INTRODUCTION TO TRIGONOMETRY WS 9

Class 10 - Mathematics

1. $2cos\theta - sin\theta = x$ ...(i)

   $cos\theta - 3sin\theta = y$

   LHS $= 2x^2 + y^2 - 2xy$

   $= 2[2cos\theta - sin\theta]^2 + (cos\theta - 3sin\theta)^2 - 2(2cos\theta - sin\theta)(cos\theta - 3sin\theta)$

   $= 2(4\cos^2\theta + \sin^2\theta - 4\cos\theta . \sin\theta) + \cos^2\theta + 9\sin^2\theta - 6\cos\theta\sin\theta - 2(2\cos^2\theta - 6\sin\theta\cos\theta - \sin\theta\cos\theta + 3\sin^2\theta)$

   $= 8\cos^2\theta + 2\sin^2\theta - 8\cos\theta . \sin\theta + \cos^2\theta + 9\sin^2\theta - 6\cos\theta . \sin\theta - 4\cos^2\theta + 14\sin\theta . \cos\theta - 6\sin^2\theta$

   $= 5(cos^2\theta + sin^2\theta)$

   $= 5 \times 1$

   $= 5$ RHS

2. L.H.S. $= (\text{cosec A} - \sin A)(\sec A - \cos A) = \left(\dfrac{1}{\sin A} - sinA\right)\left(\dfrac{1}{\cos A} - cosA\right)$

   $= \left(\dfrac{1 - \sin^2 A}{\sin A}\right)\left(\dfrac{1 - \cos^2 A}{\cos A}\right)$

   $= \dfrac{\cos^2 A}{\sin A}\dfrac{\sin^2 A}{\cos A} = \sin A \cos A$

   R.H.S. $= \dfrac{1}{\tan A + \cot A}$

   $= \dfrac{1}{\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}}$

   $= \dfrac{\cos A\sin A}{\sin^2 A + \cos^2 A}$

   $= \sin A \cos A$

   Hence, L.H.S. = R.H.S.

3. LHS $= (1 + \tan\theta + \cot\theta)(\sin\theta - \cos\theta)$

   $= \left(1 + \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)(\sin\theta - \cos\theta)$

   $= \left(\dfrac{\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)(\sin\theta - \cos\theta)$

   $= \dfrac{(\cos\theta\sin\theta + 1)}{\cos\theta\sin\theta}(\sin\theta - \cos\theta)$

   RHS $= \left(\dfrac{\sec\theta}{cosec^2\theta} - \dfrac{\cos ec\theta}{\sec^2\theta}\right) = \left(\dfrac{\frac{1}{\cos\theta}}{\frac{1}{\sin^2\theta}} - \dfrac{\frac{1}{\sin\theta}}{\frac{1}{\cos^2\theta}}\right)$

   $= \left(\dfrac{\sin^2\theta}{\cos\theta} - \dfrac{\cos^2\theta}{\sin\theta}\right) = \dfrac{\sin^3\theta - \cos^3\theta}{\cos\theta\sin\theta}$

   $= \dfrac{(\sin\theta - \cos\theta)\left(\sin^2\theta + \cos^2\theta + \cos\theta\sin\theta\right)}{\cos\theta\sin\theta}$ $[\because\ a^3 - b^3 = (a - b)(a^2 + ab + b^2)]$

   $= \dfrac{(\sin\theta - \cos\theta)(1 + \cos\theta\sin\theta)}{\cos\theta\sin\theta}$

   $\therefore$ LHS = RHS

4. $\dfrac{\sin^2\theta}{1-\cos\theta} = \dfrac{1+\sec\theta}{\sec\theta}$

   L.H.S. $= \dfrac{\sin^2\theta}{1-\cos\theta} = \dfrac{1-\cos^2\theta}{1-\cos\theta}$ $\left[\because \sin^2\theta = 1 - \cos^2\theta\right]$

   $= \dfrac{(1+\cos\theta)(1-\cos\theta)}{(1-\cos\theta)} = 1 + \cos\theta \ldots\ldots(1)$

   R. H. S. $= \dfrac{1+\sec\theta}{\sec\theta} = \dfrac{1}{\sec\theta} + \dfrac{\sec\theta}{\sec\theta}$

   $= \cos\theta + 1 = 1 + \cos\theta \ldots\ldots (2)$

   From (1) and (2), we have L.H.S. = R.H.S. proved.

5. $\operatorname{cosec}\alpha = \dfrac{1}{\sin\alpha} = \sqrt{2}$

   $\operatorname{cosec}\beta = \sqrt{1+\cot^2\beta} = \sqrt{1+3} = 2$

   $\therefore \operatorname{cosec}\alpha + \operatorname{cosec}\beta = \sqrt{2} + 2$ or $\sqrt{2}(\sqrt{2}+1)$

6. $3(\sin\theta - \cos\theta)^4 + 6(\sin\theta + \cos\theta)^2 + 4(\sin^6\theta + \cos^6\theta)$

   $= 3[(\sin\theta - \cos\theta)^2]^2 + 6(\sin^2\theta + \cos^2\theta + 2\sin\theta.\cos\theta) + 4[(\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta.\cos^2\theta(\sin^2\theta + \cos^2\theta)]$

   $= 3[\sin^2\theta + \cos^2\theta - 2\sin\theta . \cos\theta]^2 + 6[1 + 2\sin\theta . \cos\theta] + 4(1 - 3\sin^2\theta . \cos^2\theta)$

   $= 3(1 + 4\sin^2\theta . \cos^2\theta - 4\sin\theta\cos\theta) + 6 + 12\sin\theta . \cos\theta + 4 - 12\sin^2\theta\cos^2\theta$

   $= 13$

   $\Rightarrow$ independent of θ

7. LHS $= \dfrac{\cos\theta}{(1-\tan\theta)} - \dfrac{\sin^2\theta}{(\cos\theta - \sin\theta)}$

   $= \dfrac{\cos\theta}{\left(1 - \dfrac{\sin\theta}{\cos\theta}\right)} - \dfrac{\sin^2\theta}{(\cos\theta - \sin\theta)}$

   $= \dfrac{\cos^2\theta}{(\cos\theta - \sin\theta)} - \dfrac{\sin^2\theta}{(\cos\theta - \sin\theta)} = \dfrac{\cos^2\theta - \sin^2\theta}{(\cos\theta - \sin\theta)}$

   $= \dfrac{(\cos\theta - \sin\theta)(\cos\theta + \sin\theta)}{(\cos\theta - \sin\theta)} = (\cos\theta + \sin\theta) = $ RHS

   Hence proved.

8. We have,

   LHS $= \left(\sec^2\theta - 1\right)\left(1 - cosec^2\theta\right)$

   $= \left(1 + \tan^2\theta - 1\right)\left[1 - \left(1 + \cot^2\theta\right)\right]$ $\left[\because \sec^2\theta = 1 + \tan^2\theta \text{ and } cosec^2\theta = 1 + \cot^2\theta\right]$

   $= \tan^2\theta \cdot \left(-\cot^2\theta\right)$

   $= \tan^2\theta \cdot \left(\dfrac{-1}{\tan^2\theta}\right)$ $\left[\because \cot\theta = \dfrac{1}{\tan\theta}\right]$

   $= -1 = $ RHS

9. LHS $= \dfrac{1 + \tan^2 A}{1 + \cot^2 A} = \dfrac{1 + \dfrac{\sin^2 A}{\cos^2 A}}{1 + \dfrac{\cos^2 A}{\sin^2 A}}$

   $= \dfrac{\dfrac{\cos^2 A + \sin^2 A}{\cos^2 A}}{\dfrac{\sin^2 A + \cos^2 A}{\sin^2 A}}$

   $= \dfrac{\dfrac{1}{\cos^2 A}}{\dfrac{1}{\sin^2 A}} = \dfrac{\sin^2 A}{\cos^2 A} = \dfrac{1 - \cos^2 A}{\cos^2 A}$

   $= \dfrac{1}{\cos^2 A} - 1 = \sec^2 A - 1 = $ RHS

10. We have,

   LHS $= \dfrac{1 - \sin\theta}{1 + \sin\theta}$

$\Rightarrow$ LHS $= \dfrac{1-\sin\theta}{1+\sin\theta} \times \dfrac{1-\sin\theta}{1-\sin\theta}$ [Multiplying numerator and denominator by 1 - $\sin\theta$]

$\Rightarrow$ LHS $= \dfrac{(1-\sin\theta)^2}{1-\sin^2\theta}$

$\Rightarrow$ $LHS = \dfrac{(1-\sin\theta)^2}{\cos^2\theta}$ [$\because$ 1 - $\sin^2\theta = \cos^2\theta$]

$\Rightarrow$ LHS $= \left(\dfrac{1-\sin\theta}{\cos\theta}\right)^2$

$\Rightarrow$ LHS $= \left(\dfrac{1}{\cos\theta} - \dfrac{\sin\theta}{\cos\theta}\right)^2 = (\sec\theta - \tan\theta)^2 =$ RHS

11. $\because$ $\sin\theta$ - $\cos\theta = \dfrac{1}{2}$

On squaring both sides,

$(\sin\theta$ - $\cos\theta)^2 = \dfrac{1}{4}$

or, $\sin^2\theta + \cos^2\theta$ - $2\sin\theta\cos\theta = \dfrac{1}{4}$

$1 - 2sin\theta cos\theta = \dfrac{1}{4}$ ($\because$ $sin^2\theta + cos^2\theta = 1$)

$2sin\theta cos\theta = 1 - \dfrac{1}{4}$

or, $2sin\theta cos\theta = \dfrac{3}{4}$

Again, $(sin\theta + cos\theta)^2 = sin^2\theta + cos^2\theta + 2sin\theta cos\theta$

$= 1 + 2sin\theta cos\theta$

$= 1 + \dfrac{3}{4} = \dfrac{7}{4}$

$\therefore$ $\sin\theta + \cos\theta = \sqrt{\dfrac{7}{4}} = \dfrac{\sqrt{7}}{2}$

12. We have, $\tan\theta = \dfrac{1}{\sqrt{7}}$

$\cot\theta = \dfrac{1}{\tan\theta} = \sqrt{7}$

Now,

$\sec^2\theta = 1 + \tan^2\theta \Rightarrow \sec^2\theta = 1 + \left(\dfrac{1}{\sqrt{7}}\right)^2 = 1 + \dfrac{1}{7} = \dfrac{8}{7}$

and $\csc^2\theta = 1 + \cot^2\theta \Rightarrow \csc^2\theta = 1 + (\sqrt{7})^2 = 1 + 7 = 8$

$\therefore$ $\dfrac{\csc^2\theta - \sec^2\theta}{\csc^2\theta + \sec^2\theta} = \dfrac{8 - \frac{8}{7}}{8 + \frac{8}{7}} = \dfrac{48/7}{64/7} = \dfrac{48}{64} = \dfrac{3}{4}$

13. According to the question,

L.H.S. $= \dfrac{1}{(\sec\theta - \tan\theta)} - \dfrac{1}{\cos\theta}$

$= \dfrac{1}{(\sec\theta - \tan\theta)} \times \dfrac{(\sec\theta + \tan\theta)}{(\sec\theta + \tan\theta)} - \sec\theta$

$= \dfrac{(\sec\theta + \tan\theta)}{\left(\sec^2\theta - \tan^2\theta\right)} - \sec\theta$

$= (\sec\theta + \tan\theta)$ - $\sec\theta$ [$\therefore$ $sec^2\theta - tan^2\theta = 1$]

$= \tan\theta$.

R.H.S. $= \dfrac{1}{\cos\theta} - \dfrac{1}{(\sec\theta + \tan\theta)}$

$= \sec\theta - \dfrac{1}{(\sec\theta + \tan\theta)} \times \dfrac{(\sec\theta - \tan\theta)}{(\sec\theta - \tan\theta)}$

$= \sec\theta - \dfrac{(\sec\theta - \tan\theta)}{(\sec^2\theta - \tan^2\theta)}$

$= \sec\theta$ - $(\sec\theta$ - $\tan\theta)$ [$\because$ $sec^2\theta - tan^2\theta = 1$]

$= \tan\theta.$

$\therefore L.H.S. = R.H.S.$

14. L.H.S $= \dfrac{\dfrac{\cos^2\theta}{\sin^2\theta}\left(\dfrac{1}{\cos\theta}-1\right)}{1+\sin\theta} = \dfrac{\cos\theta}{(1+\cos\theta)(1+\sin\theta)}$

R.H.S $= \dfrac{1}{\cos^2\theta} \times \dfrac{(1-\sin\theta)\cos\theta}{1+\cos\theta} = \dfrac{(1-\sin\theta)\cos\theta}{\left(1-\sin^2\theta\right)(1+\cos\theta)}$

$= \dfrac{\cos\theta}{(1+\cos\theta)(1+\sin\theta)}$

$\therefore$ L.H.S = R.H.S

15. No.

$\sin^2\theta + \sin\theta = 2$

Taking $\theta = 45°$, we have

L.H.S. $= \sin^2 45° + \sin 45°$

$= \left(\dfrac{1}{\sqrt{2}}\right)^2 + \dfrac{1}{\sqrt{2}}$

$= \dfrac{1}{2} + \dfrac{1}{\sqrt{2}}$

$= \dfrac{\sqrt{2}+1}{2\sqrt{2}}$

$\neq 2$

$\neq$ R.H.S.

16. LHS $= \dfrac{1+\sec A}{\sec A} = \dfrac{1+\dfrac{1}{\cos A}}{\dfrac{1}{\cos A}}$

$= \dfrac{\dfrac{\cos A+1}{\cos A}}{\dfrac{1}{\cos A}} = \cos A + 1 = 1 + \cos A$

$= \dfrac{(1+\cos A)(1-\cos A)}{1-\cos A} = \dfrac{1-\cos^2 A}{1-\cos A}$

$= \dfrac{\sin^2 A}{1-\cos A} \cdot \; \because \sin^2 A + \cos^2 A = 1$

$= $ RHS

17. To prove : $\dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \left(\dfrac{1-\tan\theta}{1-\cot\theta}\right)^2 = \tan^2\theta$

Consider : $\dfrac{1+\tan^2\theta}{1+\cot^2\theta} = \dfrac{1+\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\cos^2\theta}{\sin^2\theta}} = \dfrac{\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}{\dfrac{\sin^2\theta+\cos^2\theta}{\sin^2\theta}}$

$= \dfrac{\dfrac{1}{\cos^2\theta}}{\dfrac{1}{\sin^2\theta}} = \dfrac{\sin^2\theta}{\cos^2\theta} \left[\because \sin^2\theta + \cos^2\theta = 1\right]$

$= \tan^2\theta$

Consider $\left(\dfrac{1-\tan\theta}{1-\cot\theta}\right)^2 = \dfrac{1+\tan^2\theta-2\tan\theta}{1+\cot^2\theta-2\cot\theta}$

$= \dfrac{\sec^2\theta-2\tan\theta}{\cos ec^2\theta-2\cot\theta} \left[\because 1+\tan^2\theta = \sec^2\theta\right]$

$= \dfrac{\dfrac{1}{\cos^2\theta}-\dfrac{2\sin\theta}{\cos\theta}}{\dfrac{1}{\sin^2\theta}-\dfrac{2\cos\theta}{\sin\theta}} = \dfrac{\dfrac{1-2\sin\theta\cos\theta}{\cos^2\theta}}{\dfrac{1-2\sin\theta\cdot\cos\theta}{\sin^2\theta}}$

$= \dfrac{\sin^2\theta}{\cos^2\theta} = \tan^2\theta$

18. We have,

$\sin\theta + \cos\theta = \sqrt{2}$

$(\sin\theta + \cos\theta)^2 = (\sqrt{2})^2$

$\sin^2 + \cos^2\theta + 2\sin\theta\cos\theta = 2$

$1 + 2\sin\theta\cos\theta = 2$

$2\sin\theta\cos\theta = 1$

$2\sin\theta\cos\theta = \sin^2\theta + \cos^2\theta \quad \ldots [\because 1 = \sin^2\theta + \cos^2\theta]$

$\Rightarrow \quad \dfrac{2\sin\theta\cos\theta}{\sin\theta\cos\theta} = \dfrac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta}$

$\Rightarrow \quad 2 = \dfrac{\sin^2\theta}{\sin\theta\cos\theta} + \dfrac{\cos^2\theta}{\sin\theta\cos\theta} \Rightarrow \quad 2 = \tan\theta + \cot\theta$

19. We have, $\dfrac{1}{\sec A + \tan A} - \dfrac{1}{\cos A} = \dfrac{1}{\cos A} - \dfrac{1}{\sec A - \tan A}$

$\Rightarrow \dfrac{1}{\sec A + \tan A} + \dfrac{1}{\sec A - \tan A} = \dfrac{1}{\cos A} + \dfrac{1}{\cos A}$

LHS $= \dfrac{1}{\sec A + \tan A} + \dfrac{1}{\sec A - \tan A}$

$= \dfrac{\sec A - \tan A + \sec A + \tan A}{(\sec A + \tan A)(\sec A - \tan A)}$

$= \dfrac{2\sec A}{\sec^2 A - \tan^2 A} \left[\because (a+b)(a-b) = (a^2 - b^2) \right]$

$= \dfrac{2\sec A}{1} \left[\because \sec^2 A - \tan^2 A = 1 \right]$

$= 2\sec A$

RHS $= \dfrac{1}{\cos A} + \dfrac{1}{\cos A}$

$= \dfrac{1+1}{\cos A}$

$= \dfrac{2}{\cos A}$

$= 2\sec A$

LHS = RHS

20. We have,

a $\sec\theta + $ b $\tan\theta + $ c $ = 0$ ......(i)

and, p $\sec\theta + $ q $\tan\theta + $ r $ = 0$ ...... (ii)

Solving equations (i) and (ii) by the cross-multiplication for $\sec\theta$ and $\tan\theta$, we get

$\dfrac{\sec\theta}{br - qc} = \dfrac{\tan\theta}{cp - ar} = \dfrac{1}{aq - bp}$

$\Rightarrow \sec\theta = \dfrac{br - cq}{aq - bp}$ and $\tan\theta = \dfrac{cp - ar}{aq - bp}$

We know that, $\sec^2\theta - \tan^2\theta = 1$

$\Rightarrow \quad \left(\dfrac{br - cq}{aq - bp}\right)^2 - \left(\dfrac{cp - ar}{aq - bp}\right)^2 = 1$

$\Rightarrow \quad \dfrac{(br - cq)^2 - (cp - ar)^2}{(aq - bp)^2} = 1$

therefore, $(br - cq)^2 - (cp - ar)^2 = (aq - bp)^2$

21. We have, $\left(\dfrac{\sin 27°}{\cos 63°}\right)^2 - \left(\dfrac{\cos 63°}{\sin 27°}\right)^2$

$= \left[\dfrac{\sin(90° - 63°)}{\cos 63°}\right]^2 - \left[\dfrac{\cos(90° - 27°)}{\sin 27°}\right]^2$

$= \left[\dfrac{\cos 63°}{\cos 63°}\right]^2 - \left[\dfrac{\sin 27°}{\sin 27°}\right]^2 \begin{bmatrix} \sin(90° - \theta) = \cos\theta \\ \cos(90° - \theta) = \sin\theta \end{bmatrix}$

$= [1]^2 - [1]^2$

$= 1 - 1 = 0$

$\therefore \left(\dfrac{\sin 27°}{\cos 63°}\right)^2 - \left(\dfrac{\cos 63°}{\sin 27°}\right)^2 = 0$

22. L.H.S. $= (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$

$= (\sin\theta + \cos\theta)\left(\dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}\right)$

$= (\sin\theta + \cos\theta)\left(\dfrac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}\right)$

$= (\sin\theta + \cos\theta) \times \dfrac{1}{\sin\theta\cos\theta}$ [$\because \sin^2\theta + \cos^2\theta = 1$]

$= \dfrac{\sin\theta + \cos\theta}{\cos\theta\sin\theta}$

$= \dfrac{\sin\theta}{\cos\theta\sin\theta} + \dfrac{\cos\theta}{\cos\theta\sin\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta}$

$= \sec\theta + \operatorname{cosec}\theta$

$= $ R.H.S.

Hence proved.

23. L.H.S $= \dfrac{\cos\theta}{1 - \sin\theta}$

Multiplying and dividing by $(1 + \sin\theta)$ we get

$= \dfrac{\cos\theta(1 + \sin\theta)}{(1 - \sin\theta)(1 + \sin\theta)}$

$= \dfrac{\cos\theta(1 + \sin\theta)}{1 - \sin^2\theta}$ $\left[\because (a - b)(a + b) = a^2 - b^2\right]$

$= \dfrac{\cos\theta(1 + \sin\theta)}{\cos^2\theta}$ $\left[\because 1 - \sin^2\theta = \cos^2\theta\right]$

$= \dfrac{\cos\theta(1 + \sin\theta)}{\cos\theta\cos\theta}$

$= \dfrac{1 + \sin\theta}{\cos\theta}$

$= $ R.H.S

Hence proved.

24. Given

m sinA + n cosA = p......(1)

m cosA - n sinA = q........(2)

Squaring (1) and (2) we get,

$m^2\sin^2 A + n^2\cos^2 A + 2mn\sin A\cos A = p^2$ ......(3)

$m^2\cos^2 A + n^2\sin^2 A - 2mn\sin A\cos A = q^2$ .... (4)

Adding (3) and (4) we get,

$m^2(\sin^2 A + \cos^2 A) + n^2(\sin^2 A + \cos^2 A) = p^2 + q^2$

$\Rightarrow m^2 + n^2 = p^2 + q^2$ [$\because \sin^2 A + \cos^2 A = 1$]

25. Given:

$cosec^2\theta(1 + \cos\theta)(1 - \cos\theta) = \lambda$

$\Rightarrow cosec^2\theta\{(1 + \cos\theta)(1 - \cos\theta)\} = \lambda$

$\Rightarrow cosec^2\theta\left(1 - \cos^2\theta\right) = \lambda$

$\Rightarrow \operatorname{cosec}^2\theta\sin^2\theta = \lambda$

$\Rightarrow \dfrac{1}{\sin^2\theta} \times \sin^2\theta = \lambda$

$\Rightarrow 1 = \lambda$

$\Rightarrow \quad \lambda = 1$

Thus, the value of λ is 1.

26. We have, $\cos\theta + \cos^2\theta = 1$

$\Rightarrow \cos\theta = 1 - \cos^2\theta$

$\Rightarrow \cos\theta = \sin^2\theta\left(\because \sin^2\theta + \cos^2\theta = 1\right)$......(1)

Now, $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2$

$= \left[\left(\sin^4\theta\right)^3 + 3\sin^4\theta \cdot \sin^2\theta\left(\sin^4\theta + \sin^2\theta\right) + \left(\sin^2\theta\right)^3\right] + 2\left(\sin^2\theta\right)^2 + 2\sin^2\theta - 2$

Using $(a + b)^3 = a^3 + b^3 + 3ab\,(a + b)$

Also, from (1) $\sin^2\theta = \cos\theta$

$= \left(\sin^4\theta + \sin^2\theta\right)^3 + 2(\cos\theta)^2 + 2\sin^2\theta - 2$

$= \left(\left(\sin^2\theta\right)^2 + \sin^2\theta\right)^3 + 2\left(\sin^2\theta + \cos^2\theta\right) - 2$

$= \left(\cos^2\theta + \sin^2\theta\right)^3 + 2 - 2\left(\because \sin^2\theta + \cos^2\theta = 1\right)$

$= (1)^3 + 0$

$= 1 = $ R.H.S

therefore, $\sin^{12}\theta + 3\sin^{10}\theta + 3\sin^8\theta + \sin^6\theta + 2\sin^4\theta + 2\sin^2\theta - 2 = 1$

Hence proved.

27. LHS $= \dfrac{\sin\theta - 2\sin^3\theta}{2\cos^3\theta - \cos\theta}$

$= \dfrac{\sin\theta\left(1 - 2\sin^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$

$= \dfrac{\sin\theta\left(1 - 2 + 2\cos^2\theta\right)}{\cos\theta\left(2\cos^2\theta - 1\right)}$

$= \tan\theta\left(\dfrac{2\cos^2\theta - 1}{2\cos^2\theta - 1}\right)$

$= \tan\theta = $ RHS

28. We have

L.H.S. $= (\text{cosec}A - \sin A)(\sec A - \cos A)$

$= \left(\dfrac{1}{\sin A} - \sin A\right)\left(\dfrac{1}{\cos A} - \cos A\right) = \left(\dfrac{1 - \sin^2 A}{\sin A}\right) \cdot \left(\dfrac{1 - \cos^2 A}{\cos A}\right)$

$= \dfrac{\cos^2 A\sin^2 A}{\cos A\sin A} = \cos A.\sin A.$

R.H.S. $= \dfrac{1}{(\tan A + \cot A)} = \dfrac{1}{\left(\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}\right)}$

$= \dfrac{\cos A\sin A}{\left(\sin^2 A + \cos^2 A\right)} = \cos A\sin A \; [\because \; \sin^2 A + \cos^2 A = 1]$

Hence, L.H.S. = R.H.S.

29. We have,

LHS $= \sin^6\theta + \cos^6\theta + 3\sin^2\theta \cos^2\theta$

$\Rightarrow$ LHS $= (\sin^2\theta)^3 + (\cos^2\theta)^3 + 3\sin^2\theta \cos^2\theta$

$\Rightarrow$ LHS $= (\sin^2\theta + \cos^2\theta)^3 - 3\sin^2\theta\cos^2\theta\,(\sin^2\theta + \cos^2\theta) + 3\sin^2\theta\cos^2\theta \; [\because \; a^3 + b^3 = (a + b)^3 - 3ab\,(a + b)]$

$\Rightarrow$ LHS $= 1 - 3\sin^2\theta \cos^2\theta + 3\sin^2\theta \cos^2\theta = 1 = $ RHS

30. LHS $= \dfrac{\tan A}{\sec A - 1} + \dfrac{\tan A}{\sec A + 1}$

$= \dfrac{\tan A(\sec A + 1) + \tan A(\sec A - 1)}{(\sec A - 1)(\sec A + 1)}$

$= \dfrac{\tan A \cdot \sec A + \tan A + \tan A \sec A - \tan A}{\sec^2 A - 1}$

$= \dfrac{2\tan A \sec A}{\tan^2 A}$ [$\because$ $(sec^2\theta - 1) = tan^2\theta$]

$= \dfrac{2\sec A}{\tan A}$

$= \dfrac{2\dfrac{1}{\cos A}}{\dfrac{\sin A}{\cos A}}$

$= 2 \times \dfrac{1}{\cos A} \times \dfrac{\cos A}{\sin A}$

$= \dfrac{2}{\sin A}$

$= 2cosec\, A$ = RHS

31. LHS $= \dfrac{1 - \tan^2\theta}{1 + \tan^2\theta}$

$= \dfrac{1 - \dfrac{\sin^2\theta}{\cos^2\theta}}{1 + \dfrac{\sin^2\theta}{\cos^2\theta}}$

$= \dfrac{\left(\dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}\right)}{\left(\dfrac{\cos^2\theta + \sin^2\theta}{\cos^2}\right)} = \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$= \dfrac{\left(\cos^2\theta - \sin^2\theta\right)}{1} = (\cos^2\theta - \sin^2\theta)$ = RHS.

Hence Proved.

32. LHS $= \tan^2 A + \cot^2 A$

$= \dfrac{\sin^2 A}{\cos^2 A} + \dfrac{\cos^2 A}{\sin^2 A}$ $\left[\because \tan A = \dfrac{\sin A}{\cos A}, \cot A = \dfrac{\cos A}{\sin A}\right]$

$= \dfrac{(\sin^2 A)^2 + (\cos^2 A)^2}{\cos^2 A \sin^2 A}$

$= \dfrac{(\sin^2 A + \cos^2 A)^2 - 2\sin^2 A\cos^2 A}{\cos^2 A \sin^2 A}$ $\left[\because a^2 + b^2 = (a + b)^2 - 2ab\right]$

$= \dfrac{(1)^2 - 2\sin^2 A\cos^2 A}{\cos^2 A \sin^2 A}$ $\left[\because \sin^2 A + \cos^2 A = 1\right]$

$= \dfrac{1}{\cos^2 A \sin^2 A} - \dfrac{2\sin^2 A\cos^2 A}{\cos^2 A \sin^2 A}$

$= \dfrac{1}{\cos^2 A} \times \dfrac{1}{\sin^2 A} - 2$

$= \sec^2 A\, \mathrm{cosec}^2 A - 2$ $\left[\because \dfrac{1}{\cos A} = \sec A, \dfrac{1}{\sin A} = \mathrm{cosec}A\right]$

= RHS

Hence proved.

33. L. HS $= \tan^4\theta + \tan^2\theta$

$= \tan^2\theta\left(\tan^2\theta + 1\right)$

$= \tan^2\theta \ \ \sec^2\theta$

$= \left(\sec^2\theta - 1\right)\sec^2\theta$ $\left[\because \tan^2\theta = \sec^2\theta - 1\right]$

$= \sec^4\theta - \sec^2\theta$

= R.H.S.

34. We have,

$\frac{\sin\theta}{\cot\theta + \cos ec\theta} = 2 + \frac{\sin\theta}{\cot\theta - \cos ec\theta}$ or, $\frac{\sin\theta}{\cot\theta + \cos ec\theta} - \frac{\sin\theta}{\cot\theta - \cos ec\theta} = 2$

Now,

LHS $= \frac{\sin\theta}{\cot\theta + \cos ec\theta} - \frac{\sin\theta}{\cot\theta - \cos ec\theta}$

$\Rightarrow$ LHS $= \frac{\sin\theta}{\cos ec\theta + \cot\theta} + \frac{\sin\theta}{\cos ec\theta - \cot\theta}$

$\Rightarrow$ LHS $= \sin\theta\left\{\frac{1}{\cos ec\theta + \cot\theta} + \frac{1}{\cos ec\theta - \cot\theta}\right\}$

$\Rightarrow$ LHS $= \sin\theta\left\{\frac{\cos ec\theta - \cot\theta + \cos ec\theta + \cot\theta}{\cos ec^2\theta - \cot^2\theta}\right\} = \sin\theta\left(\frac{2\cos ec\theta}{1}\right)$

$\Rightarrow$ LHS $= \sin\theta(2\cos ec\theta) = 2\sin\theta \times \frac{1}{\sin\theta} = 2 = $ RHS

35. Given,

R.H.S $= m^2 + n^2$

$= (a\cos\theta + b\sin\theta)^2 + (a\sin\theta - b\cos\theta)^2$ [si*nce*, $m = a\cos\theta + b\sin\theta$ *and* $n = a\sin\theta - b\cos\theta$]

$= (a^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta\sin\theta) + (a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta)$ $\left[\because (a \pm b)^2 = a^2 + b^2 \pm 2ab\right]$

$= a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta) = a^2 + b^2 = $ L.H.S $\left[\because \sin^2\theta + \cos^2\theta = 1\right]$

therefore, $m^2 + n^2 = a^2 + b^2$

Hence proved.

36. We have,

$\tan\theta + \cot\theta = 2$

$\Rightarrow$ $\tan\theta + \frac{1}{\tan\theta} = 2$

$\Rightarrow$ $\frac{\tan^2\theta + 1}{\tan\theta} = 2$

$\Rightarrow$ $\tan^2\theta + 1 = 2\tan\theta$

$\Rightarrow$ $\tan^2\theta - 2\tan\theta + 1 = 0$

$\Rightarrow$ $(\tan\theta - 1)^2 = 0$

$\Rightarrow$ $\tan\theta - 1 = 0$

$\Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45°$ (θ is acute angle, $\therefore$ $\theta = 45°$)

$\Rightarrow \theta = 45°$

$\therefore$ $\tan^7\theta + \cot^7\theta = \tan^7 45° + \cot^7 45° = \left(\tan 45°\right)^7 + \left(\cot 45°\right)^7 = (1)^7 + (1)^7 = 2$

37. We have,

L. H. S $= \frac{\sin A + \cos A}{\sin A - \cos A} + \frac{\sin A - \cos A}{\sin A + \cos A}$

$\Rightarrow$ L. H. S $= \frac{(\sin A + \cos A)^2 + (\sin A - \cos A)^2}{(\sin A - \cos A)(\sin A + \cos A)}$

$\Rightarrow$ L. H. S $= \frac{\left(\sin^2 A + \cos^2 A + 2\sin A\cos A\right) + \left(\sin^2 A + \cos^2 A - 2\sin A\cos A\right)}{\sin^2 A - \cos^2 A}$ $[\because (a \pm b)^2 = a^2 \pm 2ab + b^2]$

$\Rightarrow$ L. H. S $= \frac{(1 + 2\sin A\cos A) + (1 - 2\sin A\cos A)}{\sin^2 A - \cos^2 A}$

$\Rightarrow$ L. H. S $= \frac{2}{\sin^2 A - \cos^2 A}$

$\Rightarrow$ L. H. S $= \frac{2}{\sin^2 A - \cos^2 A} = \frac{2}{\sin^2 A - \left(1 - \sin^2 A\right)}$ $[\because \sin^2 A + \cos^2 A = 1]$

$\Rightarrow$ L. H. S $= \frac{2}{2\sin^2 A - 1} = \frac{2}{2\left(1 - \cos^2 A\right) - 1} = \frac{2}{1 - 2\cos^2 A} = $ R.H.S $[\because \sin^2 A = 1 - \cos^2 A \text{ & } \cos^2 A = 1 - \sin^2 A]$

Hence proved.

38. LHS = $(a \sin \theta + b \cos \theta)^2 + (a \cos \theta - b \sin \theta)^2$

$= a^2 (\sin^2 \theta + \cos^2 \theta) + b^2 (\cos^2 \theta + \sin^2 \theta)^2$

$= a^2 + b^2$ = RHS.

39. We have,.

$\tan \theta + \cot \theta = 2$

$\Rightarrow$ $(\tan \theta + \cot \theta)^2 = 4$ [On squaring both sides]

$\Rightarrow$ $\tan^2 \theta + \cot^2 \theta + 2 \tan \theta \cot \theta = 4$

$\Rightarrow$ $\tan^2 \theta + \cot^2 \theta + 2 = 4$ [$\because$ $\tan \theta \cot \theta = 1$]

$\Rightarrow$ $\tan^2 \theta + \cot^2 \theta = 2$

40. Given that: $\sin \theta + \cos \theta = \sqrt{3}$

or $(\sin \theta + \cos \theta)^2 = 3$

or $\sin^2 \theta + \cos^2 \theta + 2\sin\theta \cos\theta = 3$

$2\sin\theta \cos\theta = 2$ [As $\sin^2 \theta + \cos^2 \theta = 1$]

or $\sin \theta \cos \theta = 1 = \sin^2 \theta + \cos^2 \theta$

LHS=1

RHS=$\dfrac{Sin\theta}{Cos\theta} + \dfrac{Cos\theta}{Sin\theta}$

Taking LCM

Substitute the value of LHS in RHS we get,

$\tan\theta + \cot\theta = 1$

41. Taking L.H.S

$\dfrac{\cos A - \sin A + 1}{\cos A + \sin A - 1}$

Dividing Numerator and Denominator by sin A

$= \dfrac{\frac{\cos A - \sin A + 1}{\sin A}}{\frac{\cos A + \sin A - 1}{\sin A}}$

$= \dfrac{\frac{\cos A}{\sin A} - \frac{\sin A}{\sin A} + \frac{1}{\sin A}}{\frac{\cos A}{\sin A} + \frac{\sin A}{\sin A} - \frac{1}{\sin A}}$

Using the formula $\cot\theta = \dfrac{\cos \theta}{\sin \theta} = \dfrac{\cot A - 1 + cosecA}{\cot A + 1 - cosecA}$

Using the identity $cosec^2A = 1 + \cot^2A$

$= \dfrac{\cot A - \left(cosec^2A - \cot^2 A \right) + cosecA}{\cot A + 1 - cosecA}$

$= \dfrac{(\cot A + cosecA) - \left(cosec^2A - \cot^2 A \right)}{\cot A + 1 - cosecA}$

$= \dfrac{(\cot A + cosecA)(1 - cosecA + \cot A)}{\cot A + 1 - cosecA}$

$= \cot A + cosec\ A$

$=$ R.H.S

42. We have,

LHS = $\sin^4 A - \cos^4 A$

$\Rightarrow$ LHS = $(\sin^2 A)^2 - (\cos^2 A)^2$

$\Rightarrow$ LHS = $(\sin^2 A + \cos^2 A)(\sin^2 A - \cos^2 A)$

$\Rightarrow$ LHS = $\sin^2 A - \cos^2 A$ ...[$\because$ $Sin^2 A + \cos^2 A = 1$]

$\Rightarrow$ LHS = $\sin^2 A - (1 - \sin^2 A) = 2\sin^2 A - 1$

$\Rightarrow$ LHS = $2(1 - \cos^2 A) - 1 = 1 - 2\cos^2 A$ = RHS

43. **Given:** $2sin^2\theta - cos^2\theta = 2$

$\Rightarrow 2(1 - cos^2\theta) - cos^2\theta = 2$ [$\because$ $sin^2\theta + cos^2\theta = 1$]

$\Rightarrow 2 - 2cos^2\theta - cos^2\theta = 2$

$\Rightarrow 2 - 3cos^2\theta = 2$

$\Rightarrow -3cos^2\theta = 2 - 2$

$\Rightarrow -3cos^2\theta = 0$

$\Rightarrow cos^2\theta = 0$

$\Rightarrow cos\theta = 0$

$\Rightarrow cos\theta = cos90°$

$\Rightarrow \theta = 90°$

44. We have,

LHS = $(\sin\theta + \text{cosec}\theta)^2 + (\cos\theta + \sec\theta)^2$

$\Rightarrow$ LHS = $(\sin^2\theta + \text{cosec}^2\theta + 2\sin\theta\,\text{cosec}\theta) + (\cos^2\theta + \sec^2\theta + 2\cos\theta\,\sec\theta)$

$\Rightarrow$ LHS = $\left(\sin^2\theta + cosec^2\theta + 2\sin\theta\dfrac{1}{\sin\theta}\right) + \left(\cos^2\theta + \sec^2\theta + 2\cos\theta\dfrac{1}{\cos\theta}\right)$

$\Rightarrow$ LHS = $(\sin^2\theta + \text{cosec}^2\theta + 2) + (\cos^2\theta + \sec^2\theta + 2)$

$\Rightarrow$ LHS = $\sin^2\theta + \cos^2\theta + \text{cosec}^2\theta + \sec^2\theta + 4$

$\Rightarrow$ LHS = $1 + (1 + \cot^2\theta) + (1 + \tan^2\theta) + 4$ [$\because$ $\text{cosec}^2\theta = 1 + \cot^2\theta$, $\sec^2\theta = 1 + \tan^2\theta$]

$\Rightarrow$ LHS = $7 + \tan^2\theta + \cot^2\theta$ = RHS

45. W e have,

LHS = $(\text{cosec}^2\theta - 1)\tan^2\theta$

= $(1 + \cot^2\theta - 1)\tan^2\theta$ [$\because$ $\text{cosec}^2\theta = 1 + \cot^2\theta$]

= $\cot^2\theta \cdot \tan^2\theta$

= $\dfrac{1}{\tan^2\theta} \cdot \tan^2\theta$ $\left[\because \cot\theta = \dfrac{1}{\tan\theta}\right]$

= 1 = RHS

46. $\cos A + \cos^2 A = 1$ (Given)

$\Rightarrow$ $\cos A = 1 - \cos^2 A$

$\Rightarrow$ $\cos A = \sin^2 A$ [$\because$ $1 - \cos^2 A = \sin^2 A$]...... (i)

L.H.S = $\sin^2 A + \sin^4 A$

= $\sin^2 A + (\sin^2 A)^2$

= $\sin^2 A + (\cos A)^2$

= $\sin^2 A + \cos^2 A$ [$\because$ $\sin^2 A + \cos^2 A = 1$]

= 1 = R.H.S

therefore, $\sin^2 A + \sin^4 A = 1$

Hence proved.

47. Taking L.H.S. $x^2 + y^2 = (3\sin\theta + 4\cos\theta)^2 + (3\cos\theta - 4\sin\theta)^2$

= $9\sin^2\theta + 16\cos^2\theta + 2 \times 3\sin\theta \times 4\cos\theta + 9\cos^2\theta + 16\sin^2\theta - 2 \times 3\cos\theta \times 4\sin\theta$

= $9\left(\sin^2\theta + \cos^2\theta\right) + 16\left(\cos^2\theta + \sin^2\theta\right)$

= $9 \times 1 + 16 \times 1$ [$\because$ $\sin^2\theta + \cos^2\theta = 1$]

= $9 + 16$

= 25 = R.H.S

Hence proved.

48. We have,

LHS = $\dfrac{\cot A + cosec A - 1}{\cot A - cosec A + 1}$

$\Rightarrow$ LHS = $\dfrac{(\cot A + cosec A) - \left(cosec^2 A - \cot^2 A\right)}{(\cot A - cosec A) + 1}$ $\left[\because cosec^2 A - \cot^2 A = 1\right]$

$\Rightarrow$ LHS = $\dfrac{(cosec A + \cot A) - (cosec A + \cot A)(cosec A - \cot A)}{\cot A - cosec A + 1}$

$\Rightarrow$ LHS $= \dfrac{(cosecA + \cot A)\,[\,1 - (cosecA - \cot A)\,]}{\cot A - cosecA + 1}$

$\Rightarrow$ LHS $= \dfrac{(cosecA + \cot A)\,(\cot A - cosecA + 1)}{(\cot A - cosecA + 1)}$

$\Rightarrow$ LHS $= cosecA + \cot A = \dfrac{1}{\sin A} + \dfrac{\cos A}{\sin A} = \dfrac{1 + \cos A}{\sin A} =$ RHS

49. We have, $x = a\sin\theta$ and $y = b\tan\theta$

$\therefore$ LHS $= \dfrac{a^2}{x^2} - \dfrac{b^2}{y^2}$

$\Rightarrow$ LHS $= \dfrac{a^2}{a^2\sin^2\theta} - \dfrac{b^2}{b^2\tan^2\theta}$ $[\because\ x = a\sin\theta,\ y = b\tan\theta]$

$\Rightarrow$ LHS $= \dfrac{1}{\sin^2\theta} - \dfrac{1}{\tan^2\theta}$

$\Rightarrow \cosec^2\theta - \cot^2\theta = 1 =$ RHS $[\because\ 1 + \cot^2\theta = \cosec^2\theta \ \therefore\ \cosec^2\theta - \cot^2\theta = 1]$

50. Consider a right-angled triangle, $\triangle ABC$, where $\angle BAC = \theta$,
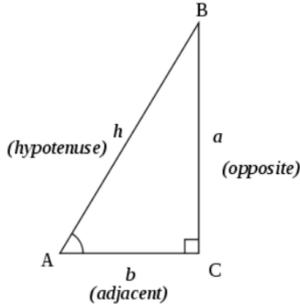
By the Pythagorean theorem,

$AC^2 + BC^2 = AB^2$

Dividing by AB^2,

$\Rightarrow \dfrac{AC^2}{AB^2} + \dfrac{BC^2}{AB^2} = \dfrac{AB^2}{AB^2}$

$\Rightarrow \left(\dfrac{\text{opposite}}{\text{hypotenuse}}\right)^2 + \left(\dfrac{\text{adjacent}}{\text{hypotenuse}}\right)^2 = \dfrac{AB^2}{AB^2} = 1$

$\Rightarrow \sin^2\theta + \cos^2\theta = 1$

51. LHS $= \dfrac{\cot A - \cos A}{\cot A + \cos A}$

$= \dfrac{\dfrac{\cos A}{\sin A} - \cos A}{\dfrac{\cos A}{\sin A} + \cos A}$

$= \dfrac{\dfrac{\cos A - \sin A\cos A}{\sin A}}{\dfrac{\cos A + \sin A\cos A}{\sin A}}$

$= \dfrac{\cos A\,(1 - \sin A)}{\cos A\,(1 + \sin A)}$

$= \dfrac{1 - \sin A}{1 + \sin A}$

$= \dfrac{\dfrac{1}{\sin A} - 1}{\dfrac{1}{\sin A} + 1}$

$= \dfrac{cosecA - 1}{cosecA + 1} =$ RHS

52. LHS $= \dfrac{1 - \cos A}{1 + \cos A}$

Multiplying numerator and denominator by 1 - cos A

$= \dfrac{(1 - \cos A)\,(1 - \cos A)}{(1 + \cos A)\,(1 - \cos A)}$

$= \dfrac{(1 - \cos A)^2}{1 - \cos^2 A}\ \left[\because (a + b)(a - b) = a^2 - b^2\right]$

$= \dfrac{(1 - \cos A)^2}{\sin^2 A}\ \left[\because 1 - \cos^2 A = \sin^2 A\right]$

$$= \left(\frac{1 - \cos A}{\sin A} \right)^2$$

$$= \left(\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right)^2$$

$$= (\text{cosec}A - \text{cot}A)^2 \left[\because \frac{1}{\sin A} = \text{cosec } A, \frac{\cos A}{\sin A} = \text{cot}A \right]$$

$$= [-1(\text{cot}A - \text{cosec}A)]^2$$

$$= (\text{cot } A - \text{cosec } A)^2$$

Hence proved.

53. We have to find the value of sin 25°cos 65°+ cos 25°sin 65°

= sin 25°cos (90°-25°) + cos 25°sin 65° {by using Cos (90 - A) = Sin A}

= sin 25°sin 25° + cos 25°sin 65°= $\sin^2$ 25° + cos 25° sin (90°-25°) {by using Sin (90-A) = Cos A}

= $\sin^2$ 25° + $\cos^2$ 25°

= 1

54. We have, $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta}$

$$= \frac{\sin^2 \theta + (1 + \cos \theta)^2}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{\sin^2 \theta + 1 + \cos^2 \theta + 2\cos \theta}{\sin \theta (1 + \cos \theta)}$$

$$= \frac{1 + 1 + 2\cos \theta}{\sin \theta (1 + \cos \theta)} = \frac{2 (1 + \cos \theta)}{\sin \theta (1 + \cos \theta)}$$

$$= 2 \text{ cosec } \theta$$

55. Given: cosecθ + cotθ = m and cosecθ - cotθ = n,

LHS = mn

= (cosec θ + cotθ)(cosecθ − cotθ)

$$= \left(\text{cosec}^2\theta - \text{cot}^2\theta \right) \left[\because (a + b)(a - b) = a^2 - b^2 \right]$$

$$= 1 \left[\because \text{cosec}^2\theta - \text{cot}^2\theta = 1 \right]$$

= RHS

Hence proved.

56. LHS=(cosecA - cotA)^2 = $\left(\frac{1}{\sin A} - \frac{\cos A}{\sin A} \right)^2$

$$= \left(\frac{1 - \cos A}{\sin A} \right)^2$$

$$= \frac{(1 - \cos A)^2}{\sin^2 A}$$

$$= \frac{(1 - \cos A)^2}{1 - \cos^2 A}$$

$$= \frac{(1 - \cos A)^2}{(1 - \cos A)(1 + \cos A)}$$

$$= \frac{1 - \cos A}{1 + \cos A}$$

Hence proved

57. We have,

$$\text{LHS} = \frac{(1 + \sin \theta)^2 + (1 - \sin \theta)^2}{\cos^2 \theta}$$

$$\Rightarrow \quad \text{LHS} = \frac{\left(1 + 2\sin \theta + \sin^2 \theta \right) + \left(1 - 2\sin \theta + \sin^2 \theta \right)}{\cos^2 \theta}$$

$$\Rightarrow \quad \text{LHS} = \frac{2 + 2\sin^2 \theta}{\cos^2 \theta} = \frac{2 \left(1 + \sin^2 \theta \right)}{1 - \sin^2 \theta} = 2 \left(\frac{1 + \sin^2 \theta}{1 - \sin^2 \theta} \right) = \text{RHS}$$

58. LHS $= \dfrac{\sin\theta + \cos\theta}{\sin\theta - \cos\theta} + \dfrac{\sin\theta - \cos\theta}{\sin\theta + \cos\theta}$

$= \dfrac{(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2}{\sin^2\theta - \cos^2\theta}$

$= \dfrac{\sin^2\theta + \cos^2\theta + 2\cos\theta\sin\theta + \sin^2\theta + \cos^2\theta - 2\cos\theta\sin\theta}{1 - \cos^2\theta - \cos^2\theta}$

$= \dfrac{1+1}{1 - 2\cos^2\theta} = \dfrac{2}{\left(1 - 2\cos^2\theta\right)}$

= RHS

therefore, LHS = RHS.

59. To prove: $\dfrac{\sin^2 63° + \sin^2 27}{\cos^2 17° + \cos^2 73°} = 1$

proof:

L.H.S. $= \dfrac{\sin^2 63° + \sin^2 27°}{\cos^2 17 + \cos^2 73°}$

$= \dfrac{\sin^2 63° + \sin^2\left(90° - 63°\right)}{\cos^2 17° + \cos^2\left(90° - 17°\right)}$

By complementary angle formula that, sin (90 - A) = cos A and cos (90 - A) = sin A

$= \dfrac{\sin^2 63° + \cos^2 63°}{\cos^2 17° + \sin^2 17°}$

(using $\sin^2 A + \cos^2 A = 1$)

$= \dfrac{1}{1}$

$= 1$

L.H.S = R.H.S

Hence proved

60. $\cos\theta + \sin\theta = \sqrt{2}\cos\theta$

$\Rightarrow \sin\theta = \sqrt{2}\cos\theta - \cos\theta$

$\Rightarrow \sin\theta = (\sqrt{2} - 1)\cos\theta$

$\Rightarrow \dfrac{\sin\theta}{\sqrt{2}-1} = \cos\theta$

$\Rightarrow \dfrac{\sin\theta(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)} = \cos\theta$

$\Rightarrow \sqrt{2}\sin\theta + \sin\theta = \cos\theta$

$\Rightarrow \cos\theta - \sin\theta = \sqrt{2}\sin\theta$ = RHS

61. According to the question,

$x^2 + y^2 = (a\sin\theta + b\cos\theta)^2 + (a\cos\theta - b\sin\theta)^2$

$= a^2\sin^2\theta + b^2\cos^2\theta + 2ab\cos\theta\sin\theta + a^2\cos^2\theta + b^2\sin^\theta - 2ab\sin\theta\cos\theta$

$= a^2(\sin^2\theta + \cos^2\theta) + b^2(\cos^2\theta + \sin^2\theta)$

$= a^2 + b^2 \ [\because \ \sin^2\theta + \cos^2\theta = 1].$

$\therefore \ x^2 + y^2 = a^2 + b^2.$

62. $\dfrac{1 - \cos\theta}{1 + \cos\theta} = (\cosec\theta - \cot\theta)^2$

L.H.S. $= \dfrac{1 - \cos\theta}{1 + \cos\theta}$

$= \dfrac{(1 - \cos\theta) \times (1 - \cos\theta)}{(1 + \cos\theta)(1 - \cos\theta)}$ [Multiplying and dividing by $(1 - \cos\theta)$]

$= \dfrac{(1 - \cos\theta)^2}{1 - \cos^2\theta} = \dfrac{(1 - \cos\theta)^2}{\sin^2\theta}\left[\ \because\ 1 - \cos^2\theta = \sin^2\theta\ \right]$

$= \left(\dfrac{1 - \cos\theta}{\sin\theta}\right)^2 = \left[\dfrac{1}{\sin\theta} - \dfrac{\cos\theta}{\sin\theta}\right]^2$

$$= (cosec\theta - \cot\theta)^2 \quad \left[\because \frac{1}{\sin\theta} = cosec\theta, \frac{\cos\theta}{\sin\theta} = \cot\theta \right]$$

= R.H.S. proved.

63. LHS-

$$\frac{\tan A}{1 - \cot A} + \frac{\cot A}{1 - \tan A}$$

$$= \frac{\tan A}{1 - \frac{1}{\tan A}} + \frac{\frac{1}{\tan A}}{1 - \tan A}$$

$$= \frac{\tan A}{\frac{\tan A - 1}{\tan A}} + \frac{1}{\tan A(1 - \tan A)}$$

$$= \frac{\tan^2 A}{\tan A - 1} + \frac{1}{\tan A(1 - \tan A)}$$

$$= \frac{\tan^3 A - 1}{\tan A(\tan A - 1)}$$

$$= \frac{(\tan A - 1)(\tan^2 A + \tan A + 1)}{\tan A(\tan A - 1)} \quad [a^3 - b^3 = (a-b)(a^2 + ab + b^2)]$$

$$= \frac{\tan^2 A + \tan A + 1}{\tan A}$$

$$= \tan A + 1 + \cot A$$

$$= \frac{\sin A}{\cos A} + \frac{\cos A}{\sin A} + 1$$

$$= \frac{\sin^2 A + \cos^2 A}{\sin A \cos A} + 1$$

$$= \frac{1}{\sin A \cos A} + 1$$

= sec A cosec A+ 1

= R.H.S

64. We have,

$$\text{LHS} = \cot^2 A \left(\frac{\sec A - 1}{1 + \sin A} \right) + \sec^2 A \left(\frac{\sin A - 1}{1 + \sec A} \right)$$

$$\Rightarrow \quad \text{LHS} = \frac{\cot^2 A(\sec A - 1)(\sec A + 1) + \sec^2 A(\sin A - 1)(1 + \sin A)}{(1 + \sin A)(1 + \sec A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\cot^2 A \left(\sec^2 A - 1 \right) + \sec^2 A \left(\sin^2 A - 1 \right)}{(1 + \sin A)(1 + \sec A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\cot^2 A \left(\sec^2 A - 1 \right) - \sec^2 A \left(1 - \sin^2 A \right)}{(1 + \sin A)(1 + \sec A)}$$

$$\Rightarrow \quad \text{LHS} = \frac{\cot^2 A \tan^2 A - \sec^2 A \cos^2 A}{(1 + \sin A)(1 + \sec A)}$$

$$= \frac{1 - 1}{(1 + \sin A)(1 + \sec A)}$$

= 0 = RHS

65. LHS $= \frac{1 + \sec\theta - \tan\theta}{1 + \sec\theta + \tan\theta}$

$$= \frac{(\sec\theta - \tan\theta) + \left(\sec^2\theta - \tan^2\theta \right)}{1 + \sec\theta + \tan\theta}$$

$$= \frac{(\sec\theta - \tan\theta)(1 + \sec\theta + \tan\theta)}{(1 + \sec\theta + \tan\theta)}$$

$$= \sec\theta - \tan\theta = \frac{1 - \sin\theta}{\cos\theta} = \text{RHS}$$

66. L.H.S. $= (\sin\theta + \cos\theta)(\tan\theta + \cot\theta)$

$$= (\sin\theta + \cos\theta)\left(\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} \right)$$

$$= (\sin\theta + \cos\theta)\left(\frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} \right)$$

$= (\sin\theta + \cos\theta)\left(\dfrac{1}{\sin\theta\cos\theta}\right)$

$= \dfrac{\sin\theta + \cos\theta}{\sin\theta\cos\theta}$

$= \dfrac{\sin\theta}{\sin\theta\cos\theta} + \dfrac{\cos\theta}{\sin\theta\cos\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{1}{\sin\theta}$

$= \sec\theta + \csc\theta$

$= $ R.H.S.

67. We have,

L.H.S $= (1 + \tan A \tan B)^2 + (\tan A - \tan B)^2$

$\Rightarrow$ L.H.S $= (1 + 2\tan A.\tan B + \tan^2 A.\tan^2 B) + (\tan^2 A + \tan^2 B - 2\tan A.\tan B)$

$\left[\because (a+b)^2 = a^2 + 2ab + b^2, \ (a-b)^2 = a^2 - 2ab + b^2 \right]$

$\Rightarrow$ L.H.S $= 1 + \tan^2 A.\tan^2 B + \tan^2 A + \tan^2 B$

$\Rightarrow$ L.H.S $= (1 + \tan^2 A) + (\tan^2 B + \tan^2 A.\tan^2 B)$

$\Rightarrow$ L.H.S $= (1 + \tan^2 A) + \tan^2 B(1 + \tan^2 A) \left[taking\ \tan^2 B\ as\ a\ common \right]$

$\Rightarrow$ L.H.S $= (1 + \tan^2 A)(1 + \tan^2 B) = \sec^2 A \sec^2 B = $ R.H.S $\left[\because 1 + \tan^2\theta = \sec^2\theta \right]$

therefore, $(1 + \tan A.\tan B)^2 + (\tan A - \tan B)^2 = \sec^2 A.\sec^2 B$
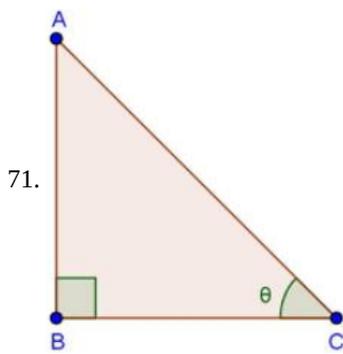
Hence proved.

68. We have,

L.H.S $= \cos^2\theta\left(1 + \tan^2\theta\right)$

$\Rightarrow$ L.H.S $= \cos^2\theta \sec^2\theta \left[\because 1 + \tan^2\theta = \sec^2\theta \right]$

$\Rightarrow$ L.H.S $= \cos^2\theta\left(\dfrac{1}{\cos^2\theta}\right) = 1 = $ R H S $\left[\because \sec\theta = \dfrac{1}{\cos\theta} \right]$

69. $\sin^6\theta + \cos^6\theta = 1 - 3\sin^2\theta\cos^2\theta$

L.H.S. $= \sin^6\theta + \cos^6\theta$

$= \left(\sin^2\theta\right)^3 + \left(\cos^2\theta\right)^3$

$= \left(\sin^2\theta + \cos^2\theta\right)\left(\sin^4\theta + \cos^4\theta - \sin^2\theta\cos^2\theta\right)\left[\because a^3 + b^3 = (a+b)\left(a^2 + b^2 - ab\right) \right]$

$= 1\left(\sin^4\theta + \cos^4\theta + 2\sin^2\theta \cdot \cos^2\theta - 2\sin^2\theta\cos^2\theta - \sin^2\theta\cos^2\theta\right]$ since, $\sin^2 A + \cos^2 A = 1$

$= \left(\sin^2\theta + \cos^2\theta\right)^2 - 3\sin^2\theta\cos^2\theta$

$= 1 - 3\sin^2\theta\cos^2\theta$

$= $ R.H.S. proved.

70. We have,

$\dfrac{\left(\sin^4\theta + \cos^4\theta\right)}{1 - 2\sin^2\theta\cos^2\theta}$

$= \dfrac{\left(\sin^2\theta\right)^2 + \left(\cos^2\theta\right)^2}{1 - 2\sin^2\theta\cos^2\theta}$

$= \dfrac{\left(\sin^2\theta + \cos^2\theta\right)^2 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta}[(a+b)^2 = a^2 + b^2 + 2ab]$

$= \dfrac{1 - 2\sin^2\theta\cos^2\theta}{1 - 2\sin^2\theta\cos^2\theta}[\sin^2\theta + \cos^2\theta = 1]$

$= 1$ Hence proved

71.

Given $\tan\theta = \dfrac{1}{\sqrt{7}} = \dfrac{AB}{BC}$

Let AB = 1K

and, $BC = \sqrt{7}K$

In ΔABC, by Pythagoras theorem

$AC^2 = AB^2 + BC^2$

$AC^2 = (1K)^2 + \left(\sqrt{7}K\right)^2$

$AC^2 = K^2 + 7K^2$

$AC^2 = 8K^2$

$AC = \sqrt{8K^2} = 2\sqrt{2}K$

$\therefore \mathrm{cosec}\theta = \dfrac{AC}{AB} = \dfrac{2\sqrt{2}K}{1K} = 2\sqrt{2}$

$\sec\theta = \dfrac{AC}{BC} = \dfrac{2\sqrt{2}K}{\sqrt{7}K} = \dfrac{2\sqrt{2}}{\sqrt{7}}$

$\therefore LHS = \dfrac{\cos ec^2\theta - \sec^2\theta}{\cos ec^2\theta + \sec^2\theta}$

$= \dfrac{\left(2\sqrt{2}\right)^2 - \left(\dfrac{2\sqrt{2}}{\sqrt{7}}\right)^2}{\left(2\sqrt{2}\right)^2 + \left(\dfrac{2\sqrt{2}}{7}\right)^2}$

$= \dfrac{8 - \dfrac{8}{7}}{8 + \dfrac{8}{7}}$

$= \dfrac{\dfrac{56 - 8}{7}}{\dfrac{56 + 8}{7}}$

$= \dfrac{\dfrac{48}{7}}{\dfrac{64}{7}}$

$= \dfrac{48}{7} \times \dfrac{7}{64} = \dfrac{3}{4} = RHS$

72. **LHS** $= \sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta}}$

$= \sqrt{\dfrac{1 + \sin\theta}{1 - \sin\theta} \times \dfrac{1 + \sin\theta}{1 + \sin\theta}}$

$= \sqrt{\dfrac{(1 + \sin\theta)^2}{1 - \sin^2\theta}}$

$= \sqrt{\dfrac{(1 + \sin\theta)^2}{\cos^2\theta}}$

$= \dfrac{1 + \sin\theta}{\cos\theta}$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= sec\theta + tan\theta$

= RHS.

73. $x = a\cos^3\theta$, $y = b\sin^3\theta$

LHS $= \left(\dfrac{x}{a}\right)^{2/3} + \left(\dfrac{y}{b}\right)^{2/3}$

$= \left(\dfrac{a\cos^3\theta}{a}\right)^{2/3} + \left(\dfrac{b\sin^3\theta}{b}\right)^{2/3}$

$= (\cos\theta)^{3\times 2/3} + (\sin\theta)^{3\times 2/3}$

$= \cos^2\theta + \sin^2\theta = 1 = $ RHS.

Hence proved.

74. L.H.S $= \sec^4\theta - \sec^2\theta$

$= \sec^2\theta\,(\sec^2\theta - 1)$

$= \sec^2\theta\,(\tan^2\theta)$ [$\because$ $1 + tan^2\theta = sec^2\theta$ or $tan^2\theta = sec^2\theta - 1$]

$= (1 + \tan^2\theta)\tan^2\theta = \tan^2\theta + \tan^4\theta = $ R.H.S

Hence proved.

75. LHS $= secA(1 - sinA)(secA + tanA)$

$= (secA - secA \times sinA)(secA + tanA)$

$= \left(secA - \dfrac{1}{\cos A} \times sinA\right)(secA + tanA)$ $\left[\because secA = \dfrac{1}{\cos A} \right]$

$= \left(secA - \dfrac{\sin A}{\cos A}\right)(secA + tanA)$

$= (secA - tanA)(secA + tanA)$ $\left[\because \dfrac{\sin A}{\cos A} = tanA \right]$

Using identity $(a - b)(a + b) = a^2 - b^2$

$= \sec^2 - \tan^2 A$

$= 1$ $\left[\because \sec^2 - \tan^2 A = 1 \right]$

$= $ RHS

Hence proved