

Solution

PAIR OF LINEAR EQUATION IN TWO VARIABLE WS 4

Class 10 - Mathematics

Section A

1. 57

Explanation:

Let the ten's digit of the required number be x and the unit's digit be y .

As per given condition the sum of the digits of a two-digit number is 12.

Then, $x + y = 12$ (i)

Required number = $(10x + y)$.

Number obtained on reversing the digits = $(10y + x)$.

As per given condition the number obtained by interchanging its digits exceeds the given number by 18.

$$\therefore (10y + x) - (10x + y) = 18$$

$$\Rightarrow 10y + x - 10x - y = 18$$

$$\Rightarrow 9y - 9x = 18$$

$$\Rightarrow y - x = 2 \text{ ... (ii)}$$

On adding (i) and (ii), we get

$$(x + y) + (y - x) = 12 + 2$$

$$\Rightarrow 2y = 14$$

$$\Rightarrow y = 7.$$

Putting $y = 7$ in (i), we get

$$x + 7 = 12$$

$$\Rightarrow x = 12 - 7 = 5$$

$\therefore x = 5$ and $y = 7$.

Hence, the required number is 57.

2. 58

Explanation:

Let the unit digit is b and ten's digit is a .

So, two digit number is $10a + b$.

As per given condition

The sum of the digits of a two digit number is 13.

So, $a + b = 13$ (i)

and the number obtained by interchanging the digits of the given number exceeds the number by 27.

$$10b + a = (10a + b) + 27$$

$$\Rightarrow 9b = 9a + 27$$

$$\Rightarrow b = a + 3 \text{ (ii)}$$

Putting (ii) in (i), we get :

$$a + a + 3 = 13$$

$$\Rightarrow 2a = 10$$

$$\Rightarrow a = 5$$

Putting $a = 5$ in (ii), we get :

$$b = 5 + 3 = 8$$

$$\text{Two digit number} = 10a + b = 5(10) + 8 = 58$$

Therefore, the number is 58.

3. 692

Explanation:

Let digit at unit's place = x and digit at hundred's place = y

$$\therefore \text{Middle digit} = x + y + 1$$

$$\text{Number} = 100y + 10(x + y + 1) + x$$

$$\text{Number obtained by reversing the digits} = 100x + 10(x + y + 1) + y$$

ATQ.,

$$x + y + (x + y + 1) = 17$$

$$\Rightarrow 2x + 2y = 16 \dots(i)$$

$$\text{and } 100x + 10(x + y + 1) = x - 396 \dots (ii)$$

$$\Rightarrow 99x - 99y = -396$$

$$\Rightarrow x - y = -4$$

By Solving equation (i) and (ii), we get

$$x = 2, y = 6 \text{ and Number} = 692$$

4. 64

Explanation:

Let unit number is y and tenth number is x .

The given number = $10x + y$

As per first condition

A number consists of two digits whose sum is 10.

$$\text{So, } x + y = 10 \dots(i)$$

The number obtained by interchanging the digits is $10y + x$.

As per second condition

If 18 is subtracted from the number, its digits are reversed.

$$\Rightarrow 10y + x = 10x + y - 18$$

$$\Rightarrow 9x - 9y = 18$$

$$\Rightarrow x - y = 2 \dots(ii)$$

Adding (i) and (ii), we get

$$x + y + x - y = 10 + 2$$

$$\Rightarrow 2x = 12$$

$$\Rightarrow x = 6$$

Substituting in (i), we get $y = 4$.

$$\text{So, the given number is } 10x + y = 10(6) + 4 = 64$$

5. 36

Explanation:

Let the tens digit be x and unit place digit be y .

$$\text{Number} = 10x + y$$

As per given condition

The sum of the digits of a two digit number is 9.

$$\text{So, } x + y = 9 \dots(i)$$

And the number obtained by reversing the order of digits of the given number exceeds the given number by 27.

$$10y + x = 10x + y + 27$$

$$\Rightarrow -9x + 9y = 27$$

$$\Rightarrow -x + y = 3 \dots(ii)$$

Adding (i) and (ii), we get

$$2y = 12$$

$$\Rightarrow y = 6$$

Putting value of y in equation (i), we get

$$x + 6 = 9$$

$$\Rightarrow x = 9 - 6$$

$$\Rightarrow x = 3$$

$$\text{Two digit number} = 10x + y = 10(3) + 6 = 30 + 6 = 36$$

So, the given number is 36.

6. 36

Explanation:

Let length of the garden = x m and breadth of the garden = y m

$$\text{A.T.Q. } x = y + 12 \Rightarrow x - y = 12 \dots (i) \text{ and}$$

$$\frac{1}{2} \text{ Perimeter} = 60$$

$$\Rightarrow \frac{1}{2} \times 2(x + y) = 60$$

$$\Rightarrow x + y = 60 \dots (ii)$$

Add (i) and (ii)

$$2x = 72$$

$$x = 36$$

then put $x = 36$ in (i)

$$y = 24$$

7. 84

Explanation:

Let digit at unit place = x and digit at tens place = y

So, two digit number is = $10y + x$

As per given condition

The result of dividing a number of two digits by the number with the digits reversed is $\frac{7}{4}$.

$$\frac{10y+x}{10x+y} = \frac{7}{4}$$

$$\Rightarrow 40y + 4x = 70x + 7y$$

$$66x - 33y = 0$$

$$\Rightarrow 2x - y = 0 \dots\dots(i)$$

And the sum of the digits is 12.

So, $x + y = 12 \dots\dots(ii)$

Adding eq (i) and (ii), we get

$$2x - y + x + y = 12 + 0$$

$$\Rightarrow 3x = 12$$

$$\Rightarrow x = 4$$

When $x = 4$, eq. (i) becomes

$$2(4) - y = 0$$

$$\Rightarrow y = 8$$

\therefore Two digit number is = $10(8) + 4 = 80 + 4 = 84$

Section B

8. (a) - (iii), (b) - (i), (c) - (iv), (d) - (ii)

Section C

9. Given, $3x - \frac{y+7}{11} + 2 = 10$

$$\Rightarrow \frac{33x - (y+7) + 22}{11} = 10$$

$$\Rightarrow 33x - y - 7 + 22 = 110$$

$$\Rightarrow 33x - y = 95 \dots(i)$$

Also, $2y + \frac{x+11}{7} = 10$

$$\Rightarrow \frac{14y+x+11}{7} = 10$$

$$\Rightarrow 14y + x + 11 = 70$$

$$14y + x = 59$$

$$14y = 59 - x$$

$$\Rightarrow y = \frac{59-x}{14}$$

eq. (i) becomes

$$33x - \left(\frac{59-x}{14}\right) = 95$$

$$\Rightarrow \frac{462x - 59 + x}{14} = 95$$

$$\Rightarrow 463x - 59 = 1330$$

$$\Rightarrow x = 3$$

When $x = 3$ eq. (i) becomes

$$33(3) - y = 95$$

$$\Rightarrow y = 4$$

10. The given system of equations is

$$3x - 2y = 4 \dots\dots (i)$$

$$2x + y = 5 \dots\dots (ii)$$

Putting $x=2$ and $y=1$ in equation (i), we have

$$\text{LHS} = 3 \times 2 - 2 \times 1$$

$$= 6 - 2$$

$$= 4$$

= RHS

Putting $x=2$ and $y=1$ in equations (ii), we have

$$\text{LHS} = 2 \times 2 + 1 \times 1$$

$$= 4 + 1$$

$$= 5$$

= RHS

Thus, $x = 2$ and $y = 1$ satisfy both the equations of the given system.

Hence, $x = 2, y = 1$ is a solution of the given system.

11. According to the question,

$$2x + 5y = \frac{8}{3} \dots\dots(i)$$

$$3x - 2y = \frac{5}{6} \dots\dots(ii)$$

Multiplying (i) by 2, we get (iii) and (ii) by 5, we get (iv)

$$4x + 10y = \frac{16}{3} \dots\dots(iii)$$

$$15x - 10y = \frac{25}{6} \dots\dots(iv)$$

Solving Eq.(iii) and (iv),

Eq. (iii) + (iv),

$$19x = \frac{25}{6} + \frac{16}{3}$$

$$\Rightarrow 19x = \frac{25+32}{6}$$

$$\Rightarrow 19x = \frac{57}{6}$$

$$\Rightarrow x = \frac{3}{6}$$

$$\Rightarrow x = \frac{1}{2}$$

Substituting $x = \frac{1}{2}$ in (iii),

$$\Rightarrow 4 \times \frac{1}{2} + 10y = \frac{16}{3}$$

$$10y = \frac{16}{3} - 2 \Rightarrow 10y = \frac{10}{3}$$

$$y = \frac{10}{3 \times 10} = \frac{1}{3}$$

\therefore Solution is $x = \frac{1}{2}, y = \frac{1}{3}$

12. $x + y = 12 \dots(i)$

$$x - y = 8 \dots(ii)$$

On adding (i) and (ii),

$$2x = 20$$

$$\Rightarrow x = 10$$

$$\therefore 10 + y = 12$$

$$\Rightarrow y = 2$$

13. Given pair of linear equations is

$$-x + py = 1 \dots(i)$$

$$\text{and } px - y - 1 = 0 \dots(ii)$$

On comparing with standard form, we get

$$a_1 = -1, b_1 = p, c_1 = -1;$$

$$\text{And } a_2 = p, b_2 = -1, c_2 = -1;$$

$$a_1/a_2 = -\frac{1}{p}$$

$$b_1/b_2 = -p$$

$$c_1/c_2 = 1$$

Since, the lines equations has no solution i.e., both lines are parallel to each other.

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

$$-\frac{1}{p} = -p \neq 1$$

Taking last two parts, we get

$$p \neq -1$$

Taking first two parts, we get

$$p^2 = 1$$

$$p = \pm 1$$

Hence, the given pair of linear equations has no solution for $p = 1$.

14. Let unit's digit be x and ten's digit be y .

$$\therefore \text{Number} = 10y + x$$

$$10y + x = 7(x + y) \Rightarrow 3y - 6x = 0$$

$$y = 2x \dots(i)$$

$$10y + x - (10x + y) = 18$$

$$9y - 9x = 18 \text{ or } y - x = 2 \dots(ii)$$

On solving (i) and (ii), $x = 2$, $y = 4$

\therefore required number is 42

15. Let the two numbers be x and y ($x > y$)

We are given that,

The difference between two numbers is 26 .

$$x - y = 26 \dots(i)$$

And one number is three times the other.

$$x = 3y \dots(ii)$$

On substituting the value x from eqn. (ii) in eqn. (i), we get

$$3y - y = 26$$

$$2y = 26$$

$$\therefore y = 13$$

Put $y = 13$ in (ii) we get

$$x = 3(13) = 39$$

Hence, the two numbers are 39 and 13.

16. Let the ten's digit of the required number be x and the unit's digit be y .

As per given condition the sum of the digits of a two-digit number is 12.

$$\text{Then, } x + y = 12. \dots(i)$$

$$\text{Required number} = (10x + y).$$

$$\text{Number obtained on reversing the digits} = (10y + x).$$

As per given condition the number obtained by interchanging its digits exceeds the given number by 18.

$$\therefore (10y + x) - (10x + y) = 18$$

$$\Rightarrow 10y + x - 10x - y = 18$$

$$\Rightarrow 9y - 9x = 18$$

$$\Rightarrow y - x = 2. \dots(ii)$$

On adding (i) and (ii), we get

$$(x + y) + (y - x) = 12 + 2$$

$$\Rightarrow 2y = 14$$

$$\Rightarrow y = 7.$$

Putting $y = 7$ in (i), we get

$$x + 7 = 12$$

$$\Rightarrow x = 12 - 7 = 5$$

$$\therefore x = 5 \text{ and } y = 7.$$

Hence, the required number is 57.

17. Given pair of equations is

$$3x + y + 4 = 0 \dots(i)$$

$$\text{and } 6x - 2y + 4 = 0 \dots(ii)$$

comparing with $ax + by + c = 0$

Here, $a_1 = 3$, $b_1 = 1$, $c_1 = 4$;

And $a_2 = 6$, $b_2 = -2$, $c_2 = 4$;

$$a_1 / a_2 = 1/2$$

$$b_1 / b_2 = -1/2$$

$$c_1 / c_2 = 1$$

since $a_1/a_2 \neq b_1/b_2$

so system of equations is consistent with a unique solution.

We have, $3x + y + 4 = 0$

$y = -4 - 3x$

When $x = 0$, then $y = -4$

When $x = -1$, then $y = -1$

When $x = -2$, then $y = 2$

x	0	-1	-2
y	-4	-1	2
Points	B	C	A

and $6x - 2y + 4 = 0$

$6x + 4 = 2y$

$y = 3x + 2$

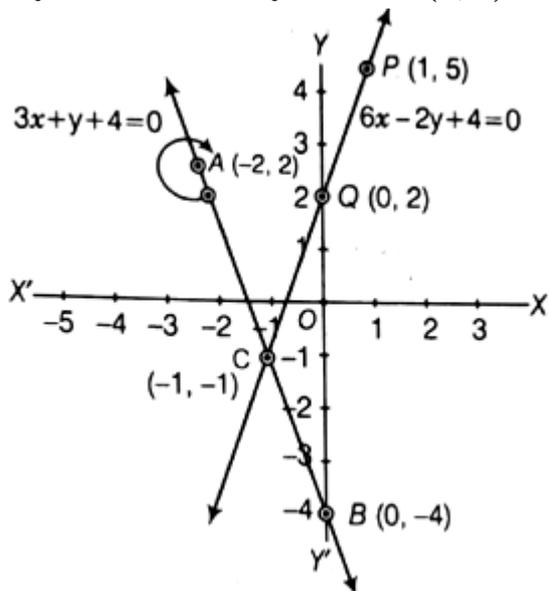
When $x = 0$, then $y = 2$

When $x = -1$, then $y = -1$

When $x = 1$, then $y = 5$

x	-1	0	1
y	-1	2	5
Points	C	Q	P

Plotting the points B(0, -4) and A(-2,2), we get the straight line AB. Plotting the points Q(0,2) and P(1,5) we get the straight line PQ. The lines AB and PQ intersect at C(-1, -1).



So, (-1, -1) is the required solution.

18. The given system of equations is:

$3x - 5y = -1$ (i)

$x - y = -1$ (ii)

From (ii), we get

$y = x + 1$

Substituting, $y = x + 1$ in (i), we get

$3x - 5(x + 1) = -1$

$\Rightarrow -2x - 5 = -1$

$\Rightarrow x = -2$

Putting $x = -2$ in $y = x + 1$ we get $y = -1$.

Hence, the solution of the given system of equations is $x = -2$ and $y = -1$.

19. The given equations are

$2x + y = 23$ (i)

and $4x - y = 19$ (ii)

Adding (i) and (ii), we get

$$6x = 42$$

$$\Rightarrow x = 7$$

Putting the value of x in (i), we get

$$2(7) + y = 23$$

$$\Rightarrow y = 9$$

$$1. 5y - 2x = 5(9) - 2(7) = 45 - 14 = 31$$

$$2. \frac{y}{x} - 2 = \frac{9}{7} - 2 = -\frac{5}{7}$$

20. let the first number = x

And second number = y

According to given condition

Sum of two numbers is 75.

$$x + y = 75 \dots\dots\dots(1)$$

And difference of two numbers is 15.

$$x - y = 15 \dots\dots\dots(2)$$

By solving using substitution method,

$$x = 75 - y \dots\dots\dots(3)$$

Now using (3) in (2), we get

$$(75 - y) - y = 15$$

$$75 - 2y = 15$$

$$75 - 15 = 2y$$

$$2y = 60$$

$$y = 30$$

Put y = 30 in (3), we get

$$x = 75 - 30$$

$$x = 45$$

So the given numbers are 30 and 45.

21. Given pair of linear equations is

$$43x + 67y = -24 \dots(i)$$

$$\text{and } 67x + 43y = 24 \dots(ii)$$

On multiplying Eq. (i) by 43 and Eq. (ii) by 67 and then subtracting both of them, we get

$$(67^2x + 43(67y)) - (43^2x + 43(67y)) = (24 \times 67) - (24 \times 43)$$

$$(67^2 - 43^2)x = 24(67 + 43)$$

$$x = 1$$

Now, put the value of x in Eq. (i), we get

$$43 \times 1 + 67y = -24$$

$$67y = -67$$

$$y = -1$$

Hence, the required values of x and y are 1 and -1, respectively.

22. $x + y = 14 \dots(1)$

$$x - y = 4 \dots(2)$$

If $x - y = 4$

$$x = 4 + y \dots\dots(3)$$

Put (3) in (1) we get

$$4 + y + y = 14$$

$$\Rightarrow 4 + 2y = 14$$

$$\Rightarrow 2y = 10$$

$$\Rightarrow y = 5$$

Putting value of y in equation (3), we get

$$x = 4 + y$$

$$\Rightarrow x = 4 + 5 = 9$$

Therefore, $x = 9$ and $y = 5$

23. Given, pair of linear equations is $-3x + 5y = 7$ and $2px - 3y = 1$

On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

Here, $a_1 = -3$, $b_1 = 5$, $c_1 = -7$; And $a_2 = 2p$, $b_2 = -3$, $c_2 = -1$;

Now,

$$a_1/a_2 = -\frac{3}{2p}$$

$$b_1/b_2 = -5/3$$

$$c_1/c_2 = 7$$

Since, the lines are intersecting at a unique point i.e., it has a unique solution

$$\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$$

$$\text{so, } -\frac{3}{2p} \neq -\frac{5}{3}$$

$$p \neq 9/10$$

Hence, the lines represented by these equations are intersecting at a unique point for all real values of p except $\frac{9}{10}$

24. Given pair of equations is

$$x - 2y = 6 \dots(i)$$

$$\text{and } 3x - 6y = 0 \dots(ii)$$

On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

Here, $a_1 = 1$, $b_1 = -2$, $c_1 = -6$;

And $a_2 = 3$, $b_2 = -6$, $c_2 = 0$;

$$a_1/a_2 = 1/3$$

$$b_1/b_2 = -2/-6 = 1/3$$

here $a_1/a_2 = b_1/b_2 \neq c_1/c_2$ (parallel lines).

Hence, the lines represented by the given equations are parallel. Therefore, it has no solution. So, the given pair of lines is inconsistent.

25. Let the supplementary angles be x° and y° ($x^\circ > y^\circ$)

$$\text{Now, } x^\circ + y^\circ = 180 \dots (i)$$

$$\text{and } x^\circ - y^\circ = 18 \dots (ii)$$

From eqn. (ii),

$$y^\circ = x^\circ - 18 \dots(iii)$$

Put (iii) in (i)

$$x^\circ + x^\circ - 18^\circ = 180^\circ$$

$$\Rightarrow 2x^\circ = 180^\circ + 18^\circ$$

$$\Rightarrow 2x^\circ = 198^\circ$$

$$\Rightarrow x^\circ = 99^\circ$$

On substituting $x^\circ = 99^\circ$ in eqn. (iii),

$$y^\circ = 99^\circ - 18^\circ = 81^\circ$$

$$\therefore y^\circ = 81^\circ$$

Hence, the angles are 99° and 81°

26. The given equations are

$$4x + \frac{6}{y} = 15 \dots(i)$$

$$6x - \frac{8}{y} = 14 \dots(ii)$$

Multiplying equation (i) by 4 and equation (ii) by 3, we get

$$16x + \frac{24}{y} = 60 \dots(iii)$$

$$18x - \frac{24}{y} = 42 \dots(iv)$$

On adding (iii) and (iv) we get

$$34x = 102$$

$$x = 3$$

On putting $x = 3$ in (i) we get

$$4 \times 3 + \frac{6}{y} = 15$$

$$12y + 6 = 15y$$

$$3y = 68, y = 2$$

Putting values of x and y in

$$y = px - 2$$

$$2 = 3p - 2$$

$$3p = 4.$$

$$\text{or } p = \frac{4}{3}$$

27. Given, pair of linear equations is $2x + 3y - 5 = 0$ and $px - 6y - 8 = 0$

On comparing with $a_1x + b_1y + c_1 = 0$ and $a_2x + b_2y + c_2 = 0$, we get

$$a_1 = 2, b_1 = 3, c_1 = -5;$$

$$\text{And } a_2 = p, b_2 = -6, c_2 = -8;$$

$$a_1/a_2 = \frac{2}{p}$$

$$b_1/b_2 = -3/6 = -1/2$$

$$c_1/c_2 = 5/8$$

Since, the pair of linear equations has a unique solution.

$$a_1/a_2 \neq b_1/b_2$$

$$\text{so } \frac{2}{p} \neq -1/2$$

$$p \neq -4$$

Hence, the pair of linear equations has a unique solution for all values of p except -4

$$\text{i.e., } p \in \mathbb{R} - \{-4\}$$

28. Let the ten's and unit's digits of the required number be x and y respectively.

Then, the number = $(10x + y)$.

The number obtained on reversing the digits = $(10y + x)$.

As per given condition

The sum of a two-digit number and the number obtained by reversing the order of its digits is 99.

$$\therefore (10y + x) + (10x + y) = 99$$

$$\Rightarrow 11(x + y) = 99$$

$$\Rightarrow x + y = 9.$$

The digits differ by 3

$$\text{So, } (x - y) = \pm 3.$$

Thus, we have

$$x + y = 9 \dots\dots (i)$$

$$x - y = 3 \dots\dots (ii)$$

$$\text{or } x + y = 9 \dots\dots (iii)$$

$$x - y = -3 \dots\dots (iv)$$

From (i) and (ii), we get $x = 6, y = 3$.

From (iii) and (iv), we get $x = 3, y = 6$.

Hence, the required number is 63 or 36.

29. $x + y = 14 \dots(1)$

$$x - y = 4 \dots(2)$$

$$x = 4 + y \text{ from equation (2)}$$

Putting this in equation (1), we get

$$4 + y + y = 14$$

$$\Rightarrow 2y = 10$$

$$\Rightarrow y = 5$$

Putting value of y in equation (1), we get

$$x + 5 = 14$$

$$\Rightarrow x = 14 - 5 = 9$$

Therefore, $x = 9$ and $y = 5$

30. $3x + 2y = 2x + y + 3 = 4x + 3y - 3$

On solving 1st and 2nd, we get

$$x + y = 3 \dots(1)$$

On solving 2nd and 3rd, we get

$$2x + 2y = 6$$

$$x + y = 3 \text{ ..(ii)}$$

The system of equations has infinitely many solutions.

31. $ax + by = a - b$ multiply by a

$$bx - ay = a + b \text{ multiply by b}$$

Now, adding the resulting equations gives

$$\begin{array}{r} a^2x + aby = a^2 - ab \\ b^2x - aby = ab + b^2 \\ \hline (a^2 + b^2)x = a^2 + b^2 \end{array}$$

$$\Rightarrow x = 1$$

$$\therefore a + by = a - b$$

$$\Rightarrow by = -b$$

$$\Rightarrow y = -1$$

$$\therefore x = 1 \text{ and } y = -1$$

32. Let the digit at unit's place be x and the digit at ten's place be y. Then,

$$\text{Number} = 10y + x$$

$$\text{Number formed by reversing the digits} = 10x + y$$

According to the given conditions

The sum of the digits of a two digit number is 8 .

$$\text{So, } x + y = 8 \text{ (i)}$$

and the difference between the number and that formed by reversing the digits is 18

$$\text{So, } (10y + x) - (10x + y) = 18$$

$$\Rightarrow 10y + x - 10x - y = 18$$

$$\Rightarrow 9y - 9x = 18$$

$$\Rightarrow 9(y - x) = 18$$

$$\Rightarrow y - x = 2 \text{ (ii)}$$

Add equations (i) and (ii), we get

$$(x + y) + (y - x) = 8 + 2$$

$$\Rightarrow x + y + y - x = 10$$

$$\Rightarrow 2y = 10$$

$$\Rightarrow y = 5$$

Put $y = 5$ in (i) we get

$$x + y = 8$$

$$\Rightarrow x + 5 = 8$$

$$\Rightarrow x = 3$$

$$\text{So, two digit number} = 10y + x = 10 \times 5 + 3 = 53$$

Hence given two digit number is 53.

33. Given linear equations are

$$x - 3y - 2 = 0 \text{ ... (i)}$$

$$\text{and } -2x + 6y - 5 = 0 \text{ ... (ii)}$$

On comparing with $ax + by + c = 0$, we get

$$\text{Here, } a_1 = 1, b_1 = -3, c_1 = -2;$$

$$\text{And } a_2 = -2, b_2 = 6, c_2 = -5;$$

$$\frac{a_1}{a_2} = \frac{-1}{-2}$$

$$\frac{b_1}{b_2} = \frac{-3}{6} = -\frac{1}{2}$$

$$\frac{c_1}{c_2} = \frac{2}{-5}$$

$$\text{i.e., } \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2} \text{ [parallel lines]}$$

Hence, two straight paths represented by the given equations never cross each other, because they are parallel to each other.

34. The given system of equations may be written as

$$9x - 10y + 12 = 0 \text{ ... (i)}$$

$$2x + 3y - 13 = 0 \text{ ... (ii)}$$

$$\text{From (ii), we get } y = \frac{13 - 2x}{3}$$

Substituting $y = \frac{13-2x}{3}$ in (i), we get

$$9x - \frac{10(13-2x)}{3} + 12 = 0$$

$$\Rightarrow 27x - 10(13 - 2x) + 36 = 0$$

$$\Rightarrow 27x - 130 + 20x + 36 = 0$$

$$\Rightarrow 47x - 94 = 0$$

$$\Rightarrow 47x = 94$$

$$\Rightarrow x = \frac{94}{47} = 2$$

Substituting $x = 2$ in (i), we get

$$9 \times 2 - 10y + 12 = 0$$

$$\Rightarrow 10y = 30$$

$$\Rightarrow y = \frac{30}{10} = 3$$

Hence, $x = 2$ and $y = 3$ is the required solution.

35. Let the numerator be 'a' and denominator be 'b'.

Given that, sum of a numerator and denominator of a fraction is 18.

$$\Rightarrow a + b = 18 \dots\dots(1)$$

Also, if the denominator is increased by 2, the fraction reduces to $\frac{1}{3}$.

$$\text{i.e. } a/(b + 2) = 1/3$$

$$\Rightarrow 3a - b = 2 \dots\dots(2)$$

Adding (1) and (2), we get

$$\Rightarrow 4a = 20$$

$$\Rightarrow a = 5$$

Thus, $b = 13$

So the required fraction is $\frac{5}{13}$.

36. It is given that $x = 2$ and $y = 1$ is a solution of the linear equation $2x + 3y = k$, substituting the respective values of x and y in the linear equation, $2x + 3y = k$

$$\therefore 2(2) + 3(1) = k$$

$$\Rightarrow k = 4 + 3$$

$$\Rightarrow k = 7$$

Therefore, we can conclude that the value of k , for which the linear equation $2x + 3y = k$ has $x = 2$ and $y = 1$ as one of its solutions is 7.

37. $3x - y = 3$

$$9x - 3y = 9$$

The given pair of linear equations is

$$3x - y = 3 \dots\dots\dots(1)$$

$$9x - 3y = 9 \dots\dots\dots(2)$$

From equation(1),

$$y = 3x - 3 \dots\dots\dots(3)$$

$$9x - 3(3x - 3) = 9$$

$$\Rightarrow 9x - 9x + 9 = 9$$

$$\Rightarrow 9 = 9$$

which is true. Therefore, equation (1) and (2) have infinitely many solutions.

38. Given pair of equations is $\frac{2xy}{x+y} = \frac{3}{2}$, Where $x + y \neq 0$

$$x + \frac{y}{2xy} = \frac{2}{3}$$

$$\frac{x}{xy} + \frac{y}{xy} = \frac{4}{3}$$

$$\frac{1}{y} + \frac{1}{x} = \frac{4}{3}$$

And

$$\frac{xy}{2x-y} = -\frac{3}{10}$$

$$\frac{2x-y}{xy} = -\frac{10}{3}$$

$$\frac{2x}{xy} - \frac{y}{xy} = -\frac{10}{3}$$

$$\frac{2}{y} - \frac{1}{x} = -\frac{10}{3}$$

Now, put $\frac{1}{x} = u$ and $\frac{1}{y} = v$ then the pair of equations becomes

$$u + v = \frac{4}{3} \dots\dots(iii)$$

$$\text{and } 2v - u = -\frac{10}{3} \dots(\text{iv})$$

On adding both equations, we get

$$3v = \frac{4}{3} - \frac{10}{3} = -\frac{6}{3}$$

$$3v = -2$$

$$v = -\frac{2}{3}$$

Now, put the value of v in Eq. (iii), we get

$$-\frac{2}{3} + u = \frac{4}{3}$$

$$u = \frac{4}{3} + \frac{2}{3} = \frac{6}{3} = 2$$

$$x = \frac{1}{u} = \frac{1}{2}$$

$$\text{and } y = \frac{1}{v} = -\frac{3}{2}$$

Hence, the required values of x and y are $\frac{1}{2}$ and $-\frac{3}{2}$ respectively.

39. $x + y = a - b$ (i)

and $ax - by = a^2 + b^2$ (ii)

Multiply (i) by b and subtract by (ii)

$$bx + by = ab - b^2$$

$$ax - by = a^2 + b^2$$

$$\hline (a+b)x = a(a+b)$$

$$x = a$$

Put $x = a$ in (i)

$$x + y = a - b$$

$$a + y = a - b$$

$$y = -b$$

40. The given system of equations is

$$2x + 7y = 11 \dots\dots(i)$$

$$x - 3y = 5 \dots\dots(ii)$$

Putting $x = 2, y = 1$ in equation (i), we have

$$\text{LHS} = 2 \times 2 + 7 \times 1$$

$$= 4 + 7$$

$$= 11$$

$$= \text{RHS}$$

So, $x = 2$ and $y = 1$ satisfy equation (i)

Putting $x=2, y=1$ in equation (ii), we have,

$$\text{LHS} = 2 \times 1 - 3 \times 1$$

$$= 2 - 3$$

$$= -1$$

$$\neq \text{RHS}$$

So, $x=2$ and $y=1$ is not a solution of the given system of equations.

41. Given that, x, y and 40° are the angles of a triangle

$$\text{So, } x + y + 40 = 180$$

[since, the sum of all the angles of a triangle is 180°]

$$\text{And hence } x + y = 140 \dots(i)$$

$$\text{Also, difference of angles} = x - y = 30 \dots(ii)$$

On adding Eqs. (i) and (ii), we get

$$2x = 170$$

$$\text{So, } x = 85$$

On putting in Eq. (i), we get

$$y = 55$$

Hence, the required values of x and y are 85° and 55° , respectively.

42. Given, $2x - y = 2$..(i)

$$x + 3y = 15 \dots(ii)$$

From eqn. (i), we get $y = 2x - 2$... (iii)

Substituting the value of y in eqn. (ii),

$$x + 3y = 15$$

$$x + 3(2x - 2) = 15$$

$$x + 6x - 6 = 15$$

$$7x = 15 + 6$$

$$\text{or, } 7x = 21$$

$$\therefore x = 3$$

Substituting this value of x in (iii), we get

$$y = 2x - 2$$

$$y = 2 \times 3 - 2$$

$$y = 6 - 2$$

$$y = 4$$

Hence the value of x and y of given equations are 3 and 4 respectively.

43. Given pair of linear equations are is

$$x + y = 3.3$$

$$\text{and } \frac{0.6}{3x - 2y} = -1$$

$$0.6 = -3x + 2y$$

$$3x - 2y = -0.6$$

Now, multiplying Eq. (i) by 2 and then adding with Eq. (ii), we get

$$2x + 2y = 6.6$$

$$3x - 2y = -0.6$$

$$5x = 6$$

$$x = \frac{6}{5}$$

Now, put the value of x in Eq. (i), we get

$$1.2 + y = 3.3$$

$$y = 2.1$$

Hence, the required values of x and y are 1.2 and 2.1, respectively.

44. The given equations are

$$37x + 43y = 123 \dots (i)$$

$$43x + 37y = 117 \dots (ii)$$

Clearly, the coefficients of x and y in one equation are interchanged in the other.

Adding (i) and (ii), we get

$$(37x + 43y) + (43x + 37y) = 123 + 117$$

$$(37 + 43)x + (43 + 37)y = (123 + 117)$$

$$\Rightarrow 80x + 80y = 240$$

$$\Rightarrow 80(x + y) = 240$$

$$\Rightarrow x + y = 3 \dots\dots (iii)$$

Subtracting (i) from (ii), we get

$$(37x + 43y) - (43x + 37y) = 123 - 117$$

$$6x - 6y = -6$$

$$\Rightarrow 6(x - y) = -6$$

$$\Rightarrow x - y = -1 \dots (iv)$$

Adding (iii) and (iv), we get

$$(x + y) + (x - y) = 3 + (-1)$$

$$\Rightarrow x + y + x - y = 2$$

$$\Rightarrow 2x = 2$$

$$\Rightarrow x = 1.$$

Subtracting (iv) from (iii), we get

$$(x + y) - (x - y) = 3 - (-1)$$

$$\Rightarrow x + y - x + y = 4$$

$$\Rightarrow 2y = 4$$

$$\Rightarrow y = 2.$$

Hence, x = 1 and y = 2.

45. Let the numerator and denominator of fraction be x and y respectively.

Then, the fraction is $\frac{x}{y}$.

As per first condition

The sum of the numerator and denominator of a fraction is 4 more than twice the numerator.

$$x + y = 2x + 4$$

$$\Rightarrow -x + y = 4 \dots\dots(i)$$

According to the second condition,

If the numerator and denominator are increased by 3, they are in the ratio 2 : 3.

$$\frac{x+3}{y+3} = \frac{2}{3}$$

$$\Rightarrow 3x + 9 = 2y + 6$$

$$\Rightarrow 3x - 2y = -3 \dots\dots(ii)$$

Multiply (i) by -2, we get

$$-2x + 2y = 8 \dots\dots(iii)$$

Adding (ii) and (iii), we get

$$\text{and } 3x - 2x = -3 + 8$$

$$\Rightarrow x = 5$$

Substituting $x = 5$ in (i), we get

$$5 - y = 4$$

$$y = 9$$

Hence, the required fraction is $\frac{5}{9}$

46. Let the larger number is x and smaller number is y .

We know that, $Dividend = (Divisor \times Quotient) + Remainder \dots\dots\dots(i)$

When $3x$ is divided by y , we get 4 as quotient and 3 as remainder.

\therefore by using (i), we get

$$3x = 4y + 3$$

$$\Rightarrow 3x - 4y = 3 \dots\dots(ii)$$

When $7y$ is divided by x , we get 5 as quotient and 1 as remainder.

\therefore by using (i), we get

$$7y = 5x + 1$$

$$\Rightarrow 5x - 7y + 1 = 0 \dots\dots\dots(iii)$$

By Solving equations (ii) and (iii), we get

$$\frac{x}{-4-21} = \frac{-y}{3+15} = \frac{1}{-21+20}$$

$$\Rightarrow \frac{x}{-25} = \frac{-y}{18} = \frac{1}{-1}$$

$$\Rightarrow x = 25 \text{ and } y = 18$$

47. $3x + 2y - 7 = 0 \dots(i)$

$$4x + y - 6 = 0 \dots(ii)$$

From eqn. (ii)

$$4x + y - 6 = 0$$

$$4x + y = 6$$

$$y = 6 - 4x \dots(iii)$$

On putting this value of y in eqn. (i)

$$3x + 2(6 - 4x) - 7 = 0$$

$$\Rightarrow 3x + 12 - 8x - 7 = 0$$

$$\Rightarrow -5x + 5 = 0$$

$$\Rightarrow 5x - 5 = 0$$

$$\Rightarrow 5x = 5$$

$$\Rightarrow x = 5/5$$

$$\therefore x = 1$$

Substituting this value of x in (ii), we get,

$$4(1) + y - 6 = 0$$

$$\Rightarrow y = 6 - 4 \times 1$$

$$\Rightarrow y = 6 - 4$$

$$\Rightarrow y = 2$$

Hence, values of x and y of given linear equations are 1 and 2 respectively.

48. Let the two numbers be x and y.

Then, by first Condition, ratio of these two numbers = 5:6.

$$x : y = 5 : 6$$

$$\Rightarrow \frac{x}{y} = \frac{5}{6}$$

$$\Rightarrow y = \frac{6x}{5} \dots\dots\dots (i)$$

As per second condition if 8 is subtracted from each of the numbers, then ratio becomes 4:5.

$$\Rightarrow \frac{x-8}{y-8} = \frac{4}{5}$$

$$5x - 40 = 4y - 32$$

$$5x - 4y = 8 \dots\dots\dots (ii)$$

Now Put the value of y from (i) to (ii), we get

$$5x - 4\left(\frac{6x}{5}\right) = 8$$

$$\Rightarrow 25x - 24x = 40$$

$$\Rightarrow x = 40$$

Put the value of x in equation (i), we get

$$y = \frac{6}{5} \times 40$$

$$= 6 \times 8$$

$$= 48$$

49. Let the common ratio term of income be x and expenditure be y.

So, the income of first person is Rs.9x and the income of second person is Rs.7x.

And the expenditures of first and second person is 4y and 3y respectively.

Then, Saving of first person = 9x - 4y

and saving of second person = 7x - 3y

As per given condition

$$9x - 4y = 200$$

$$\Rightarrow 9x - 4y - 200 = 0 \dots (i)$$

$$\text{and, } 7x - 3y = 200$$

$$\Rightarrow 7x - 3y - 200 = 0 \dots\dots (ii)$$

Solving equation (i) and (ii) by cross-multiplication, we have

$$\frac{x}{800-600} = \frac{-y}{-1800+1400} = \frac{1}{-27+28}$$

$$\frac{x}{200} = \frac{-y}{-400} = \frac{1}{1}$$

$$\Rightarrow x = 200 \text{ and } y = 400$$

So, the solution of equations is x = 200 and y = 400.

Thus, monthly income of first person = Rs.9x = Rs.(9 × 200) = Rs.1800

and, monthly income of second person = Rs.7x = Rs.(7 × 200) = Rs. 1400

50. Given pair of linear equations is

$$3x - y - 5 = 0 \dots(i)$$

$$\text{and } 6x - 2y - p = 0 \dots(ii)$$

On comparing with standard form, we get

$$\text{Here, } a_1 = 3, b_1 = -1, c_1 = -5;$$

$$\text{And } a_2 = 6, b_2 = -2, c_2 = -p;$$

$$a_1/a_2 = 3/6 = 1/2$$

$$b_1/b_2 = 1/2$$

$$c_1/c_2 = 5/p$$

Since, the lines represented by these equations are parallel, then

$$a_1/a_2 = b_1/b_2 \neq c_1/c_2$$

$$\text{Taking last two parts, we get } \frac{1}{2} \neq \frac{5}{p}$$

$$\text{So, } p \neq 10$$

Hence, the given pair of linear equations are parallel for all real values of p except 10 i.e.,

$$p \in \mathbb{R} - \{10\}$$

51. $152x - 378y = -74$ (1)

$$-378x + 152y = -604$$

Adding the equations (1) and (2), we obtain:

$$(152x - 378y) + (-378x + 152y) = -74 + (-604)$$

$$152x - 378y - 378x + 152y = -74 - 604$$

$$-226x - 226y = -678$$

$$\Rightarrow x + y = 3$$
 ... (3)

Subtracting the equation (2) from equation (1), we obtain:

$$(152x - 378y) - (-378x + 152y) = -74 - (-604)$$

$$152x - 378y + 378x - 152y = -74 + 604$$

$$530x - 530y = 530$$

$$x - y = 1$$
 (4)

Adding equations (3) and (4), we obtain:

$$(x + y) + (x - y) = 3 - 1$$

$$x + y + x - y = 2$$

$$2x = 4$$

$$\Rightarrow x = 2$$

Substituting the value of x in equation (3), we obtain:

$$x + y = 3$$

$$2 + y = 3$$

$$y = 1$$

Hence the value of x and y are 2 and 1 respectively.

52. The given equations are

$$3x - 2y = 5$$
(i)

$$2x + y = 7$$
(ii)

Putting $x = 3$ and $y = 2$ in (i), we get

$$\text{LHS} = 3x - 2y$$

$$= (3 \times 3 - 2 \times 2)$$

$$= 9 - 4$$

$$= 5$$

$$= \text{RHS}$$
 (iii)

Putting $x = 3$ and $y = 2$ in (ii), we get

$$\text{LHS} = 2x + y$$

$$= (2 \times 3 + 2)$$

$$= 6 + 2$$

$$= 8$$

$$\neq \text{RHS}$$
..... (iv)

From (iii) and (iv), we observe that the values $x = 3, y = 2$ do not satisfy (ii).

Hence, $x = 3, y = 2$ is not a solution of the given system of equations.

53. Let digit at unit's place = x and digit at hundred's place = y

$$\therefore \text{Middle digit} = x + y + 1$$

$$\text{Number} = 100y + 10(x + y + 1) + x$$

$$\text{Number obtained by reversing the digits} = 100x + 10(x + y + 1) + y$$

ATQ.,

$$x + y + (x + y + 1) = 17$$

$$\Rightarrow 2x + 2y = 16$$
 ..(i)

$$\text{and } 100x + 10(x + y + 1) = x - 396$$
 (ii)

$$\Rightarrow 99x - 99y = -396$$

$$\Rightarrow x - y = -4$$

By Solving equation (i) and (ii), we get

$$x = 2, y = 6 \text{ and Number} = 692$$

54. Let $\frac{x}{y}$ be the fraction, where x and y are positive integers.

Given,

A fraction becomes $\frac{9}{11}$, if 2 is added to both the numerator and the denominator.

$$\frac{x+2}{y+2} = \frac{9}{11}$$

$$11 \times (x + 2) = 9 \times (y + 2)$$

$$11x + 22 = 9y + 18$$

$$11x - 9y + 4 = 0 \dots\dots (i)$$

If 3 is added to both the numerator and the denominator it becomes $\frac{5}{6}$.

$$\frac{x+3}{y+3} = \frac{5}{6}$$

$$6 \times (x + 3) = 5 \times (y + 3)$$

$$6x + 18 = 5y + 15$$

$$6x - 5y + 3 = 0 \dots\dots (ii)$$

$$\text{If } 6x - 5y + 3 = 0$$

$$\text{or, } y = \frac{6x+3}{5} \dots(iii)$$

On substituting y from eqn. (iii) in eqn. (i),

$$11x - 9 \times \left(\frac{6x+3}{5}\right) + 4 = 0$$

$$55x - 9(6x + 3) + 20 = 0$$

$$55x - 54x - 27 + 20 = 0$$

$$x - 7 = 0$$

$$\therefore x = 7$$

On substituting x = 7 in eqⁿ. (iii),

$$y = \frac{6 \times 7 + 3}{5}$$

$$y = \frac{45}{5}$$

$$\therefore y = 9$$

Hence, the required fraction = $\frac{7}{9}$

55. The given system of equations is

$$2x + y = 7 \dots (i)$$

$$4x - 3y = -1 \dots(ii)$$

From (i), we get

$$2x + y = 7$$

$$\Rightarrow y = (7 - 2x).$$

Substituting y = (7 - 2x) in (ii), we get

$$4x - 3y = -1$$

$$\Rightarrow 4x - 3(7 - 2x) = -1$$

$$\Rightarrow 4x - 21 + 6x = -1$$

$$\Rightarrow 10x - 21 = -1$$

$$\Rightarrow 10x = -1 + 21$$

$$\Rightarrow 10x = 20$$

$$\Rightarrow x = \frac{20}{10}$$

$$\Rightarrow x = 2.$$

Substituting x = 2 in y = (7 - 2x), we get

$$y = (7 - 2x)$$

$$\Rightarrow y = 7 - 2(2)$$

$$\Rightarrow y = 7 - 4$$

$$\Rightarrow y = 3.$$

Hence, the solution is x = 2, y = 3.

56. The given equations are

$$2x + 3y = 16 \dots\dots\dots(i)$$

$$x - 2y = 1 \dots\dots\dots (ii)$$

Putting x = 5 and y = 2 in (i), we get

$$\text{LHS} = 2x + 3y$$

$$= (2 \times 5 + 3 \times 2)$$

$$= 10 + 6$$

$$= 16$$

$$= \text{RHS. (iii)}$$

Putting $x = 5$ and $y = 2$ in (ii), we get

$$\text{LHS} = x - 2y$$

$$= (5 - 2 \times 2)$$

$$= 5 - 4$$

$$= 1$$

$$= \text{RHS. (iv)}$$

From (iii) and (iv), $x = 5$ and $y = 2$ satisfy both (i) and (ii).

Hence, $x = 5, y = 2$ is a solution of the given system of equations.

57. $x + y = a + b$(i)

$$ax + by = a^2 + b^2$$
.....(ii)

multiply (i) by b we have

$$bx + by = ab + bb$$
.....(iii)

subtracting (ii) and (iii)

$$\Rightarrow (a - b)x = a(a - b)$$

$$\Rightarrow x = a$$
 put $x = a$ in (i) we get $y = b$.

58. Let the two numbers be x and y .

Given condition: sum of two numbers is 35.

$$\text{Then, } x + y = 35$$
(i)

and Difference of two numbers is 13.

$$x - y = 13$$
.....(ii)

Adding equations (i) and (ii), we get

$$(x + y) + (x - y) = 35 + 13$$

$$\Rightarrow x + y + x - y = 48$$

$$\Rightarrow 2x = 48$$

$$\Rightarrow x = 24$$

Subtracting equation (ii) from equation (i), we get

$$(x + y) - (x - y) = 35 - 13$$

$$x + y - x + y = 22$$

$$2y = 22$$

$$\Rightarrow y = 11$$

Hence, the two numbers are 24 and 11.

$$(a - b)x + (a + b)y = a^2 - b^2 - 2ab$$

59.
$$\frac{(a + b)x + (a + b)y = a^2 + b^2}{-2bx \qquad \qquad \qquad = -2b(b + a)}$$

$$x = a + b$$

$$\therefore (a - b)(a + b) + (a + b)y = a^2 - b^2 - 2ab$$

$$a^2 - b^2 + (a + b)y = a^2 - b^2 - 2ab$$

$$y = \frac{-2ab}{a + b}$$

60. $x + y = 6$, ... (i)

$$2x - 3y = 4$$
 ... (ii)

Multiplying by 3 in equation (i)

$$3x + 3y = 18$$
 ... (iii)

Adding equation (ii) and (iii)

$$5x = 22$$

$$x = \frac{22}{5}$$

Putting in equation (i)

$$y = 6 - \frac{22}{5} = \frac{30 - 22}{5} = \frac{8}{5}$$

61. **Step 1:** By substitution method, we pick either of the equations and write one variable in terms of the other.

$$7x - 15y = 2$$
 ... (1)

and $x + 2y = 3 \dots(2)$

Let us consider the Equation (2):

$$x + 2y = 3$$

and write it as $x = 3 - 2y \dots(3)$

Step 2: Now substitute the value of x in Equation (1)

We get $7(3 - 2y) - 15y = 2$

i.e., $21 - 14y - 15y = 2$

i.e., $-29y = -19$

Therefore $y = \frac{19}{29}$

Step 3: Substituting this value of y in Equation (3), we get

$$x = 3 - 2\left(\frac{19}{29}\right) = \frac{49}{29}$$

Therefore, the solution is $x = \frac{49}{29}, y = \frac{19}{29}$

62. Let the digit at units place be x and the digit at ten's place be y .

Then, Number = $10y + x$

If a two digit number is obtained by either multiplying sum of the digits by 8 and adding 1 or by multiplying the difference of the digits by 13 and adding 2.

According to the given conditions, we have

$$10y + x = 8(x + y) + 1$$

$$\Rightarrow 10y + x = 8x + 8y + 1$$

$$\Rightarrow 10y - 8y + x - 8x - 1 = 0$$

$$\Rightarrow 7x - 2y + 1 = 0$$

and, $10y + x = 13(y - x) + 2$

$$10y + x = 13y - 13x + 2$$

$$\Rightarrow 14x - 3y - 2 = 0$$

By using cross-multiplication, we have

$$\frac{x}{-2 \times -2 - (-3) \times 1} = \frac{-y}{7 \times -2 - 14 \times 1} = \frac{1}{7 \times -3 - 14 \times -2}$$

$$\Rightarrow \frac{x}{4+3} = \frac{-y}{-14-14} = \frac{1}{-21+28}$$

$$\Rightarrow \frac{x}{7} = \frac{y}{28} = \frac{1}{7}$$

$$\Rightarrow x = \frac{7}{7} = 1 \text{ and } y = \frac{28}{7} = 4$$

Hence, the number = $10y + x = 10 \times 4 + 1 = 41$.

63. Let the unit digit is b and ten's digit is a .

So, two digit number is $10a + b$.

As per given condition

The sum of the digits of a two digit number is 13.

So, $a + b = 13 \dots(i)$

and the number obtained by interchanging the digits of the given number exceeds the number by 27.

$$10b + a = (10a + b) + 27$$

$$\Rightarrow 9b = 9a + 27$$

$$\Rightarrow b = a + 3 \dots(ii)$$

Putting (ii) in (i), we get :

$$a + a + 3 = 13$$

$$\Rightarrow 2a = 10$$

$$\Rightarrow a = 5$$

Putting $a = 5$ in (ii), we get :

$$b = 5 + 3 = 8$$

Two digit number = $10a + b = 5(10) + 8 = 58$

Therefore, the number is 58.

64. Let the required numbers be x and y .

As per given condition the sum of two numbers is 8 .

Then, $x + y = 8 \dots(i)$

And the sum of their reciprocals is $\frac{8}{15}$

And, $\frac{1}{x} + \frac{1}{y} = \frac{8}{15}$

$$\Rightarrow \frac{x+y}{xy} = \frac{8}{15}$$

$$\Rightarrow \frac{8}{xy} = \frac{8}{15} \text{ [Using (i)]}$$

$$\Rightarrow xy = 15$$

$$\therefore (x - y) = \pm \sqrt{(x + y)^2 - 4xy}$$

$$= \pm \sqrt{8^2 - 4 \times 15} = \pm \sqrt{64 - 60} = \pm \sqrt{4} = \pm 2$$

Thus, we have

$$x + y = 8 \dots\dots\dots(i)$$

$$x - y = 2 \dots\dots\dots(ii)$$

or

$$x + y = 8 \dots\dots\dots(iii)$$

$$x - y = -2 \dots\dots\dots(iv)$$

On solving (i) and (ii), we get $x = 5$ and $y = 3$.

On solving (iii) and (iv), we get $x = 3$ and $y = 5$.

Hence, the required numbers are 5 and 3.

65. Given, pair of linear equations is $\frac{x}{3} + \frac{y}{4} = 4$

$$\Rightarrow 4x + 3y = 48 \dots(i)$$

$$\text{and } \frac{5x}{6} - \frac{y}{8} = 4$$

$$\Rightarrow 20x - 3y = 96 \dots(ii)$$

Now, adding Eqs. (i) and (ii), we get

$$24x = 144$$

$$x = 6$$

Now, put the value of x in Eq. (i), we get

$$4 \times 6 + 3y = 48$$

$$3y = 24$$

$$y = 8$$

Hence, the required values of x and y are 6 and 8, respectively.

66. Let walking speeds be x km/hr. and y km/hr. ($x > y$)

In case of walking towards each other

Relative speed = speed of 1st person + speed of 2nd person = $x + y$

$$\text{Time} = \frac{\text{Distance between them}}{\text{Relative speed}}$$

$$2 = \frac{16}{x+y}$$

$$2x + 2y = 16$$

$$x + y = 8 \dots(i)$$

In case of walking in same direction

Relative speed = $x - y$

$$\text{Time} = \frac{16}{x-y}$$

$$8x - 8y = 16$$

$$x - y = 2 \dots(ii)$$

on adding (i) and (ii)

$$2x = 10$$

$$x = 5$$

Put $x = 5$ in equation (i), we get $y = 3$

\therefore speed of first person and 2nd person are 5km/hr and 3km/hr.

67. Let the present age of Aftab and his daughter be x and y years respectively. Then, the pair of linear equations that represent the situation is

$$x - 7 = 7(y - 7), \text{ i.e., } x - 7y + 42 = 0 \dots(1)$$

$$\text{and } x + 3 = 3(y + 3), \text{ i.e., } x - 3y = 6 \dots(2)$$

from equation (2), we get $x = 3y + 6$

By putting this value of x in equation (1), we get

$$(3y + 6) - 7y + 42 = 0,$$

$$\text{i.e., } -4y = -48, \text{ which gives } y = 12$$

Again by putting this value of y in equation (2), we get

$$x = 3 \times 12 + 6 = 42$$

So, the present age of Aftab and his daughter are 42 and 12 years respectively.

68. We have,

$$2x - y = 4 \dots\dots\dots(i)$$

$$y - z = 6 \dots\dots\dots(ii)$$

$$x - z = 10 \dots\dots\dots(iii)$$

From equation (iii), we get $z = x - 10$

Substituting the value of z in equation (ii), we get

$$y - (x-10) = 6$$

$$\Rightarrow y - x + 10 = 6$$

$$\Rightarrow -x + y = 6 - 10$$

$$\Rightarrow -x + y = -4 \dots\dots\dots(iv)$$

Adding equations (i) and (iv), we get

$$(2x - y) + (-x + y) = 4 + (-4)$$

$$\Rightarrow 2x - y - x + y = 0$$

$$\Rightarrow x = 0$$

Putting $x = 0$ equation in (i) and (iii) we get

$$2x - y = 4$$

$$\Rightarrow 2(0) - y = 4$$

$$\Rightarrow y = -4$$

And $x - z = 10$

$$\Rightarrow 0 - z = 10$$

$$\Rightarrow z = -10$$

So, $y = -4$ and $z = -10$

Hence, $x = 0, y = -4, z = -10$ is the solution of the given system of equations.

69. The pair of linear equations are given as:

$$x + 2y - 4 = 0 \dots(i)$$

$$2x + 4y - 12 = 0 \dots(ii)$$

We express x in terms of y from equation (i), to get

$$x = 4 - 2y$$

Now, we substitute this value of x in equation (ii), to get

$$2(4 - 2y) + 4y - 12 = 0$$

$$\text{i.e., } 8 - 12 = 0$$

$$\text{i.e., } -4 = 0$$

Which is a false statement. Therefore, the equations do not have a common solution. So, the two rails will not cross each other.

70. The given system of equations is

$$3x - 2y = 4 \dots\dots(i)$$

$$6x - 4y = 8 \dots\dots(ii)$$

Putting $x = 2$ and $y = 1$ in equation (i) and (ii) respectively, we get

$$\text{LHS} = 3x - 2y$$

$$= 3 \times 2 - 2 \times 1$$

$$= 6 - 2$$

$$= 4$$

$$= \text{RHS}$$

$$\text{LHS} = 6x - 4y$$

$$= 6 \times 2 - 4 \times 1$$

$$= 12 - 4$$

$$= 8$$

$$= \text{RHS}$$

So, $x = 2, y = 1$ is a solution of the given system of equations.

Putting $x = 4$ and $y = 4$ in equation (i) and (ii) respectively, we get

$$\text{LHS} = 3x - 2y$$

$$= 3 \times 4 - 2 \times 4$$

$$= 12 - 8$$

$$= 4$$

$$= \text{RHS}$$

$$\text{LHS} = 6x - 4y$$

$$= 6 \times 4 - 4 \times 4$$

$$= 24 - 16$$

$$= 8$$

$$= \text{RHS}$$

So, $x = 4$, $y = 4$ is a solution of the given system of equations.

Hence $x = 2$, $y = 1$ and $x = 4$, $y = 4$ are solutions of the given system of equations.

71. Given pair of linear equations is

$$2x + 3y = 7$$

$$\text{and } 2px + py = 28 - qy$$

$$\text{or } 2px + (p + q)y - 28 = 0$$

On comparing with $ax + by + c = 0$ we get

$$\text{Here, } a_1 = 2, b_1 = 3, c_1 = -7;$$

$$\text{And } a_2 = 2p, b_2 = (p + q), c_2 = -28;$$

$$\frac{a_1}{a_2} = \frac{2}{2p}$$

$$\frac{b_1}{b_2} = \frac{3}{p+q}$$

$$\frac{c_1}{c_2} = \frac{1}{4}$$

Since, the pair of equations has infinitely many solutions i.e., both lines are coincident.

$$a_1/a_2 = b_1/b_2 = c_1/c_2$$

$$\frac{1}{p} = \frac{3}{p+q} = \frac{1}{4}$$

Taking first and third parts, we get

$$p = 4$$

Again, taking last two parts, we get

$$\frac{3}{p+q} = \frac{1}{4}$$

$$p + q = 12$$

$$\text{Since } p = 4$$

$$\text{So, } q = 8$$

Here, we see that the values of $p = 4$ and $q = 8$ satisfies all three parts.

Hence, the pair of equations has infinitely many solutions for all values of $p = 4$ and $q = 8$.

72. The given equations may be written as

$$\frac{x+1}{2} + \frac{y-1}{3} = 8$$

$$\Rightarrow 3(x+1) + 2(y-1) = 48$$

$$\Rightarrow 3x + 3 + 2y - 2 = 48$$

$$\Rightarrow 3x + 2y + 1 = 48$$

$$\Rightarrow 3x + 2y = 47 \dots (i)$$

$$\frac{x-1}{3} + \frac{y+1}{2} = 9$$

$$\Rightarrow 2(x-1) + 3(y+1) = 54$$

$$\Rightarrow 2x - 2 + 3y + 3 = 54$$

$$\Rightarrow 2x + 3y + 1 = 54$$

$$\Rightarrow 2x + 3y = 53 \dots (ii)$$

Multiplying (i) by 2 and (ii) by 3 and subtracting, we get

$$(4 - 9)y = 94 - 159$$

$$\Rightarrow -5y = -65$$

$$\Rightarrow y = \frac{-65}{-5}$$

$$\Rightarrow y = 13$$

Putting $y = 13$ in (i), we get

$$3x + (2 \times 13) = 47$$

$$\Rightarrow 3x + 26 = 47$$

$$\Rightarrow 3x = (47 - 26)$$

$$\Rightarrow 3x = 21$$

$$\Rightarrow x = \frac{21}{3} = 7$$

$$\text{So, } x = 7$$

$$\text{Hence, } x = 7 \text{ and } y = 13$$

73. Opposite sides of rectangle are equal.

$$AB = DC \text{ and } BC = AD$$

$$\Rightarrow x + y = 30 \dots (i)$$

$$\text{and } x - y = 14 \dots (ii)$$

Adding (i) and (ii), we get

$$2x = 44$$

$$x = 22$$

Put $x = 22$ in (i), we get

$$22 + y = 30$$

$$y = 30 - 22 = 8$$

$$\text{So, } x = 22 \text{ and } y = 8$$