CBSE

INTRODUCTION TO TRIGONOMETRY WS 7

Class 10 - Mathematics

Section A

1. $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$ is equal to **[1]**

   a) $tan\ \theta\ -\ sec\,\theta$ b) $-sec\,\theta\ -\ tan\ \theta$

   c) $sec\,\theta\ +\ tan\ \theta$ d) $sec\,\theta$ - $tan\ \theta$

2. If a sinθ + b cosθ = c, then the value of a cosθ - b sinθ is **[1]**

   a) $\sqrt{a^2+b^2-c^2}$ b) $\sqrt{a^2+b^2+c^2}$

   c) $\sqrt{a^2-b^2+c^2}$ d) $\sqrt{a^2-b^2-c^2}$

3. $\sin^2A + \sin^2A\tan^2A$ = **[1]**

   a) $\tan^2A$ b) $\cos^2A$

   c) $\cot^2A$ d) $\sin^2A$

4. The value of $\sin^6A + \cos^6A + 3\cos^2A\ \sin^2A$ is **[1]**

   a) 0 b) – 1

   c) 2 d) 1

5. $(\ cosec\theta - \cot\theta)^2 = ?$ **[1]**

   a) $\frac{1+sin\,\theta}{1-sin\,\theta}$ b) $\frac{1-cos\,\theta}{1+cos\,\theta}$

   c) $\frac{1-sin\,\theta}{1+sin\,\theta}$ d) $\frac{1+cos\,\theta}{1-cos\,\theta}$

6. The value of $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}}$ is **[1]**

   a) $\text{cosec}^2\,\theta + \cot^2\,\theta$ b) $\cot\,\theta$ - $\text{cosec}\,\theta$

   c) $\text{cosec}\,\theta + \cot\,\theta$ d) $(\cot\,\theta + \text{cosec}\,\theta)^2$

7. $(1 + \tan\,\theta + \sec\,\theta)\,(1 + \cot\theta - \text{cosec}\theta)$ = **[1]**

   a) 0 b) 2

   c) 1 d) –1

8. $\left(\frac{\sqrt{3}+2\cos A}{1-2\sin A}\right)^{-3} + \left(\frac{1+2\sin A}{\sqrt{3}-2\cos A}\right)^{-3}$ = _____. **[1]**

   a) $\sqrt{3}$ b) 0

   c) -1 d) 1

9. The value of $\text{cosec}^4A - 2\,\text{cosec}^2A + 1$ is **[1]**

   a) $\tan^4A$ b) $\sec^4A$

c) cosec^4A d) $\cot^4\text{A}$

10. If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A$ = **[1]**

a) – 1 b) 1

c) 0 d) 2

11. If tan A = n tan B and sin A = m sin B, then $\cos^2\text{A}$ = **[1]**

a) $\frac{m^2-1}{n^2-1}$ b) $\frac{m^2+1}{n^2-1}$

c) $\frac{m^2+1}{n^2+1}$ d) $\frac{m^2-1}{n^2+1}$

12. If x = a cos θ and y = b sin θ, then $b^2x^2 + a^2y^2$ = **[1]**

a) $a^2 + b^2$ b) ab

c) a^4b^4 d) a^2b^2

13. If $\tan\theta = \frac{m}{n}$, then $\frac{m\sin\theta - n\cos\theta}{m\sin\theta + n\cos\theta}$ = **[1]**

a) $\frac{m^2-n^2}{m^2+n^2}$ b) $\frac{m^2+n^2}{m^2-n^2}$

c) 1 d) $\frac{n^2-m^2}{n^2+m^2}$

14. If $\sin\theta = \frac{a}{b}$, then sec θ is equal to ($0 \leq \theta \leq 90^o$): **[1]**

a) $\frac{\text{b}}{\sqrt{\text{b}^2-\text{a}^2}}$ b) $\frac{\text{a}}{\sqrt{\text{b}^2-\text{a}^2}}$

c) $\frac{\sqrt{\text{b}^2-\text{a}^2}}{\text{b}}$ d) $\frac{\sqrt{\text{b}^2-\text{a}^2}}{\text{a}}$

15. If (cos θ + sec θ) = $\frac{5}{2}$ then ($\cos^2\theta + \sec^2\theta$) = ? **[1]**

a) $\frac{33}{4}$ b) $\frac{21}{4}$

c) $\frac{17}{4}$ d) $\frac{29}{4}$

16. Find the value of $\frac{1}{\tan\theta} + \frac{\sin\theta}{1+\cos\theta}$, if $1 + \cot^2\theta = (\sqrt{3+2\sqrt{2}} - 1)^2$. **[1]**

a) $\sqrt{2}$ b) 1

c) 3 d) -2

17. **Assertion (A):** $\sin^2\theta = 1 - \cos^2\theta$ for any value of θ. **[1]**

Reason (R): Value of sinθ is always more than 1.

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

18. **Assertion (A):** If cosA + $\cos^2$A = 1 then $\sin^2$A + $\sin^4$A = 2 **[1]**

Reason (R): 1 - $\sin^2$A = $\cos^2$A, for any value of A

a) Both A and R are true and R is the correct explanation of A. b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false. d) A is false but R is true.

19. **Assertion (A):** For $0 < \theta \leq 90^o$, cosec θ - cot θ and cosec θ + cot θ are reciprocal of each other. **[1]**

Reason (R): $\text{cosec}^2\theta - \cot^2\theta = 1$.

a) Both A and R are true and R is the correct explanation of A.

b) Both A and R are true but R is not the correct explanation of A.

c) A is true but R is false.

d) A is false but R is true.

20. Prove the trigonometric identity: $(1 - \sin^2\theta)\sec^2\theta = 1$ **[1]**

21. What is the value of $6\tan^2\theta - \frac{6}{\cos^2\theta}$? **[1]**

22. Prove that: $\sqrt{\frac{\sec A-1}{\sec A+1}} + \sqrt{\frac{\sec A+1}{\sec A-1}} = 2 \text{ cosec } A$ **[1]**

23. Prove the trigonometric identity: **[1]**

$\frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta} = \sec\theta\cos ec\theta + \cot\theta$

24. Prove the trigonometric identity: **[1]**

$\cot\theta - \tan\theta = \frac{2\cos^2\theta-1}{\sin\theta\cos\theta}$

25. Prove the trigonometric identity: **[1]**

$\tan^2\theta - \frac{1}{\cos^2\theta} = -1$

26. Show that $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$ **[1]**

27. Prove that: $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$ **[1]**

28. Prove that: $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$ **[1]**

29. Prove that: $(1 + \cot^2 A)\sin^2 A = 1$ **[1]**

30. Show that $\tan^2\theta + \cot^2\theta + 2 = \sec^2\theta + \cos ec^2\theta$. **[1]**

31. If $\sin^2\theta\cos^2\theta(1 + \tan^2\theta)(1 + \cot^2\theta) = \lambda$, then find the value of λ. **[1]**

32. Prove that: $\frac{1}{(cosec\theta-\cot\theta)} = (\text{cosec}\theta + \cot\theta)$ **[1]**

33. What is the value of $\left(\frac{1}{1+\cot^2\theta} + \frac{1}{1+\tan^2\theta}\right)$? **[1]**

34. Prove the trigonometric identity: $\sin^2\theta + \frac{1}{(1+\tan^2\theta)} = 1$ **[1]**

35. Prove the trigonometric identity: $\text{cosec}^2\theta + \sec^2\theta = \text{cosec}^2\theta\sec^2\theta$ **[1]**

36. Prove the trigonometric identity: **[1]**

$\cos^2\theta + \frac{1}{1+\cot^2\theta} = 1$

37. Prove the trigonometric identity: **[1]**

$\frac{1}{\sec\theta-\tan\theta} = \sec\theta + \tan\theta$

38. Prove that $(\sin^4\theta - \cos^4\theta + 1)\text{cosec}^2\theta = 2$ **[1]**

39. Prove the trigonometric identity: **[1]**

$\cot^2\theta - \frac{1}{\sin^2\theta} = -1$

Section B

40. **Fill in the blanks:** **[2]**

(a) The value of trigonometric function $\cot^2\theta - \frac{1}{\sin^2\theta} =$ _____. **[1]**

(b) The value of $\sin 23^o \cos 67^o + \cos 23^o \sin 67^o$ is _____. **[1]**

Section C

41. **State True or False:** **[4]**

(a) If $\cos A + \cos^2 A = 1$, then $\sin^2 A + \sin^4 A = 1$ **[1]**

(b) $(\tan\theta + 2)(2\tan\theta + 1) = 5\tan\theta + \sec^2\theta$. **[1]**

(c) $\sqrt{(1-\cos^2\theta)\sec^2\theta} = \tan\theta$ **[1]**

(d) The value of $\sin\theta + \cos\theta$ is always greater than 1 **[1]**

Section D

42. If cosec θ + cot θ = p, then prove that cos $\theta = \frac{p^2-1}{p^2+1}$. **[5]**

43. Prove that: $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{cosec^2\theta - \sin^2\theta}\right)\sin^2\theta\cos^2\theta = \frac{1-\sin^2\theta\cos^2\theta}{2+\sin^2\theta\cos^2\theta}$ **[5]**

44. Find the value of θ, if $\frac{cos\theta}{1-sin\theta} + \frac{cos\theta}{1+sin\theta} = 4$, $\theta \leqslant 90^o$. **[5]**

45. If cos A - sin A = m and cos A + sin A = n. Show that: $\frac{m^2-n^2}{m^2+n^2}$ = -2 sin A. cos A $= -\frac{2}{\tan A + \cot A}$ **[5]**

46. Prove the following identity: $\frac{\sin A}{\sec A + \tan A - 1} + \frac{\cos A}{cosecA + \cot A - 1} = 1$ **[5]**

47. Prove that: $\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ = 1 + tanA + cotA = 1 + secA cosecA **[5]**

48. If 2 cos θ - sin θ = x and cos θ - 3 sin θ = y, prove that $2x^2 + y^2 - 2xy = 5$. **[5]**

49. Prove that: $\sqrt{\sec^2\theta + \text{cosec}^2\theta}$ = tan θ + cot θ **[5]**

50. If sec θ + tan θ = p, show that sec θ - tan θ = $\frac{1}{p}$. Hence, find the values of cos θ and sin θ. **[5]**

51. If a sin θ + b cos θ = c, then prove that a cos θ – b sinθ = $\sqrt{a^2 + b^2 - c^2}$. **[5]**

52. Prove the following identity: $\left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{cosec^2\theta - \sin^2\theta}\right)\sin^2\theta \,.\, \cos^2\theta = \frac{1-\sin^2\theta\cdot\cos^2\theta}{2+\sin^2\theta\cdot\cos^2\theta}$ **[5]**

53. Prove that : (sin θ + 1 + cos θ) (sin θ – 1 + cos θ) . sec θ cosec θ = 2 **[5]**

54. Prove that: $(1 - \sin\theta + \cos\theta)^2 = 2(1 + \cos\theta)(1 - \sin\theta)$. **[5]**

55. If a cos θ +b sin θ =m and a sin θ - b cos θ =n, prove that $m^2 + n^2 = a^2 + b^2$. **[5]**

56. If x = γ cos α sin β ; y = γ cos α cos β and $z = \gamma \sin\alpha$, show that $x^2 + y^2 + z^2 = \gamma^2$. **[5]**

57. Prove that: $\frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$ = sec θ cosec θ - 2 sin θ cos θ. **[5]**

58. Prove the trigonometric identity: **[5]**

$2\sec^2\theta - \sec^4\theta - 2\cos ec^2\theta + \cos ec^4\theta = \cot^4\theta - \tan^4\theta$

59. Prove the trigonometric identity: **[5]**

$\left(\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right)^2 = \frac{1-\cos\theta}{1+\cos\theta}$

60. If cosec θ + cot θ = p, then prove that cos θ = $\frac{p^2-1}{p^2+1}$. **[5]**

61. Prove that : $(\sin A + \sec A)^2 + (\cos A + \text{cosec } A)^2 = (1 + \sec A \text{ cosec } A)^2$. **[5]**

62. Prove the trigonometric identity: **[5]**

$\frac{\cot^2 A(\sec A - 1)}{1+\sin A} = \sec^2 A\left(\frac{1-\sin A}{1+\sec A}\right)$

63. Prove the trigonometric identity: **[5]**

If cosecθ - sinθ = a^3, secθ - cosθ = b^3, prove that $a^2\, b^2(a^2 + b^2) = 1$

64. If cosec A - cot A = q, show that $\frac{q^2-1}{q^2+1} + \cos A = 0$ **[5]**

65. Prove that: $\frac{\sec A - \tan A}{\sec A + \tan A} = \frac{\cos^2 A}{(1+\sin A)^2}$ **[5]**

66. If sec $\theta = x + \frac{1}{4x}$, x $\neq$ 0, find (sec θ + tan θ). **[5]**

67. Prove that: $\frac{1+\tan^2 A}{1+\cot^2 A} = \left(\frac{1-\tan A}{1-\cot A}\right)^2 = \tan^2 A$ **[5]**

68. Prove that: $\frac{1+\sin\theta}{1-\sin\theta} - \frac{1-\sin\theta}{1+\sin\theta}$ = 4 tan θ sec θ **[5]**

69. Prove that: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}} = 2\sec\theta$. **[5]**

70. Prove that $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{(\sec^3\theta - \csc^3\theta)} = \sin^2\theta\cos^2\theta$. **[5]**

71. If cosecθ - sinθ = l and secθ - cosθ = m, prove that $l^2m^2(l^2 + m^2 + 3) = 1$. **[5]**