

Solution

INTRODUCTION TO TRIGONOMETRY WS 6

Class 10 - Mathematics

1. 30

Explanation:

As per the question we have,

$$\cos x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ.$$

$$\Rightarrow \cos x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\Rightarrow \cos x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4}$$

$$\Rightarrow \cos x = \frac{\sqrt{3}}{2} \Rightarrow \cos x = \cos 30^\circ \Rightarrow x = 30^\circ$$

2. 48

Explanation:

$$\text{Here, } \tan\left(\frac{5\theta}{2}\right) = \sqrt{3}$$

$$\text{Also } \tan 60^\circ = \sqrt{3}$$

$$\Rightarrow \tan\left(\frac{5\theta}{2}\right) = \tan 60^\circ$$

$$\Rightarrow \frac{5\theta}{2} = 60^\circ$$

$$\Rightarrow 5\theta = 120^\circ$$

$$\Rightarrow \theta = 24^\circ$$

$$\text{and } 2\theta = 2 \times 24^\circ$$

$$= 48^\circ$$

3. 5

Explanation:

$$= \frac{3 \tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$$

$$= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$$

$$= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1}$$

$$= 1 + 3 + 2 - 1$$

$$= 6 - 1$$

$$= 5$$

4. (a) - (iv), (b) - (i), (c) - (iii), (d) - (ii)

5. (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

6. (a) - (iii), (b) - (ii), (c) - (iv), (d) - (i)

7. (a) - (iii), (b) - (i), (c) - (ii), (d) - (iv)

8. (a) - (iii), (b) - (i), (c) - (iv), (d) - (ii)

9. Here, we will use the values of known angles of different trigonometric ratios.

$$\text{Therefore, } 4(\sin^4 30^\circ + \cos^4 60^\circ) - \frac{2}{3}(\sin^2 60^\circ - \cos^2 45^\circ) + \frac{1}{2} \tan^2 60^\circ$$

$$= 4\left(\frac{1}{16} + \frac{1}{16}\right) - \frac{2}{3}\left(\frac{3}{4} - \frac{1}{2}\right) + \frac{1}{2}(3)$$

$$= 4\left(\frac{1}{8}\right) - \frac{2}{3}\left(\frac{1}{4}\right) + \frac{3}{2}$$

$$= \frac{11}{6}$$

10. According to the question,

$$\sec \theta = \operatorname{cosec} 60^\circ$$

$$\Rightarrow \sec \theta = \operatorname{cosec} (90^\circ - 30^\circ)$$

$$\Rightarrow \sec \theta = \sec 30^\circ \quad [\operatorname{cosec} (90^\circ - \theta) = \sec \theta]$$

$$\Rightarrow \theta = 30^\circ$$

$$\therefore 2\cos^2 \theta - 1$$

$$= 2\cos^2 30^\circ - 1$$

$$= 2\left(\frac{\sqrt{3}}{2}\right)^2 - 1$$

$$\begin{aligned}
&= 2 \times \frac{3}{4} - 1 \\
&= \frac{3}{2} - 1 \\
&= \frac{3-2}{2} \\
&= \frac{1}{2}
\end{aligned}$$

11. Consider L.H.S. = $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$
 $= (\sqrt{3}/2) \times (\sqrt{3}/2) - (1/2)(1/2)$
 $= (3/4) - (1/4)$
 $= 2/4$
 $= 1/2$

Consider R.H.S. = $\sin 30^\circ = 1/2$

L.H.S. = R.H.S.

Hence, verified.

12. L.H.S. = $\frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ}$
 $= \frac{\frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2}}{1 + \frac{1}{2} + \frac{1}{2}} = \frac{\sqrt{3}}{2}$

R.H.S = $\cos 30^\circ = \frac{\sqrt{3}}{2}$

Hence, $\frac{\cos 30^\circ + \sin 60^\circ}{1 + \sin 30^\circ + \cos 60^\circ} = \cos 30^\circ$

13. We have

$$\cos 2x = \cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

$$\cos 2x = \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2}$$

$$\cos 2x = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{2} = \cos 30^\circ \dots \left(\because \cos 30^\circ = \frac{\sqrt{3}}{2} \right)$$

Comparing the angles, we get

$$2x = 30^\circ \Rightarrow x = 15^\circ$$

$$\therefore x = 15^\circ$$

14. L.H.S. = $\cos 2\theta = \cos 2 \times 30^\circ = \cos 60^\circ = \frac{1}{2}$

$$\text{R. H. S.} = \frac{1 - \tan^2 \theta}{1 + \tan^2 \theta}$$

$$= \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \frac{1 - \left(\frac{1}{\sqrt{3}}\right)^2}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}} = \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{3} \times \frac{3}{4}$$

$$= \frac{1}{2}$$

$$= \frac{1}{2}$$

Clearly, L.H.S = R.H.S

15. Consider L.H.S. = $\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$

$$= (1/2) \times (\sqrt{3}/2) + (\sqrt{3}/2)(1/2)$$

$$= (\sqrt{3}/4) + (\sqrt{3}/4)$$

$$= \sqrt{3}/2 = \cos 30^\circ = \text{R.H.S.}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Hence, verified.

16. $\frac{4}{\cot^2 30^\circ} + \frac{1}{\sin^2 30^\circ} - 2\cos^2 45^\circ - \sin^2 0^\circ$

$$= \frac{4}{(\sqrt{3})^2} + \frac{1}{\left(\frac{1}{2}\right)^2} - 2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - 0$$

$$= \frac{4}{3} + \frac{4}{1} - \frac{2}{2} - 0$$

$$= \frac{8+24-6-0}{6}$$

$$= \frac{26}{6} = \frac{13}{3}$$

17. We have,

$$x = 30^\circ$$

$$\Rightarrow 3x = 90^\circ$$

$$\therefore \text{L. H. S} = \cos 3x = \cos 90^\circ = 0$$

$$\begin{aligned}
&\text{and, R. H. S} = 4 \cos^3 x - 3 \cos x \\
&= 4 \cos^3 30^\circ - 3 \cos 30^\circ \\
&= 4 \left(\frac{\sqrt{3}}{2} \right)^3 - 3 \left(\frac{\sqrt{3}}{2} \right) \\
&= 4 \times \frac{3\sqrt{3}}{8} - \frac{3\sqrt{3}}{2} = \frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} = 0
\end{aligned}$$

18. Here, $A = 30^\circ, B = 60^\circ$

$$\begin{aligned}
\text{L. H.S.} &= \sin(A + B) = \sin(30^\circ + 60^\circ) \\
&= \sin 90^\circ = 1
\end{aligned}$$

$$\begin{aligned}
\text{R. H. S.} &= \sin A \cos B + \cos A \sin B \\
&= \sin 30^\circ \cos 60^\circ + \cos 30^\circ \sin 60^\circ \\
&= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} = \frac{1}{4} + \frac{3}{4} = \frac{4}{4} = 1
\end{aligned}$$

\therefore LHS = RHS

19. $\sin(A + B) = 1 = \sin 90^\circ$

$$A + B = 90^\circ \quad (1)$$

$$\cos(A - B) = 1 = \cos 0^\circ$$

$$\Rightarrow A - B = 0^\circ \quad (2)$$

Solving (1) and (2) we get

$$A = 45^\circ, \quad B = 45^\circ$$

20. $(\sin^2 30^\circ + 4 \cot^2 45^\circ - \sec^2 60^\circ)(\operatorname{cosec}^2 45^\circ \sec^2 30^\circ)$

$$\begin{aligned}
&= \left(\frac{1}{2} \right)^2 + 4 \times (1)^2 - (2)^2 \left[(\sqrt{2})^2 \times \left(\frac{2}{\sqrt{3}} \right)^2 \right] \\
&= \left(\frac{1}{4} + 4 - 4 \right) \left(2 \times \frac{4}{3} \right) \\
&= \frac{1}{4} \times \frac{8}{3} = \frac{2}{3}
\end{aligned}$$

21. Here, L.H.S. = $\cos(A + B) = \cos(30^\circ + 60^\circ) = \cos 90^\circ = 0$

$$\text{R. H. S} = \cos A \cos B - \sin A \sin B$$

$$= \cos 30^\circ \cos 60^\circ - \sin 30^\circ \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

\therefore LHS = R.H.S

22. To verify: $\cos(A + B) = \cos A \cos B - \sin A \sin B$

If $A = 60^\circ$ and $B = 30^\circ$, then

To verify: $\cos(90^\circ) = \cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

Consider R.H.S. = $\cos 60^\circ \cos 30^\circ - \sin 60^\circ \sin 30^\circ$

$$= (1/2) \times (\sqrt{3}/2) - (\sqrt{3}/2)(1/2)$$

$$= (\sqrt{3}/4) - (\sqrt{3}/4)$$

$$= 0 = \cos 90^\circ = \text{L.H.S.}$$

\therefore L.H.S. = R.H.S.

Hence, verified.

23. Given, $2 \sin 2\theta = \sqrt{3}$

$$\text{or, } \sin 2\theta = \frac{\sqrt{3}}{2}$$

$$\sin 2\theta = \sin 60^\circ$$

$$2\theta = 60^\circ$$

$$\text{Hence, } \cos 2\theta = \cos 60^\circ = \frac{1}{2}$$

24. Given: $\cos A = \sqrt{\frac{1 + \cos 2A}{2}}, \dots (1)$

$$\cos 60^\circ = \frac{1}{2}$$

To find: $\cos 30^\circ$

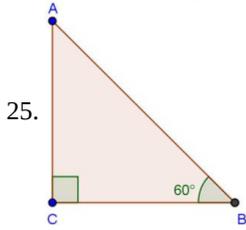
By putting $A = 30^\circ$ in equation (1), we get the following:

$$\cos 30^\circ = \sqrt{\frac{1 + \cos 60^\circ}{2}}$$

$$= \sqrt{\frac{1 + (1/2)}{2}}$$

$$= \sqrt{\frac{3}{4}} = \frac{\sqrt{3}}{2}$$

$$\therefore \cos 30^\circ = \frac{\sqrt{3}}{2}$$



Given $\angle C = 90^\circ$, $\angle B = 60^\circ$, $AB = 15 \text{ units}$.

In $\triangle ACB$, $\angle A + \angle B + \angle C = 180^\circ$

$$\angle A + 60^\circ + 90^\circ = 180^\circ$$

$$\angle A = 180^\circ - 60^\circ - 90^\circ$$

$$\angle A = 180^\circ - 150^\circ$$

therefore, $\angle A = 30^\circ$

In $\triangle ACB$

$$\sin 60^\circ = \frac{AC}{AB} \text{ and } \cos 60^\circ = \frac{BC}{AB}$$

$$\frac{\sqrt{3}}{2} = \frac{AC}{15} \text{ and } \frac{1}{2} = \frac{BC}{15} \text{ [} \sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2} \text{]}$$

$$AC = \frac{15\sqrt{3}}{2} \text{ units} = 7.5\sqrt{3} \text{ units and } BC = \frac{15}{2} = 7.5 \text{ units}$$

Hence, $\angle A = 30^\circ$, $\angle B = 60^\circ$, $\angle C = 90^\circ$ and, $AB = 15 \text{ units}$, $BC = 7.5 \text{ units}$, $CA = 7.5\sqrt{3} \text{ units}$

26. L.H.S. = $\tan(A - B) = \tan(60^\circ - 60^\circ) = \tan 0^\circ = 0$

$$\begin{aligned} \text{R. H. S.} &= \frac{\tan A - \tan B}{1 + \tan A \tan B} \\ &= \frac{\tan 60^\circ - \tan 60^\circ}{1 + \tan 60^\circ \tan 60^\circ} \\ &= \frac{\sqrt{3} - \sqrt{3}}{1 + \sqrt{3} \times \sqrt{3}} = \frac{0}{1+3} = 0 \end{aligned}$$

\therefore L.H.S. = R.H.S.

27. $\therefore \sin(A + B) = \frac{\sqrt{3}}{2}$

$$\sin(A + B) = \sin 60^\circ$$

On comparing angles

$$\therefore A + B = 60 \dots(i)$$

$$\text{Also, } \sin(A - B) = \frac{1}{2}$$

$$\sin(A - B) = \sin 30^\circ$$

On comparing angles

$$A - B = 30 \dots(ii)$$

Adding equation (i) & (ii)

$$2A = 90$$

$$A = 45^\circ$$

Putting in (i)

$$45 + B = 60$$

$$B = 15^\circ$$

28. Here, we will use the values of known angles of different trigonometric ratios.

$$\begin{aligned} &= \frac{2}{3} (\cos^4 30^\circ - \sin^4 45^\circ) - 3 (\sin^2 60^\circ - \sec^2 45^\circ) + \frac{1}{4} \cot^2 30^\circ \\ &= \frac{2}{3} \left(\frac{9}{16} - \frac{1}{4} \right) - 3 \left(\frac{3}{4} - 2 \right) + \frac{1}{4} (3) \\ &= \frac{2}{3} \left(\frac{5}{16} \right) + 3 \left(\frac{5}{4} \right) + \frac{3}{4} \\ &= \frac{113}{24} \end{aligned}$$

29. To verify: $\sin(A + B) = \sin A \cos B + \cos A \sin B$

If $A = 60^\circ$ and $B = 30^\circ$, then

$$\text{To verify: } \sin 90^\circ = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$\text{Consider R.H.S.} = \sin 60^\circ \cos 30^\circ + \cos 60^\circ \sin 30^\circ$$

$$= (\sqrt{3}/2) \times (\sqrt{3}/2) + (1/2)(1/2)$$

$$= (3/4) + (1/4)$$

$$= 4/4$$

$$= 1$$

Consider L.H.S. = $\sin 90^\circ = 1$

\therefore L.H.S. = R.H.S.

Hence, verified.

$$30. \text{ L. H.S.} = \sin 2\theta = \sin 2 \times 30^\circ = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\text{R.H.S.} = \frac{2 \tan \theta}{1 + \tan^2 \theta}$$

$$= \frac{2 \times \frac{1}{\sqrt{3}}}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}} = \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \frac{\sqrt{3}}{2}$$

\therefore L. H. S = R. H. S

$$31. \text{ Given: } \frac{5 \cos^2 60^\circ + 4 \sec^2 30^\circ - \tan^2 45^\circ}{\sin^2 30^\circ + \cos^2 30^\circ}$$

$$= \frac{5\left(\frac{1}{2}\right)^2 + 4\left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2}{\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2}$$

$$= \frac{5 \times \frac{1}{4} + 4 \times \frac{4}{3} - 1}{\frac{1}{4} + \frac{3}{4}}$$

$$= \frac{\frac{5}{4} + \frac{16}{3} - 1}{1}$$

$$= \frac{1+3}{4}$$

$$= \frac{15+64-12}{12}$$

$$= \frac{67}{12}$$

$$= \frac{1}{12}$$

$$= \frac{67}{12}$$

$$32. \cos 30^\circ = \frac{\sqrt{3}}{2}, \Rightarrow \cos^2 30^\circ = 3/4$$

$$\sin 30^\circ = 1/2$$

$$\therefore \operatorname{cosec} 30^\circ = 1/\sin 30^\circ = 2 \Rightarrow \operatorname{cosec}^2 30^\circ = 4$$

$$\cot 30^\circ = (\cos 30^\circ / \sin 30^\circ) = \sqrt{3} \Rightarrow \cot^2 30^\circ = 3$$

$$\cos 45^\circ = 1/\sqrt{2}$$

$$\therefore \sec 45^\circ = 1/\cos 45^\circ = \sqrt{2} \Rightarrow \sec^2 45^\circ = 2$$

$$\therefore \cot^2 30^\circ - 2\cos^2 30^\circ - (3/4)\sec^2 45^\circ + (1/4)\operatorname{cosec}^2 30^\circ$$

$$= 3 - 2(3/4) - (3/4) \times 2 + (1/4) \times 4$$

$$= 3 - 1.5 - 1.5 + 1$$

$$= 1$$

$$33. \text{ Given } \tan A = 1 \text{ and } \sin B = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \tan A = \tan 45^\circ \text{ and } \sin B = \sin 45^\circ$$

$$\Rightarrow A = 45^\circ \text{ and } B = 45^\circ$$

$$\text{Now } \cos(A+B) = \cos(45^\circ + 45^\circ) = \cos 90^\circ = 0.$$

$$34. \cos 2\theta = \cos 60^\circ = \frac{1}{2}$$

$$2 \cos^2 \theta - 1 = 2 \times \cos^2 30^\circ - 1$$

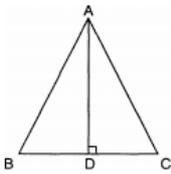
$$= \left[2 \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1 \right] = \left(2 \times \frac{3}{4} - 1 \right) = \left(\frac{3}{2} - 1 \right) = \frac{1}{2}$$

$$1 - 2 \sin^2 \theta = 1 - 2 \sin^2 30^\circ$$

$$= \left[1 - 2 \times \left(\frac{1}{2}\right)^2 \right] = \left(1 - 2 \times \frac{1}{4} \right) = \left(1 - \frac{1}{2} \right) = \frac{1}{2}$$

$$\therefore \cos 2\theta = 2\cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$$

35.



Let a triangle ABC with each side equal to $2a$

$$\angle A = \angle B = \angle C = 60^\circ$$

Draw AD perpendicular to BC

In $\triangle BDA$ and $\triangle CDA$

$$AB = AC, AD = AD \text{ and } \angle BDA = \angle CDA$$

$\triangle BDA \cong \triangle CDA$ by RHS

$$BD = CD$$

$$\angle BAD = \angle CAD = 30^\circ \text{ by CPCT}$$

$$\text{In } \triangle BDA, \operatorname{cosec} 30^\circ = \frac{H}{P} = \frac{AB}{BD} = \frac{2a}{a} = 2 \text{ and } \cos 60^\circ = \frac{B}{H} = \frac{BD}{AB} = \frac{a}{2a} = \frac{1}{2}$$

$$36. \text{ L.H.S.} = \sin(A - B) = \sin(60^\circ - 60^\circ)$$

$$= \sin 0^\circ = 0$$

$$\text{R. H. S.} = \sin A \cos B - \cos A \sin B$$

$$= \sin 60^\circ \cos 60^\circ - \cos 60^\circ \sin 60^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{2} - \frac{1}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{\sqrt{3}}{4} - \frac{\sqrt{3}}{4} = 0$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

$$37. \frac{3}{2} \left(\frac{1}{\sqrt{3}} \right)^2 - 2(0)^2 - \frac{1}{2}(2)^2$$

$$= -\frac{3}{2}$$

38. We have,

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 45^\circ = \frac{1}{\sqrt{2}}, \cot 30^\circ = \sqrt{3}, \cos 90^\circ = 0$$

therefore,

$$\frac{\sin 60^\circ}{\cos^2 45^\circ} - \cot 30^\circ + 15 \cos 90^\circ$$

$$= \frac{\frac{\sqrt{3}}{2}}{(1/\sqrt{2})^2} - \sqrt{3} + 15 \times 0$$

$$= \sqrt{3} - \sqrt{3} + 0 = 0$$

$$39. \sin 30^\circ \cdot \cos 60^\circ + \cos 30^\circ \cdot \sin 60^\circ$$

$$= \frac{1}{2} \times \frac{1}{2} + \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= \frac{1}{4} + \frac{3}{4} = \frac{1+3}{4}$$

$$= \frac{4}{4}$$

$$= 1 = \sin 90^\circ = \cos 90^\circ$$

40. We have,

$$\cos 60^\circ = \frac{1}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \sin 60^\circ = \frac{\sqrt{3}}{2} \text{ and } \sin 30^\circ = \frac{1}{2}$$

therefore,

$$\cos 60^\circ \cos 30^\circ + \sin 60^\circ \sin 30^\circ$$

$$= \frac{1}{2} \times \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \times \frac{1}{2} = \frac{\sqrt{3}}{4} + \frac{\sqrt{3}}{4} = 2 \left(\frac{\sqrt{3}}{4} \right) = \frac{\sqrt{3}}{2}$$

$$41. \frac{\tan^2 60^\circ + 4 \sin^2 45^\circ + 3 \sec^2 30^\circ + 5 \cos^2 90^\circ}{\operatorname{cosec} 30^\circ + \sec 60^\circ - \cot^2 30^\circ}$$

$$= \frac{(\sqrt{3})^2 + 4 \times \left(\frac{1}{\sqrt{2}} \right)^2 + 3 \times \left(\frac{2}{\sqrt{3}} \right)^2 + 5 \times 0^2}{2 + 2 - (\sqrt{3})^2}$$

$$= \left(\frac{3 + 4 \times \frac{1}{2} + 3 \times \frac{4}{3} + 5 \times 0}{4 - 3} \right)$$

$$= \left(\frac{3 + 2 + 4 + 0}{1} \right) = 9$$

42. We have,

$$\cos(A - B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos(A-B) = \cos 30^\circ$$

$$A - B = 30^\circ \dots (i)$$

$$\text{Again, } \sin(A+B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \sin(A+B) = \sin 60^\circ$$

$$A + B = 60^\circ \dots (ii)$$

Adding, (i) and (ii),

$$2A = 90^\circ$$

$$\therefore A = 45^\circ$$

Put $A = 45^\circ$ in (ii),

$$B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$$

Therefore, $A = 45^\circ$ and $B = 15^\circ$

43. Given that: $A = B = 60^\circ$

$$\text{L.H.S.} = \cos(A - B) = \cos(60^\circ - 60^\circ) = \cos 0^\circ = 1$$

$$\text{R.H.S.} = \cos A \cos B + \sin A \sin B$$

$$= \cos 60^\circ \cos 60^\circ + \sin 60^\circ \sin 60^\circ$$

$$= \cos^2 60^\circ + \sin^2 60^\circ$$

$$= \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

$\therefore \text{LHS} = \text{RHS}$

44. To verify: $\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$

If $A = 60^\circ$ and $B = 30^\circ$, then

$$\text{To verify: } \tan 90^\circ = \frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ}$$

$$\text{Consider R.H.S.} = \frac{\tan 60^\circ + \tan 30^\circ}{1 - \tan 60^\circ \tan 30^\circ} = \frac{\sqrt{3} + \frac{1}{\sqrt{3}}}{1 - \sqrt{3} \times \frac{1}{\sqrt{3}}}$$

$$= \frac{3+1}{0} = 4/0 = \infty$$

$$= \tan 90^\circ = \text{L.H.S.}$$

$\therefore \text{R.H.S.} = \text{L.H.S.}$

Hence, verified.

45. We have

$$4 - \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$= 4 - \frac{\frac{1}{2} + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1}$$

$$= 4 - \frac{\frac{3\sqrt{3}-4}{2}}{\frac{4+3\sqrt{3}}{2}}$$

$$= 4 - \frac{3\sqrt{3}-4}{4+3\sqrt{3}}$$

$$= \frac{16+12\sqrt{3}-3\sqrt{3}+4}{4+3\sqrt{3}}$$

$$= \frac{20+9\sqrt{3}}{4+3\sqrt{3}}$$

46. We have,

$$\cos 30^\circ = \frac{\sqrt{3}}{2}, \sin 30^\circ = \frac{1}{2}$$

therefore,

$$\text{R.H.S.} = \cos^2 30^\circ - \sin^2 30^\circ$$

$$= \left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{2}\right)^2 = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\text{and, L.H.S.} = \cos 60^\circ = \frac{1}{2}$$

$\therefore \text{L.H.S.} = \text{R.H.S.}$

$$\text{i.e., } \cos 60^\circ = \cos^2 30^\circ - \sin^2 30^\circ = \frac{1}{2}$$

47. We have

$$\sin(A+B) = 1$$

$$\Rightarrow \sin(A+B) = \sin 90^\circ \quad (\sin 90^\circ = 1)$$

$$\therefore A+B = 90^\circ \dots (i)$$

and $\cos(A-B) = 1$

$$\Rightarrow \cos(A-B) = \cos 0^\circ \quad (\cos 0^\circ = 1)$$

$$\therefore A-B = 0^\circ \dots (ii)$$

Adding equation (i) and (ii), we get

$$A + B + A - B = 90^\circ + 0^\circ$$

$$2A = 90^\circ$$

$$A = \frac{90^\circ}{2} = 45^\circ$$

Putting value of A in equation (i)

$$45^\circ + B = 90^\circ$$

$$\Rightarrow B = 90^\circ - 45^\circ = 45^\circ$$

$$\begin{aligned} 48. & 4(\sin^4 30^\circ + \cos^4 60^\circ) - \frac{2}{3}(\sin^2 60^\circ - \cos^2 45^\circ) + \frac{1}{2}\tan^2 60^\circ \\ &= 4\left[\left(\frac{1}{2}\right)^4 + \left(\frac{1}{2}\right)^4\right] - \frac{2}{3}\left[\left(\frac{\sqrt{3}}{2}\right)^2 - \left(\frac{1}{\sqrt{2}}\right)^2\right] + \frac{1}{2}(\sqrt{3})^2 \\ &= 4\left[\frac{1}{16} + \frac{1}{16}\right] - \frac{2}{3}\left[\frac{3}{4} - \frac{1}{2}\right] + \frac{1}{2} \times 3 \\ &= 4 \times \frac{2}{16} - \frac{2}{3}\left[\frac{3-2}{4}\right] + \frac{3}{2} \\ &= \frac{1}{2} - \frac{2}{3} \times \frac{1}{4} + \frac{3}{2} \\ &= \frac{1}{2} - \frac{1}{6} + \frac{3}{2} \\ &= \frac{3-1+9}{6} = \frac{11}{6} \end{aligned}$$

49. We have to prove that: $(\sqrt{3} + 1)(3 - \cot 30^\circ) = \tan^3 60^\circ - 2 \tan 60^\circ$

Here, LHS = $(\sqrt{3} + 1)(3 - \cot 30^\circ)$

$$= (\sqrt{3} + 1)(3 - \sqrt{3})$$

$$= \sqrt{3}(3 - \sqrt{3}) + 1(3 - \sqrt{3})$$

$$= 3\sqrt{3} - 3 + 3 - \sqrt{3}$$

$$= 2\sqrt{3}$$

$$\text{RHS} = \tan^3 60^\circ - 2 \sin 60^\circ$$

$$= (\sqrt{3})^3 - 2 \times \frac{\sqrt{3}}{2}$$

$$= 3\sqrt{3} - \sqrt{3}$$

$$= 2\sqrt{3}$$

$$\Rightarrow \text{LHS} = \text{RHS}$$

Hence, proved.

$$50. \frac{\cos 45^\circ}{\sec 30^\circ} + \frac{1}{\sec 60^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}}} + \frac{1}{2}$$

$$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2}$$

Rationalise the denominator,

$$= \frac{\sqrt{3}}{2\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}} + \frac{1}{2}$$

$$= \frac{\sqrt{6}}{4} + \frac{1}{2}$$

$$51. \text{ We have } \frac{\cos(45^\circ)}{\sec(30^\circ) + \operatorname{cosec}(30^\circ)}$$

$$= \frac{\frac{1}{\sqrt{2}}}{\frac{2}{\sqrt{3}} + 2}$$

$$= \frac{\frac{1}{\sqrt{2}}}{2\left(\frac{1}{\sqrt{3}} + 1\right)}$$

$$= \frac{1}{2\sqrt{2}\left(\frac{1+\sqrt{3}}{\sqrt{3}}\right)}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}(1+\sqrt{3})} \times \frac{\sqrt{2}(1-\sqrt{3})}{\sqrt{2}(1-\sqrt{3})}$$

$$\begin{aligned}
&= \frac{\sqrt{6}-3\sqrt{2}}{4(1-3)} \\
&= \frac{\sqrt{6}-3\sqrt{2}}{4(-2)} \\
&= \frac{\sqrt{6}-3\sqrt{2}}{-8} \\
&= \frac{3\sqrt{2}-\sqrt{6}}{8}
\end{aligned}$$

52. We have,

$$\tan(A+B) = \sqrt{3}$$

$$\Rightarrow \tan(A+B) = \tan 60^\circ$$

$$A+B = 60^\circ \dots\dots(i)$$

$$\text{Again, } \tan(A-B) = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \tan(A-B) = \tan 30^\circ$$

$$A-B = 30^\circ \dots\dots(ii)$$

Adding, (i) and (ii)

$$2A = 90^\circ$$

$$\therefore A = \frac{90^\circ}{2} = 45^\circ$$

Putting $A=45^\circ$ in equation (i),

$$B = 60^\circ - A = 60^\circ - 45^\circ = 15^\circ$$

Therefore,

$$A = 45^\circ \text{ and } B = 15^\circ.$$

53. $2(\sin^2 45^\circ + \cot^2 30^\circ) - 6(\cos^2 45^\circ - \tan^2 30^\circ)$

$$\left[\left(\frac{1}{\sqrt{2}}\right)^2 + (\sqrt{3})^2 \right] - 6 \left[\left(\frac{1}{\sqrt{2}}\right)^2 - \left(\frac{1}{\sqrt{3}}\right)^2 \right]$$

$$= 7 - 1 = 6$$

54. $\sin(A+B) = 1 \Rightarrow \sin(A+B) = \sin 90^\circ$

$$\Rightarrow A+B = 90^\circ \dots\dots(i)$$

$$\cos(A-B) = \frac{\sqrt{3}}{2}$$

$$\Rightarrow \cos(A-B) = \cos 30^\circ$$

$$\Rightarrow A-B = 30^\circ \dots\dots(ii)$$

Solving (i) and (ii), we get $A = 60^\circ$ and $B = 30^\circ$.

Hence, $A = 60^\circ$ and $B = 30^\circ$.

55. To find:- $\tan 60^\circ$

$$\text{Given: } \tan 2A = \frac{2 \tan A}{1 - \tan^2 A} \dots(1)$$

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

\therefore Putting $A = 30^\circ$ in equation (1), we have the following:

$$\tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$\therefore \tan 60^\circ = \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ} = \frac{2 \times \frac{1}{\sqrt{3}}}{1 - \frac{1}{3}} = \frac{2/\sqrt{3}}{2/3}$$

$$= \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\therefore \tan 60^\circ = \sqrt{3}$$

56. $= \frac{3 \tan^2 30^\circ + \tan^2 60^\circ + \operatorname{cosec} 30^\circ - \tan 45^\circ}{\cot^2 45^\circ}$

$$= \frac{3 \times \left(\frac{1}{\sqrt{3}}\right)^2 + (\sqrt{3})^2 + 2 - 1}{(1)^2}$$

$$= \frac{3 \times \frac{1}{3} + 3 + 2 - 1}{1}$$

$$= 1 + 3 + 2 - 1$$

$$= 6 - 1$$

$$= 5$$

57. To prove: $\sin 2A = \frac{2 \tan A}{1 + \tan^2 A}$

$$A = 30^\circ$$

$$\therefore \text{To show: } \sin 60^\circ = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ}$$

$$\text{Consider L.H.S.} = \frac{2 \tan 30^\circ}{1 + \tan^2 30^\circ} = \frac{2 \times \left(\frac{1}{\sqrt{3}}\right)}{1 + \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 + \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{4}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{4}$$

$$= \sqrt{3}/2$$

$$= \sin 60^\circ = \text{R.H.S.}$$

$$\therefore \text{R.H.S.} = \text{L.H.S.}$$

Hence, verified.

$$58. \cos 60^\circ = 1/2 \Rightarrow \cos^2 60^\circ = 1/4$$

$$\sin 45^\circ = 1/\sqrt{2} \Rightarrow \sin^2 45^\circ = 1/2$$

$$\sin 30^\circ = 1/2 \Rightarrow \sin^2 30^\circ = 1/4$$

$$\cos 90^\circ = 0 \Rightarrow \cos^2 90^\circ = 0$$

$$\therefore 2 \cos^2 60^\circ + 3 \sin^2 45^\circ - 3 \sin^2 30^\circ + 2 \cos^2 90^\circ$$

$$= 2(1/4) + 3(1/2) - 3(1/4) + 2(0)$$

$$= (1/2) + (3/2) - (3/4) = 2 - (3/4)$$

$$= 5/4$$

$$59. \frac{\sin 30^\circ + \tan 45^\circ - \operatorname{cosec} 60^\circ}{\sec 30^\circ + \cos 60^\circ + \cot 45^\circ}$$

$$\frac{1/2 + 1 - \frac{2}{\sqrt{3}}}{\frac{2}{\sqrt{3}} + \frac{1}{2} + 1} = \frac{\frac{\sqrt{3} + 2\sqrt{3} - 4}{2\sqrt{3}}}{\frac{4 + \sqrt{3} + 2\sqrt{3}}{2\sqrt{3}}}$$

$$= \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} = \frac{3\sqrt{3} - 4}{3\sqrt{3} + 4} \times \frac{3\sqrt{3} - 4}{3\sqrt{3} - 4}$$

$$= \frac{(3\sqrt{3} - 4)^2}{(3\sqrt{3})^2 - (4)^2}$$

$$= \frac{27 + 16 - 24\sqrt{3}}{27 - 16} = \frac{43 - 24\sqrt{3}}{11}$$

60. We have,

$$\sin 30^\circ = \frac{1}{2}, \tan 60^\circ = \sqrt{3}, \cos 60^\circ = \frac{1}{2} \text{ and } \sec 30^\circ = \frac{2}{\sqrt{3}}$$

therefore,

$$2 \sin^2 30^\circ \tan 60^\circ - 3 \cos^2 60^\circ \sec^2 30^\circ$$

$$= 2(\sin 30^\circ)^2 \tan 60^\circ - 3(\cos 60^\circ)^2 (\sec 30^\circ)^2$$

$$= 2 \times \left(\frac{1}{2}\right)^2 \times \sqrt{3} - 3 \times \left(\frac{1}{2}\right)^2 \times \left(\frac{2}{\sqrt{3}}\right)^2$$

$$= 2 \times \frac{1}{4} \times \sqrt{3} - 3 \times \frac{1}{4} \times \frac{4}{3} = \frac{\sqrt{3}}{2} - 1 = \frac{\sqrt{3} - 2}{2}$$

$$61. \text{ Given: } \sin A = \sqrt{\frac{1 - \cos 2A}{2}} \dots (1)$$

$$\cos 60^\circ = 1/2$$

To find: $\sin 30^\circ$

By putting $A = 30^\circ$ in equation (1), we get the following:

$$\sin 30^\circ = \sqrt{\frac{1 - \cos 60^\circ}{2}}$$

$$= \sqrt{\frac{1 - (1/2)}{2}}$$

$$= \sqrt{\frac{1}{4}} = 1/2$$

$$\therefore \sin 30^\circ = 1/2$$

$$62. \text{ Consider L.H.S.} = 2 \sin 30^\circ \cos 30^\circ$$

$$= 2 \times (1/2) \times (\sqrt{3}/2)$$

$$= \sqrt{3}/2 = \sin 60^\circ = \text{R.H.S.}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Hence, verified.

$$63. \tan(A + B) = \sqrt{3}$$

$$\tan(A + B) = \tan 60^\circ$$

$$A + B = 60^\circ \dots(i)$$

$$\tan(A - B) = 1$$

$$\tan(A - B) = \tan 45^\circ$$

$$A - B = 45^\circ \dots(ii)$$

Solving (i) and (ii), we get

$$A = (52.5)^\circ \text{ and } B = (7.5)^\circ.$$

Hence, $A = (52.5)^\circ$ and $B = (7.5)^\circ$.

$$64. \sin 60^\circ \cos 30^\circ + \sin 30^\circ \cos 60^\circ$$

$$= \frac{\sqrt{3}}{2} \cdot \frac{\sqrt{3}}{2} + \frac{1}{2} \cdot \frac{1}{2}$$

$$= \frac{3}{4} + \frac{1}{4} = 1$$

65. We have,

$$\sin 60^\circ = \frac{\sqrt{3}}{2}, \cos 30^\circ = \frac{\sqrt{3}}{2}, \cos 60^\circ = \frac{1}{2}, \sin 30^\circ = \frac{1}{2}$$

therefore,

$$\sin 2x = \sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ$$

$$\Rightarrow \sin 2x = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2}$$

$$\Rightarrow \sin 2x = \frac{3}{4} - \frac{1}{4} = \frac{2}{4} = \frac{1}{2}$$

$$\Rightarrow \sin 2x = \sin 30^\circ \Rightarrow 2x = 30^\circ \Rightarrow x = 15$$

66. According to question, we have to solve $\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$ in the simplest form.

We have,

$$\sin 60^\circ \cos 45^\circ + \cos 60^\circ \sin 45^\circ$$

$$= \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{2}} + \frac{1}{2} \times \frac{1}{\sqrt{2}} = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}}$$

$$67. \text{L.H.S.} = \tan 2\theta$$

$$= \tan(2 \times 30^\circ)$$

$$= \tan 60^\circ$$

$$= \sqrt{3}$$

$$\text{R. H. S.} = \frac{2 \tan \theta}{1 - \tan^2 \theta}$$

$$= \frac{2 \tan 30^\circ}{1 - \tan^2 30^\circ}$$

$$= \frac{2 \cdot \frac{1}{\sqrt{3}}}{1 - \left(\frac{1}{\sqrt{3}}\right)^2}$$

$$= \frac{\frac{2}{\sqrt{3}}}{1 - \frac{1}{3}}$$

$$= \frac{\frac{2}{\sqrt{3}}}{\frac{2}{3}} = \frac{2}{\sqrt{3}} \times \frac{3}{2} = \frac{3}{\sqrt{3}} = \sqrt{3}$$

$$\therefore \text{L. H. S} = \text{R. H. S}$$

68. We have,

$$\operatorname{cosec} 30^\circ = 2, \sin 45^\circ = \frac{1}{\sqrt{2}} \text{ and } \sec 60^\circ = 2$$

therefore,

$$\operatorname{cosec}^2 30^\circ \sin^2 45^\circ - \sec^2 60^\circ$$

$$= (\operatorname{cosec} 30^\circ)^2 (\sin 45^\circ)^2 - (\sec 60^\circ)^2 = (2)^2 \times \left(\frac{1}{\sqrt{2}}\right)^2 - (2)^2 = 4 \times \frac{1}{2} - 4 = 2 - 4 = -2$$

69. To show: $\cos 2A = 2 \cos^2 A - 1 = 1 - 2 \sin^2 A$

$$A = 45^\circ$$

$$\therefore \text{We show: } \cos 90^\circ = 2 \cos^2 45^\circ - 1 = 1 - 2 \sin^2 45^\circ$$

$$\text{Consider } 2 \cos^2 45^\circ - 1 = 1 - 2 \sin^2 45^\circ$$

$$= 2 \times (1/\sqrt{2})^2 - 1$$

$$= 1 - 1$$

$$= 0$$

$$= \cos 90^\circ = \text{R.H.S.}$$

$$\text{Consider } 1 - 2 \sin^2 45^\circ = 1 - 2(1/2)$$

$$= 1 - 1$$

$$= 0$$

$$= \cos 90^\circ = \text{R.H.S.}$$

\therefore L.H.S. = R.H.S.

Hence, verified.

70. We have,

$$\tan 30^\circ = \frac{1}{\sqrt{3}}$$

$$\text{R.H.S} = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

$$= \frac{1 - (1/\sqrt{3})^2}{1 + (1/\sqrt{3})^2}$$

$$= \frac{1 - \frac{1}{3}}{1 + \frac{1}{3}}$$

$$= \frac{2/3}{4/3} = \frac{1}{2}$$

$$\text{and, L.H.S} = \cos 60^\circ = \frac{1}{2}$$

therefore, L.H.S = R.H.S

$$71. \tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\Rightarrow \tan(A + B) = \frac{\frac{1}{2} + \frac{1}{3}}{1 - \frac{1}{2} \times \frac{1}{3}}$$

$$\Rightarrow \tan(A + B) = \frac{\frac{3+2}{6}}{\frac{3-1}{6}} = \frac{5}{1} = 5$$

$$\Rightarrow \tan(A + B) = 1 = \tan 45^\circ$$

$$\Rightarrow A + B = 45^\circ$$

$$72. \text{To show: } \cos 2A = \frac{1 - \tan^2 A}{1 + \tan^2 A}$$

$$A = 30^\circ$$

$$\therefore \text{To show: } \cos 60^\circ = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ}$$

$$\text{Consider R.H.S.} = \frac{1 - \tan^2 30^\circ}{1 + \tan^2 30^\circ} = \frac{1 - (\frac{1}{\sqrt{3}})^2}{1 + (\frac{1}{\sqrt{3}})^2}$$

$$= \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{1}{2} = \text{RHS}$$

$$\therefore \text{LHS} = \cos 60^\circ = \frac{1}{2}$$

Hence proved.

73. To show: $\sin 2A = 2 \sin A \cos A$

$$A = 45^\circ$$

$$\therefore \text{To show: } \sin 90^\circ = 2 \sin 45^\circ \cos 45^\circ$$

$$\text{Consider R.H.S.} = 2 \sin 45^\circ \cos 45^\circ$$

$$= 2 \times (1/\sqrt{2}) \times (1/\sqrt{2})$$

$$= (2 \times 1/2)$$

$$= 1 = \sin 90^\circ = \text{L.H.S.}$$

$$\therefore \text{L.H.S.} = \text{R.H.S.}$$

Hence, verified.

$$74. \tan^2 60^\circ - 2 \operatorname{cosec}^2 30^\circ - 2 \tan^2 30^\circ$$

$$= (\sqrt{3})^2 - 2(\sqrt{2})^2 - 2\left[\frac{1}{\sqrt{3}}\right]^2$$

$$= -\frac{17}{3}$$

$$75. \sin(A + B) = 1$$

$$\sin(A + B) = \sin 90^\circ$$

$$\text{or, } A + B = 90^\circ \dots (i)$$

$$\sin(A - B) = \frac{1}{2}$$

$$\sin(A - B) = \sin 30^\circ$$

$$\text{or, } A - B = 30^\circ \dots (ii)$$

Solving eq. (i) and (ii),

$$2A = 120^\circ$$

$$A = 60^\circ \text{ and } B = 30^\circ$$