CBSE

INTRODUCTION TO TRIGONOMETRY WS 8

Class 10 - Mathematics

Section A

1. If cosec$^2\theta$(1 + cosθ)(1 - cosθ) = λ, then find the value of λ. **[2]**
2. Find the value of θ, if, $\frac{\cos\theta}{1-\sin\theta} + \frac{\cos\theta}{1+\sin\theta} = 4; \theta \leq 90^\circ$. **[2]**
3. If tan θ + cot θ = 2, find the value of tan^2 θ + cot^2 θ. **[2]**
4. If 2 sin^2 θ – cos^2 θ = 2, then find the value of θ. **[2]**
5. Match the following **[2]**

(a) $\sqrt{1-tan^2A}$	(i) cot A
(b) $\sqrt{tan^2(90^\circ - A)}$	(ii) sec A
(c)$\sqrt{sec^2A\ -\ 1}$	(iii) cos A
(d) $\sqrt{sin^2(90-A)}$	(iv) tan A

6. Match the following **[2]**

(a) $sin^2A + cos^2A$	(i) $\frac{1}{tan^2A}$
(b) $cosec^2A\ -\ 1$	(ii) $cosecA$
(c) $cosecA.tanA$	(iii) $sec^2A - tan^2A$
(d) $secA.cotA$	(iv) $secA$

7. Match the following: **[2]**

(a) If x = p.sinA; y = p.cosA; z = p.tanA; then $x^2 + y^2$	(i) p^2sinA
(b) If x = p.sinA; y = p.cosA; z = p.tanA; then $y \times z$	(ii) $cosA$
(c) If x = p.sinA; y = p.cosA; z = p.tanA; then $\frac{x}{z}$	(iii) p^2
(d) If x = p.sinA; y = p.cosA; z = p.tanA; then x/y	(iv) $tanA$

Section B

8. If tan θ + sec θ = l, then prove that secθ = $\frac{l^2+1}{2l}$. **[3]**
9. If $\tan\theta + \frac{1}{\tan\theta} = 2$, find the value of $\tan^2\theta + \frac{1}{\tan^2\theta}$ **[3]**
10. A man on the deck of a ship, 12 m above water level, observes that the angle of elevation of the top of a cliff is 60° and the angle of depression of the base of the cliff is 30°. Find the distance of the cliff from the ship and the height of the cliff. [Use $\sqrt{3}$ = 1.732] **[3]**
11. Prove that : (sin A - sec A)^2 + (cos A - cosec A)^2 = (1 - sec A . cosec A)^2 **[3]**
12. Evaluate : $\frac{\cos^2(45^\circ+\theta)+\cos^2(45^\circ-\theta)}{\tan(60^\circ+\theta)\tan(30^\circ-\theta)}$ + cosec(75° + θ) - sec (15° - θ). **[3]**
13. If sinθ + cosθ = p and secθ + cosecθ = q, show that q (p^2 - 1) = 2p. **[3]**
14. If cotθ + tanθ = x and secθ - cosθ = y, prove that (x^{2}y)$^{2/3}$ - (xy^2)$^{2/3}$ = 1. **[3]**

15. If $\sin\theta + \cos\theta = x$, prove that $\sin^6\theta + \cos^6\theta = \frac{4-3(x^2-1)^2}{4}$ **[3]**

16. If 3sinθ + 5cosθ = 5, prove that 5sinθ - 3cosθ = ± 3. **[3]**

17. Prove the trigonometric identity: **[3]**

$\sqrt{\sec^2\theta + cosec^2\theta}$ = tan θ + cot θ.

18. Prove: $\frac{1}{(\cot \text{A})(\sec \text{A})-\cot \text{A}} - \text{cosec A}$ = $\text{cosec A} - \frac{1}{(\cot A)(\sec \text{A})+\cot A}$ **[3]**

19. If,cot $B = \frac{12}{5}$ prove that $\tan^2 B - \sin^2 B = \sin^4 B \cdot \sec^2 B$ **[3]**

20. Prove the trigonometric identity: $\frac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta} = \tan\theta$ **[3]**

21. If tan A = n tanB and sin A = m sinB, then prove that cos^2 A = $\frac{m^2-1}{n^2-1}$ **[3]**

22. If $\sec\theta = x + \frac{1}{4x}$, prove that: $\sec\theta + \tan\theta = 2x$ or, $\frac{1}{2x}$ **[3]**

23. If 4 tan θ = 3, evaluate $\left(\frac{4\sin\theta - \cos\theta + 1}{4\sin\theta + \cos\theta - 1}\right)$ **[3]**

24. Prove the following identity: $\frac{\sin\theta}{1-\cos\theta} + \frac{\tan\theta}{1+\cos\theta}$ = $sec\theta \cdot \cos ec\theta + cot\theta$ **[3]**

25. Prove that: **[3]**

$\frac{\tan\theta + \sec\theta - 1}{\tan\theta - \sec\theta + 1} = \frac{1+\sin\theta}{\cos\theta}$

26. If a cos$^3\theta$ + 3 a cosθ sin$^2\theta$ = m, a sin$^3\theta$ + 3 a cos$^2\theta$ sinθ = n, prove that (m + n)$^{2/3}$ + (m - n)$^{2/3}$ = 2 a$^{2/3}$ **[3]**

27. Prove the identity: **[3]**

(tanA + cosecB)^2 - (cotB - secA)^2 = 2tanA cotB (cosecA + secB)

28. Prove the following identity : $\frac{1}{\cot^2\theta} + \frac{1}{1+\tan^2\theta} = \frac{1}{1-\sin^2\theta} - \frac{1}{cosec^2\theta}$ **[3]**

29. Prove that: $1 + \frac{\cot^2\alpha}{1+ cosec\alpha} = cosec\ \alpha$ **[3]**

30. Prove that: (sin θ + 1 + cos θ) (sin θ - 1 + cos θ) · sec θ cosec θ = 2 **[3]**

31. Given that: (1 + cosα) (1 + cosβ) (1 + cosλ) = (1 - cosα) (1 - cosβ) (1 - cosλ) **[3]**

Show that one of the values of each member of this equality is sinα sinβ sin λ

32. Prove that: $\frac{\cos^3\theta + \sin^3\theta}{\cos\theta + \sin\theta} + \frac{\cos^3\theta - \sin^3\theta}{\cos\theta - \sin\theta}$ = 2. **[3]**

33. If sin θ + cos θ = p and sec θ + cosec θ = q, show that q(p^2 - 1) = 2p **[3]**

34. If tan θ + sin θ = m and tan θ - sin θ = n, show that m^2 - n^2 = 4$\sqrt{mn}$. **[3]**

35. If $\frac{\cos\alpha}{\cos\beta}$ = m and $\frac{\cos\alpha}{\sin\beta}$ = n show that (m^2 + n^2) cos$^2\beta$= n^2 **[3]**

36. Prove: sin^6 A + 3 sin^2 A cos^2 A = 1 - cos^6 A **[3]**

37. Prove that $\left(\frac{1+\tan^2 A}{1+\cot^2 A}\right)$ = $\left(\frac{1-\tan A}{1-\cot A}\right)^2$ = tan^2 A **[3]**

38. Write all the other trigonometric ratios of ∠A in terms of sec A. **[3]**

39. Prove that secθ(1 - sinθ)(secθ + tanθ) = 1 **[3]**

40. Prove that (sin A + cosec A)^2 + (cos A + sec A)^2 = 7 + tan^{2}A + cot^{2}A. **[3]**

41. Express the trigonometric ratios sin A, sec A and tan A in terms of cot A. **[3]**

42. If (2 sin θ + 3 cos θ) = 2, prove that (3 sin θ - 2 cos θ) = $\pm$ 3 **[3]**

43. Prove the identity: **[3]**

2(sin$^6\theta$ + cos$^6\theta$) - 3(sin$^4\theta$ + cos$^4\theta$) + 1 = 0

44. Prove that: sec A (1 - sin A) (sec A + tan A) = 1 **[3]**

45. Prove the trigonometric identity: **[3]**

sec$^6\theta$ = tan$^6\theta$ + 3tan$^2\theta$ sec$^2\theta$ + 1

46. If sin θ+ cos θ = $\sqrt{2}$, then evaluate tan θ + cot θ. **[3]**

47. If (tan θ + sin θ) = m and (tan θ - sin θ) = n, prove that (m^2 - n^2)^2 = 16mn **[3]**

48. When is an equation called 'an identity'. Prove the trigonometric identity $1 + \tan^2 A = \sec^2 A$. **[3]**

49. Prove that (cosec A - sin A) (sec A - cos A) = $\frac{1}{\cot A+\tan A}$. **[3]**

50. Prove that: $\left(1+\tan^2 A\right)+\left(1+\frac{1}{\tan^2 A}\right)=\frac{1}{\sin^2 A-\sin^4 A}$ **[3]**

51. Prove that $\frac{\sin\theta-\cos\theta+1}{\sin\theta+\cos\theta-1}=\frac{1}{\sec\theta-\tan\theta}$, using identity $sec^2\theta = 1 + tan^2\theta$. **[3]**

52. If secθ + tanθ = p, show that $\frac{p^2-1}{p^2+1}=\sin\theta$ **[3]**

53. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, then prove that tan θ = 1, or $\frac{1}{2}$. **[3]**

54. If secθ + tanθ = p, prove that $\tan\theta=\frac{1}{2}\left(p-\frac{1}{p}\right)$ **[3]**

55. If $1 + \sin^2\theta = 3\sin\theta\cos\theta$, prove that tan$\theta$ = 1 or $\frac{1}{2}$ **[3]**

56. If cosec A + cot A = m. show that $\frac{m^2-1}{m^2+1}$ = cos A. **[3]**

57. Verify: $\sqrt{\frac{1-\cos\theta}{1+\cos\theta}}=\frac{\sin\theta}{1+\cos\theta}$, for θ=60°. **[3]**

58. Prove that: $(cosec\theta-\sin\theta)(\sec\theta-\cos\theta)=\frac{1}{\tan\theta+\cot\theta}$ **[3]**

59. If $2\sin^2\theta-\cos^2\theta=2,$ then find the value of θ. **[3]**

60. Prove that: $2\left(\sin^6\theta+\cos^6\theta\right)-3\left(\sin^4\theta+\cos^4\theta\right)+1=0$ **[3]**

61. Prove the trigonometric identity: $\frac{\cot^2\theta(\sec\theta-1)}{(1+\sin\theta)}+\frac{\sec^2\theta(\sin\theta-1)}{(1+\sec\theta)}=0$ **[3]**

62. If secθ + tanθ = p, obtain the values of secθ, tanθ and sinθ in terms of p. **[3]**

63. Find the acute angle θ, when $\frac{\cos\theta-\sin\theta}{\cos\theta+\sin\theta}=\frac{1-\sqrt{3}}{1+\sqrt{3}}$. **[3]**

64. If (cotθ + tanθ) = m and (secθ - cosθ) = n, prove that $(m^2n)^{2/3} - (mn^2)^{2/3} = 1$. **[3]**

65. If $\sin\theta + \sin^2\theta + \sin^3\theta = 1$, then prove that $\cos^6\theta - 4\cos^4\theta + 8\cos^2\theta = 4$ **[3]**

66. Prove the identity: **[3]**

$$\frac{(1+\cot A+\tan A)(\sin A-\cos A)}{\sec^3 A- cosec^3 A}=\sin^2 A\cos^2 A$$

67. If sinθ + 2 cosθ = 1 prove that 2 sinθ - cosθ = 2. **[3]**

68. If $\left(\frac{x}{a}\sin\theta-\frac{y}{b}\cos\theta\right)$ = 1 and $\left(\frac{x}{a}\cos\theta+\frac{y}{b}\sin\theta\right)$ = 1, prove that $\left(\frac{x^2}{a^2}+\frac{y^2}{b^2}\right)=2$ **[3]**

69. If $x\sin^3\theta + y\cos^3\theta = \sin\theta\cos\theta$ and $x\sin\theta = y\cos\theta$, prove that $x^2 + y^2 = 1$ **[3]**

70. Prove that: $\frac{\tan A}{1+\sec A}-\frac{\tan A}{1-\sec A}=2cosecA$ **[3]**

71. Prove the identity: **[3]**

$(\sin^8\theta - \cos^8\theta) = (\sin^2\theta - \cos^2\theta)(1 - 2\sin^2\theta\cos^2\theta)$

72. Prove that: $\frac{1}{\text{cosec}A-\cot A}-\frac{1}{\sin A}=\frac{1}{\sin A}-\frac{1}{\text{cosec}A+\cot A}$. **[3]**

73. Prove the trigonometric identity: $\frac{\cos\theta\, cosec\theta-\sin\theta\sec\theta}{\cos\theta+\sin\theta}=cosec\theta-\sec\theta$ **[3]**

74. Prove the identity: **[3]**

$$\frac{\cos A}{1-\sin A}+\frac{\sin A}{1-\cos A}+1=\frac{\sin A\cos A}{(1-\sin A)(1-\cos A)}$$