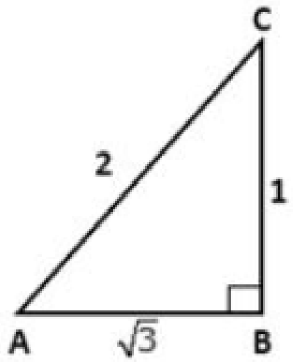
Solution

INTRODUCTION TO TRIGONOMETRY WS 5

Class 10 - Mathematics

Section A

1. Given condition of question is
   $\sin(2\theta + 45^\circ) = \cos(30^\circ - \theta)$
   $\Rightarrow \sin(2\theta + 45^\circ) = \cos[90^\circ - 60^\circ - \theta]$
   $\Rightarrow \sin(2\theta + 45^\circ) = \cos[90^\circ - (60^\circ - \theta)]$
   $\Rightarrow \sin(2\theta + 45^\circ) = \sin(60^\circ + \theta)$ $[\cos(90^\circ - \theta) = \sin\theta]$
   $\Rightarrow 2\theta + 45^\circ = 60^\circ + \theta$
   $\Rightarrow 2\theta - \theta = 60^\circ - 45^\circ$
   $\Rightarrow \theta = 15^\circ$
2. We have to find the value of,
   $cosec^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$ ...(1)
   Now,
   $cosec 30^\circ = 2,\ cos 60^\circ = \frac{1}{2},\ \sec 45^\circ = \sqrt{2},\ \tan 45^\circ = 1,\ \sin 90^\circ = 1,\ \cot 30^\circ = \sqrt{3}$
   So by substituting above values in equation (1)
   We get,
   $cosec^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ$
   $= (2)^3 \times \left(\frac{1}{2}\right) \times (1)^3 \times (1)^2 \times (\sqrt{2})^2 \times (\sqrt{3})$
   $= 8 \times \left(\frac{1}{2}\right) \times 1 \times 1 \times 2 \times (\sqrt{3})$
   $= \frac{8}{2} \times 2 \times (\sqrt{3})$
   Now, 2 gets cancelled and we get,
   $cosec^3 30^\circ \cos 60^\circ \tan^3 45^\circ \sin^2 90^\circ \sec^2 45^\circ \cot 30^\circ = 8\sqrt{3}$
3. Let us draw a triangle ABC in which ∠B = 90°.

   Let ∠A = θ°.
   $\tan A = \frac{1}{\sqrt{3}}$
   Then, $\tan A = \frac{\text{Perpendicular}}{\text{Base}} = \frac{BC}{AB} = \frac{1}{\sqrt{3}}$
   Let BC = 1 and AB = $\sqrt{3}$,
   By Pythagoras' theorem, we have
   $AC^2 = AB^2 + BC^2$
   $=(\sqrt{3})^2 + 1^2$ = 3 + 1 = 4
   $\Rightarrow \quad AC = 2$
   Now,
   $\sin A = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{BC}{AC} = \frac{1}{2}$ and $\cos A = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$
   $\sin C = \frac{\text{Perpendicular}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{\sqrt{3}}{2}$ and $\cos C = \frac{\text{Base}}{\text{Hypotenuse}} = \frac{AB}{AC} = \frac{1}{2}$
   i. LHS = sin A cos C + cos A sin C
      $= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$
      $= \frac{1}{4} + \frac{3}{4}$

$= \frac{4}{4}$
= 1
= R.H.S

ii. cos A cos C - sin A sin C
= $\frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$
$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4}$
= 0
= R.H.S

4. We have $\frac{\sin 30^\circ + \tan 45^\circ - cosec 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$
after putting values,we get
$= \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$
$= \frac{\frac{3}{2} - \frac{2}{\sqrt{3}}}{\frac{3}{2} + \frac{2}{\sqrt{3}}}$
$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4}$ Rationalise it, we get
$= \frac{3\sqrt{3}-4}{3\sqrt{3}+4} \times \frac{3\sqrt{3}-4}{3\sqrt{3}-4}$
$= \frac{(3\sqrt{3}-4)^2}{(3\sqrt{3})^2 - (4)^2}$
$= \frac{27+16-24\sqrt{3}}{27-16}$
$= \frac{43-24\sqrt{3}}{11}$

5. We have to find the value of:
$4\left(\sin^4 30^\circ + \cos^2 60^\circ\right) - 3\left(\cos^2 45^\circ - \sin^2 90^\circ\right) - \sin^2 60^\circ$
Now,
$= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^2\right] - 3\left[\left(\frac{1}{\sqrt{2}}\right)^2 - (1)^2\right] - \left(\frac{\sqrt{3}}{2}\right)^2$
$= 4\left[\frac{1}{16} + \frac{1}{4}\right] - 3\left[\frac{1}{2} - 1\right] - \frac{3}{4}$
$= 4\left(\frac{1+4}{16}\right) - 3\left(\frac{-1}{2}\right) - \frac{3}{4} = \frac{5}{4} + \frac{3}{2} - \frac{3}{4}$
$= \frac{5+6-3}{4} = \frac{8}{4} = 2$

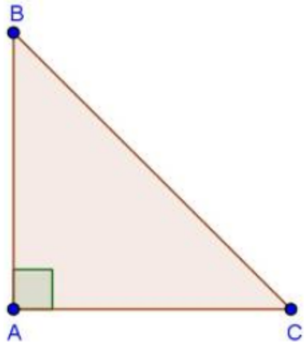
6. We have
tan A + B = 1 = tan 45
tan A - B = $\frac{1}{\sqrt{3}}$ = tan 30
From the above we get,
A + B = 45^o ...(1)
A - B = 30^o ..(2)
Adding (1) And (2)
2A = 75
$\Rightarrow$ A = 37.5
Subtract (1) By (2)
$\Rightarrow$ 2B = 15
$\Rightarrow$ B = 7.5

7. We have to find the value of the following expression
$\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45$ …… (1)
Now,
$\tan 30^\circ = \frac{1}{\sqrt{3}},\ \tan 60^\circ = \sqrt{3},\ \tan 45^\circ = 1$
So, by substituting above values in equation (1)
We get,
$\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45$
$= \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + (1)^2$

$= \frac{1^2}{(\sqrt{3})^2} + (\sqrt{3})^2 + 1$
$= \frac{1}{3} + 3 + 1$
$= \frac{1}{3} + 4$
$= \frac{1}{3} + \frac{4\times3}{1\times3}$
$= \frac{1}{3} + \frac{12}{3}$
$= \frac{1+12}{3}$
$= \frac{13}{3}$
Therefore,
$\tan^2 30^\circ + \tan^2 60^\circ + \tan^2 45^\circ = \frac{13}{3}$

8. In $\triangle ABC$,
$\tan A = 1$
$\Rightarrow \quad \frac{BC}{AC} = 1$
$\Rightarrow$ BC = x and AC = x
Using Pythagoras theorem,
$\Rightarrow AB^2 = AC^2 + BC^2$
$\Rightarrow \quad AB^2 = x^2 + x^2$
$\Rightarrow \quad AB = \sqrt{2}x$
$\therefore \quad \sin A = \frac{BC}{AB} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$ and $\cos A = \frac{AC}{\sqrt{2}x} = \frac{x}{\sqrt{2}x} = \frac{1}{\sqrt{2}}$
2 sin A cos A $= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} = 1$

9. We have to find the value following expression
$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90 \ \ldots(1)$
Now,
$\cos 45^\circ = \frac{1}{\sqrt{2}},\ \cos 30^\circ = \frac{\sqrt{3}}{2},\ \cos 60^\circ = \frac{1}{2},\ and\ \cos 90^\circ = 0$
So, by substituting above values in equation (1)
We get,
$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ$
$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 + (0)^2$
$= \frac{(\sqrt{3})^2}{2^2} + \frac{1^2}{(\sqrt{2})^2} + \frac{1^2}{2^2} + 0$
$= \frac{3}{4} + \frac{1}{2} + \frac{1}{4}$
$= \frac{3}{4} + \frac{1}{4} + \frac{1}{2}$
$= \frac{3+1}{4} + \frac{1}{2}$
$= \frac{4}{4} + \frac{1}{2}$
$= 1 + \frac{1}{2}$
$= \frac{1\times2}{1\times2} + \frac{1}{2}$
$= \frac{2}{2} + \frac{1}{2}$
$= \frac{2+1}{2}$
$= \frac{3}{2}$
Therefore,
$\cos^2 30^\circ + \cos^2 45^\circ + \cos^2 60^\circ + \cos^2 90^\circ = \frac{3}{2}$

10. 

Given $\tan C = \sqrt{3}$

$\Rightarrow$ tanC = tan60°

$\therefore$ C =60°.......(1)

In ΔABC , ∠ A = 90°, ∠C = 60°

We know that,

$\angle A+\angle B+\angle C=180^{\circ}$ (angle sum property of triangle)

$\Rightarrow$ 90° + $\angle B$ + 60° = 180°

$\therefore$ $\angle B$ = 180° - (90° + 60°) = 30°

Now,

sinB cosC + cosB sinC

= sin30° cos60° + cos30°sin60°

$=\frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}$

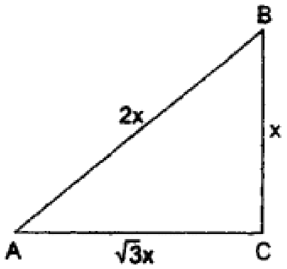
$=\frac{1}{4}+\frac{3}{4}$

$=\frac{4}{4}=1$

11. Let us draw a triangle ABC in which ∠C = 90° and tan A= $\frac{1}{\sqrt{3}}$.

Then tan A= $\frac{1}{\sqrt{3}}\Rightarrow\frac{BC}{AC}=\frac{1}{\sqrt{3}}$

Let BC = x. Then, AC = $\sqrt{3}$x



By Pythagoras' theorem, we have

$AB^2 = AC^2 + BC^2$

$=(\sqrt{3}x)^2+x^2=\left(3x^2+x^2\right)=4x^2$

$\Rightarrow\quad AB=\sqrt{4x^2}=2x$

For Trigonometric - ratios of ∠A, we have,

base = AC = $\sqrt{3}x$, perpendicular = BC = x and hypotenuse = AB = 2x

$\therefore\quad\sin A=\frac{BC}{AB}=\frac{x}{2x}=\frac{1}{2}$ and $\cos A=\frac{AC}{AB}=\frac{\sqrt{3}x}{2x}=\frac{\sqrt{3}}{2}$

For Trigonometric - ratios of ∠B, we have,

base = BC = x, perpendicular = AC = $\sqrt{3}$x and hypotenuse = AB = 2x

$\therefore\quad\sin B=\frac{AC}{AB}=\frac{\sqrt{3}x}{2x}=\frac{\sqrt{3}}{2}$ and $\cos B=\frac{BC}{AB}=\frac{x}{2x}=\frac{1}{2}$

i. (sin A cosB + cos A sin B)

$=\left(\frac{1}{2}\times\frac{1}{2}+\frac{\sqrt{3}}{2}\times\frac{\sqrt{3}}{2}\right)$

$=\left(\frac{1}{4}+\frac{3}{4}\right)=1$

ii. (cos A cos B - sin A sin B)

$=\left(\frac{\sqrt{3}}{2}\times\frac{1}{2}-\frac{1}{2}\times\frac{\sqrt{3}}{2}\right)$

= 0

12. We have to find the value of the following expression

$2\sin^2 30^{\circ}-3\cos^2 45^{\circ}+\tan^2 60\ \ldots(1)$

Now,

$\sin 30^{\circ}=\frac{1}{2},\ \cos 45^{\circ}=\frac{1}{\sqrt{2}},\ \tan 60^{\circ}=\sqrt{3}$

So, by substituting above values in equation (1)

We get,

$2\sin^2 30^{\circ}-3\cos^2 45+\tan^2 60^{\circ}$

$=2\times\left(\frac{1}{2}\right)^2-3\times\left(\frac{1}{(\sqrt{2})}\right)^2+(\sqrt{3})^2$

$=2\times\frac{1^2}{2^2}-3\times\frac{1^2}{(\sqrt{2})^2}+3$

$= \frac{2}{4} - \frac{3}{2} + 3$
$= \frac{1-3}{2} + 3$
$= \frac{-2}{2} + 3$
= -1 + 3
= 2
Therefore,
$2\sin^2 30^\circ - 3\cos^2 45^\circ + \tan^2 60^\circ = 2$

13. We have, sin (A+B) =1
$\Rightarrow$ sin (A+B)$= \sin 90^\circ$
$\Rightarrow$ A+B = 90°...(i)
and, cos (A - B)$= \cos 30^\circ$
$\Rightarrow \quad A - B = 30^\circ$ ...(ii)
Adding (i) and (ii), we get
(A+B) + (A-B)
$= 90^\circ + 30^\circ \Rightarrow 2A = 120^\circ \Rightarrow A = 60^\circ$
Putting $A = 60^\circ$ in (i), we get
$60^\circ + B = 90^\circ$
$\Rightarrow B = 30^\circ$
Hence, A = 60^o and B = 30^o
14. It is given that $\sin(A-B) = \frac{1}{2}, \cos(A+B) = \frac{1}{2}, 0^o < A + B \leqslant 90^o, A > B$, we have to find the values of A and B.
Now, $\sin(A-B) = \frac{1}{2}$
$\Rightarrow \sin(A-B) = \sin 30^o$ $[\because \sin 30^o = \frac{1}{2}]$
On equating both sides , we get
$A - B = 30^o$ ...(i)
Also, $\cos(A+B) = \frac{1}{2}$
$\Rightarrow \cos(A+B) = \cos 60^o$ $[\because \cos 60^o = \frac{1}{2}]$
On equating both sides, we get
$A + B = 60^o$ ...(ii)
On Adding Eq(i) and Eq(ii), we get
$2A = 90^o$
$\Rightarrow A = 45^o$
From Eq(i) , we get $45^o - B = 30^o$
$\Rightarrow B = 45^o - 30^o$
$\therefore B = 15^o$
Hence, $A = 45^o \; and \; B = 15^o$
15. According to the question,
tan (A+B-C) = 1
$\Rightarrow$ tan (A+B-C) = $\tan 45^\circ$
$\Rightarrow$ A + B - C = 45° ....(1)
Also given, sec (B+C-A) = 2
$\Rightarrow$ sec (B + C - A) = sec 60°
$\therefore B + C - A = 60^\circ$ ....(2)
Adding equation (1) & (2);
$(A + B - C) + (B + C - A) = 45^\circ + 60^\circ$
$\Rightarrow \quad 2B = 105^\circ$
$\Rightarrow \quad B = 52\frac{1^\circ}{2}$
Putting $B = 52\frac{1^\circ}{2}$ in equation (2); we get :-
$52\frac{1^\circ}{2} + C - A = 60^\circ$
$\Rightarrow C - A = 7\frac{1^\circ}{2}$ ......(3)
Also, in ΔABC, we have
$A + B + C = 180^\circ$

$\Rightarrow \quad A + 52\frac{1}{2}^{\circ} + C = 180^{\circ} \quad \left[\because B = 52\frac{1}{2}^{\circ}\right]$

$\Rightarrow \quad C + A = 127\frac{1}{2}^{\circ}$ ......(4)

Adding and subtracting (3) and (4), we get

2C = 135^{o} and 2A = 120^{o}

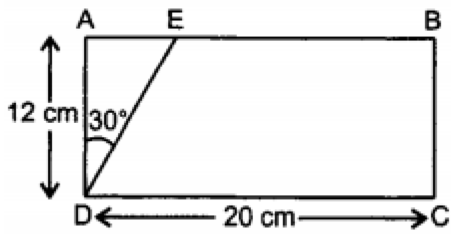
$\Rightarrow C = 67\frac{1}{2}^{\circ}$ and A = 60^{o}

Hence, we get the values of A = 60^{o},

$B = 52\frac{1}{2}^{\circ}$

and $C = 67\frac{1}{2}^{\circ}$.

16. In right $\triangle$DAE,



$\frac{\text{AE}}{\text{AD}} = \tan 30^{\circ}$

$\Rightarrow \frac{\text{AE}}{12} = \frac{1}{\sqrt{3}}$

$\Rightarrow \text{AE} = \frac{12}{\sqrt{3}}$

= $4\sqrt{3}$ cm

Also $\frac{\text{DE}}{\text{AD}} = \sec 30^{\circ}$

$\Rightarrow \frac{\text{DE}}{12} = \frac{2}{\sqrt{3}}$

$\Rightarrow \text{DE} = \frac{2}{\sqrt{3}} \times 12$

$= \frac{24}{\sqrt{3}}$

$= 8\sqrt{3}$ cm

17. L.H.S = $\frac{1-\sin 60^{\circ}}{\cos 60^{\circ}} = \frac{1-\frac{\sqrt{3}}{2}}{\frac{1}{2}} = \frac{2-\sqrt{3}}{1}$

R.H.S = $\frac{\tan 60^{\circ}-1}{\tan 60^{\circ}+1} = \frac{\sqrt{3}-1}{\sqrt{3}+1} \times \frac{\sqrt{3}-1}{\sqrt{3}-1}$

$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2}$

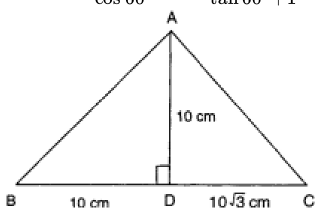
$= \frac{3+1-2\sqrt{3}}{3-1}$

$= \frac{4-2\sqrt{3}}{2}$

$= \frac{2(2-\sqrt{3})}{2}$

$= 2 - \sqrt{3}$

Hence, $\frac{1-\sin 60^{\circ}}{\cos 60^{\circ}} = \frac{\tan 60^{\circ}-1}{\tan 60^{\circ}+1}$

18. 

Given that,

ABC is a right triangle, right angled at D in which, AD = 10 cm and BD = 10 cm.

$\therefore \quad \tan \angle BAD = \frac{BD}{AD}$

$\Rightarrow \quad \tan \angle BAD = \frac{10}{10} = 1$

$\Rightarrow \quad \tan \angle BAD = \tan 45^{\circ}$

$\Rightarrow \quad \angle BAD = 45^{\circ}$ ......(i)

ACD is a right triangle right angled at D in which AD = 10 cm and DC $= 10\sqrt{3}\,cm$

$\therefore \quad \tan \angle CAD = \frac{CD}{AD}$

$\Rightarrow \quad \tan \angle \text{CAD} = \frac{10\sqrt{3}}{10} = \sqrt{3}$

$\Rightarrow \quad \tan \angle \mathrm{CAD} = \tan 60^\circ$

$\therefore \quad \angle \mathrm{CAD} = 60^\circ$ ......(ii)

Adding (i) and (ii), we get

$\angle BAD + \angle CAD = 45^\circ + 60^\circ$

$\therefore \angle BAC = 105^\circ$

Section B

19. cos θ = $\frac{60}{120}$

cos θ = $\frac{1}{2}$

θ = 60^o

20. tan 60^o = $\sqrt{3}$

21. tan θ = $\frac{AB}{BC}$

tan 60^o = $\frac{AB}{60}$

$\sqrt{3}$ = $\frac{AB}{60}$

AB = 60 $\times$ 1.732

AB = 103.9 m

22. $\angle$A = 90 - θ

$\angle$A = 90 - 60

$\angle$A = 30^o

tan 30^o = $\frac{1}{\sqrt{3}}$

23. Sin θ = $\frac{2}{4}$

Sin θ = $\frac{1}{2}$

θ = 30^o

i.e., the dock makes an angle of 30^o with the street.

24. tan 30^o = $\frac{2}{BC}$

BC = $\frac{2}{tan 30^o}$

B = $\frac{2}{\frac{1}{\sqrt{3}}}$

BC = $2\sqrt{3}$

BC = 3.5 m

$\therefore$ the length of base of ramp = 3.5 m

25. tan 45^o = $\frac{AB}{BC}$

1 = $\frac{AB}{BC}$

1 $\times$ 3.5 = AB

AB = 3.5 m

$\therefore$ height of ramp becomes 3.5 m

26. sin 45^o = $\frac{3.5}{AC}$

$\frac{1}{\sqrt{2}} = \frac{3.5}{AC}$

AC = 3.5 $\times$ 1.41

= 4.93 m

27. cos 60^o = $\frac{10}{a}$

$\frac{1}{2} = \frac{10}{a}$

a = 20 m

28. tan 60^o = $\frac{b}{10}$

$\sqrt{3}$ = $\frac{b}{10}$

b = $10\sqrt{3}$

b = 17.32 m

29. Sin 30^o = $\frac{b}{d}$

$\frac{1}{2} = \frac{17.32}{d}$

d = 34.64 m

30. Cos 30^o = $\frac{c}{d}$
$\frac{\sqrt{3}}{2} = \frac{c}{34.64}$
c = $\frac{34.64 \times \sqrt{3}}{2}$
c = 29.99
c $\approx$ 30 m
31. In $\triangle$ ABC
tan 45^o = $\frac{5}{BC}$
BC = 5 m
In $\triangle$ DEF
tan 30^o = $\frac{6}{EF}$
$\frac{1}{\sqrt{3}} = \frac{6}{EF}$
EF = $6\sqrt{3}$
length of flat part = 30 - (5 + $6\sqrt{3}$)
= 30 - 15.392
= 14.60 m
32. Upper inclination
33. Length of slide = AB + DE
AB = $\sqrt{5^2 + 5^2}$
AB = $5\sqrt{2}$ m
DC = $\sqrt{6^2 + 6\sqrt{3}^2}$
$= \sqrt{36 + 108}$
$= \sqrt{144}$
= 12 m
$\therefore$ Length of slide = $5\sqrt{2}$ + 12
= 8.66 + 12
= 20.66 m
34. Length of single slide = $\sqrt{30^2 + 11^2}$
$= \sqrt{900 + 121}$
$= \sqrt{1021}$ m
= 31.95 m