Solution

INTRODUCTION TO TRIGONOMETRY WS 7

Class 10 - Mathematics

Section A

1.

(c) $sec\,\theta\ +\ tan\,\theta$

Explanation: Given: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}}$

= $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} \times \sqrt{\frac{1+\sin\theta}{1+\sin\theta}}$

= $\sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}}$

= $\sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}}$

= $\frac{1+\sin\theta}{\cos\theta}$

= $\frac{1}{\cos\theta} + \frac{\sin\theta}{\cos\theta}$

= $\sec\theta + \tan\theta$

2. **(a)** $\sqrt{a^2 + b^2 - c^2}$

Explanation: Given: a $\sin\theta$ + b $\cos\theta$ = c

Squaring both sides, we get

⇒ $a^2\sin^2\theta + b^2\cos^2\theta + 2ab\sin\theta\cos\theta = c^2$

⇒ $a^2(1 - \cos^2\theta) + b^2(1 - \sin^2\theta) + 2ab\sin\theta\cos\theta = c^2$

⇒ $a^2 - a^2\cos^2\theta + b^2 - b^2\sin^2\theta + 2ab\sin\theta\cos\theta = c^2$

⇒ $a^2\cos^2\theta - b^2\sin\theta - 2ab\sin\theta\cos\theta = a^2 + b^2 - c^2$

⇒ $(a\cos\theta - b\sin\theta)^2 = a^2 + b^2 - c^2$

⇒ $a\cos\theta - b\sin\theta = \sqrt{a^2 + b^2 - c^2}$

3. **(a)** $\tan^2A$

Explanation: Given: $\sin^2A + \sin^2A\tan^2A$

= $\sin^2A(1 + \tan^2A)$

= $\sin^2A(\sec^2A)$

= $\sin^2A \times \frac{1}{\cos^2A}$

= $\frac{\sin^2A}{\cos^2A}$

= $\tan^2A$

4.

(d) 1

Explanation: $\sin^6A + \cos^6A + 3\cos^2A\sin^2A$

= $(\sin^2A)^3 + (\cos^2A)^3 + 3\cos^2A\sin^2A$

= $(\sin^2A + \cos^2A)(\sin^4A + \cos^4A - \sin^2A\cos^2A + 3\cos^2A\sin^2A)$

= $(\sin^4A + \cos^4A + 2\cos^2A\sin^2A)$

= $(\sin^2A + \cos^2A)^2$

= $1^2 = 1$

5.

(b) $\frac{1-cos\,\theta}{1+cos\,\theta}$

Explanation: $(\,cosec\theta - \cot\theta)^2 = \left(\frac{1}{\sin\theta} - \frac{\cos\theta}{\sin\theta}\right)^2 = \frac{(1-\cos\theta)^2}{\sin^2\theta} = \frac{(1-\cos\theta)^2}{(1-\cos^2\theta)} = \frac{(1-\cos\theta)}{(1+\cos\theta)}$

6.

(c) cosec θ + cot θ

Explanation: $\sqrt{\frac{1+\cos\theta}{1-\cos\theta}} = \sqrt{\frac{(1+\cos\theta)(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}}$
$= \sqrt{\frac{(1+\cos\theta)^2}{1-\cos^2\theta}} = \sqrt{\frac{(1+\cos\theta)^2}{\sin^2\theta}}$
$= \frac{1+\cos\theta}{\sin\theta} = cosec\theta + \cot\theta$

7.

(b) 2

Explanation: By applying formulae
$\tan\theta = \frac{\sin\theta}{\cos\theta}, \quad \cot\theta = \frac{\cos\theta}{\sin\theta}, \quad \sec\theta = \frac{1}{\cos\theta}, \quad \csc\theta = \frac{1}{\sin\theta}$
$=\left(1 + \frac{\sin\theta}{\cos\theta} + \frac{1}{\cos\theta}\right)\left(1 + \frac{\cos\theta}{\sin\theta} - \frac{1}{\sin\theta}\right)$
$= \left(\frac{1+\cos\theta+\sin\theta}{\cos\theta}\right)\left(\frac{\sin\theta+\cos\theta-1}{\sin\theta}\right)$
Multiplying both terms, we get
$= \frac{\sin\theta+\sin\theta\cos\theta+\sin^2\theta+\cos\theta+\cos^2\theta+\sin\theta\cos\theta-1-\cos\theta-\sin\theta}{\cos\theta\sin\theta}$
$= \frac{\sin^2\theta+\cos^2\theta+2\sin\theta\cos\theta-1}{\cos\theta\sin\theta}$
$= \frac{1+2\sin\theta\cos\theta-1}{\cos\theta\sin\theta}$
$= \frac{2\sin\theta\cos\theta}{\cos\theta\sin\theta}$
= 2
Therefore, (1 + tanθ + secθ)(1 + cotθ - cosecθ) = 2

8.

(b) 0

Explanation: We have, $\left(\frac{\sqrt{3}+2\cos A}{1-2\sin A}\right)^{-3} + \left(\frac{1+2\sin A}{\sqrt{3}-2\cos A}\right)^{-3}$
$= \left(\frac{1-2\sin A}{\sqrt{3}+2\cos A}\right)^3 + \left(\frac{\sqrt{3}-2\cos A}{1+2\sin A}\right)^3$
$= \frac{[(1-2\sin A)(1+2\sin A)]^3+[(\sqrt{3}-2\cos A)(\sqrt{3}+2\cos A)]^3}{[(\sqrt{3}+2\cos A)(1+2\sin A)]^3}$
$= \frac{[1-4\sin^2 A]^3+[3-4\cos^2 A]^3}{(\sqrt{3}+2\cos A)^3(1+2\sin A)^3} = 0$

9.

(d) $\cot^4$A

Explanation: Given: $cosec^4$A - 2 $cosec^2$A + 1

= $(cosec^2$A - 1$)^2$

= $(\cot^2$A$)^2$

= $\cot^4$A

10.

(b) 1

Explanation: Given: $\cos A + \cos^2 A = 1$
$\Rightarrow \cos A = 1 - \cos^2 A$
$\Rightarrow \cos A = \sin^2 A$
Squaring both sides, we get
$\Rightarrow \cos^2 A = \sin^4 A$
$\Rightarrow 1 - \sin^2 A = \sin^4 A$
$\Rightarrow \sin^2 A + \sin^4 A = 1$

11. **(a)** $\frac{m^2-1}{n^2-1}$

Explanation: Given: tanA = n tanB
$\Rightarrow \frac{1}{\tan B} = \frac{n}{\tan A}$
$\Rightarrow \cot B = \frac{n}{\tan A}$
And sin A = m sin B
$\Rightarrow \frac{1}{\sin B} = \frac{m}{\sin A}$
$\Rightarrow$ cosec B = $\frac{n}{\sin A}$

Now, $\text{cosec}^2B - \cot^2B = 1$

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2}{\tan^2 A} = 1$

$\Rightarrow \frac{m^2}{\sin^2 A} - \frac{n^2\cos^2 A}{\sin^2 A} = 1$

$\Rightarrow m^2 - n^2\cos^2A = \sin^2A$

$\Rightarrow m^2 - n^2\cos^2A = 1 - \cos^2A$

$\Rightarrow m^2 - 1 = (n^2 - 1)\cos^2A$

$\Rightarrow \cos^2A = \frac{m^2-1}{n^2-1}$

12.

(d) a^2b^2

Explanation: $x = a \cos \theta$, $y = b \sin \theta$

$bx = ab \cos \theta$ ...(i)

$ay = ab \sin \theta$ ...(ii)

Squaring and adding (i) and (ii) we get,

$b^2x^2 + a^2y^2 = a^2b^2 \cos^2 \theta + a^2b^2 \sin^2 \theta$

$= a^2b^2 (\cos^2 \theta + \sin^2 \theta)$

$= a^2b^2 \times 1$

$= a^2b^2$

13. **(a)** $\frac{m^2-n^2}{m^2+n^2}$

Explanation: Given: $\tan\theta = \frac{m}{n}$

Dividing all the terms of $\frac{m \sin \theta - n \cos \theta}{m \sin \theta + n \cos \theta}$ by $\cos\theta$,

$= \frac{m \tan \theta - n}{m \tan \theta + n}$

$= \frac{m \times \frac{m}{n} - n}{m \times \frac{m}{n} + n}$

$= \frac{m^2-n^2}{m^2+n^2}$

14. **(a)** $\frac{b}{\sqrt{b^2-a^2}}$

Explanation: $\sin \theta = \frac{a}{b}$

$\cos \theta = \sqrt{1 - sin^2\theta}$

$= \sqrt{1 - \frac{a^2}{b^2}}$

$= \sqrt{\frac{b^2-a^2}{b^2}}$

Now,

$\sec \theta = \frac{1}{cos\theta}$

$= \sqrt{\frac{b^2}{b^2-a^2}}$

$= \frac{b}{\sqrt{b^2-a^2}}$

15.

(c) $\frac{17}{4}$

Explanation: $(\cos \theta + \sec \theta)^2 = \frac{25}{4} \Rightarrow \cos^2 \theta + \sec^2 \theta + 2 = \frac{25}{4} \Rightarrow \cos^2 \theta + \sec^2 \theta = \frac{25}{4} - 2 = \frac{17}{4}$

16. **(a)** $\sqrt{2}$

Explanation: We have, $1 + \cot^2\theta = (\sqrt{3 + 2\sqrt{2}} - 1)^2$

$\Rightarrow \text{cosec}^2\theta = (\sqrt{2 + 1 + 2\sqrt{2}} - 1)^2$

$= \left(\sqrt{(\sqrt{2} + 1)^2} - 1\right)^2 = (\sqrt{2} + 1 - 1)^2 = (\sqrt{2})^2$

$\Rightarrow \text{cosec}\theta = \sqrt{2}$

$\Rightarrow$ sinθ = $\frac{1}{\sqrt{2}}$, cosθ = $\frac{1}{\sqrt{2}}$, tanθ = 1

Now, $\frac{1}{\tan\theta} + \frac{\sin\theta}{1+\cos\theta}$ = $\frac{1}{1}$ + $\frac{\frac{1}{\sqrt{2}}}{1+\frac{1}{\sqrt{2}}}$ = $\sqrt{2}$

17.

(c) A is true but R is false.

Explanation: We know that, identity sin$^2\theta$ + cos$^2\theta$ = 1 is true for every real value of θ so sin$^2\theta$ = 1 – cos$^2\theta$ is always true.

18.

(d) A is false but R is true.

Explanation: cos A + cos^{2}A = 1

cos A = 1 - cos^{2}A = sin^{2}A

sin^{2}A + sin^{4}A = cos A + cos^{2}A = 1

sin^{2}A + sin^{4}A = 1

19. **(a)** Both A and R are true and R is the correct explanation of A.

Explanation: $\because$ cosec$^2\theta$ - cot$^2\theta$ = 1

$\Rightarrow$ (cosecθ - cotθ) (cosecθ + cotθ) = 1

$\Rightarrow$ (cosecθ - cotθ) = $\frac{1}{\operatorname{cosec}\theta+\cot\theta}$

$\therefore$ It is clear that cosecθ - cotθ and cosecθ + cotθ are reciprocal of each other.

20. L.H.S = (1 - sin$^2\theta$) sec$^2\theta$

$\Rightarrow$ L.H.S = cos$^2\theta$ sec$^2\theta$ $\left[\because 1-\sin^2\theta=\cos^2\theta\right]$

$\Rightarrow$ L.H.S = $\cos^2\theta\left(\frac{1}{\cos^2\theta}\right)$ = 1 = R.H.S $\left[\because \sec\theta=\frac{1}{\cos\theta} : \sec^2\theta=\frac{1}{\cos^2\theta}\right]$

21. We have

$6\tan^2\theta-\frac{6}{\cos^2\theta}=6\tan^2\theta-6\sec^2\theta=-6\left(\sec^2\theta-\tan^2\theta\right)$

We know that, $\sec^2\theta-\tan^2\theta=1$

Therefore, $6\tan^2\theta-\frac{6}{\cos^2\theta}=-6$

22. LHS = $\frac{\sqrt{\sec A-1}}{\sqrt{\sec A+1}}$ + $\frac{\sqrt{\sec A+1}}{\sqrt{\sec A-1}}$

= $\frac{\sec A-1+\sec A+1}{\sqrt{\sec^2 A-1}}$

= $\frac{2\sec A}{\tan A}$

= 2 cosec A = RHS

23. To prove: $\frac{\sin\theta}{1-\cos\theta}+\frac{\tan\theta}{1+\cos\theta}=\sec\theta\cos ec\theta+\cot\theta$

We have,

$\text{LHS}=\frac{\sin\theta}{1-\cos\theta}+\frac{\tan\theta}{1+\cos\theta}$

$LHS=\frac{\sin\theta(1+\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}+\frac{\tan\theta(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$=\frac{\sin\theta(1+\cos\theta)}{1-\cos^2\theta}+\frac{\tan\theta(1-\cos\theta)}{1-\cos^2\theta}$

$=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}+\frac{\tan\theta(1-\cos\theta)}{\sin^2\theta}$

$=\frac{\sin\theta(1+\cos\theta)}{\sin^2\theta}+\frac{\sin\theta(1-\cos\theta)}{\cos\theta\sin^2\theta}$

$=\frac{1+\cos\theta}{\sin\theta}+\frac{1-\cos\theta}{\cos\theta\sin\theta}$

$=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}+\frac{1}{\cos\theta\sin\theta}-\frac{\cos\theta}{\cos\theta\sin\theta}$

$=\frac{1}{\sin\theta}+\frac{\cos\theta}{\sin\theta}+\frac{1}{\cos\theta\sin\theta}-\frac{1}{\sin\theta}$

$=\cot\theta+\sec\theta\cos ec\theta=RHS$

24. We have,

LHS = cotθ - tanθ

$\Rightarrow \quad \text{LHS}=\frac{\cos\theta}{\sin\theta}-\frac{\sin\theta}{\cos\theta}$

$\Rightarrow \quad \text{LHS}=\frac{\cos^2\theta-\sin^2\theta}{\sin\theta\cos\theta}=\frac{\cos^2\theta-\left(1-\cos^2\theta\right)}{\sin\theta\cos\theta} \quad \left[\because \sin^2\theta=1-\cos^2\theta\right]$

$\Rightarrow \quad \text{LHS}=\frac{\cos^2\theta-1+\cos^2\theta}{\sin\theta\cos\theta}=\frac{2\cos^2\theta-1}{\sin\theta\cos\theta}=\text{RHS}$

25. We have,

LHS = $\tan^2\theta - \frac{1}{\cos^2\theta}$

⇒ LHS = $\tan^2\theta$ - $\sec^2\theta$ $\left[\because \frac{1}{\cos\theta} = \sec\theta \therefore \frac{1}{\cos^2\theta} = \sec^2\theta\right]$

⇒ LHS = - ($\sec^2\theta$ - $\tan^2\theta$) = -1 = RHS

26. L.H.S = $\tan^4\theta$ + $\tan^2\theta$

= $\tan^2\theta$ ($\tan^2\theta$ + 1)

= ($\sec^2\theta$ - 1) ($\sec^2\theta$) = $\sec^4\theta$ - $\sec^2\theta$ = R.H.S

27. L.H.S. = $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \frac{1-\frac{\sin^2\theta}{\cos^2\theta}}{1+\frac{\sin^2\theta}{\cos^2\theta}} = \frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$

= $\cos^2\theta$ - $\sin^2\theta$

28. We know that $\sin^2\theta + \cos^2\theta = 1$

Therefore, $(\sin^2\theta + \cos^2\theta)^3 = 1$

or, $(\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta) = 1$

or, $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

29. To prove: $(1 + \cot^2 A)\sin^2 A = 1$

LHS = $(1 + \cot^2 A)\sin^2 A$

= $\text{cosec}^2 A \sin^2 A$ $[\because 1 + \cot^2 A = \cos ec^2 A]$

$= \frac{1}{\sin^2 A}\sin^2 A\ \left[\because \cos ecA = \frac{1}{\sin A}\right]$

$= 1 = RHS$

30. L.H.S. $= tan^2\theta + cot^2\theta + 2$

$= tan^2\theta + cot^2\theta + 1 + 1$

$= (1 + \tan^2\theta) + (1 + cot^2\theta)$

$(\because sec^2\theta = 1 + tan^2\theta\ and\ cosec^2\theta = 1 + cot^2\theta)$

$= sec^2\theta + cosec^2\theta$

= R.H.S.

31. Given:

$\sin^2\theta\cos^2\theta\left(1+\tan^2\theta\right)\left(1+\cot^2\theta\right) = \lambda$

$\Rightarrow \quad \sin^2\theta\cos^2\theta\sec^2\theta\csc^2\theta = \lambda$

$\Rightarrow \left(\sin^2\theta\csc^2\theta\right) \times \left(\cos^2\theta\sec^2\theta\right) = \lambda$

$\Rightarrow \left(\sin^2\theta \times \frac{1}{\sin^2\theta}\right)\left(\cos^2\theta \times \frac{1}{\cos^2\theta}\right) = \lambda$

$\Rightarrow \lambda = 1 \times 1 = 1$

Hence, the value of λ is 1.

32. We have,

L.H.S. = $\frac{1}{(\ cosec\theta - \cot\theta)} = \frac{1}{(\ cosec\theta - \cot\theta)} \times \frac{(\ cosec\theta + \cot\theta)}{(\ cosec\theta + \cot\theta)}$

$= \frac{(\ cosec\theta + \cot\theta)}{(cosec^2\theta - \cot^2\theta)} = \ cosec\theta + \cot\theta$ = R.H.S. [∴ $\text{cosec}^2\theta$ - $\cot^2\theta$ = 1]

∴ L.H.S. = R.H.S.

33. Given expression = $\sin^2\theta$ + $\cos^2\theta$

= 1

34. LHS = $\sin^2\theta + \frac{1}{1+\tan^2\theta}$ [∵ 1 + $\tan^2\theta$ = $\sec^2\theta$]

$= \sin^2\theta + \frac{1}{\sec^2\theta}$

= $\sin^2\theta$ + $\cos^2\theta$ = 1 = RHS

Hence Proved

35. LHS = $\sec^2\theta$ + $\text{cosec}^2\theta$

$= \frac{1}{\cos^2\theta} + \frac{1}{\sin^2\theta} = \frac{\sin^2\theta+\cos^2\theta}{\cos^2\theta\sin^2\theta}$

$= \frac{1}{\cos^2\theta\sin^2\theta}$ [∵ $\sin_2\theta$ + $\cos_2\theta$ = 1]

= $\sec^2\theta \text{cosec}^2\theta$ = R.H.S.

Hence Proved.

36. We have,

LHS = $\cos^2\theta + \frac{1}{1+cot^2\theta}$

$\Rightarrow$ LHS = $\cos^2\theta + \frac{1}{cosec^2\theta}$ [$\because 1 + \cot^2\theta = \text{cosec}^2\theta$]

$\Rightarrow$ LHS = $\cos^2\theta + \sin^2\theta = 1$ = RHS $\left[\because \frac{1}{cosec\theta} = \sin\theta\right]$

37. To prove: $\frac{1}{\sec\theta - \tan\theta} = \sec\theta + \tan\theta$

We have,

$\text{LHS} = \frac{1}{\sec\theta - \tan\theta}$

$= \frac{1}{\sec\theta - \tan\theta} \times \frac{\sec\theta + \tan\theta}{\text{sed}\,\theta + \tan\theta}$

$= \frac{\sec\theta + \tan\theta}{\sec^2\theta - \tan^2\theta} = \frac{\sec\theta + \tan\theta}{1}$ [$\because \sec^2\theta - \tan^2\theta = 1$]

$= sec\theta + tan\theta = RHS$

38. L.H.S. = $(\sin^4\theta - \cos^4\theta + 1)\,\text{cosec}^2\theta$

= $[(\sin^2\theta - \cos^2\theta)(\sin^2\theta + \cos^2\theta) + 1]\,\text{cosec}^2\theta$

= $(\sin^2\theta - \cos^2\theta + 1)\,\text{cosec}^2\theta$ [Because $\sin^2\theta + \cos^2\theta = 1$]

= $2\sin^2\theta\,\text{cosec}^2\theta$ [Because $1 - \cos^2\theta = \sin^2\theta$]

= 2 = RHS

39. We have,

LHS = $\cot^2\theta - \frac{1}{\sin^2\theta}$

$\Rightarrow \quad \text{LHS} = \cot^2\theta - \ cosec^2\theta \quad [\because \frac{1}{\sin\theta} = \ cosec\theta \therefore \frac{1}{\sin^2\theta} = \ cosec^2\theta]$

$\Rightarrow$ LHS = $-(\text{cosec}^2\theta - \cot^2\theta) = -1$ = RHS

Section B

40. Fill in the blanks:

(i) 1. -1

(ii) 1. 1

Section C

41. State True or False:

(i) **(a)** True

Explanation: True

Given: $\cos A + \cos^2 A = 1$

$\Rightarrow \cos A = 1 - \cos^2 A$

$\Rightarrow \cos A = \sin^2 A$ …(i)

Squaring both sides,

we get $\cos^2 A = \sin^4 A$ …(ii)

So, adding eq. (i) and eq. (ii), we get

$\sin^2 A + \sin^4 A = \cos A + \cos^2 A$

$\therefore \sin^2 A + \sin^4 A = 1$

(ii) **(b)** False

Explanation: False

L.H.S= $(\tan\theta + 2)(2\tan\theta + 1)$

= $2\tan^2\theta + \tan\theta + 4\tan\theta + 2$

= $2\tan^2\theta + 5\tan\theta + 2$

= $2(\sec^2\theta - 1) + 5\tan\theta + 2$

($\because \sec^2\theta - \tan^2\theta = 1 \Rightarrow \tan^2\theta = \sec^2\theta - 1$)

= $2\sec^2\theta - 2 + 5\tan\theta + 2$

$= 5\tan\theta + 2\sec^2\theta \neq$ R.H.S

$\therefore$ L.H.S $\neq$ R.H.S

(iii) **(a)** True

Explanation: True

LHS: $\sqrt{((1-\cos^2\theta)\sec^2\theta}\Big)$

$= \sqrt{\sin^2\theta\sec^2\theta}$

$\left(\because \sin^2\theta + \cos^2\theta = 1 \Rightarrow \sin^2\theta = 1 - \cos^2\theta\right)$

$= \sqrt{\frac{\sin^2\theta}{\cos^2\theta}} \left(\text{since}, \sec^2\theta = \frac{1}{\cos^2\theta}\right)$

$= \frac{\sin\theta}{\cos\theta}$

$= \tan\theta$

= RHS

(iv) **(b)** False

Explanation: False. The value of (sinθ + cosθ) for θ = 0° is 1

Section D

42. Given, $cosec\theta + cot\theta = p$

Squaring the above equation,

$p^2 = (cosec\theta + cot\theta)^2$

RHS= $\frac{p^2-1}{p^2+1}$

$= \frac{(\text{cosec}\theta+\cot\theta)^2-1}{(\text{cosec}\theta+\cot\theta)^2+1}$

$= \frac{\text{cosec}^2\theta+\cot^2\theta+2\text{cosec}\theta\cot\theta-1}{\text{cosec}^2\theta+\cot^2\theta+2\text{cosec}\theta\cot\theta+1}$

$= \frac{1+\cot^2\theta+\cot^2\theta+2\text{cosec}\theta\cot\theta-1}{\text{cosec}^2\theta+\text{cosec}^2\theta-1+2\text{cosec}\theta\cot\theta+1}$

$= \frac{2\cot\theta(\cot\theta+\text{cosec}\theta)}{2\text{cosec}\theta(\text{cosec}\theta+\cot\theta)}$

$= \frac{\cos\theta}{\sin\theta} \times \sin\theta$

$= \cos\theta$

= LHS

43. LHS$= \left(\frac{1}{\sec^2\theta-\cos^2\theta} + \frac{1}{\cos ec^2\theta-\sin^2\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{1}{\frac{1}{\cos^2\theta}-\cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta}-\sin^2\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{1}{\frac{1-\cos^4\theta}{\cos^2\theta}} + \frac{1}{\frac{1-\sin^4\theta}{\sin^2\theta}}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta}{1-\cos^4\theta} + \frac{\sin^2\theta}{1-\sin^4\theta}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta(1-\sin^4\theta)+\sin^2\theta(1-\cos^4\theta)}{(1-\cos^4\theta)(1-\sin^4\theta)}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta(1-\sin^2\theta)(1+\sin^2\theta)+\sin^2\theta(1-\cos^2\theta)(1+\cos^2\theta)}{(1-\cos^2\theta)(1+\cos^2\theta)(1-\sin^2\theta)(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^2\theta\cdot\cos^2\theta(1+\sin^2\theta)+\sin^2\theta\sin^2\theta(1+\cos^2\theta)}{\sin^2\theta(1+\cos^2\theta)\cos^2\theta(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \left(\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{\sin^2\theta\cos^2\theta(1+\cos^2\theta)(1+\sin^2\theta)}\right)\sin^2\theta\cos^2\theta$

$= \left(\frac{\cos^4\theta(1+\sin^2\theta)+\sin^4\theta(1+\cos^2\theta)}{(1+\cos^2\theta)(1+\sin^2\theta)}\right)$

$= \left(\frac{\cos^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta+\sin^4\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}\right)$

$= \left(\frac{\cos^4\theta+\sin^4\theta+\cos^4\theta\sin^2\theta+\sin^4\theta\cos^2\theta}{(1+\cos^2\theta)(1+\sin^2\theta)}\right)$

$= \frac{(\cos^2\theta+\sin^2\theta)^2-2\cos^2\theta\sin^2\theta+\cos^2\theta\sin^2\theta(\cos^2\theta+\sin^2\theta)}{(1+\cos^2\theta)(1+\sin^2\theta)}$

$= \frac{1-2\cos^2\theta\sin^2\theta+\cos^2\theta\sin^2\theta\times 1}{1+\sin^2\theta+\cos^2\theta+\cos^2\theta\sin^2\theta} \left[\sin^2\theta + \cos^2\theta = 1\right]$

$= \frac{1-\cos^2\theta\sin^2\theta}{1+1+\cos^2\theta\sin^2\theta}$
$= \frac{1-\cos^2\theta\sin^2\theta}{2+\cos^2\theta\sin^2\theta}$
= RHS

Hence proved

44. Given, $\frac{cos\theta}{1-sin\theta}$ + $\frac{cos\theta}{1+sin\theta}$ = 4.........(1)

Simplifying L.H.S :-
$\frac{cos\theta}{1-sin\theta}$ + $\frac{cos\theta}{1+sin\theta}$
= $\frac{\cos\theta(1+\sin\theta)+\cos\theta(1-\sin\theta)}{(1-\sin\theta)(1+\sin\theta)}$
= $\frac{\cos\theta((1-\sin\theta)+(1+\sin\theta))}{1-\sin^2\theta}$
= $\frac{\cos\theta(1-\sin\theta+1+\sin\theta)}{1-\sin^2\theta}$
= $\frac{2\cos\theta}{1-\sin^2\theta}$
= $\frac{2cos\theta}{cos^2\theta}$ [Since,$\sin^2$A + $\cos^2$A = 1]
= $\frac{2}{cos\theta}$

Hence , we get :-
$\frac{cos\theta}{1-sin\theta} + \frac{cos\theta}{1+sin\theta} = \frac{2}{cos\theta}$
$\Rightarrow 4 = \frac{2}{cos\theta}$ [from (1)]
$\Rightarrow$ $\frac{cos\theta}{2}$ = $\frac{1}{4}$
Or, cos θ = $\frac{1}{2}$
$\Rightarrow$ $cos\theta$ = cos($\frac{\pi}{3}$)
$\therefore$ θ = $\frac{\pi}{3}$, when $0 \leqslant \theta \leqslant \pi$.

In degree, θ = 60^o when $0^o \leqslant \theta \leqslant 180^o$.

45. Given,

cos A - sin A = m

$\Rightarrow$ (cos A - sin A)^2 = m^2

$\Rightarrow$ cos^2 A + sin^2 A - 2 cos A sin A = m^2

$\Rightarrow$ 1 -2 cos A sin A = m^2 ...(i)

Also given,

cos A + sin A = n

$\Rightarrow$ (cos A + sin A)^2 = n^2

$\Rightarrow$ cos^2 A + sin^2 A + 2 cos A sin A= n^2

$\Rightarrow$ 1 + 2 cos A sin A = n^2 ...(ii)

Adding (i) & (ii), we get :-

(1 - 2cosAsinA) + (1 + 2cosAsinA) = m^2 + n^2

$\Rightarrow$ m^2 + n^2 = 2 .....(iii)

Similarly, on subtracting equation (ii) from (i) we get :-

- 4 cos A sin A = m^2 - n^2 ...(iv)

Now, L.H.S.
= $\frac{m^2-n^2}{m^2+n^2}$
$= \frac{-4\cos A\sin A}{2}$ [from (iii) & (iv)]
= - 2 sin A cos A
So, $\frac{m^2-n^2}{m^2+n^2} = -2SinACosA$ ......(v)
Now,

-2sinAcosA

$= \frac{-2\sin A\cos A}{1}$

$= \frac{-2\sin A\cdot\cos A}{\sin^2 A+\cos^2 A}$ (∵ $\sin^2$ A + $\cos^2$ A = 1)

$= \frac{\frac{-2\sin A\cos A}{\sin A\cos A}}{\frac{\sin^2 A+\cos^2 A}{\sin A\cdot\cos A}}$

$= \frac{-2}{\frac{\sin^2 A}{\sin A\cdot\cos A}+\frac{\cos^2 A}{\sin A\cdot\cos A}}$

$= \frac{-2}{\tan A+\cot A}$

So, -2sinAcosA $= \frac{-2}{\tan A+\cot A}$ ......(vi)

Now, from (v) & (vi) ,

$\frac{m^2-n^2}{m^2+n^2}$ = - 2 sin A . cos A $= \frac{-2}{\tan A+\cot A}$ Hence, Proved.

46. LHS

$= \frac{\sin A}{\sec A+\tan A-1} + \frac{\cos A}{cosecA+\cot A-1}$

$= \frac{\sin A}{\frac{1}{\cos A}+\frac{\sin A}{\cos A}-1} + \frac{\cos A}{\frac{1}{\sin A}+\frac{\cos A}{\sin A}-1}$

$= \frac{\sin A}{\frac{1+\sin A-\cos A}{\cos A}} + \frac{\cos A}{\frac{1+\cos A-\sin A}{\sin A}}$

$= \frac{\sin A\cos A}{1+\sin A-\cos A} + \frac{\sin A\cos A}{1+\cos A-\sin A}$

= sin A cos A $\left[\frac{1}{1+\sin A-\cos A} + \frac{1}{1+\cos A-\sin A}\right]$

= sin A. cos A $\left[\frac{1+\cos A-\sin A+1+\sin A-\cos A}{(1+\sin A-\cos A)(1+\cos A-\sin A)}\right]$

= sin A. cos A $\left[\frac{2}{(1+\sin A-\cos A)(1+\cos A-\sin A)}\right]$

$= \left[\frac{2\sin A\cdot\cos A}{\{1+(\sin A-\cos A)\}\{1-(\sin A-\cos A)\}}\right]$

$= \frac{2\sin A\cdot\cos A}{(1)^2-(\sin A-\cos A)^2}$

$= \frac{2\sin A\cdot\cos A}{1-\left(\sin^2 A+\cos^2 A-2\sin A\cdot\cos A\right)}$

$= \frac{2\sin A\cdot\cos A}{1-(1-2\sin A\cdot\cos A)}$ [Since, Sin^2A + Cos^2A = 1]

$= \frac{2\sin A\cdot\cos A}{2\sin A\cdot\cos A}$

= 1

= RHS. Hence proved

47. We have,

$\text{LHS} = \frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$

$\Rightarrow \quad \text{LHS} = \frac{\tan A}{1-\frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1-\tan A}$

$\Rightarrow \quad \text{LHS} = \frac{\tan A}{\frac{\tan A-1}{\tan A}} + \frac{1}{\tan A(1-\tan A)}$

$\Rightarrow \quad \text{LHS} = \frac{\tan^2 A}{\tan A-1} + \frac{1}{\tan A(1-\tan A)}$

$\Rightarrow \quad \text{LHS} = \frac{\tan^2 A}{\tan A-1} - \frac{1}{\tan A(\tan A-1)}$

$\Rightarrow \quad \text{LHS} = \frac{\tan^3 A-1}{\tan A(\tan A-1)}$ [Taking LCM]

$\Rightarrow \quad \text{LHS} = \frac{(\tan A-1)\left(\tan^2 A+\tan A+1\right)}{\tan A(\tan A-1)}$ [∵ $a^3 - b^3 = (a - b)(a^2 + ab + b^2)$]

$\Rightarrow \quad \text{LHS} = \frac{\tan^2 A+\tan A+1}{\tan A}$

$\Rightarrow \quad \text{LHS} = \frac{\tan^2 A}{\tan A} + \frac{\tan A}{\tan A} + \frac{1}{\tan A}$

⇒ LHS = tanA + 1 + cotA [since (1/tanA) =cotA].

= (1 + tanA + cotA)

$\therefore \quad \frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ = 1 + tanA + cotA ....(1)

Now, 1 + tanA + cotA

= 1 + $\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}$

$= 1 + \frac{\sin^2 A+\cos^2 A}{\sin A \cos A} = 1 + \frac{1}{\sin A \cos A}$ [∵Sin^2A + Cos^2 A = 1]

= 1 + cosecAsecA

So, 1 + tanA + cotA = 1+ cosecAsecA.......(2)

From (1) and (2), we obtain

$\frac{\tan A}{1-\cot A} + \frac{\cot A}{1-\tan A}$ = 1 + tanA + cotA = 1 + cosecAsecA

48. Given, 2 cosθ - sin$\theta = x$ and

cosθ - 3 sin$\theta = y$

Put the values of x and y in $2x^2 + y^2 - 2xy$ (LHS) , we get

= 2(2 cosθ − sinθ)^2 + (cosθ − 3 sinθ)^2 − 2(2 cosθ − sinθ)(cosθ − 3 sinθ)

= 2(4cos^2 θ− 4cosθ sinθ + sin$^2\theta$) + (cos^2 θ− 6cosθ sinθ + 9sin^2 θ) − 2(2cos^2 θ− 7cosθ sinθ + 3sin^2 θ)

= 8cos$^2\theta$ − 8cosθ sinθ + 2sin$^2\theta$ + cos$^2\theta$ − 6cosθ sinθ + 9sin$^2\theta$ − 2(2cos^2 θ− 7cosθ sinθ + 3sin^2 θ)

= 8cos$^2\theta$ − 8cosθ sinθ + 2sin$^2\theta$ + cos$^2\theta$ − 6cosθ sinθ + 9sin$^2\theta$ − 4cos$^2\theta$ + 14cosθ sinθ − 6sin$^2\theta$

= 5cos$^2\theta$ + 5sin$^2\theta$

= 5(cos$^2\theta$ + sin$^2\theta$)

$= 5$

= RHS

49. LHS = $\sqrt{1 + \tan^2\theta + 1 + \cot^2\theta}$

= $\sqrt{\tan^2\theta + \cot^2\theta + 2 \times \tan\theta \times \cot\theta}$

= $\sqrt{(\tan\theta + \cot\theta)^2}$

= tanθ + cotθ = RHS

50. We know that,

$Sec^2\theta - tan^2\theta = 1$

$(sec\theta + tan\theta)(sec\theta - tan\theta) = 1$

$(p)(sec\theta - tan\theta) = 1$

$sec\theta - tan\theta = \frac{1}{p}$ Hence proved

$sec\theta + tan\theta = p$ ....(i) and $sec\theta - tan\theta = \frac{1}{p}$ ....(ii)

Add equation i and ii

$sec\theta + tan\theta + sec\theta - \tan\theta = p + \frac{1}{p}$

$2sec\theta = \frac{p^2+1}{p}$

$\frac{1}{cos\theta} = \frac{p^2+1}{2p}$

$cos\theta = \frac{2p}{p^2+1}$

Now, $sin\theta = \sqrt{1 - cos^2\theta}$

$sin\theta = \sqrt{1 - (\frac{2p}{p^2+1})^2}$

$sin\theta = \sqrt{1 - \frac{4p^2}{1+p^4+2p^2}}$

$sin\theta = \sqrt{\frac{1+p^4-2p^2}{1+p^4+2p^2}}$

$sin\theta = \sqrt{(\frac{1-p^2}{1+p^2})^2}$

$sin\theta = \frac{1-p^2}{1+p^2}$

51. We have, $asin\theta + bcos\theta = c$

On squaring both sides, we get

$(asin\theta + bcos\theta)^2 = c^2$

(a sin θ)^2 + (b cos θ)^2 + 2(a sin θ) (b cos θ) = c^2

⇒ a^2 sin^2 θ + b^2 cos^2 θ + 2ab sin θ cos θ = c^2

⇒ a^2(1 – cos^2 θ) + b^2 (1 – sin^2 θ) + 2 ab sin θ cos θ = c^2 [$\because sin^2\theta + cos^2\theta = 1$]

⇒ a^2 – a^2 cos^2 θ + b^2 – b^2 sin^2 θ + 2ab sin θ cos θ = c^2

⇒ –a^2 cos^2 θ – b^2 sin^2 θ + 2ab sin θ cos θ = c^2 – a^2 – b^2

Taking Negative common,

⇒ $a^2 \cos^2 θ + b^2 \sin^2 θ – 2ab \sin θ \cos θ = a^2 + b^2 – c^2$

⇒ $(a \cos θ)^2 + (b \sin θ)^2 – 2(a \cos θ) (b \sin θ) = a^2 + b^2 – c^2$

⇒ $(acos\theta - bsin\theta)^2 = a^2 + b^2 - c^2$

⇒$acos\theta - bsin\theta = \pm\sqrt{a^2 + b^2 - c^2}$

Hence proved, $acos\theta - bsin\theta = \sqrt{a^2 + b^2 - c^2}$

52. LHS $= \left(\frac{1}{\sec^2\theta - \cos^2\theta} + \frac{1}{cosec^2\theta - \sin^2\theta}\right) \sin^2\theta . \cos^2\theta$

$= \left(\frac{1}{\frac{1}{\cos^2\theta} - \cos^2\theta} + \frac{1}{\frac{1}{\sin^2\theta} - \sin^2\theta}\right) \sin^2\theta \cos^2\theta$

$= \left(\frac{\cos^2\theta}{1 - \cos^4\theta} + \frac{\sin^2\theta}{1 - \sin^4\theta}\right) \sin^2\theta \cos^2\theta$

$= \left(\frac{\cos^2\theta}{(1 - \cos^2\theta)(1 + \cos^2\theta)} + \frac{\sin^2\theta}{(1 - \sin^2\theta)(1 + \sin^2\theta)}\right) \sin^2\theta \cos^2\theta$

$= \left(\frac{\cos^2\theta}{\sin^2\theta(1 + \cos^2\theta)} + \frac{\sin^2\theta}{\cos^2\theta \cdot (1 + \sin^2\theta)}\right) \sin^2\theta \cos^2\theta$ [Since, $\text{Sin}^2A + \text{Cos}^2A=1$]

$= \left(\frac{\cos^4\theta(1 + \sin^2\theta) + \sin^4\theta(1 + \cos^2\theta)}{\sin^2\theta(1 + \cos^2\theta) \cos^2\theta(1 + \sin^2\theta)}\right) \sin^2\theta \cos^2\theta$

$= \frac{\cos^4\theta + \cos^4\theta \sin^2\theta + \sin^4\theta + \sin^4\theta \cos^2\theta}{(1 + \cos^2\theta)(1 + \sin^2\theta)}$

$= \frac{\cos^4\theta + \cos^4\theta \sin^2\theta + \sin^4\theta + \sin^4\theta \cos^2\theta}{(1 + \cos^2\theta)(1 + \sin^2\theta)}$

$= \frac{\cos^4\theta + \cos^2\theta \sin^2\theta(\sin^2\theta + \cos^2\theta) + \sin^4\theta}{(1 + \cos^2\theta)(1 + \sin^2\theta)}$

$= \frac{\cos^4\theta + \cos^2\theta \sin^2\theta + \sin^4\theta}{(1 + \cos^2\theta)(1 + \sin^2\theta)}$

$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta + \sin^2\theta\cos^2\theta}{1 + 1 + \sin^2\theta\cos^2\theta}$

$= \frac{1 - \sin^2\theta \cdot \cos^2\theta}{2 + \sin^2\theta \cdot \cos^2\theta}$ = RHS

Hence, Proved.

53. $L.H.S = [(sin\theta + cos\theta) + 1][(sin\theta + cos\theta) - 1]sec\theta.cosec\theta$

$= [(sin\theta + cos\theta)^2 - 1]\frac{1}{cos\theta sin\theta}$

$= [sin^2\theta + cos^2\theta + 2sin\theta cos\theta - 1]\frac{1}{cos\theta sin\theta}$

$= [1 + 2sin\theta cos\theta - 1]\frac{1}{cos\theta sin\theta}$

$= 2sin\theta cos\theta\frac{1}{cos\theta sin\theta}$

$= 2$

$= R.H.S$

54. **L.H.S** = $(1 - \sin\theta + \cos\theta)^2$

$Using\, identity\ (a + b + c)^2 = a^2 + b^2 + c^2 + 2ab + 2bc + 2ac$

= $1 + sin^2\theta + cos^2\theta - 2sin\theta - 2sin\theta cos\theta + 2cos\theta$

= $1 + 1 - 2sin\theta - 2sin\theta cos\theta + 2cos\theta$

= $2 + 2cos\theta - 2sin\theta - 2sin\theta cos\theta$

=$2(1 + cos\theta) - 2sin\theta(1 + cos\theta)$

= $(1 + cos\theta)(2 - 2sin\theta)$

=$2(1 + cos\theta)(1 - sin\theta)$

= R.H.S

55. Given, $a\cos\theta + b\sin\theta = m$ and $a\sin\theta - b\cos\theta = n$

Squaring both sides, we get

$(acos\theta + bsin\theta)^2 = m^2\ and\ (asin\theta - bcos\theta)^2 = n^2$

$m^2 = a^2cos^2\theta + b^2sin^2\theta + 2absin\theta cos\theta$....(i)

and $n^2 = a^2sin^2\theta + b^2cos^2\theta - 2sin\theta cos\theta$...(ii)

Adding equations (i) and (ii),

$m^2 + n^2 = a^2(\cos^2\theta + \sin^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$= a^2(1) + b^2(1)$

= $a^2 + b^2$

= RHS.

56. $x = \gamma\ cos\alpha\ .\ sin\ \beta$

$\Rightarrow x^2 = \gamma^2 . \cos^2 \alpha . \sin^2\beta$ ...(i)

$y = \gamma\ cos\ \alpha . cos\ \beta$

$\Rightarrow y^2 = \gamma^2 . \cos^2 \alpha . \cos^2 \beta$ ...(ii)

$z = \gamma . \sin \alpha$

$\Rightarrow z^2 = \gamma^2 , \sin^2 \alpha$ ...(iii)

By Adding (i), (ii) and (iii), we get

$x^2 + y^2 + z^2 = \gamma^2 \cos^2 \alpha . \sin^2 \beta + \gamma^2 \cos^2 \alpha . \cos^2 \beta + \gamma^2 \sin^2 \alpha$

$\Rightarrow x^2 + y^2 + z^2 = \gamma^2 \cos^2 \alpha (\sin^2 \beta + \cos^2 \beta) + \gamma^2 \sin^2 \alpha$

$\Rightarrow x^2 + y^2 + z^2 = \gamma^2 \cos^2 \alpha . 1 + \gamma^2 . \sin^2 \alpha$

$= \gamma^2 (\cos^2 \alpha + \sin^2 \alpha)$

$= \gamma^2$

Therefore, $x^2 + y^2 + z^2 = \gamma^2$

57. LHS $= \frac{\tan^3\theta}{1+\tan^2\theta} + \frac{\cot^3\theta}{1+\cot^2\theta}$

$= \frac{\tan^3\theta}{\sec^2\theta} + \frac{\cot^3\theta}{\cos ec^2\theta} \begin{bmatrix} \because 1 + \tan^2\theta = \sec^2\theta \\ 1 + \cot^2\theta = \cos ec^2\theta \end{bmatrix}$

$= \frac{\frac{\frac{\sin^3\theta}{\cos^3\theta}}{1}}{\cos^2\theta} + \frac{\frac{\frac{\cos^3\theta}{\sin^3\theta}}{1}}{\sin^2\theta} \begin{bmatrix} \because \tan\theta = \dfrac{\sin\theta}{\cos\theta}, \sec\theta = \dfrac{1}{\cos\theta} \\ \cot\theta = \dfrac{\cos\theta}{\sin\theta}, \cos ec\theta = \dfrac{1}{\sin\theta} \end{bmatrix}$

$= \frac{\sin^3\theta}{\cos^3\theta} \times \frac{\cos^2\theta}{1} + \frac{\cos^3\theta}{\sin^3\theta} \times \frac{\sin^2\theta}{1}$

$= \frac{\sin^3\theta}{\cos^2\theta \cos\theta} \times \frac{\cos^2\theta}{1} + \frac{\cos^3\theta}{\sin^2\theta \times \sin\theta} \times \frac{\sin^2\theta}{1}$

$= \frac{\sin^3\theta}{\cos\theta} + \frac{\cos^3\theta}{\sin\theta}$

$= \frac{\sin^4\theta + \cos^4\theta}{\cos\theta \sin\theta}$

$= \frac{(\sin^2\theta)^2 + (\cos^2\theta)^2}{\cos\theta \sin\theta}$

$= \frac{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta}{\cos\theta \sin\theta} \left[\because a^2 + b^2 = (a+b)^2 - 2ab\right]$

$= \frac{1 - 2\sin^2\theta\cos^2\theta}{\cos\theta \sin\theta} \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \frac{1}{\cos\theta \sin\theta} - \frac{2\sin^2\theta\cos^2\theta}{\cos\theta \sin\theta}$

$= \frac{1}{\cos\theta} \frac{1}{\sin\theta} - 2\sin\theta\cos\theta$

$= \sec\theta \cos ec\theta - 2\sin\theta\cos\theta \left[\because \frac{1}{\cos\theta} = \sec\theta, \frac{1}{\sin\theta} = \cos ec\theta\right]$

= RHS

Hence proved.

58. We have,

$\text{LHS} = 2\sec^2\theta - \sec^4\theta - 2\ cosec^2\theta + cosec^4\theta$

$\Rightarrow \quad \text{LHS} = \left(cosec^4\theta - 2cosec^2\theta\right) - \left(\sec^4\theta - 2\sec^2\theta\right)$

$\Rightarrow \quad \text{LHS} = \left(cosec^4\theta - 2\ cosec^2\theta + 1\right) - \left(\sec^4\theta - 2\sec^2\theta + 1\right)$

$\Rightarrow \quad LHS = \left(cosec^2\theta - 1\right)^2 - \left(\sec^2\theta - 1\right)^2$

$\Rightarrow \quad \text{LHS} = \left(\cot^2\theta\right)^2 - \left(\tan^2\theta\right)^2$

$\Rightarrow \quad \text{LHS} = \cot^4\theta - \tan^4\theta = \text{RHS}$

Hence,proved LHS = RHS

59. We have, $\left[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}\right]^2$

$= \left[\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta} \times \frac{1+\sin\theta-\cos\theta}{1+\sin\theta-\cos\theta}\right]^2$

$= \left[\frac{(1+\sin\theta-\cos\theta)^2}{(1+\sin\theta)^2 - \cos^2\theta}\right]^2$

$= \left[\frac{(1)^2+\sin^2\theta+(-\cos\theta)^2+2\times1\times\sin\theta+2\times\sin\theta(-\cos\theta)+2(-\cos\theta)\times1}{(1)^2+\sin^2\theta+2\sin\theta-\cos^2\theta}\right]^2$

$= \left[\frac{1+\sin^2\theta+\cos^2\theta+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{1-\cos^2\theta+\sin^2\theta+2\sin\theta}\right]^2$

$= \left[\frac{1+1+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{\sin^2\theta+\sin^2\theta+2\sin\theta}\right]^2 \quad [\mathrm{Sin}\,ce,\ \sin^2\theta+\cos^2\theta=1]$

$= \left[\frac{2+2\sin\theta-2\sin\theta\cos\theta-2\cos\theta}{2\sin^2\theta+2\sin\theta}\right]^2$

$= \left[\frac{2(1+\sin\theta)-2\cos\theta(\sin\theta+1)}{2\sin\theta(\sin\theta+1)}\right]^2$

$= \left[\frac{(1+\sin\theta)(2-2\cos\theta)}{2\sin\theta(\sin\theta+1)}\right]^2$

$= \left[\frac{2-2\cos\theta}{2\sin\theta}\right]^2$

$= \left[\frac{2(1-\cos\theta)}{2\sin\theta}\right]^2$

$= \left[\frac{1-\cos\theta}{\sin\theta}\right]^2$

$= \frac{(1-\cos\theta)^2}{\sin^2\theta}$

$= \frac{(1-\cos\theta)^2}{1-\cos^2\theta}$

$= \frac{(1-\cos\theta)^2}{(1)^2-\cos^2\theta}$

$= \frac{(1-\cos\theta)^2}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{(1-\cos\theta)(1-\cos\theta)}{(1+\cos\theta)(1-\cos\theta)}$

$= \frac{1-\cos\theta}{1+\cos\theta} = RHS$

60. Given, cosec θ + cot θ = p...(i)

We know that, $cosec^2\theta - cot^2\theta = 1$

$\Rightarrow (cosec\theta + cot\theta)(cosec\theta - cot\theta) = 1$

$\Rightarrow p(cosec\theta - cot\theta) = 1$

$\Rightarrow cosec\theta - cot\theta = \frac{1}{p}$ ....(ii)

Adding i and ii, we get

$2cosec\theta = p + \frac{1}{p}$

$cosec\theta = \frac{p^2+1}{2p}$

$\Rightarrow sin\theta = \frac{1}{cosec\theta} = \frac{2p}{p^2+1}$

We know that,$cos\theta = \sqrt{1-sin^2\theta} = \sqrt{1-\frac{4p^2}{(p^2+1)^2}} = \sqrt{\frac{p^4+1-2p^2}{(p^2+1)^2}}$

$cos\theta = \sqrt{\frac{(p^2-1)^2}{(p^2+1)^2}} = \frac{p^2-1}{p^2+1}$

61. LHS = (sin A + sec A)^2 + (cos A + cosec A)^2

$= \left(\sin A + \frac{1}{\cos A}\right)^2 + \left(\cos A + \frac{1}{\sin A}\right)^2$

$= \sin^2 A + \frac{1}{\cos^2 A} + 2\frac{\sin A}{\cos A} + \cos^2 A + \frac{1}{sin^2 A} + 2\frac{cos A}{sin A}$

$= sin^2 A + cos^2 A + \frac{1}{\sin^2 A} + \frac{1}{\cos^2 A} + 2\left(\frac{\sin A}{\cos A} + \frac{\cos A}{\sin A}\right)$

$= 1 + \frac{\sin^2 A+\cos^2 A}{\sin^2 A\cos^2 A} + 2\left(\frac{\sin^2 A+\cos^2 A}{\sin A\cos A}\right)$

$= 1 + \frac{1}{\sin^2 A\cos^2 A} + \frac{2}{\sin A\cos A}$

$= \left(1 + \frac{1}{\sin A\cos A}\right)^2$

$= (1 + secAcosecA)^2$ = RHS

62. LHS $= \frac{\cot^2 A(\sec A-1)}{1+\sin A}$

$= \frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1}{\cos A}-1\right)}{1+\sin A}$

$= \frac{\frac{\cos^2 A}{\sin^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$

$= \frac{\frac{\cos A \times \cos A}{1-\cos^2 A}\left(\frac{1-\cos A}{\cos A}\right)}{1+\sin A}$ $\left[\because \sin^2 A + \cos^2 A = 1\right]$

$= \frac{\frac{\cos A}{(1)^2-\cos^2 A}(1-\cos A)}{1+\sin A}$

$= \frac{\frac{\cos A}{(1+\cos A)(1-\cos A)}(1-\cos A)}{(1+\sin A)}$

$= \frac{\cos A}{(1+\cos A)(1+\sin A)}$

$\text{RHS} = \sec^2 A\left[\frac{1-\sin A}{1+\sec A}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{1+\frac{1}{\cos A}}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{1-\sin A}{\frac{\cos A+1}{\cos A}}\right]$

$= \frac{1}{\cos^2 A}\left[\frac{(1-\sin A)\cos A}{(1+\cos A)}\right]$

$= \frac{1}{\cos A \times \cos A}\left[\frac{(1-\sin A)\cos A}{(1+\cos A)}\right]$

$= \frac{1-\sin A}{\cos A(1+\cos A)}$

Multiplying numerator and denominator by (1 + sinA)

$= \frac{(1-\sin A)}{\cos A(1+\cos A)} \times \frac{(1+\sin A)}{1+\sin A}$

$= \frac{(1)^2-\sin^2 A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos^2 A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos A \times \cos A}{\cos A(1+\cos A)(1+\sin A)}$

$= \frac{\cos A}{(1+\cos A)(1+\sin A)}$

$\therefore$ LHS = RHS

63. We have, $\cos ec\ \theta - \sin\theta = a^3$

$\frac{1}{\sin\theta} - \sin\theta = a^3$

$\frac{1-\sin^2\theta}{\sin\theta} = a^3$

$\frac{\cos^2\theta}{\sin\theta} = a^3$ $\left[\because \sin^2\theta + \cos^2 = 1\right]$

$\frac{\cos^{\frac{2}{3}}\theta}{\sin^{\frac{1}{3}}} = a$

$\Rightarrow a^2 = \frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta}$

$\sec\theta - \cos\theta = b^3$

$\frac{1}{\cos\theta} - \cos\theta = b^3$

$\frac{1-\cos^2\theta}{\cos\theta} = b^3$

$\frac{\sin^2\theta}{\cos\theta} = b^3$

$\frac{\sin^{\frac{2}{3}}\theta}{\cos^{\frac{1}{3}}\theta} = b$

$\Rightarrow b^2 = \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}$

Now, $a^2b^2(a^2 + b^2)$

$\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} \times \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\left[\frac{\cos^{\frac{4}{3}}\theta}{\sin^{\frac{2}{3}}\theta} + \frac{\sin^{\frac{4}{3}}\theta}{\cos^{\frac{2}{3}}\theta}\right]$

$\cos^{\frac{4}{3}-\frac{2}{3}}\theta \times \sin^{\frac{4}{3}-\frac{2}{3}}\theta\left[\frac{\cos^{\frac{4}{3}} \times \cos^{\frac{2}{3}}\theta + \sin^{\frac{4}{3}}\theta \times \sin^{\frac{2}{3}}\theta}{\sin^{\frac{2}{3}}\theta \times \cos^{\frac{2}{3}}\theta}\right]$

$\cos^{\frac{2}{3}}\theta \times \sin^{\frac{2}{3}}\theta\left[\frac{\cos^2\theta + \sin^2\theta}{\sin^{\frac{2}{3}}\theta \times \cos^{\frac{2}{3}}\theta}\right]$

$\cos^2\theta + \sin^2\theta = 1$

LHS = RHS

64. We have,

q = cosec A - cot A

$\Rightarrow$ q^2 = (cosec A - cot A)^2

$\Rightarrow$ q^2 = cosec^{2}A + cot^{2}A - 2 cosec A.cot A

$\Rightarrow \mathrm{q}^2=\frac{1}{\sin^2 \mathrm{~A}}+\frac{\cos^2 \mathrm{~A}}{\sin^2 \mathrm{~A}}-2 \frac{\cos \mathrm{A}}{\sin^2 \mathrm{~A}}$

$\Rightarrow \mathrm{q}^2=\frac{1+\cos^2 \mathrm{~A}-2 \cos \mathrm{A}}{\sin^2 \mathrm{~A}}$

$\Rightarrow \mathrm{q}^2=\frac{(1-\cos \mathrm{A})^2}{1-\cos^2 \mathrm{~A}}=\frac{(1-\cos A)(1-\cos A)}{(1-\cos A)(1+\cos A)}=\frac{(1-\cos A)}{(1+\cos A)}$

Now,

$\mathrm{LHS}=\frac{\mathrm{q}^2-1}{\mathrm{q}^2+1}+\cos \mathrm{A}$

$=\frac{\frac{1-\cos A}{1+\cos A}-1}{\frac{1-\cos A}{1+\cos A}+1}+\cos A$

$=\frac{1-\cos A-1-\cos A}{1-\cos A+1+\cos A}+\cos A$

$=\frac{-2 \cos A}{2}+\cos A$

= -cos A + cos A

= 0

= RHS

65. The given trigonometric identity is,

$\frac{\sec A-\tan A}{\sec A+\tan A}=\frac{\cos^2 A}{(1+\sin A)^2}$

Now, take LHS $=\frac{\sec A-\tan A}{\sec A+\tan A}$

Multiplying numerator and denominator by (secA - tanA)

$=\frac{(\sec A-\tan A)(\sec A-\tan A)}{(\sec A+\tan A)(\sec A-\tan A)}$

$=\frac{(\sec A-\tan A)^2}{\sec^2 A-\tan^2 A}\left[\because(a+b)(a-b)=a^2-b^2\right]$

$=\frac{(\sec A-\tan A)^2}{1}\left[\because \sec^2 A-\tan^2 A=1\right]$

$=(\sec A-\tan A)^2$

$=\left(\frac{1}{\cos A}-\frac{\sin A}{\cos A}\right)^2$

$=\left(\frac{1-\sin A}{\cos A}\right)^2$

$=\frac{(1-\sin A)^2}{\cos^2 A}$

$=\frac{(1-\sin A)^2}{1-\sin^2 A}\left[\because \cos^2 A=1-\sin^2 A\right]$

$=\frac{(1-\sin A)(1-\sin A)}{(1+\sin A)(1-\sin A)}\left[\because a^2-b^2=(a+b)(a-b)\right]$

$=\frac{1-\sin A}{1+\sin A}$

Multiplying numerator and denominator by 1 + sinA

$=\frac{(1-\sin A)(1+\sin A)}{(1+\sin A)(1+\sin A)}$

$=\frac{1-\sin^2 A}{(1+\sin A)^2}\left[\because(a-b)(a+b)=a^2-b^2\right]$

$=\frac{\cos^2 A}{(1+\sin A)^2}\left[\because 1-\sin^2 A=\cos^2 A\right]$

= RHS

Hence proved

66. $\sec^2 \theta=\left(x+\frac{1}{4 x}\right)^2=x^2+\frac{1}{16 x^2}+\frac{1}{2}$

$\therefore \quad \tan^2 \theta=\sec^2 \theta-1=x^2+\frac{1}{16 x^2}-\frac{1}{2}$

$\Rightarrow \quad \tan^2 \theta=\left(x-\frac{1}{4 x}\right)^2$

$\Rightarrow \quad \tan \theta=\left(x-\frac{1}{4 x}\right)$ or $\left(\frac{1}{4 x}-x\right)$

Hence $\sec \theta+\tan \theta=2 x$ or $\frac{1}{2 x}$

67. Given $\frac{1+tan^2A}{1+cot^2A}$ = $(\frac{1-tanA}{1-cotA})^2$ = $\tan^2$ A.

LHS = $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$ = $\frac{\sec^2 A}{cosec^2A}$

$= \frac{\frac{1}{\cos^2 A}}{\frac{1}{\sin^2 A}}$ = $\frac{\sin^2 A}{\cos^2 A}$ = $\tan^2$ A

RHS = $(\frac{1-tanA}{1-cotA})^2$ = $\left(\frac{1-\tan A}{1-\frac{1}{\tan A}}\right)^2$

$= \left(\frac{1-\tan A}{\frac{\tan A-1}{\tan A}}\right)^2$ = $(\frac{1-tanA}{tanA-1} \times \tan A)^2$ = $(-\tan A)^2$ = $\tan^2$ A

LHS = RHS.

68. LHS = $\frac{(1+\sin\theta)^2-(1-\sin\theta)^2}{(1+\sin\theta)(1-\sin\theta)}$

= $\frac{4\sin\theta}{1-\sin^2\theta}$

= $\frac{4\sin\theta}{\cos^2\theta}$

= 4 $\tan\theta$ $\sec\theta$ = RHS

69. LHS = $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} + \sqrt{\frac{1-\sin\theta}{1+\sin\theta}}$

$= \sqrt{\frac{(1+\sin\theta)}{(1-\sin\theta)} \times \frac{(1+\sin\theta)}{(1+\sin\theta)}} + \sqrt{\frac{(1-\sin\theta)}{(1+\sin\theta)} \times \frac{(1-\sin\theta)}{(1-\sin\theta)}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{1-\sin^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{1-\sin^2\theta}}$

$= \sqrt{\frac{(1+\sin\theta)^2}{\cos^2\theta}} + \sqrt{\frac{(1-\sin\theta)^2}{\cos^2\theta}}$ $[\because sin^2\theta + cos^2\theta = 1]$

$= \frac{1+\sin\theta}{\cos\theta} + \frac{1-\sin\theta}{\cos\theta}$

$= \frac{1+\sin\theta+1-\sin\theta}{\cos\theta}$

$= \frac{2}{\cos\theta}$

= $2sec\theta$

= RHS

70. L.H.S = $\frac{(1+\cot\theta+\tan\theta)(\sin\theta-\cos\theta)}{\sec^3\theta-\csc^3\theta}$

$= \frac{\left(1+\frac{\cos\theta}{\sin\theta}+\frac{\sin\theta}{\cos\theta}\right)(\sin\theta-\cos\theta)}{\frac{1}{\cos^3\theta}-\frac{1}{\sin^3\theta}}$ ....($\because$ $\cot\theta$ = $\frac{cos\theta}{\sin\theta}$)

$= \frac{\left(\cos\theta\sin\theta+\cos^2\theta+\sin^2\theta\right)(\sin\theta-\cos\theta)}{\frac{\cos\theta\sin\theta}{\frac{\sin^3\theta-\cos^3\theta}{\cos^3\theta\sin^3\theta}}}$

$= \frac{\sin^3\theta-\cos^3\theta}{\sin\theta\cos\theta} \times \frac{\cos^3\theta\sin^3\theta}{\sin^3\theta-\cos^3\theta}$ ...$\{\because \left(\cos\theta\sin\theta+\cos^2\theta+\sin^2\theta\right)(\sin\theta-\cos\theta) = \sin^3\theta-\cos^3\theta\ \}$

$= \cos^2\theta\sin^2\theta$ = RHS.

71. We have, l = cosecθ - sinθ and m = secθ - cosθ

$\therefore$ LHS = l^2m^2 $(l^2 + m^2 + 3)$

$\Rightarrow$ LHS = $(\text{cosec}\theta - \sin\theta)^2(\sec\theta - \cos\theta)^2$ $\{(\text{cosec}\theta - \sin\theta)^2 + (\sec\theta - \cos\theta)^2 + 3\}$

$\Rightarrow \text{LHS} = \left(\frac{1}{\sin\theta} - \sin\theta\right)^2\left(\frac{1}{\cos\theta} - \cos\theta\right)^2\left\{\left(\frac{1}{\sin\theta} - \sin\theta\right)^2 + \left(\frac{1}{\cos\theta} - \cos\theta\right)^2 + 3\right\}$

$\Rightarrow \text{LHS} = \left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2\left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{1-\sin^2\theta}{\sin\theta}\right)^2 + \left(\frac{1-\cos^2\theta}{\cos\theta}\right)^2 + 3\right\}$

$\Rightarrow \text{LHS} = \left(\frac{\cos^2\theta}{\sin\theta}\right)^2\left(\frac{\sin^2\theta}{\cos\theta}\right)^2\left\{\left(\frac{\cos^2\theta}{\sin\theta}\right)^2 + \left(\frac{\sin^2\theta}{\cos\theta}\right)^2 + 3\right\}$

$\Rightarrow \text{LHS} = \frac{\cos^4\theta}{\sin^2\theta} \times \frac{\sin^4\theta}{\cos^2\theta}\left\{\frac{\cos^4\theta}{\sin^2\theta} + \frac{\sin^4\theta}{\cos^2\theta} + 3\right\}$

$\Rightarrow \text{LHS} = \cos^2\theta \times \sin^2\theta\left\{\frac{\cos^6\theta+\sin^6\theta+3\cos^2\theta\sin^2\theta}{\cos^2\theta\sin^2\theta}\right\}$

$\Rightarrow$ LHS = $\cos^6\theta + \sin^6\theta + 3\cos^2\theta\sin^2\theta$

$\Rightarrow$ LHS = $\{(\cos^2\theta)^3 + (\sin^2\theta)^3\} + 3\cos^2\theta\sin^2\theta$

$\Rightarrow$ LHS = $\{(\cos^2\theta + \sin^2\theta)^3 - 3\cos^2\theta \sin^2\theta (\cos^2\theta + \sin^2\theta)\} + 3\sin^2\theta\cos^2\theta$ [$\because a^3 + b^3 = (a + b)^2 - 3ab(a + b)$]

$\Rightarrow$ LHS = $\{1 - 3\cos^2\theta \sin^2\theta\} + 3\cos^2\theta\sin^2\theta$ = RHS [$\because \cos^2\theta + \sin^2\theta = 1$]