CBSE

INTRODUCTION TO TRIGONOMETRY WS 9

Class 10 - Mathematics

1. If 2 cos θ - sin θ = x and cos θ - 3 sin θ = y, prove that $2x^2 + y^2 - 2xy = 5$. **[2]**
2. Prove that: (cosec A – sin A) (sec A – cos A) = $\frac{1}{\tan A+\cot A}$ **[2]**
3. Prove the trigonometric identity: $(1+\tan\theta+\cot\theta)(\sin\theta-\cos\theta)=\left(\frac{\sec\theta}{cosec^2\theta}-\frac{cosec\theta}{\sec^2\theta}\right)$ **[2]**
4. Prove the identity: $\frac{\sin^2\theta}{1-\cos\theta}=\frac{1+\sec\theta}{\sec\theta}$ **[2]**
5. If sin $\alpha=\frac{1}{\sqrt{2}}$ and cot $\beta=\sqrt{3}$, then find the value of cosec α + cosec β. **[2]**
6. Show that $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4(\sin^6\theta + \cos^6\theta)$ is independent of θ **[2]**
7. Prove the trigonometric identity: $\frac{\cos\theta}{(1-\tan\theta)}-\frac{\sin^2\theta}{(\cos\theta-\sin\theta)}=(\cos\theta+\sin\theta)$ **[2]**
8. Prove that: $\left(\sec^2\theta-1\right)\left(1-cosec^2\theta\right)=-1$ **[2]**
9. Prove that $\frac{1+\tan^2 A}{1+\cot^2 A}$ = $\sec^2$ A - 1 **[2]**
10. Prove the trigonometric identity: $\frac{1-\sin\theta}{1+\sin\theta}=(\sec\theta-\tan\theta)^2$ **[2]**
11. If sinθ - cosθ = $\frac{1}{2}$, then find the value of sinθ + cosθ. **[2]**
12. If $\tan\theta=\frac{1}{\sqrt{7}}$, find the value of $\frac{\csc^2\theta-\sec^2\theta}{\csc^2\theta+\sec^2\theta}$ **[2]**
13. Prove that: $\frac{1}{(\sec\theta-\tan\theta)}-\frac{1}{\cos\theta}=\frac{1}{\cos\theta}-\frac{1}{(\sec\theta+\tan\theta)}$ **[2]**
14. Prove that: $\frac{\cot^2\theta(\sec\theta-1)}{1+\sin\theta}$ = $\sec^2\theta\frac{(1-\sin\theta)}{(\sec\theta+1)}$. **[2]**
15. Is $\sin^2\theta$ + sinθ = 2 is an identity? Justify your answer. **[2]**
16. Prove $\frac{1+\sec A}{\sec A}=\frac{\sin^2 A}{1-\cos A}$, where the angles involved are acute angles for which the expressions are defined. **[2]**
17. Prove that: $\frac{1+\tan^2\theta}{1+\cot^2\theta}=\left(\frac{1-\tan\theta}{1-\cot\theta}\right)^2=\tan^2\theta$ **[2]**
18. If $\sin\theta+\cos\theta=\sqrt{2}$, then prove that $\tan\theta+\cot\theta=2$. **[2]**
19. Prove that: $\frac{1}{\sec A+\tan A}-\frac{1}{\cos A}=\frac{1}{\cos A}-\frac{1}{\sec A-\tan A}$ **[2]**
20. If a secθ + b tanθ + c = 0 and p secθ + q tanθ + r = 0, prove that $(br - qc)^2 - (pc - ar)^2 = (aq - bp)^2$ **[2]**
21. Evaluate: $\left(\frac{\sin 27^\circ}{\cos 63^\circ}\right)^2-\left(\frac{\cos 63^\circ}{\sin 27^\circ}\right)^2$. **[2]**
22. Prove that (sin θ + cos θ) (tan θ + cot θ) = sec θ + cosec θ. **[2]**
23. Prove that: $\frac{\cos\theta}{1-\sin\theta}=\frac{1+\sin\theta}{\cos\theta}$ **[2]**
24. If m sinA + n cosA = p and m cosA - n sinA = q, prove that $m^2 + n^2 = p^2 + q^2$. **[2]**
25. If $cosec^2\theta(1 + \cos\theta)(1 - \cos\theta) = \lambda$, then find the value of λ. **[2]**
26. If $\cos\theta + \cos^2\theta = 1$, prove that $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2 = 1$ **[2]**
27. Prove that $\frac{\sin\theta-2\sin^3\theta}{2\cos^3\theta-\cos\theta}$ = tan θ. **[2]**
28. Prove that (cosecA - sinA)(secA - cosA) = $\frac{1}{\tan A+\cot A}$. **[2]**
29. Prove the identity: **[2]**

    $\sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$
30. Prove that : $\frac{\tan A}{\sec A-1}+\frac{\tan A}{\sec A+1}$ = 2 cosec A. **[2]**
31. Prove the trigonometric identity: $\frac{1-\tan^2\theta}{1+\tan^2\theta}=\left(\cos^2\theta-\sin^2\theta\right)$ **[2]**

32. Prove the trigonometric identity: [2]

$\tan^2 A + \cot^2 A = \sec^2 A \cos ec^2 A - 2$

33. Show that: $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$ for $0^\circ \leq \theta \geq 90^\circ$ [2]

34. Prove that: $\frac{\sin\theta}{\cot\theta + \operatorname{cosec}\theta} = 2 + \frac{\sin\theta}{\cot\theta - \operatorname{cosec}\theta}$ [2]

35. If a cosθ + b sinθ = m and a sinθ - b cosθ = n, prove that $a^2 + b^2 = m^2 + n^2$ [2]

36. If θ is an acute angle and $\tan\theta + \cot\theta = 2$, then find the value of $\tan^7\theta + \cot^7\theta$ · [2]

37. Prove the trigonometric identity: [2]

$\frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A} = \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{2\sin^2 A - 1} = \frac{2}{1 - 2\cos^2 A}$

38. Prove that if x = a sin θ + b cos θ and y = a cos θ - b sin θ, then $x^2 + y^2 = a^2 + b^2$. [2]

39. If tan θ + cot θ = 2, find the value of $\tan^2\theta + \cot^2\theta$**.** [2]

40. If sinθ + cos θ = $\sqrt{3}$, then prove that tan θ + cot θ = 1 [2]

41. Prove $\frac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$ = cosec A + cot A, using the identity $\operatorname{cosec}^2 A = 1 + \cot^2 A$. where the angles involved are acute angles for which the expressions are defined. [2]

42. Prove that: $\sin^4 A - \cos^4 A = \sin^2 A - \cos^2 A = 2\sin^2 A - 1 = 1 - 2\cos^2 A$ [2]

43. If $2\sin^2\theta - \cos^2\theta = 2$, then find the value of θ. [2]

44. Prove the trigonometric identity: [2]

$(\sin\theta + \operatorname{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$

45. Prove that: $\left(cosec^2\theta - 1\right)\tan^2\theta = 1$ [2]

46. If $\cos A + \cos^2 A = 1$, prove that $\sin^2 A + \sin^4 A = 1$ [2]

47. If x = 3 sinθ + 4 cosθ and y = 3 cosθ – 4 sinθ then prove that $x^2 + y^2 = 25$. [2]

48. Prove that: $\frac{\cot A + cosec A - 1}{\cot A - cosec A + 1} = \frac{1 + \cos A}{\sin A}$ [2]

49. If x = a sinθ and y = b tanθ, then prove that $\frac{a^2}{x^2} - \frac{b^2}{y^2} = 1$ [2]

50. Show that $\sin^2\theta + \cos^2\theta = 1$ is an identity. [2]

51. Prove that: $\frac{\cot A - \cos A}{\cot A + \cos A} = \frac{cosec A - 1}{cosec A + 1}$ [2]

52. Prove that: $\frac{1 - \cos A}{1 + \cos A} = (\cot A - \ cosec A)^2$ [2]

53. Find the value of sin 25° cos 65° + cos 25° sin 65° [2]

54. Prove $\frac{\sin\theta}{1 + \cos\theta} + \frac{1 + \cos\theta}{\sin\theta} = 2 cosec\theta$ [2]

55. If cosecθ + cotθ = m and cosecθ - cotθ = n, prove that mn = 1 [2]

56. Prove that $(\operatorname{cosec}A - \cot A)^2 = \frac{(1 - \cos A)}{(1 + \cos A)}$ [2]

57. Prove that: $\frac{(1 + \sin\theta)^2 + (1 - \sin\theta)^2}{\cos^2\theta} = 2\left(\frac{1 + \sin^2\theta}{1 - \sin^2\theta}\right)$ [2]

58. Prove the trigonometric identity: $\frac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \frac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta} = \frac{2}{(1 - 2\cos^2\theta)}$ [2]

59. Prove that $\frac{\sin^2 63^\circ + \sin^2 27^\circ}{\cos^2 17^\circ + \cos^2 73^\circ} = 1$ [2]

60. If cos θ + sin θ = $\sqrt{2}$ cos θ, show that cos θ - sin θ = $\sqrt{2}$ sin θ. [2]

61. If x = asinθ + bcosθ and y = acosθ - bsinθ , prove that $x^2 + y^2 = a^2 + b^2$. [2]

62. Prove the trigonometric identity: $\frac{1 - \cos\theta}{1 + \cos\theta} = (\cos ec\,\theta - \cot\theta)^2$ [2]

63. Prove $\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$ = 1 + sec A cos ecA, where the angles involved are acute angles for which the expressions are defined. [2]

64. Prove the identity: [2]

$\cot^2 A\left(\frac{\sec A - 1}{1 + \sin A}\right) + \sec^2 A\left(\frac{\sin A - 1}{1 + \sec A}\right) = 0$

[2]

65. Prove that $\frac{1+\sec\theta-\tan\theta}{1+\sec\theta+\tan\theta} = \frac{1-\sin\theta}{\cos\theta}$.

66. Prove that: $(\sin\theta + \cos\theta)(\tan\theta + \cot\theta) = \sec\theta + \cos ec\theta$ **[2]**

67. Prove the identity: **[2]**

$(1 + \tan A.\tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A.\sec^2 B$

68. Prove the trigonometric identity: $\cos^2\theta\left(1+\tan^2\theta\right) = 1$ **[2]**

69. Prove that: $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$ **[2]**

70. Prove that : $\frac{\left(\sin^4\theta+\cos^4\theta\right)}{1-2\sin^2\theta\cos^2\theta} = 1$. **[2]**

71. If $\tan\theta = \frac{1}{\sqrt{7}}$, then show that $\frac{\text{cosec}^2\theta-\sec^2\theta}{\text{cosec}^2\theta+\sec^2\theta} = \frac{3}{4}$. **[2]**

72. Prove the trigonometric identity: $\sqrt{\frac{1+\sin\theta}{1-\sin\theta}} = (\sec\theta + \tan\theta)$ **[2]**

73. If $x = a\cos^3\theta$, $y = b\sin^3\theta$, prove that $\left(\frac{x}{a}\right)^{2/3} + \left(\frac{y}{b}\right)^{2/3} = 1$ **[2]**

74. Prove the trigonometric identity: $\sec^4\theta - \sec^2\theta = \tan^4\theta + \tan^2\theta$. **[2]**

75. Prove the trigonometric identity: **[2]**

secA (1 - sinA) (secA + tanA) = 1