Q = \dfrac{PR}{PQ}$

$\because \quad \sin B = \sin Q$

$\Rightarrow \quad \dfrac{AC}{AB} = \dfrac{PR}{PQ}$

$\Rightarrow \quad \dfrac{AC}{PR} = \dfrac{AB}{PQ} = k(\text{say}) \ \text{......} \ (i)$

$\Rightarrow \ AC = kPR \text{ and } AB = kPQ \ \text{.....(ii)}$

Using Pythagoras theorem in triangles ABC and PQR, we obtain

$AB^2 = AC^2 + BC^2 \text{ and } PQ^2 = PR^2 + QR^2$

$\Rightarrow \quad BC = \sqrt{AB^2 - AC^2} \text{ and } QR = \sqrt{PQ^2 - PR^2}$

$\Rightarrow \quad \dfrac{BC}{QR} = \dfrac{\sqrt{AB^2 - AC^2}}{\sqrt{PQ^2 - PR^2}} = \dfrac{\sqrt{k^2PQ^2 - k^2PR^2}}{\sqrt{PQ^2 - PR^2}} \ [\text{ using (ii) }]$

$\Rightarrow \quad \dfrac{BC}{QR} = \dfrac{k\sqrt{PQ^2 - PR^2}}{\sqrt{PQ^2 - PR^2}} = k...(iii)$

From (i) and (iii), we get

$\dfrac{AC}{PR} = \dfrac{AB}{PQ} = \dfrac{BC}{QR}$

$\Rightarrow \quad \Delta ACB - \Delta PRQ \ [\text{By S.A.S similarity}]$

$\therefore \quad \angle B = \angle Q$

Hence proved.

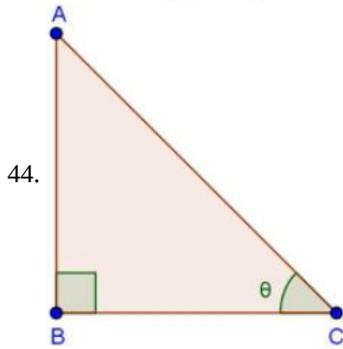43. We have, $16\cot A = 12 \Rightarrow \cot A = \dfrac{12}{16} \Rightarrow \cot A = \dfrac{3}{4}$

Now, $\dfrac{\sin A + \cos A}{\sin A - \cos A} = \dfrac{\frac{\sin A + \cos A}{\sin A}}{\frac{\sin A - \cos A}{\sin A}}$ [Dividing Numerator Denominator by $\sin A$]

$= \dfrac{\frac{\sin A}{\sin A} + \frac{\cos A}{\sin A}}{\frac{\sin A}{\sin A} - \frac{\cos A}{\sin A}} \ [\ \because \ \dfrac{cosA}{sinA} = cotA \ ]$

$= \dfrac{1 + \cot A}{1 - \cot A}$

$= \dfrac{1 + \frac{3}{4}}{1 - \frac{3}{4}} = \dfrac{\frac{7}{4}}{\frac{1}{4}} = 7 \left[\ \because \cot A = \dfrac{3}{4} \right]$

therefore, $\dfrac{\sin A + \cos A}{\sin A - \cos A} = 7$

44.

Given $\tan\theta = \dfrac{20}{21} = \dfrac{AB}{BC}$

Let AB = 20K

and, BC = 21K

In ΔABC, by Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = (20K)^2 + (21K)^2$

$AC^2 = 400K^2 + 441K^2$

$AC^2 = 841K^2$

$AC = \sqrt{841K^2} = 29K$

$\therefore \sin\theta = \dfrac{AB}{AC} = \dfrac{20K}{29K} = \dfrac{20}{29}$

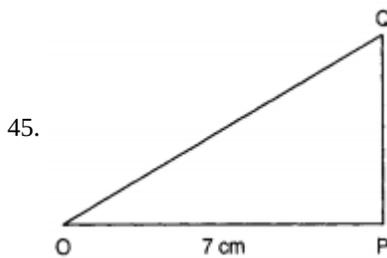$\cos\theta = \dfrac{BC}{AC} = \dfrac{21K}{29K} = \dfrac{21}{29}$

$LHS = \dfrac{1 - \sin\theta + \cos\theta}{1 + \sin\theta + \cos\theta}$

$= \dfrac{1 - \dfrac{20}{29} + \dfrac{21}{29}}{1 + \dfrac{20}{29} + \dfrac{21}{29}}$

$= \dfrac{\dfrac{29 - 20 + 21}{29}}{\dfrac{29 + 20 + 21}{29}}$

$= \dfrac{\dfrac{30}{29}}{\dfrac{70}{29}}$

$= \dfrac{30}{29} \times \dfrac{29}{70} = \dfrac{3}{7} = RHS$

45.

In $\triangle OPQ$, by Pythagoras theorem

$OQ^2 = OP^2 + PQ^2$

$\Rightarrow (PQ + 1)^2 = OP^2 + PQ^2 \; [\because OQ - PQ = 1 \Rightarrow OQ = 1 + PQ]$

$\Rightarrow PQ^2 + 2PQ + 1 = 7^2 + PQ^2$

$\Rightarrow 2PQ + 1 = 49$

$\Rightarrow 2PQ = 48$

$\Rightarrow PQ = 24$ cm

$\therefore OQ - PQ = 1cm \Rightarrow OQ - 24 = 1 \Rightarrow OQ = 25$ cm

Now, $\sin Q = \dfrac{OP}{OQ} = \dfrac{7}{25}$

and, $\cos Q = \dfrac{PQ}{OQ} = \dfrac{24}{25}$

46. Given: $\cos\theta = \dfrac{7}{25}$ …(1)
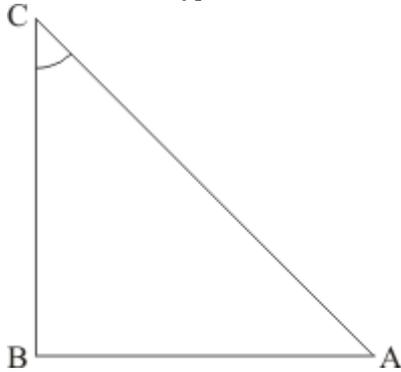
By definition,

$\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$ ... (2)

By Comparing (1) and (2)

We get,

Base = 7k and Hypotenuse = 25k

Therefore,

By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

Now we substitute the value of base side (*AB*) and hypotenuse (*AC*) and get the perpendicular side (*BC*)

$(25k)^2 = (7k)^2 + BC^2$

$BC^2 = (25k)^2 - (7k)^2$

$BC^2 = (625 - 49)\,k^2$

$BC^2 = 576\,k^2$

$BC = 24\,k$

Hence, Perpendicular side = 24 k

Now, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore

$\sin\theta = \dfrac{24}{25}$

Next, $cosec\theta = \dfrac{1}{\sin\theta}$

Therefore, $cosec\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

$cosec\theta = \dfrac{25}{24}$

Next, $\sec\theta = \dfrac{1}{\cos\theta}$

Therefore, $\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$

$\sec\theta = \dfrac{25}{7}$

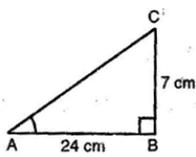Next, $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$

Therefore, $\tan\theta = \dfrac{24}{7}$

Finally, $\cot\theta = \dfrac{1}{\tan\theta}$

Therefore, $\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$

$\cot\theta = \dfrac{7}{24}$

47. Let us draw a right angled triangle ABC, right angled at B.

Using Pythagoras theorem,

Given, AB = 24cm and BC = 7cm

Using Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

$AC^2 = 24^2 + 7^2 = 576 + 49 = 625$

∴ AC = 25 cm

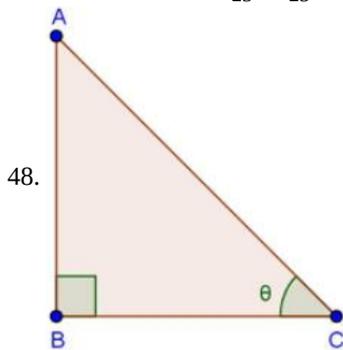i. $\sin A = \frac{P}{H} = \frac{BC}{AC} = \frac{7}{25}$, $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{24}{25}$

⇒ $\sin A . \cos A = \frac{7}{25} \times \frac{24}{25} = \frac{168}{625}$

ii. $\sin C = \frac{P}{H} = \frac{AB}{AC} = \frac{24}{25}$, $\cos C = \frac{B}{H} = \frac{BC}{AC} = \frac{7}{25}$

⇒ $\sin C . \cos C = \frac{24}{25} \times \frac{7}{25} = \frac{168}{625}$

48.

Given $\cos\theta = \frac{12}{13} = \frac{BC}{AC}$

Let BC = 12K

and, AC = 13K

In ΔABC, By Pythagoras theorem

$AB^2 + BC^2 = AC^2$

$AB^2 + (12K)^2 = (13K)^2$

$AB^2 + 144K^2 = 169K^2$

$AB^2 = 169K - 144K^2 = 25K^2$

$AB = \sqrt{25K^2} = 5K$

∴ $\sin\theta = \frac{AB}{AC} = \frac{5K}{13K} = \frac{5}{13}$

$\tan\theta = \frac{AB}{BC} = \frac{5K}{12K} = \frac{5}{12}$

∴ $LHS = \sin\theta(1 - \tan\theta)$

$= \frac{5}{13}\left(1 - \frac{5}{12}\right)$
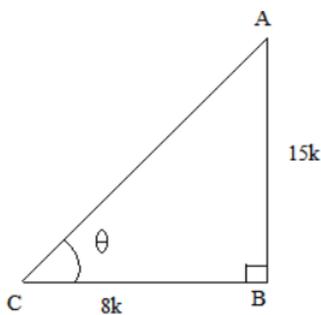
$= \frac{5}{13}\left(\frac{12 - 5}{12}\right)$

$= \frac{5}{13} \times \frac{7}{12}$

$= \frac{35}{156} = RHS$

49. Let us first draw a right △ABC, right angled at B and ∠C = θ.

Now, we know that $\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{AB}{BC} = \frac{15}{8}$

So, if BC = 8k, then AB = 15k, where k is a positive number.

Now, using Pythagoras theorem, we have:

$AC^2 = AB^2 + BC^2 = (15k)^2 + (8k)^2$

$\Rightarrow AC^2 = 225k^2 + 64k2 = 289k^2$

$\Rightarrow AC = 17k$

Now, finding the other T-ratios using their definitions, we get:

$\sin\theta = \dfrac{AB}{AC} = \dfrac{15k}{17k} = \dfrac{15}{17}$

$\cos\theta = \dfrac{BC}{AC} = \dfrac{8k}{17k} = \dfrac{8}{17}$

$\therefore \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{8}{15}, cosec\theta = \dfrac{1}{\sin\theta} = \dfrac{17}{15}$ and $\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{17}{8}$

50. $\cos\theta = \dfrac{3}{5} \Rightarrow \cos^2\theta = \dfrac{9}{25}$

$\therefore \sin^2\theta = 1 - \cos^2\theta$

$= 1 - \dfrac{9}{25} = \dfrac{25-9}{25} = \dfrac{16}{25}$

$\Rightarrow \sin\theta = \dfrac{4}{5}$

$\Rightarrow \tan\theta = \dfrac{\sin\theta}{\cos\theta} = \dfrac{4/5}{3/5} = \dfrac{4}{3}$

$\Rightarrow \cot\theta = \dfrac{1}{\tan\theta} = \dfrac{3}{4}$

Thus,

$\text{L.H.S} = \dfrac{(\sin\theta - \cot\theta)}{2\tan\theta}$

$= \dfrac{4/5 - 3/4}{2 \times 4/3}$

$= \dfrac{\frac{16-15}{20}}{8/3}$

$= \dfrac{1}{20} \times \dfrac{3}{8}$

$= \dfrac{3}{160}$

$= \text{R.H.S}$

51. Given: $\tan\theta = \dfrac{8}{15}$ …(1)

By definition,
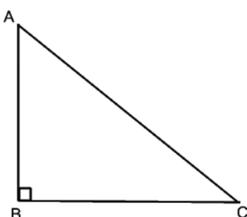
$\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$ …... (2)

By Comparing (1) and (2)

We get,

Base = 15k and

Perpendicular = 8k

Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Now we substitute the value of base side (*AB*) and the perpendicular side (*BC*) and get hypotenuse (*AC*)

$AC^2 = (15k)^2 + (8k)^2$

$AC^2 = 225k^2 + 64k^2$

$AC^2 = 289k^2$

$AC = \sqrt{289}k$

$AC = 17k$

Hence, Hypotenuse = 17k

Now, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

$\sin\theta = \dfrac{8}{17}$

Next, $cosec\theta = \dfrac{1}{\sin\theta}$

Therefore,

$cosec\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

$cosec\theta = \dfrac{17}{8}$

Next, $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

$\cos\theta = \dfrac{15}{17}$

Next, $\sec\theta = \dfrac{1}{\cos\theta}$

Therefore,

$\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$

$\sec\theta = \dfrac{17}{15}$

Finally, $\cot\theta = \dfrac{1}{\tan\theta}$

Therefore,

$\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$

$\cot\theta = \dfrac{15}{8}$

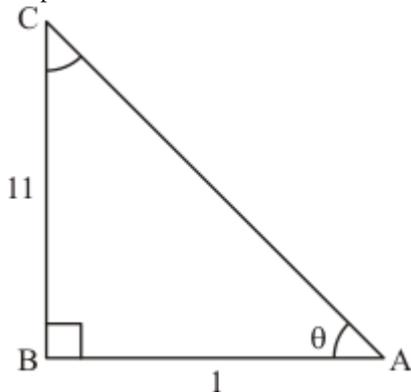52. Given: $\tan\theta = \dfrac{11}{1}$ …(1)

By definition,

$\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$ ... (2)

By Comparing (1) and (2)

We get,

Base = 1k and

Perpendicular side = 11k

Therefore,

By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

Now we substitute the value of base side (*AB*) and the perpendicular side (*BC*) and get hypotenuse (*AC*)

$AC^2 = (1k)^2 + (11k)^2$

$AC^2 = 1k + 121k$

$AC^2 = 122k$

$AC = \sqrt{122}k$

Hence, Hypotenuse = $\sqrt{122}k$

Now, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

$\sin\theta = \dfrac{11}{\sqrt{122}}$

Now, $cosec\theta = \dfrac{1}{\sin\theta}$

Therefore,

$cosec\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

$cosec\theta = \dfrac{\sqrt{122}}{11}$

Next, $\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$

Therefore,

$\cos\theta = \dfrac{1}{\sqrt{122}}$

Next, $\sec\theta = \dfrac{1}{\cos\theta}$

Therefore,

$\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$

$\sec\theta = \dfrac{\sqrt{122}}{1}$

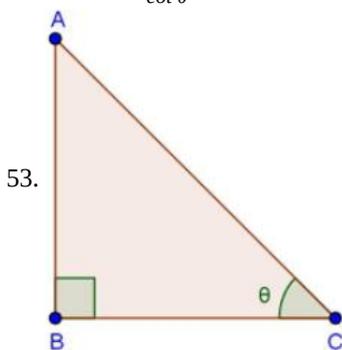$\sec\theta = \sqrt{122}$

Next, $\cot\theta = \dfrac{1}{\tan\theta}$

Therefore,

$\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$

$\cot\theta = \dfrac{1}{11}$

and,

$\tan\theta = \dfrac{1}{\cot\theta} = 11$

53.

Let θ is $\angle C$. Given $\sin\theta = \dfrac{12}{13} = \dfrac{AB}{AC}$ ......(1)

Let AB = 12K and AC = 13K ,where K is positive integer.

In ΔABC, By using Pythagoras theorem :-

$AB^2 + BC^2 = AC^2$

Or, $(12K)^2 + BC^2 = (13K)^2$

Or, $144K^2 + BC^2 = 169K^2$

Or, $BC^2 = 169K^2 - 144K^2$

Or, $BC^2 = 25K^2$

$\therefore BC = \sqrt{25K^2} = 5K$

Now,

$\cos\theta = \dfrac{BC}{AC} = \dfrac{5K}{13K} = \dfrac{5}{13}$ ....(2)

$\tan\theta = \dfrac{AB}{BC} = \dfrac{12K}{5K} = \dfrac{12}{5}$ ....(3)

Now,

$\dfrac{\sin^2\theta - \cos^2\theta}{2\sin\theta \times \cos\theta} \times \dfrac{1}{\tan^2\theta}$

$= \dfrac{\left(\dfrac{12}{13}\right)^2 - \left(\dfrac{5}{13}\right)^2}{2 \times \dfrac{12}{13} \times \dfrac{5}{13}} \times \dfrac{1}{\left(\dfrac{12}{5}\right)^2}$ [from (1),(2) & (3)]

$= \dfrac{\dfrac{144}{169} - \dfrac{25}{169}}{\dfrac{120}{169}} \times \dfrac{1}{\dfrac{144}{25}}$

$= \dfrac{\dfrac{144-25}{169}}{\dfrac{120}{169}} \times \dfrac{25}{144}$

$= \dfrac{119}{169} \times \dfrac{169}{120} \times \dfrac{25}{144}$

$= \dfrac{595}{3456}$
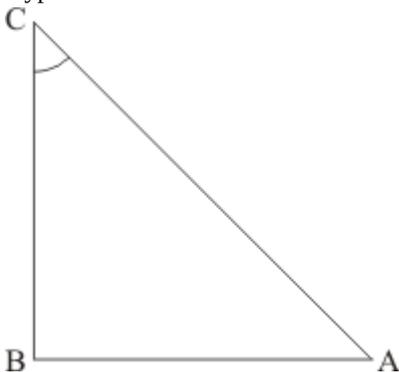
54. Given: $\cos\theta = \dfrac{12}{15}$ ...(1)

By definition,

$\cos\theta = \dfrac{\text{Base}}{\text{Hypotenuse}}$ .... (2)

Without loss of generality, Comparing (1) and (2), gives

Base = 12 and

Hypotenuse = 15

Therefore,

By Pythagoras theorem,

$AC^2 = AB^2 + BC^2$

Now we substitute the value of base side (AB) and hypotenuse (AC) and get the perpendicular side (BC)

$15^2 = 12^2 + BC^2$

$BC^2 = 15^2 - 12^2$

$BC^2 = 225 - 144$

$BC^2 = 81$

$BC = \sqrt{81}$

BC = 9

Hence, Perpendicular side = 9

Next, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}}$

Therefore,

$\sin\theta = \dfrac{9}{15}$

Next, $cosec\theta = \dfrac{1}{\sin\theta}$

Therefore,

$cosec\theta = \dfrac{\text{Hypotenuse}}{\text{Perpendicular}}$

$cosec\theta = \dfrac{15}{9}$

Next, $\sec\theta = \dfrac{1}{\cos\theta}$

Therefore,

$\sec\theta = \dfrac{\text{Hypotenuse}}{\text{Base}}$

$\sec\theta = \dfrac{15}{12}$

Next, $\tan\theta = \dfrac{\text{Perpendicular}}{\text{Base}}$
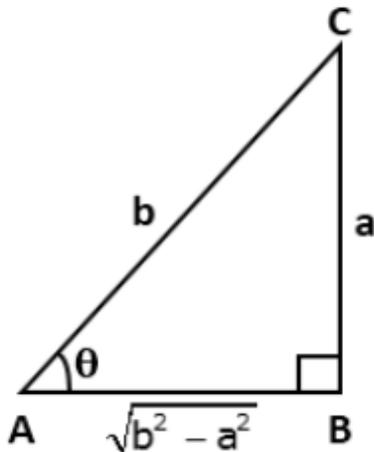
Therefore,

$\tan\theta = \dfrac{9}{12}$

Finally, $\cot\theta = \dfrac{1}{\tan\theta}$

Therefore,

$\cot\theta = \dfrac{\text{Base}}{\text{Perpendicular}}$

$\cot\theta = \dfrac{12}{9}$

55. Let us draw a triangle ABC in which ∠B = 90°.

Let ∠A = $\theta°$.

$\sin\theta = \dfrac{a}{b}$

Then, $\sin\theta = \dfrac{\text{Perpendicular}}{\text{Hypotenuse}} = \dfrac{BC}{AC} = \dfrac{a}{b}$

Let BC = a and AC = b,

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$\Rightarrow AB^2 = AC^2 - BC^2$

$= b^2 - a^2$

$\Rightarrow AB = \sqrt{b^2 - a^2}$

Now,

$$\sec\theta = \frac{\text{Hypotenuse}}{\text{Base}} = \frac{AC}{AB} = \frac{b}{\sqrt{b^2-a^2}}$$

$$\tan\theta = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{a}{\sqrt{b^2-a^2}}$$

LHS = $(\sec\theta + \tan\theta)$

$$= \frac{b}{\sqrt{b^2-a^2}} + \frac{a}{\sqrt{b^2-a^2}}$$

$$= \frac{b+a}{\sqrt{b^2-a^2}}$$

$$= \frac{b+a}{\sqrt{(b+a)(b-a)}}$$

$$= \frac{\sqrt{b+a} \times \sqrt{b+a}}{\sqrt{(b+a)} \times \sqrt{b-a}}$$
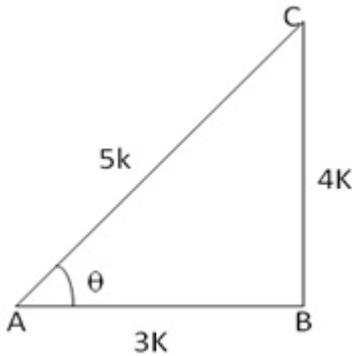
$$= \sqrt{\frac{b+a}{b-a}}$$

= RHS

56. Given : $3\tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$

Let BC = 4k and AB = 3k

where k is positive.

Let us draw a triangle ABC in which $\angle B = 90°$ and $\angle A = \theta°$.

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$= (3k)^2 + (4k)^2 = 9k^2 + 16k^2 = 25k^2$

$\Rightarrow \quad AC = \sqrt{25k^2} = 5k$

$\sin\theta = \frac{4k}{5k} = \frac{4}{5}$

$\cos\theta = \frac{3k}{5k} = \frac{3}{5}$

LHS $= \frac{(4\cos\theta - \sin\theta)}{(2\cos\theta + \sin\theta)}$

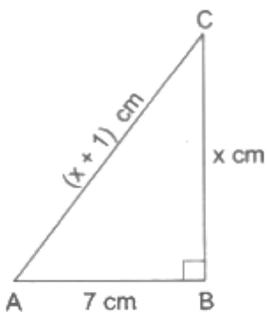$$= \frac{4 \times \frac{3}{5} - \frac{4}{5}}{2 \times \frac{3}{5} + \frac{4}{5}}$$

$$= \frac{\frac{8}{5}}{\frac{10}{5}}$$

$$= \frac{8}{5} \times \frac{1}{2}$$

$$= \frac{4}{5}$$

= RHS

57.

Let BC = x cm . Then, AC = (x + 1) cm.

By Pythagoras' theorem, we have

$AB^2 + BC^2 = AC^2$

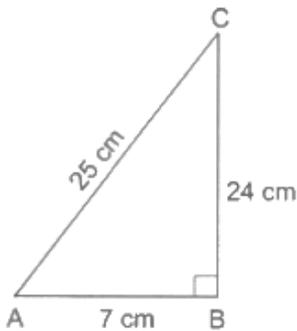$\Rightarrow 7^2 + x^2 = (x + 1)^2$

$\Rightarrow 49 + x^2 = x^2 + 2x + 1$

$\Rightarrow 2x = 48$

x = 24.

∴ BC = 24 cm, AC = 25 cm and AB = 7 cm.

For T-ratios of ∠A, we have

$\sin A = \dfrac{BC}{AC} = \dfrac{24}{25}$ and $\cos A = \dfrac{AB}{AC} = \dfrac{7}{25}$

For T-ratios of ∠C, we have

$\sin C = \dfrac{AB}{AC} = \dfrac{7}{25}$ and $\cos C = \dfrac{BC}{AC} = \dfrac{24}{25}$

58. Given: $\sin\theta = \dfrac{4}{5}$

We know that,

$\sin^2\theta + \cos^2\theta = 1$

$\Rightarrow \left(\dfrac{4}{5}\right)^2 + \cos^2\theta = 1$

$\Rightarrow \dfrac{16}{25} + \cos^2\theta = 1$

$\Rightarrow \cos^2\theta = 1 - \dfrac{16}{25}$

$\Rightarrow \cos^2\theta = \dfrac{9}{25}$

$\Rightarrow \cos\theta = \dfrac{3}{5}$

We have,

$\cot\theta + \csc\theta = \dfrac{\cos\theta}{\sin\theta} + \dfrac{1}{\sin\theta}$

$= \dfrac{\left(\dfrac{3}{5}\right)}{\left(\dfrac{4}{5}\right)} + \dfrac{1}{\left(\dfrac{4}{5}\right)}$
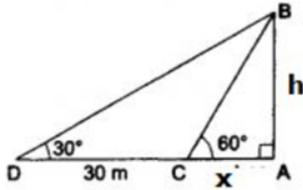
$= \dfrac{3}{4} + \dfrac{5}{4}$

$= 2$

Hence, the value of $\cot\theta + cosec\theta$ is 2.

Section B

59. Let AB be the tree of height h meter and AC = x be the width of river.

Assuming that C be the position of a man standing on the opposite bank of the river. After moving 30 m away from point C. Let new position of man be D,

Thus CD = 30 m, $\angle ACB = 60°$, $\angle ADB = 30°$ and $\angle DAB = 90°$

From right ΔABC we have

$\dfrac{AC}{AB} = \cot 60° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{x}{h} = \dfrac{1}{\sqrt{3}}$

$\Rightarrow x = \dfrac{h}{\sqrt{3}}$ ...(i)

From right $\Delta DAB,$ we have

$\dfrac{AD}{AB} = \cot 30° = \sqrt{3}$

$\Rightarrow \dfrac{x+30}{h} = \sqrt{3}$

$\Rightarrow x = \sqrt{3}h - 30$ ...(ii)
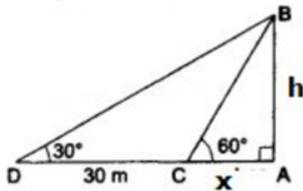
Equating the values of x from (i) and (ii), we get

$\Rightarrow 2h = 30\sqrt{3} \Rightarrow h = 15\sqrt{3} = 15 \times 1.732 = 25.98$ m

Thus the height of the tree h = 25.98 m

60. Let AB be the tree of height h meter and AC = x be the width of river.

Assuming that C be the position of a man standing on the opposite bank of the river. After moving 30 m away from point C. Let new position of man be D, Thus CD = 30 m, $\angle ACB = 60°$, $\angle ADB = 30°$ and $\angle DAB = 90°$

From right ΔABC we have

$\dfrac{AC}{AB} = \cot 60° = \dfrac{1}{\sqrt{3}}$

$\Rightarrow \dfrac{x}{h} = \dfrac{1}{\sqrt{3}}$
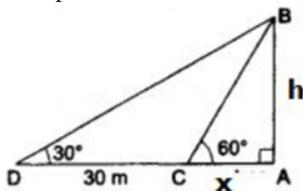
$\Rightarrow x = \dfrac{h}{\sqrt{3}}$ ...(i)

Putting the value of h in (i) we get
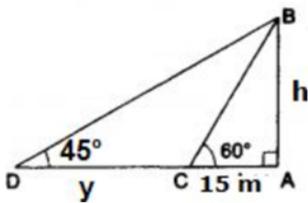
$x = \dfrac{15\sqrt{3}}{\sqrt{3}} = 15\ m$

Hence the width of the river is 15 m.

61. Let AB be the tree of height h meter and AC=x be the width of river.

Assuming that C be the position of a man standing on the opposite bank of the river. After moving 30 m away from point C. Let new position of man be D, Thus CD = 30 m, $\angle ACB = 60°$, $\angle ADB = 30°$ and $\angle DAB = 90°$

Suppose after moving y meters away from the river the angle of elevation becomes 45º.

Then in the $\triangle ABD$

$\dfrac{y + 15}{h} = \cot 45° = 1$

$y + 15 = h$
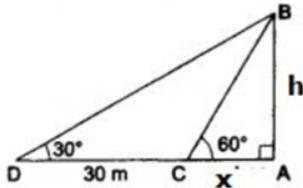
$y = h - 15 = 25.98 - 15$

$y = 10.98$ m

Hence after moving 10.98 meters away from the river the angle of elevation would become.

62. Let AB be the tree of height h meter and AC = x be the width of river.

Assuming that C be the position of a man standing on the opposite bank of the river. After moving 30 m away from point C. Let new position of man be D, Thus CD = 30 m, $\angle ACB = 60°$, $\angle ADB = 30°$ and $\angle DAB = 90°$

From right ΔDAB, we have

$\dfrac{AD}{AB} = \cot 30° = \sqrt{3}$

$\Rightarrow \dfrac{x + 30}{h} = \sqrt{3}$

$\Rightarrow x = \sqrt{3}h - 30$ ...(ii)

From (ii) we are having

$x = h\sqrt{3} - 30$

Given that width of river x = 40 m

Thus $40 = h\sqrt{3} - 30$

$h = \dfrac{70}{\sqrt{3}}$

h = 40.41 m

Thus in this case height of the tree would be 40.41 m.

63. In $\triangle$BRS

$\text{Sin}\theta = \dfrac{RS}{RB}$

$\text{Sin}\theta = \dfrac{8}{16} = \dfrac{1}{2}$

$\therefore \ \theta = 30°$

Now, $\text{Cos}\theta = \dfrac{BS}{RB}$

$\text{Cos } 30° = \dfrac{BS}{16}$

$\dfrac{\sqrt{3}}{2} = \dfrac{BS}{16}$

BS = $8\sqrt{3}$ m

64. In $\triangle$BRS

use, $\text{Sin } \theta = \dfrac{RS}{RB}$

$\text{Sin } \theta = \dfrac{8}{16}$

$\theta = 30°$

Hence, jib B makes an angle of 30° with the horizontal.

65. $AS = \sqrt{AB^2 + BS^2}$

$= \sqrt{24^2 + (8\sqrt{3})^2}$

$= \sqrt{576 + 192}$

$= \sqrt{768}$

$= 27.71$ m

66. $\tan \alpha = \dfrac{8\sqrt{3}}{24}$

$\alpha = 30^o$