**Solution**

**INTRODUCTION TO TRIGONOMETRY WS 3**

**Class 10 - Mathematics**

1. 16

   Explanation:

   Given:

   $\cos\theta = \frac{3}{4}$

   $\Rightarrow \frac{1}{\cos\theta} = \frac{4}{3}$

   $\Rightarrow \sec\theta = \frac{4}{3}$

   We know that

   $\sec^2\theta - \tan^2\theta = 1$

   $\Rightarrow \left(\frac{4}{3}\right)^2 - \tan^2\theta = 1$

   $\Rightarrow \tan^2\theta = \frac{16}{9} - 1$

   $\Rightarrow \tan^2\theta = \frac{7}{9}$

   Therefore,

   $9\tan^2\theta + 9 = 9 \times \frac{7}{9} + 9$

   = 7 + 9

   = 16

2. 3

   Explanation:

   $3\,tan\,\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$

   Given,

   $= \frac{3\sin\theta+2\cos\theta}{3\sin\theta-2\cos\theta}$

   $= \frac{3\tan\theta+2}{3\tan\theta-2}$ [Dividing numerator and denominator by cos $\theta$]

   $= \frac{\left(3\times\frac{4}{3}+2\right)}{\left(3\times\frac{4}{3}-2\right)} = \frac{6}{2} = 3$

3. (a) - (iii), (b) - (i), (c) - (iv), (d) - (ii)
4. (a) - (iv), (b) - (iii), (c) - (ii), (d) - (i)
5. (a) - (iv), (b) - (i), (c) - (ii), (d) - (iii)
6. (a) - (iii), (b) - (ii), (c) - (iv), (d) - (i)
7. (a) - (ii), (b) - (i), (c) - (iv), (d) - (iii)
8. (a) - (ii), (b) - (iii), (c) - (iv), (d) - (i)
9. Given: tan (A +B) = $\frac{\tan A+\tan B}{1-\tan A\tan B}$ and tan A = 1/3, tan B = 1/2

   $\therefore$ tan(A +B) = $\frac{\tan A+\tan B}{1-\tan A\tan B} = \frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\times\frac{1}{2}} = \frac{5/6}{5/6}$ = 1

   $\therefore$ tan(A + B) = 1

   Also, A and B are acute angles, therefore both A and B are less than 90°. So A + B must be less than 180°.

   Therefore, the only possible case for which tan (A+B) = 1 will be when (A + B) equals 45°.

   Thus, A + B = 45°

10. Let us first draw a right $\triangle$ABC

    Now, we have given that, tan $A = \frac{BC}{AB} = \frac{4}{3}$

    Therefore, if BC = 4k, then AB = 3k, where k is any positive integer.

    Now, by using the Pythagoras Theorem, we have

    $AC^2 = AB^2 + BC^2 = (4k)^2 + (3k)^2 = 25k$

    So, AC = 5k

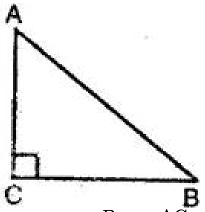
    Now, we can write all the trigonometric ratios using their definitions

    $\sin A = \frac{BC}{AC} = \frac{4k}{5k} = \frac{4}{5}$

    $\cos A = \frac{AB}{AC} = \frac{3k}{5k} = \frac{3}{5}$

    Therefore, cot A = $\frac{1}{\tan A} = \frac{3}{4}, cosecA = \frac{1}{\sin A} = \frac{5}{4}$ and sec $A = \frac{1}{\cos A} = \frac{5}{3}$

11. In right triangle ABC,



$\cos A = \frac{B}{H} = \frac{\text{AC}}{\text{AB}}$ and $\cos B = \frac{B}{H} = \frac{BC}{AB}$

Given, $cosA = cosB$

$\Rightarrow \frac{\text{AC}}{\text{AB}} = \frac{\text{BC}}{\text{AB}}$

$\Rightarrow AC = BC$

$\Rightarrow \angle A = \angle B$ [Angles opposite to equal sides are equal]

12. Here $\sqrt{3}sin\theta - cos\theta$ = 0 and 0°< $\theta$ < 90°,

or, $\sqrt{3}sin\theta = cos\theta$

or, $\frac{\sin\theta}{\cos\theta} = \frac{1}{\sqrt{3}}$

or, $\tan\theta = \frac{1}{\sqrt{3}}$ $\left[\because \tan\theta = \frac{\sin\theta}{\cos\theta}\right]$

$tan\theta = tan30^\circ$

$\therefore \quad \theta = 30^\circ$

13. We have, $\cot\theta = \frac{9}{40}$

$\therefore \quad \cos ec\theta = \sqrt{1+\cot^2\theta} \Rightarrow \cos ec\theta = \sqrt{1+\left(\frac{9}{40}\right)^2} = \sqrt{1+\frac{81}{1600}} = \sqrt{\frac{1681}{1600}} = \frac{41}{40}$

Again, $\cot\theta = \frac{9}{40} \Rightarrow \tan\theta = \frac{1}{\cot\theta} = \frac{40}{9}$

$\therefore \quad \sec\theta = \sqrt{1+\tan^2\theta} \Rightarrow \sec\theta = \sqrt{1+\left(\frac{40}{9}\right)^2} = \sqrt{\frac{1681}{81}} = \frac{41}{9}$

14. We have,

$\sin\theta = \frac{3}{5}$

$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta} \Rightarrow \cos\theta = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

$\therefore \quad cosec\ \theta = \frac{1}{\sin\theta} = \frac{5}{3}, \sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3/5}{4/5} = \frac{3}{4}$ and $\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$

15. Given:

$\cos\theta = \frac{3}{4}$

$\Rightarrow \frac{1}{\cos\theta} = \frac{4}{3}$

$\Rightarrow \sec\theta = \frac{4}{3}$

We know that

$\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \left(\frac{4}{3}\right)^2 - \tan^2\theta = 1$

$\Rightarrow \tan^2\theta = \frac{16}{9} - 1$

$\Rightarrow \tan^2\theta = \frac{7}{9}$

Therefore,

$9\tan^2\theta + 9 = 9 \times \frac{7}{9} + 9$

= 7 + 9

= 16

16. In $\triangle ABC$, $\angle C$ = $90^\circ$ (Angle in a semi-circle)

$tanA$= $\frac{P}{B}$ = $\frac{BC}{AC} = \frac{2}{3}$

and $tanB = \frac{P}{B}$ = $\frac{AC}{BC} = \frac{3}{2}$

$\therefore \tan A.\tan B = \frac{2}{3} \times \frac{3}{2} = 1$

17. $\frac{\tan A}{1+\tan^2 A} = \frac{\sqrt{2}-1}{1+(\sqrt{2}-1)^2}$

$= \frac{\sqrt{2}-1}{1+2+1-2\sqrt{2}} = \frac{\sqrt{2}-1}{4-2\sqrt{2}} = \frac{\sqrt{2}-1}{2\sqrt{2}(\sqrt{2}-1)} = \frac{1}{2\sqrt{2}}$

$= \frac{1}{2\sqrt{2}}\frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{4}$

18. Given: $\cot\theta = \frac{7}{8}$

To Evaluate: $\frac{(1+\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\sin^2\theta}{1-\cos^2\theta} = \frac{\cos^2\theta}{\sin^2\theta}$

= $\cot^2\theta$

$= \left(\frac{7}{8}\right)^2$

$= \frac{49}{64}$

19. Given, $5cosec\theta = 7$

or, $cosec\theta$= $\frac{7}{5}$

or, $\sin\theta = \frac{5}{7}$ $[\because \text{cosec}\theta = \frac{1}{\sin\theta}]$

$\sin\theta + \cos^2\theta - 1 = \sin\theta - \left(1 - \cos^2\theta\right)$

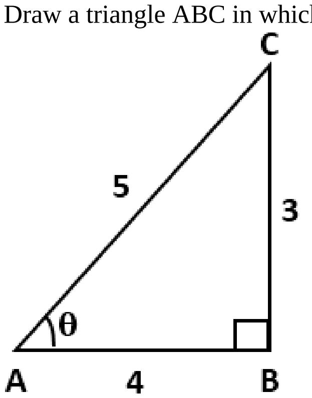
$= \sin\theta - \sin^2\theta\left(\because \sin^2\theta + \cos^2\theta = 1\right)$

=$\frac{5}{7} - \left(\frac{5}{7}\right)^2$

$= \frac{5}{7} - \frac{25}{49}$

$= \frac{35-25}{49} = \frac{10}{49}$

20. Draw a triangle ABC in which ∠B = 90°.




Let ∠A = $\theta$°.

$3\cot\theta = 4 \Rightarrow \cot\theta = \frac{4}{3} \Rightarrow \tan\theta = \frac{3}{4}$

Then, $\cot\theta = \frac{\text{Base}}{\text{Perpendicular}} = \frac{AB}{BC} = \frac{4}{3}$

Let AB = 4 and BC = 3,

By Pythagoras' theorem, we have

$AC^2 = AB^2 + BC^2$

$= 4^2 + 3^2$

$= 16 + 9$

$= 25$

$\Rightarrow \quad AC = 5$

Now,

$\sin\theta = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{3}{5}$

$\cos\theta = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{4}{5}$

LHS $= \frac{(1-\tan^2\theta)}{(1+\tan^2\theta)}$

$= \frac{1-(3/4)^2}{1+(3/4)^2} = \frac{1-9/16}{1+9/16} = \frac{7/16}{25/16} = \frac{7}{25}$

RHS = $\cos^2\theta - \sin^2\theta$

$= \left(\frac{4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 = \frac{16}{25} - \frac{9}{25} = \frac{7}{25}$

Hence, $\frac{(1-\tan^2\theta)}{(1+\tan^2\theta)} = \left(\cos^2\theta - \sin^2\theta\right)$

21. In right triangle ABC,

$\sin 30^\circ = \frac{\text{BC}}{AB} \Rightarrow \frac{1}{2} = \frac{\text{BC}}{12}$

$\Rightarrow BC = \frac{12}{2} = 6\text{cm}$

Again, $\cos 30^\circ = \frac{AC}{AB}$

$\Rightarrow \frac{\sqrt{3}}{2} = \frac{AC}{12} \Rightarrow AC = 6\sqrt{3}\text{cm}$

22. LHS = $\frac{1+\tan A}{2\sin A} + \frac{1+\cot A}{2\cos A}$

= $\frac{\cos A+\sin A}{2\sin A\cos A} + \frac{\sin A+\cos A}{2\cos A\sin A}$

= $\frac{2(\cos A+\sin A)}{2\sin A\cos A}$

= cosec A + sec A

= RHS

23. Given, m cot A = n

$\Rightarrow cotA = \frac{n}{m}$

$\frac{m\sin A-n\cos A}{n\cos A+m\sin A}$ = $\frac{m-n\frac{cosA}{sinA}}{n\frac{cosA}{sinA}+m}$ [dividing numerator and denominator by sin A]

$cotA = \frac{cosA}{sinA}$

$\frac{m-ncotA}{ncotA+m}$=$\frac{m-n\times\frac{n}{m}}{n\times\frac{n}{m}+m}$

$= \frac{\frac{m^2-n^2}{n}}{\frac{n^2+m^2}{n}} = \frac{m^2-n^2}{m^2+n^2}$

24. Given,

3 cot A = 4

$\Rightarrow$ cot A = $\frac{4}{3}$

We know that,

$cosec^2A - cot^2A = 1$

$cosec^2A - \left(\frac{4}{3}\right)^2 = 1$

$cosec^2A = 1 + \frac{16}{9} = \frac{9+16}{9} = \frac{25}{9}$

Thus,

$\frac{cosec^2A+1}{cosec^2A-1} = \frac{25/9+1}{25/9-1} = \frac{34}{16}$

25. We have, $\sin\theta = \frac{a}{\sqrt{a^2+b^2}}$

$\therefore \quad \cos\theta = \sqrt{1-\sin^2\theta}$

$\Rightarrow \cos\theta = \sqrt{1-\frac{a^2}{a^2+b^2}}$

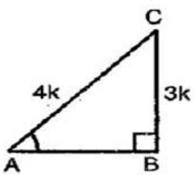
$= \sqrt{\frac{a^2+b^2-a^2}{a^2+b^2}} = \sqrt{\frac{b^2}{a^2+b^2}} = \frac{\sqrt{b^2}}{\sqrt{a^2+b^2}} = \frac{b}{\sqrt{a^2+b^2}}$

and, we know that

$\tan\theta = \frac{\sin\theta}{\cos\theta} \Rightarrow \tan\theta = \frac{a/\sqrt{a^2+b^2}}{b/\sqrt{a^2+b^2}} = \frac{a}{b}$

$\therefore \tan\theta = \frac{a}{b}$

26. Given: A triangle ABC in which ∠ B = 90°




Let BC = 3k and AC = 4k

Then, Using Pythagoras theorem,

AB = $\sqrt{(AC)^2-(BC)^2}$ = $\sqrt{(4k)^2-(3k)^2}$

=$\sqrt{16k^2-9k^2}$

=$k\sqrt{7}$

cos A = $\frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$

tan A = $\frac{Sin\ A}{Cos\ A} = \frac{BC}{AB} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

27. We have

$5\tan\alpha = 4 \Rightarrow \tan\alpha = \frac{4}{5}$

Now, $\frac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha} = \frac{\frac{5\sin\alpha - 3\cos\alpha}{\cos\alpha}}{\frac{5\sin\alpha + 2\cos\alpha}{\cos\alpha}}$ [ Dividing Numerator and Denominator by cos $\alpha$]

$= \frac{\frac{5\sin\alpha}{\cos\alpha} - \frac{3\cos\alpha}{\cos\alpha}}{\frac{5\sin\alpha}{\cos\alpha} + \frac{2\cos\alpha}{\cos\alpha}} = \frac{5\tan\alpha - 3}{5\tan\alpha + 2} = \frac{5 \times \frac{4}{5} - 3}{5 \times \frac{4}{5} + 2}$ $\left[\because \tan\alpha = \frac{4}{5}\right]$

$= \frac{4-3}{4+2} = \frac{1}{6}$

Hence,

$\frac{5\sin\alpha - 3\cos\alpha}{5\sin\alpha + 2\cos\alpha} = \frac{1}{6}$

28. $secA$ = $\frac{1}{\cos A} = \frac{1}{\sqrt{1 - \sin^2 A}}$ $(cos^2A = 1 - sin^2A)$

$tanA$= $\frac{\sin A}{\cos A} = \frac{\sin A}{\sqrt{1 - \sin^2 A}}$

29. $sin\phi$= $\frac{1}{2}$

$sin\phi = sin30^\circ$

or, $\phi$ = $30^\circ$

Substituting this value of $\phi$ in LHS

LHS = $3cos30^\circ - 4cos^3 30^\circ$

$= 3\left(\frac{\sqrt{3}}{2}\right) - 4\left(\frac{\sqrt{3}}{2}\right)^3$

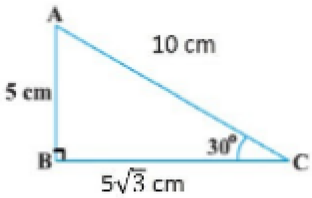
$= \frac{3\sqrt{3}}{2} - 4 \times \frac{3\sqrt{3}}{8}$

$= \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2}$

= 0 = RHS

30. Given AB = 5 cm

$\angle$ACB = $30^o$




According to diagram,

tan C = $\frac{side\ opposite\ to\ angle\ C}{side\ adjacent\ to\ angle\ C}$

tan $30^o$ = $\frac{AB}{BC}$

$\frac{1}{\sqrt{3}}$ = $\frac{5}{BC}$

BC = 5$\sqrt{3}$ cm

sin C = $\frac{side\ of\ angle\ C}{hypotenuse}$

sin $30^o$ = $\frac{AB}{AC}$

$\frac{1}{2}$ = $\frac{5}{AC}$

AC = 10 cm.

31. Given,a $\triangle PQR$ in which $\angle Q = 90^o$ , QR = 3 cm and PR - PQ = 1 ...(i)

On applying Pythagoras theorem In $\triangle PQR$,

$H^2 = P^2 + B^2$ (By Pythagoras theorem)

$PR^2 = PQ^2 + QR^2$

$\Rightarrow QR^2 = PR^2 - PQ^2$

$\Rightarrow (3)^2 = PR^2 - PQ^2$

$\Rightarrow PR^2 - PQ^2 = 9$

$\Rightarrow (PR + PQ)(PR - PQ) = 9$ [$\because a^2 - b^2 = (a+b)(a-b)$ ]

$\Rightarrow (PR + PQ)(1) = 9$ [From Eq(i)]

$\Rightarrow PR + PQ = 9$ ...(ii)

On adding Eq(i) and Eq(ii), we get

PR - PQ + PR + PQ = 1 + 9 $\Rightarrow 2PR = 10 \Rightarrow PR = 5\ cm$

On substituting PR = 5 cm in Eq(i), we get

$5 - PQ = 1 \Rightarrow PQ = 4\ cm$

$\therefore PR = 5\ and\ PQ = 4$

So the perpendicular is PQ = 4 cm

Base is QR = 3cm and hypotenuse is PR = 5cm

Now, $\sin R = \frac{P}{H} = \frac{PQ}{PR} = \frac{4}{5}$ ,

$\cos R = \frac{B}{H} = \frac{QR}{PR} = \frac{3}{5}$ and $\tan R = \frac{P}{B} = \frac{PQ}{QR} = \frac{4}{3}$

32. We have, $\cos\theta = \frac{3}{5}$

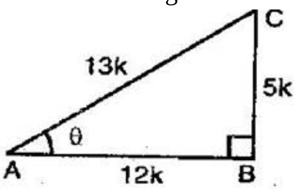
we know that,

$\therefore \quad \sin\theta = \sqrt{1-\cos^2\theta} \Rightarrow \sin\theta = \sqrt{1-\left(\frac{3}{5}\right)^2} = \sqrt{1-\frac{9}{25}} = \sqrt{\frac{25-9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$

and, $\cot\ \theta = \frac{\cos\theta}{\sin\theta} \Rightarrow \cot\ \theta = \frac{3/5}{4/5} = \frac{3}{4}, \quad \cos ec\ \theta = \frac{1}{\sin\theta} \Rightarrow \cos ec\ \theta = \frac{5}{4}$

$\therefore \quad \cot\theta + \cos ec\theta = \frac{3}{4} + \frac{5}{4} = \frac{8}{4} = 2$

33. Consider a triangle ABC in which $\angle A = \theta$ and $\angle B = 90^o$




Let AB = 12k and AC = 13k

Then, using Pythagoras theorem,

$BC = \sqrt{(\text{AC})^2 - (\text{AB})^2} = \sqrt{(13k)^2 - (12k)^2}$

$= \sqrt{169k^2 - 144k^2} = \sqrt{25k^2} = 5k$

$\therefore \sin\theta = \frac{BC}{AC} = \frac{5k}{13k} = \frac{5}{13}$

$\cos\theta = \frac{AB}{AC} = \frac{12k}{13k} = \frac{12}{13}$

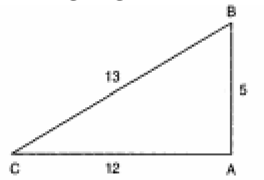
$\tan\theta = \frac{BC}{AB} = \frac{5k}{12k} = \frac{5}{12}$

$\cot\theta = \frac{AB}{BC} = \frac{12k}{5k} = \frac{12}{5}$

$\cos ec\theta = \frac{AC}{BC} = \frac{13k}{5k} = \frac{13}{5}$

34. A + 2B = $60^o$ and A + 4B = $90^o$

Solving to get B = $15^o$ and A = $30^o$

35. 


With reference to $\angle B$, we have given that

In $\triangle ABC$

$Base = AB = 5$ , $Perpendicular = AC = 12$ and, $Hypotenuse = BC = 13$.

$\therefore \quad \sin B = \frac{AC}{BC} = \frac{12}{13}$

and, $\tan B = \frac{AC}{AB} = \frac{12}{5}$

With reference to $\angle C$, we have

$Base = AC = 12$ , $Perpendicular = AB = 5$ and,

$Hypotenuse = BC = 13$

$\therefore \quad \cos C = \frac{AC}{BC} = \frac{12}{13}$

36. In right triangle ABC,

$\cos A = \frac{AC}{AB}\ and \cos B = \frac{BC}{AB}$

But cos A = cos B [Given]

$\Rightarrow \frac{AC}{AB} = \frac{BC}{AB} \Rightarrow_{AC} = BC$

$\Rightarrow \angle A = \angle B$

[Angles opposite to equal sides are equal]

37. We have, $\tan\theta = \frac{3}{4}$

$\therefore \quad \sec\theta = \sqrt{1+\tan^2\theta} \Rightarrow \sec\theta = \sqrt{1+\left(\frac{3}{4}\right)^2} = \sqrt{1+\frac{9}{16}} = \frac{5}{4}$

and, $\cos\theta = \frac{1}{\sec\theta} \Rightarrow \cos\theta = \frac{4}{5}$

$\therefore \quad \frac{1-\cos\theta}{1+\cos\theta} = \frac{1-\frac{4}{5}}{1+\frac{4}{5}} = \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{9}$

38. According to the question,

$\sin\theta = \frac{1}{\sqrt{2}}$

$\because \cos\theta = \sqrt{1-\sin^2\theta}$

$\Rightarrow \cos\theta = \sqrt{1-\left(\frac{1}{\sqrt{2}}\right)^2}$

$\Rightarrow \cos\theta = \sqrt{1-\frac{1}{2}}$

$\Rightarrow \cos\theta = \frac{1}{\sqrt{2}}$

$\therefore \sec\theta = \frac{1}{\cos\theta} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\cos ec\theta = \frac{1}{\sin\theta} = \frac{\sqrt{2}}{1} = \sqrt{2}$

$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}}} = 1$

$\cot\theta = \frac{1}{\tan\theta} = \frac{1}{1} = 1$

39. We have

tanA = 1

$\Rightarrow \frac{\sin A}{\cos A} = 1$

$\Rightarrow$ sin A = cos A

$\Rightarrow$ sin A - cos A = 0

Squaring both sides, we get

$(\sin A - \cos A)^2 = 0$

$\Rightarrow \sin^2 A + \cos^2 A - 2\sin A \cdot \cos A = 0$

$\Rightarrow$ 1 - 2sinA $\cdot$ cosA = 0

$\therefore$ 2sinA$\cdot$cosA = 1

40. Here,

$3\cot\theta = 4 \Rightarrow \cot\theta = \frac{4}{3}$

Now, $\frac{4\cos\theta-\sin\theta}{2\cos\theta+\sin\theta} = \frac{4\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\sin\theta}}{2\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\sin\theta}}$ ..(Dividing throughout by $\sin\theta$)

$= \frac{4\cot\theta-1}{2\cot\theta+1}$

$= \frac{4\times\frac{4}{3}-1}{2\times\frac{4}{3}+1} = \frac{\frac{16}{3}-1}{\frac{8}{3}+1}$

$= \frac{\frac{13}{3}}{\frac{11}{3}} = \frac{13}{3} \times \frac{3}{11}$

$= \frac{13}{11}$

41. It is given that $\cot\theta = \frac{2}{3}$ .

LHS $= \frac{4\sin\theta-3\cos\theta}{2\sin\theta+6\cos\theta}$

Dividing the above expressions by sin $\theta$, we get:

$\frac{4-3\cot\theta}{2+6\cot\theta} \quad \left[\because \cot\theta = \frac{\cos\theta}{\sin\theta}\right]$

Now, substituting the values of cot θ in the above expression, we get:

$\frac{4-3\left(\frac{2}{3}\right)}{2+6\left(\frac{2}{3}\right)}$

$= \frac{4-2}{2+4} = \frac{2}{6} = \frac{1}{3}$

i.e. LHS = RHS

Hence proved.

42. Given $\cot\theta = \frac{15}{8}$

To evaluate: $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

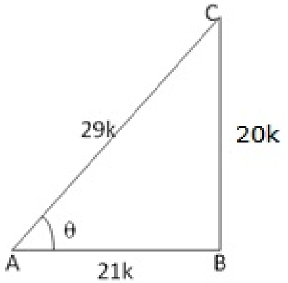
$= \frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$

$= \frac{(1-\sin^2\theta)}{(1-\cos^2\theta)} = \frac{\cos^2\theta}{\sin^2\theta} = \cot^2\theta$

$= (\cot\theta)^2 = \left(\frac{15}{8}\right)^2 = \frac{225}{64}$

Hence, the value of the given expression is $\frac{225}{64}$.

43. Let us draw a triangle ABC in which ∠B = 90°.




Let ∠A = $\theta$°.

Then, $\tan\theta = \frac{20}{21} = \frac{20k}{21k}$

Using Pythagoras' theorem,

$AC^2 = AB^2 + BC^2$

$= (21k)^2 + (20k)^2$

$= 441k^2 + 400k^2$

$= 841k^2$

$\Rightarrow \quad AC = 29k$

$\sin\theta = \frac{BC}{AC} = \frac{20k}{29k} = \frac{20}{29}$

$\cos\theta = \frac{AB}{AC} = \frac{21k}{29k} = \frac{21}{29}$

LHS = $\frac{1-\sin\theta+\cos\theta}{1+\sin\theta+\cos\theta}$

= $\frac{1-\frac{20}{29}+\frac{21}{29}}{1+\frac{20}{29}+\frac{21}{29}} = \frac{\frac{29-20+21}{29}}{\frac{29+20+21}{29}}$

= $\frac{30}{70} = \frac{3}{7}$ = RHS

44. $2cot^2 A - 1$= $2(cosec^2 A - 1) - 1\left(\because \cot^2\theta = -1 + \text{cosec}^2\theta\right)$

$= 2cosec^2 A - 2 - 1$

$= \frac{2}{\sin^2 A} - 3$ $(\because cosec\theta = \frac{1}{sin\theta})$

$= \frac{2}{\left(\frac{\sqrt{3}}{2}\right)^2} - 3$

$2cot^2 A - 1$= $\frac{8}{3} - 3$= $\frac{8-9}{3} = \frac{-1}{3}$

45. Given :$4cos\theta = 11sin\theta$

or, $cos\theta$ = $\frac{11}{4}\sin\theta$

Now, $\frac{11\cos\theta-7\sin\theta}{11\cos\theta+7\sin\theta}$= $\frac{11\times\frac{11}{4}\sin\theta-7\sin\theta}{11\times\frac{11}{4}\sin\theta+7\sin\theta}$

= $\frac{\sin\theta\left(\frac{121}{4}-7\right)}{\sin\theta\left(\frac{121}{4}+7\right)}$

= $\frac{121-28}{121+28} = \frac{93}{149}$

46. We have

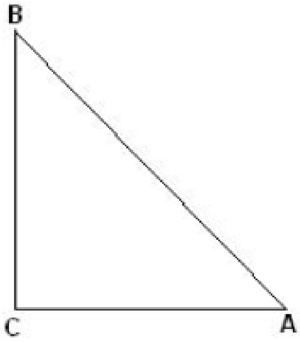
$\because\ \sin\theta = \sqrt{1-\cos^2\theta} \Rightarrow \sin\theta = \sqrt{1-\left(\frac{4}{5}\right)^2}$

$\Rightarrow \sin\theta = \sqrt{1-\frac{16}{25}}$

$\Rightarrow \sin\theta = \sqrt{\frac{9}{25}}$

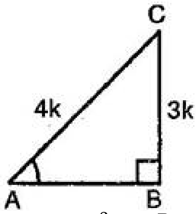
$\sin\theta = \frac{3}{5}$

$\because \cos ec\theta = \frac{1}{\sin\theta} = \frac{5}{3}$
$\sec\theta = \frac{1}{\cos\theta} = \frac{5}{4}$
$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{3}{4}$
$\cot\theta = \frac{1}{\tan\theta} = \frac{4}{3}$

47. 


Given cos A = cos B
Hence, $\frac{AC}{AB} = \frac{BC}{AB}$
$\Rightarrow AC = BC$
Since angle opposite to equal sides in a $\Delta$ are equal
$\therefore \angle B = \angle A$
Hence proved

48. Given: A triangle ABC in which $\angle B = 90^\circ$




$SinA = \frac{3}{4} = \frac{P}{H}$
Let BC = 3k and AC = 4k where k is a positive integer.|
Using Pythagoras theorem,
$AB^2 = AC^2 - BC^2$
$AB = \sqrt{(AC)^2 - (BC)^2} = \sqrt{(4k)^2 - (3k)^2}$
$= \sqrt{16k^2 - 9k^2} = \sqrt{7k^2} = k\sqrt{7}$
Therefore, $\cos A = \frac{B}{H} = \frac{AB}{AC} = \frac{k\sqrt{7}}{4k} = \frac{\sqrt{7}}{4}$
$\tan \mathrm{A} = \frac{\mathrm{P}}{\mathrm{B}} = \frac{\mathrm{BC}}{\mathrm{AB}} = \frac{3k}{k\sqrt{7}} = \frac{3}{\sqrt{7}}$

49. Given, $\sqrt{2}$sin$\theta$ = 1
sin $\theta$ = $\frac{1}{\sqrt{2}}$
sin 45°=$\frac{1}{\sqrt{2}}$
$\therefore sin\theta$ = sin45°
$\theta = 45^\circ$
put $\theta = 45^\circ in$
sec$^2\theta$ - cosec$^2$ $\theta$
= sec$^2$45° - cosec$^2$45°
= $(\sqrt{2})^2 - (\sqrt{2})^2$
=$2 - 2$
= 0

50. $3tan\theta = 4 \Rightarrow \tan\theta = \frac{4}{3}$
Given,
$= \frac{3\sin\theta + 2\cos\theta}{3\sin\theta - 2\cos\theta}$
$= \frac{3\tan\theta + 2}{3\tan\theta - 2}$ [Dividing numerator and denominator by cos $\theta$]
$= \frac{\left(3\times\frac{4}{3}+2\right)}{\left(3\times\frac{4}{3}-2\right)} = \frac{6}{2} = 3$

51. Given: $\cot\theta = \frac{15}{8}$,

To Evaluate: $\frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)}$

$= \frac{2(1+\sin\theta)(1-\sin\theta)}{2(1+\cos\theta)(1-\cos\theta)}$

$= \frac{2\left(1-\sin^2\theta\right)}{2\left(1-\cos^2\theta\right)} = \frac{\cos^2\theta}{\sin^2\theta} = \left(\frac{\cos\theta}{\sin\theta}\right)^2 = (\cot\theta)^2$

$\therefore \quad \cot\theta = \frac{15}{8} \Rightarrow \frac{(2+2\sin\theta)(1-\sin\theta)}{(1+\cos\theta)(2-2\cos\theta)} = (\cot\theta)^2 = \left(\frac{15}{8}\right)^2 = \frac{225}{64}$

52. Here, $\tan\theta = \frac{4}{5}$

Let $I = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}$

Dividing Numerator and denominator by $\cos\theta$

$I = \frac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta} = \frac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}$

$= \frac{1-\tan\theta}{1+\tan\theta} = \frac{1-\frac{4}{5}}{1+\frac{4}{5}}$

$= \frac{\frac{1}{5}}{\frac{9}{5}} = \frac{1}{5} \times \frac{5}{9} = \frac{1}{9}$