Solution

PAIR OF LINEAR EQUATION IN TWO VARIABLE WS 3

Class 10 - Mathematics

Section A

1. The given system of equations is
   35x + 23y = 209 ....(1)
   23x + 35y = 197 ....(2)
   Adding equation (1) and equation (2), we get
   58x + 58y = 406
   ⇒ x+ y = 7 ....(3) ....Dividing throughout by 58
   Subtracting equation (2) from equation (1), we get
   12x - 12y = 12
   ⇒ x - y = 1 .....(4) ....Dividing throughout by 12
   Adding equation (3) and equation (4), we get 2x = 8
   $\Rightarrow\ x = \frac{8}{2} = 4$
   Subtracting equation (4) from equation (3), we get 2y = 6
   $\Rightarrow\ y = \frac{6}{2} = 3$
   ⇒ hence, the solution of the given pair of equations is
   x = 4, y = 3.
   Verification : Substituting x = 4, y = 3
   We find that both the equations (1) and (2) are satisfied as shown below:
   35x + 23y = 35(4) + 23(3) = 140 + 69 = 209
   23x + 35y = 23(4) + 35(3) = 92 + 105 = 197
   Hence, the solution is correct.
2. s - t = 3; $\frac{s}{3} + \frac{t}{2} = 6$
   The given pair of linear equations is :
   s - t = 3..............(1)
   $\frac{s}{3} + \frac{t}{2} = 6$..........(2)
   From equation(1),
   s = t + 3.............(3)
   Substitute this value of s in equation(2), we get
   $\frac{t+3}{3} + \frac{t}{2} = 6$
   $\Rightarrow \quad \frac{2(t+3)+3t}{6} = 6$
   ⇒2(t + 3) + 3t = 36
   ⇒ 2t + 6 + 3t = 36
   ⇒ 5t + 6 = 36
   ⇒ 5t = 30
   $\Rightarrow \quad t = \frac{30}{5} = 6$
   Substituting this value of t in equation (3), we get
   s = 6 + 3= 9
   therefore the solution is
   s = 9, t = 6
   Verification : Substituting s = 9 and t = 6, we find that both equation (1) and (2) are satisfied as shown below:
   s - t = 9 - 6 = 3
   $\frac{s}{3} + \frac{t}{2} = \frac{9}{3} + \frac{6}{2} = 3 + 3 = 6$
   This verifies the solution.
3. Let us suppose that the digit in the unit's place be x and the digit at the ten's place be y.
   Therefore, Number = 10y + x
   The number obtained by reversing the order of the digits is 10x + y.

According to the given conditions, we have
(10y + x) + (10x + y)=121
10y + x + 10x + y = 121
11y + 11x = 121
$\Rightarrow$ 11(x + y)=121
$\Rightarrow$ x + y=11
Again, according to question the difference between the digits is 3
i.e, x - y=± 3
Thus, we have the following sets of simultaneous equations
x + y = 11 ......(i)
& x - y = 3.......(ii)
Or x + y = 11.......(iii)
x - y = -3......(iv)
Adding equation (i) and (ii), we get
x + y + x - y = 11 + 3
2x = 14
x = 7
Similarly on subtracting equation (ii) from (I) we get y = 4
On solving equations (iii) and (iv), we get x = 4,y = 7
When x =7, y = 4, we have
Number =10y + x =10 $\times$4 + 7=47
When x = 4, y =7, we have
Number = 10y + x = 1 0 $\times$7 + 4 = 74
Therefore, the required number is either 47 or, 74.

4. We have
217x + 131y = 913 ....(i)
131x + 217y = 827 ....(ii)
Adding equations (i) and (ii), we get
(217x + 131y) + (131x + 217y) = 913 + 827
217x + 131y + 131x + 217y = 1740
348x + 348y = 1740
$\Rightarrow$ x + y = 5 ....(iii)
Subtracting equation (ii) from equation (i), we get
(217x + 131y) - (131x + 217y) = 913 - 827
⇒ 217x + 131y - 131x - 217y = 86
⇒ 86x - 86y = 86
⇒ x - y = 1 .... (iv)
Adding equation (iii) and (iv), we get
(x + y) + (x - y) = 5 + 1
⇒ x + y + x - y = 6
⇒ 2x = 6
⇒ x = 3
Putting x = 3 in equation (iii), we get y = 2.
Hence, x= 3 and y = 2 is the solution of the given system of equations.

5. The given equations are
$\sqrt{2}x-\sqrt{3}y=0$ ....(i)
$\sqrt{3}x-\sqrt{8}y=0$ ....(ii)
From equation (i), we obtain:
$x=\frac{\sqrt{3}y}{\sqrt{2}}$ ...(iii)
Substituting this value in equation (ii), we obtain:
$\sqrt{3}\left(\frac{\sqrt{3}y}{\sqrt{2}}\right)-\sqrt{8}y=0$
$\frac{3y}{\sqrt{2}}-2\sqrt{2}y=0$

$y\left(\frac{3}{\sqrt{2}}-2\sqrt{2}\right)=0$

y = 0

Substituting the value of y in equation (iii), we obtain:

x = 0

$\therefore$ x = 0, y = 0

Hence the solution of given equation is (0,0).

6. Let number of questions answered correctly be x
   and number of questions answered wrong be y
   Therefore 3x - y = 40 ...(i)
   and 4x - 2y = 40 ...(ii)
   solving, x = 20, y = 20
   Total number of questions = x + y = 40
7. Let the ten's digit of required number be x and its unit digit be y respectively.
   Then, As per given condition
   The sum of digits of a two digit number is 15.
   x + y = 15 ....(i)
   Required number = 10x + y
   Number formed on reversing the digits = 10y + x
   So, as per given condition, the number obtained by reversing the order of digits of the given number exceeds the given number by 9.
   $\therefore$ 10y + x -(10x + y) = 9
   $\therefore$ 10y + x - 10x - y = 9
   9y - 9x = 9
   - x + y = 1.............(ii)
   Adding (i) and (ii), we get
   $2y=16$
   $\Rightarrow y=\frac{16}{2}=8$
   Putting y = 8 in (i), we get
   x + 8 = 15
   x = 15 - 8 = 7
   Number = 10x + y
   = 10 $\times$ 7 + 8
   = 70 + 8
   = 78
   Hence the given two digit number is 78.
8. Let the digits at units and tens place of the given number be x and y respectively.
   Then,
   Number =$10y+x$ ....(i)
   Number obtained by reversing the order of the digits = $10x+y$
   According to the question,
   $(10y+x)+(10x+y)=165$
   $\Rightarrow x+y=15$
   and, $x-y=3$
   Thus, we obtain the following systems of linear equations.
   i. $x+y=15$
      $x-y=3$
   ii. $x+y=15$
      $y-x=3$

   Solving first system of equations, we get
   $x=9, y=6$
   Solving second system of equation, we. get
   $x=6, y=9$

Substituting the values of x and y in equation (i), we have

Number = $69\ or,\ 96$.

9. Let the two numbers be x and y.

According to question

x + y = 16.....(i)

and,

$\frac{1}{x}+\frac{1}{y}=\frac{1}{3}$

$\Rightarrow \frac{x+y}{xy}=\frac{1}{3}$

⇒ 3x + 3y = xy.....(ii)

From equation (i), we get

x = 16 - y.....(iii)

Substitute the value of x in equation (ii), we get

3(16 - y) + 3y = (16 - y)y

⇒ 48 = 16y - y^2

⇒ y^2 - 16y + 48 = 0

⇒ y^2 - 12y - 4y + 48 = 0

⇒ y(y - 12) - 4(y - 12) = 0

⇒ (y - 4)(y - 12) = 0

⇒ y = 4 or y = 12

Case 1. When y = 4

x = 12 [from equation (iii)]

Case 2. When y = 12

x = 4 [from equation (iii)]

Thus, the possible values are 12 and 4.

10. The given equations are

$6x+5y=7x+3y+1=2(x+6y-1)$

Therefore,we have

$6x+5y=2(x+6y-1)$

$6x+5y=2x+12y-2$

$6x-2x+5y-12y=-2$

$4x-7y=-2$ ......(i)

Also,

$7x+3y+1=2(x+6y-1)$

$7x+3y+1=2x+12y-2$

$7x-2x+3y-12y=-2-1$

$5x-9y=-3$ ....(ii)

Multiplying (i) by 9 and (ii) by 7, we get

$36x-63y=-18$ ......(iii)

$35x-63y=-21$ .....(iv)

Subtracting (iii) and (iv),we get

$x=3$

Substituting x = 3 in (i),we get

$\Rightarrow 4\times 3-7y=-2$

$\Rightarrow -7y=-2-12$

$\Rightarrow -7y=-14$

⇒y = 2

∴ Solution is x = 3, y = 2

11. The given pair of linear equations

2x + y = 4

and 2x-y = 4

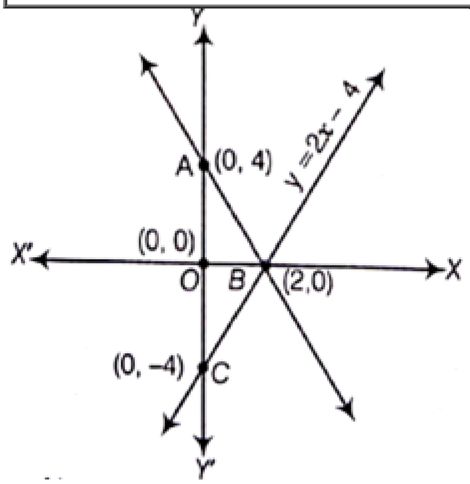
Table for line 2x + y = 4

x	0	2

y	4	0
Points	A	B

and table for line 2x - y = 4

x	0	2
y	- 4	0
Points	C	B



So the Graphical representation of both lines is as above.

Here, both lines and Y - axis from a $\triangle \mathrm{ABC}$.

Hence, the vertices of a $\triangle \mathrm{ABC}$ are A (0,4), B(2,0) and C(0,- 4) where A and C are obtained by putting x = 0 in the given equations abd B is obtained by solving them together.

$\therefore$ Required area of $\triangle \mathrm{ABC}$= 2 $\times$ Area of $\triangle \mathrm{AOB}$

$\triangle \mathrm{ABC}$ = 2 $\times$ $\left(\frac{1}{2} \times 4 \times 2\right) = 8$ sq. units.

Hence, the required area of the triangle is 8 sq units.

12. The given system of equations is

$2x + 3y = 9$ ....(i)

$3x + 4y = 5$ ....(ii)

From equation (i), we get

$3y = 9 - 2x$

$\Rightarrow y = \frac{9-2x}{3}$

Substituting $y = \frac{9-2x}{3}$ in equation (ii), we get

$3x + 4\left(\frac{9-2x}{3}\right) = 5$

$\Rightarrow \quad \frac{9x+36-8x}{3} = 5$

$\Rightarrow x + 36 = 15$

$\Rightarrow x = -21$

Putting, $x = -21$ in $y = \frac{9-2x}{3}$, we get

$y = \frac{9+42}{3} = 17$

Hence, the solution of the given system of equations is $x = -21$, $y = 17$

13. As the opposite sides of a rectangle are equal so by figure,

3x + y = 7 .... (i)

x + 3y = 13 ..... (ii)

multiplying (i) by 3

$9x + 3y = 21 \quad$ [From (i)]

$x + 3y = 13$

$- \quad - \quad -$

$8x \quad = 8 \quad$ [Substracting (ii) from (i)]

$x = 1$

Now, $3(1) + y = 7$ [From (i)]

$\Rightarrow y = 7 - 3$
$\Rightarrow y = 4$
$x = 1$
The required values of x and y are 1 and 4 respectively.

14. Since $(x+1)$ is a factor of $2x^3 + ax^2 + 2bx + 1$
$\Rightarrow x = -1$ is a zero of $2x^3 + ax^2 + 2bx + 1$
$\Rightarrow 2(-1)^3 + a(-1)^2 + 2b(-1) + 1 = 0$
$\Rightarrow a - 2b - 1 = 0$
$\Rightarrow$ a - 2b = 1 ...(i)
Given that $2a - 3b = 4$ ...(ii)
Multiplying equation (i) by 2, we get
$2a - 4b = 2$ ...(iii)
Subtracting equation (iii) from (ii), we get
b = 2
Substituting b = 2 in equation (i), we have
a - 2(2) = 1
$\Rightarrow$ a - 4 = 1
$\Rightarrow$ a = 5
Hence, a = 5 and b = 2.

15. Let us suppose that the ten's digit of required number be x and its unit digit be y respectively.
Therefore, required number = 10x + y
According to the given conditions
10x + y = 4(x + y) + 3
$\Rightarrow$ 10x + y = 4x + 4y + 3
or, 10x +y - 4x - 4y = 3
$\Rightarrow$ 6x - 3y = 3
$\Rightarrow$ 2x - y = 1...........(i)
and
10x + y + 18 = 10y + x
$\Rightarrow$10x + y - 10y - x = - 18
$\Rightarrow$9x - 9y = -18
$\Rightarrow$9(x - y) = -18
$\Rightarrow (x - y) = \frac{-18}{9}$
$\Rightarrow$x - y = - 2 ....(ii)
Subtracting equation (ii) from equation (i), we get
x - y - (2x - y) = - 2 - 1
x - y - 2x +y = - 3
- x = -3
$\therefore$x = 3
Put the value of x = 3 in equation (i), we get
2×3 - y = 1
$\Rightarrow$y = 6 - 1 = 5
$\therefore$ x = 3, y = 5
Required number = 10x + y
= 10 $\times$3 + 5
= 30 + 5
= 35
Therefore the required number is 35.

16. Suppose the first and second number be x and y respectively.
According to the question,
$2x + 3y = 92$ ....(i)
$4x - 7y = 2$ ....(ii)
Multiplying equation (i) by 7 and (ii) by 3,

$\Rightarrow 14x + 21y = 644$ ....(iii)

$12x - 21y = 6$ ....(iv)

Adding equations (iii) and (iv),

$\Rightarrow 26x = 650$

$\Rightarrow x = \frac{650}{26} = 25$

Putting $x = 25$ in equation (i),

$\Rightarrow 2 \times 25 + 3y = 92$

$\Rightarrow 50 + 3y = 92$

$\Rightarrow 3y = 92 - 50$

$y = \frac{42}{3} = 14$

y = 14

∴ the first number is 25 and second is 14

17. 0.2 x + 0.3 y = 1.3 ; 0.4 x + 0.5 y = 2.3

The given system of linear equations is:

0.2 x + 0.3 y = 1.3.............(1)

0.4 x + 0.5 y = 2.3..................(2)

From equation (1),

0.3 y = 1.3 - 0.2 x

$\Rightarrow \quad y = \frac{1.3 - 0.2x}{0.3}$ ....(3)

Substituting this value of y in equation(2), we get

$0.4x + 0.5\left(\frac{1.3 - 0.2x}{0.3}\right) = 2.3$

⇒0.12 x + 0.65 - 0.1 x = 0.69

⇒0.12 x - 0.1 x = 0.69 - 0.65

⇒0.02 x = 0.04

$\Rightarrow \text{x} = \frac{0.04}{0.02} = 2$

Substituting this value of x in equation(3), we get

$y = \frac{1.3 - 0.2(2)}{0.3} = \frac{1.3 - 0.4}{0.3} = \frac{0.9}{0.3} = 3$

Therefore, the solution is x = 2, y = 3, we find that both equation (1) and (2) are satisfied as shown below:

0.2 x + 0.3 y = (0.2)(2)+(0.3)(3) = 0.4 + 0.9 = 1.3

0.4 x + 0.5 y= (0.4)(2) + (0.5)(3) } = 0.8 + 1.5 = 2.3

This verifies the solution.

18. According to question the given system of equations are

$\frac{x}{10} + \frac{y}{5} - 1 = 0$

$\Rightarrow \frac{x + 2y - 10}{10} = 0$

⇒ x + 2y = 10 ....(i)

And, $\frac{x}{8} + \frac{y}{6} = 15$

$\Rightarrow \frac{3x + 4y}{24} = 15$

⇒ 3x + 4y = 360 ....(ii)

Multiplying equation (i) by 3, we get

3x + 6y = 30 ....(iii)

Subtracting equation (iii) from (ii), we get

3x + 4y - 3x - 6y = 360 - 30

-2y = 330

⇒ y = -165

Substitute the value of y = -165 in (i), we get

x + 2(-165) = 10

⇒ x - 330 = 10

⇒ x = 340

Now it is given that $y = \lambda x + 5$

$\Rightarrow -165 = \lambda \times 340 + 5$

$\Rightarrow 340\lambda = -170$

$\Rightarrow \lambda = \frac{-170}{340} = -\frac{1}{2}$

Hence, x = 340, y = -165 and $\lambda = -\frac{1}{2}$.

19. In cyclic quadrilateral

$\angle A + \angle C = 180 \text{ and } \angle B + \angle D = 180$

$\Rightarrow$ 2x + 4 + 2y + 10 = 180

$\Rightarrow$ x + y = 83 ... (i)

and $\angle B + \angle D = 180$

$\Rightarrow$ y + 3 + 4x - 5 = 180

$\Rightarrow$ 4x + y = 182 ...(ii)

on solving eq. (i) and (ii), we get,

x = 33 and y = 50

$\therefore$ Angles are

$\angle A = 2x + 4 = 2 \times 33 + 4 = 70^\circ$

$\angle B = y + 3 = 50 + 3 = 53^\circ$

$\angle C = 2y + 10 = 2 \times 50 + 10 = 110^\circ$

$\angle D = 4x - 5 = 4 \times 33 - 5 = 127^\circ$

20. Let us suppose that the numbers are x and y.

According to question it is given that

The sum of the two numbers is 1000.

Thus, we have x + y = 1000

The difference between the squares of the two numbers is 256000.

Therefore, we have $x^2 - y^2 = 256000$

$\Rightarrow$ (x + y)(x - y) = 256000

$\Rightarrow$ 1000(x - y) = 256000

$\Rightarrow x - y = \frac{256000}{1000}$

$\Rightarrow$ x - y = 256

Therefore, we have two equations

x + y = 1000 ......(1)

x - y = 256 .....(2)

Here x and y are unknowns.

We have to solve the above equations for x and y.

Adding equation (1) and (2), we get

(x + y) + (x - y) = 1000 + 256

$\Rightarrow$ x + y + x - y = 1256

$\Rightarrow$ 2x = 1256

$\Rightarrow x = \frac{1256}{2}$

x = 628

Substituting the value of x in the equation (1) we get,

628 + y = 1000

$\Rightarrow$ y = 1000 - 628

$\Rightarrow$ y = 372

Therefore the numbers are 628 and 372.

21. Let the numerator be ‘a’ and denominator be ‘b’.

According to the first condition in the question, we have

$\Rightarrow \frac{2a}{b-5} = \frac{6}{5}$

$\Rightarrow 5a - 3b = -15$ ..... (1)

Also, according to the second condition, we have

$\Rightarrow \frac{a+8}{2b} = \frac{2}{5}$

$\Rightarrow 5a - 4b = -40$ ......(2)

Subtracting (2) from (1), gives

b = 25

Using this value of b in (1) gives

a = $\frac{-15+3b}{5} = \frac{-15+3\times25}{5} = 12$

So, the required fraction is $\frac{12}{25}$.

22. We know, by property of cyclic quadrilateral that

Sum of opposite angles = 180^o

So, $\angle A + \angle C = (6x + 10)^\circ + (x + y)^\circ = 180^\circ$

Since $\angle A = (6x + 10)^\circ$

and $\angle C = (x + y)^\circ$

So, 7x + y = 170 …(i)

Also, $B + \angle D = (5x)^\circ + (3y - 10)^\circ = 180^\circ$

Since $\angle B = (5x)^\circ$

and $\angle D = (3y - 10)^\circ$

So, 5x + 3y = 190 …(ii)

On multiplying Eq. (i) by 3 and then subtracting, we get

3(7x + y) – (5x + 3y) = 3(170) – 190

16x = 320

x = 20^o

On putting x = 20^o in Eq. (i), we get

7(20) + y = 170

So, y = 30^o

And hence $\angle A = (6x + 10)^\circ = (6(20) + 10)^\circ = 130^\circ$

$\angle B = (5x)^\circ = 5 \times 20 = 100^\circ$

$\angle C = (x + y)^\circ$ = 20 + 30 = 50^o

$\angle D = (3y - 10)^\circ = 3(30) - 10 = 80^\circ$

Hence, the required values of x and y are 20^o and 30^o respectively and the values of the four angles i.e., and$\angle A$, $\angle B, \angle C$ and $\angle D$ are 130^o, 100^o, 50^o, and 80^o, respectively.

23. The given system of equations is:

x + y = 7......(1)

2x - 3y = 11....(2)

From, equation(1),

y = 7 - x.....(3)

Substitute this value of y in equation(2), we get

2x - 3(7 - x) = 11

2x - 21 + 3x = 11

$\Rightarrow$ 5x - 21 =11

$\Rightarrow$ 5x = 32

$\Rightarrow$ x = $\frac{32}{5}$

Substituting this values of x in equation(3), we get

y = $7 - \frac{35}{2} = \frac{35-32}{5} = \frac{3}{5}$

Therefore, the solution is

$x = \frac{32}{5}, y = \frac{3}{5}$

Verification:

Substituting,$x = \frac{32}{5}, y = \frac{3}{5}$ we find that both the equations (1) and (2) are satisfied as shown below:

$x + y = \frac{32}{5} + \frac{3}{5} = \frac{32+3}{5} = \frac{35}{5} = 7$

$2x - 3y = 2\left(\frac{32}{5}\right) - 3\left(\frac{3}{5}\right)$

$= \frac{64}{5} - \frac{9}{5} = \frac{64-9}{5} = \frac{55}{5} = 11$.

This verifies the solution.

24. It is given that

$\frac{x}{2} + \frac{2y}{3} = -1$

$\frac{3x+4y}{6} = -1$

3x + 4y = - 6 .....(i)

and $\frac{x}{1}-\frac{y}{3}=3$

$\frac{3x-y}{3}=3$

3 x - y = 9 ....(ii)

We have to find out the values of x and y from these two given equations

On subtracting eqn (ii) from eqn (i),

$$\begin{array}{r} 3x+4y=-6 \\ 3x-y=\ \ 9 \\ -\ \ +\ \ \ \ \ \ - \\ \hline 5y=-15 \\ y=-3 \end{array}$$

Putting y = - 3 in eq (i), we get

3x + 4 (- 3) = - 6

3x -12 = - 6

3x = 12 - 6

3x = 6

∴x = 2

Hence x = 2 and y = - 3

25. The given equations are:

x - y + z = 4 ...(i)

x + y + z =2 ...(ii)

2x + y - 3z = 0 ...(iii)

First of all we find the value of x from (i). So,

x = 4 + y - z

Put the value of x in equation (i), we get

4 + y - z + y + z = 2

⇒ 2 y = -2

⇒ y = -1

Put the value of x and y in equation (iii) we get

2(4 + y - z) + y - 3z = 0

⇒ 8 - 2 - 2z -1 - 3z = 0

⇒ -5z = -5

⇒ $z = 1$

Put the value of y and z in equation (i), we get

x - (-1) + 1 = 4

⇒ x = 2

Hence the value of x = 2, y = 1 and z = 1

26. Let the ten's and unit digit be y and x respectively.

So the number is 10y + x

The number when digits are reversed becomes 10x + y

So, 7(10x + y) = 4(10y + x)

or, 70x+ 7y= 40y + 4x

or, 70x - 4x = 40y - 7y

or 66x = 33y

⇒ 2x = y .....(i)

The difference of the digits is 3

y - x= 3

2x - x = 3

x = 3

x = 3 and y = 6

Hence the number is 63

27. Let numerator of fraction is x and denominator is y.

Let the fraction be $\frac{x}{y}$.

Then, according to the given conditions, we have

If 1 is added to both numerator and denominator then fraction becomes $\frac{4}{5}$.

$\frac{x+1}{y+1}=\frac{4}{5}$

$\Rightarrow$ 5x +5 =4y + 4

$\Rightarrow$ 5x - 4y + 1=0 ...... (i)

If 5 is subtracted from both numerator and denominator, the fraction becomes $\frac{1}{2}$.

$\frac{x-5}{y-5}=\frac{1}{2}$

$\Rightarrow$ 2x - 10 = y - 5

$\Rightarrow$ 2x - y - 5=0 .... (ii)

From (i) and (ii)

By using cross-multiplication, we have

$\frac{x}{-4\times-5-(-1)\times1}=\frac{-y}{5\times-5-2\times1}=\frac{1}{5\times-1-2\times-4}$

$\Rightarrow \quad \frac{x}{20+1}=\frac{y}{25+2}=\frac{1}{-5+8}$

$\Rightarrow \quad \frac{x}{21}=\frac{y}{27}=\frac{1}{3}$

$\Rightarrow x=\frac{21}{3}=7$ and $y=\frac{27}{3}=9$

Hence, the given fraction is 7/9.

28. The given system of linear equations is:

4 x + 7 y = 20....................(1)

21 x - 13 y = 21...................(2)

From equation (2), 13 y = 21x - 21

$\Rightarrow \quad y=\frac{21x-21}{13}$ ....(3)

Substitute this value of y in equation (1), we get

$4x+7\left(\frac{21x-21}{13}\right)=20$

$\Rightarrow$52 x + 147 x - 147 = 260

$\Rightarrow$199 x = 147 + 260

$\Rightarrow$ 199 x = 407

$\Rightarrow x=\frac{407}{199}$

Substituting this value of x in equation (3), we get

$y=\frac{21\left(\frac{407}{199}\right)-21}{13}=\frac{8547-4179}{2587}=\frac{4368}{2587}=\frac{336}{199}$

Therefore, the solution is

$\mathrm{x}=\frac{407}{199}, \quad \mathrm{y}=\frac{336}{199}$

Verification: Substituting, $\mathrm{x}=\frac{407}{199}, \quad \mathrm{y}=\frac{336}{199}$ we find that both the equations (1) and (2) are satisfied as shown below:

$4x+7y=4\left(\frac{407}{199}\right)+7\left(\frac{336}{199}\right)=\frac{1628+2352}{199}=\frac{3980}{199}=20$

21x - 13y $=21\left(\frac{407}{199}\right)-13\left(\frac{336}{199}\right)=21$

This verifies the solution.

29. We have to solve the following systems of equations by using the method of substitution

$\frac{2x}{a}+\frac{y}{b}=2$ ......(i)

$\frac{x}{a}-\frac{y}{b}=4$ ....(ii)

From equation (i), we get

$\frac{y}{b}=2-\frac{2x}{a}$

$\Rightarrow y=b\left(2-\frac{2x}{a}\right)$

Substituting $y=b\left(2-\frac{2x}{a}\right)$ in equation (ii), we get

$\frac{x}{a}-\frac{b}{b}\left(2-\frac{2x}{a}\right)=4$

$\Rightarrow \frac{x}{a}-2+\frac{2x}{a}=4$

$\Rightarrow \frac{3x}{a}=6$

$\Rightarrow$ 3x = 6a

$\Rightarrow$ x = 2a

Putting x = 2a in equation (i), we get

$4 + \frac{y}{b} = 2$
$\Rightarrow \frac{y}{b} = -2$
$\Rightarrow y = -2b$
Hence, the solution of the given system of equations is x = 2a and y = -2b.

30. The vertex of a triangle is the common solution of the two equations forming its two sides. So, solving the given equations pairwise will give the vertices of the triangle.
From the given equations, we will have the following three pairs of equations:
5x – y = 5 and x + 2y = 1
x + 2y = 1 and 6x + y = 17
5x – y = 5 and 6x + y = 17
Solving the pair of equations
5x – y = 5
x + 2y = 1
we get, x = 1, y = 0
So, one vertex of the triangle is (1, 0)
Solving the second pair of equations
x + 2y = 1
6x + y = 17
we get x = 3, y = –1
So, another vertex of the triangle is (3, –1)
Solving the third pair of equations
5x – y = 5
6x + y = 17,
we get x = 2, y = 5.
So, the third vertex of the triangle is (2, 5). So, the three vertices of the triangle are (1, 0), (3, –1) and (2, 5).

31. The given pair of linear equations
2x + 3y = 11 ...... (1)
2x - 4y = -24 .... (2)
From equation (1), 3y = 11 - 2x
$\Rightarrow \quad y = \frac{11-2x}{3}$
Substituting this value of y in equation (2), we get
$2x - 4\left(\frac{11-2x}{3}\right) = -24$
$\Rightarrow 6x - 44 + 8x = -72$
$\Rightarrow 14x - 44 = -72$
$\Rightarrow 14x = 44 - 72$
$\Rightarrow 14x = -28$
$\Rightarrow x = -\frac{28}{14} = -2$
Substituting this value of x in equation (3), we get
$y = \frac{11-2(-2)}{3} = \frac{11+4}{3} = \frac{15}{3} = 5$
Verification, Substituting x = -2 and y = 5, we find that both the equations (1) and (2) are satisfied as shown below:
$2x + 3y = 2(-2) + 3(5) = -4 + 15 = 11$
$2x - 4y = 2(-2) - 4(5) = -4 - 20 = -24$
This verifies the solution,
Now, y = axe + 3
$\Rightarrow 5 = m(-2) + 3$
$\Rightarrow -2m = 5 - 3$
$\Rightarrow -2m = 2$
$\Rightarrow \mathrm{m} = \frac{2}{-2} = -1$

32. The given systems of equations is
$x + \frac{y}{2} = 4$ ....(i)
$\frac{x}{3} + 2y = 5$ ....(ii)
From (i), we get

$\frac{2x+y}{2} = 4$

$2x + y = 8$

$y = 8 - 2x$

From (ii), we get

$\frac{x+6y}{3} = 5$

$x + 6y = 15$ ....(iii)

Substituting y = 8 - 2x in (iii), we get

x + 6(8 - 2x) = 15

⇒ -11x = 15 - 48

⇒ -11x = -33

$\Rightarrow x = \frac{-33}{-11} = 3$

Putting x = 3, in y = 8 - 2x, we get

y = 8 - 2 $\times$ 3

y = 2

Hence, solution of the given system of equations is x = 3, y = 2.

33. x + y = 14; x - y = 4

the given pair of linear equations is

x + y = 14.................(1)

x - y = 4....................(2)

From equation(1),

y = 14 - x...................(3)

Substitute this value of y in equation(2), we get

x - (14 - x) = 4

⇒ x - 14 + x = 4

⇒ 2x - 14 = 4

⇒ 2x = 4 + 14

⇒ 2x = 18

$\Rightarrow x = \frac{18}{2} = 9$

Substituting this value of x in equation (3), we get y = 14 - 9 = 5

Therefore, the solution is x = 9, y = 5

verification: Substituting x = 9 and y = 5, we find that both the equations (1) and (2) are satisfied as shown below:

x + y = 9 + 5 = 14

x - y = 9 - 5=4

This verifies the solution.

34. 2x +3y =11 .... (1)

2x−4y =−24 .... (2)

Using equation (2), we can say that

2x =−24 + 4y

⇒ x =−12 + 2y

Putting this in equation (1), we get

2(−12 + 2y)+3y =11 ⇒ −24 + 4y + 3y =11

⇒ 7y =35 ⇒ y =5

Putting value of y in equation (1), we get

2x +3(5)=11 ⇒ 2x +15=11

⇒ 2x =11 - 15=−4 ⇒ x =−2

Therefore, x =−and y =5

Putting values of x and y in y =mx +3, we get

5=m (−2) + 3 ⇒ 5 =−2m +3

⇒ −2m =2 ⇒ m =−1

35. The given system of equations is

$3x - \frac{y+7}{11} + 2 = 10$ ...(i)

$2y + \frac{x+11}{7} = 10$ ....(ii)

From (i), we get

$\frac{33x-y-7+22}{11}=10$

$\Rightarrow 33x-y+15=10\times 11$

$\Rightarrow 33x+15-110=y$

$\Rightarrow y=33x-95$

From (ii), we get

$\frac{14y+x+11}{7}=10$

$\Rightarrow 14y+x+11=10\times 7$

$\Rightarrow 14y+x+11=70$

$\Rightarrow 14y+x=70-11$

$\Rightarrow 14y+x=59$ ....(iii)

Substituting y = 33x - 95 in (iii), we get

14(33x - 95) + x = 59

⇒ 462x - 1330 + x = 59

⇒ 463x = 59 + 1330

⇒ 463x = 1389

$\Rightarrow x=\frac{1389}{463}=3$

Putting x = 3, in y = 33x - 95, we get

y = 33 × 3 - 95

⇒ y = 99 - 95 = 4

⇒ y = 4

Hence, Solution of the given system of equation is x = 3, y = 4.

36. We have to solve 2x + 3y = 11 and 2x − 4y = −24 and also we have to find the value of 'm' for which y = mx + 3.

2x + 3y = 11 …. (1)

2x - 4y = −24 …. (2)

Using equation (2), we can say that

2x = −24 + 4y

⇒ x = −12 + 2y

Putting this in equation (1), we get

2 (−12 + 2y) + 3y = 11

⇒ −24 + 4y + 3y = 11

⇒ 7y = 11+ 24

⇒ 7y = 35

or, y = 5

Putting value of y in equation (1), we get

2x + 3 (5) = 11

⇒ 2x + 15 = 11

⇒ 2x = 11 – 15 = −4⇒ x = −2

Therefore, x = −2 and y = 5

Putting values of x and y in y = mx + 3, we get

5 = m (−2) + 3

⇒ 5 = −2m + 3

or, 5 - 3 = -2m

⇒ −2m = 2 ⇒ m = −1

37. $31x+29y=33$ ----------(1)

$29x+31y=27$ ------------ (2)

Multiply (1) by 29 and (2) by 31 (Since 29,31 are primes and Lcm is 29×31)

(1) becomes 31x×29 + 29×29y = 33×29 ------- (3)

(2) becomes 29x×31+31×31y=27×31 ------- (4)

Subtracting (3) from (4),

$(31^2-29^2)y=27\times 31-33\times 29=-120$

$(31-29)(31+29)y=-120$

$120y=-120$

$y = -1$
Substituting in (1),
$31x - 29 = 33$
$31x = 62$
Hence,
$x = 2\ and\ y = -1$

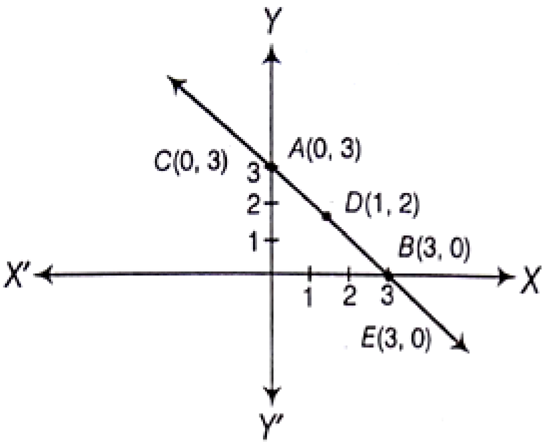
38. Let us suppose that the numerator be x and denominator be y
Therefore, the fraction is $\frac{x}{y}$.
Then, according to the given conditions, we have
$\frac{3x}{y-3} = \frac{18}{11}$ and $\frac{x+8}{2y} = \frac{2}{5}$
⇔ 11x = 6y -18 and 5x + 40 =4y
⇔11x - 6y + 18 =0 and 5x - 4y + 40 =0
By cross multiplication, we have
$\frac{x}{(-6)\times40-(-4)\times18} = \frac{-y}{11\times40-5\times18} = \frac{1}{11\times(-4)-5\times(-6)}$
$\Rightarrow \quad \frac{x}{-240+72} = \frac{-y}{440-90} = \frac{1}{-44+30}$
$\Rightarrow \quad \frac{x}{-168} = \frac{y}{-350} = \frac{1}{-14}$
$\Rightarrow \quad x = \frac{-168}{-14}$ and $y = \frac{-350}{-14}$
⇒x =12 and y =25
Therefore, the fraction is $\frac{12}{25}$.

39. Given pair of equations is
x + y = 3 …(i)
and 3x + 3y = 9 …(ii)
On comparing with standard form we get
$a_1 = 1$, $b_1 = 1$, $c_1 = -3$;
And $a_2 = 3$, $b_2 = 3$, $c_2 = -9$;
$a_1 / a_2 = 1/3$
$b_1 / b_2 = 1/3$
$c_1 / c_2 = 1/3$
Here, $a_1/a_2 = b_1/b_2 = c_1/c_2$, i.e. coincident lines
Hence, the given pair of linear equations is coincident and having infinitely many solutions.
The given pair of linear equations is consistent.
Now, x + y = 3 or y = 3 - x
If x = 0 then y = 3, If x = 3, then y = 0.

x	0	3
y	3	0
Points	A	B

and 3x + 3y = 9 or $y = \frac{9-3x}{3}$
If x = 0 then y = 3, if x = 1, then y = 2, and if x = 3, then y = 0.

x	0	1	3
y	3	2	0
Points	C	D	E



Plotting the points A(0, 3) and B(3, 0), we get the line AB. Again, plotting the points C(0, 3) and D(1, 2) and E(3, 0), we get the line CDE.

40. Let the numerator and denominator of the fraction be x and y respectively.

Then,

Fraction= $\frac{x}{y}$

It is given that

Denominator = 2 (Numerator) + 4

$\Rightarrow y = 2x + 4$

$\Rightarrow 2x - y + 4 = 0$

According to the given condition, we have

$y - 6 = 12(x - 6)$

$\Rightarrow y - 6 = 12x - 72$

$\Rightarrow 12x - y - 66 = 0$

Thus, we have the following system of equations

2x - y + 4 =0 ....(i)

12x - y -66 =0 ....(ii)

Subtracting equation (i) from equation (ii), we get

10x - 70 =0 $\Rightarrow$ x = 7

Putting x = 7 in equation (i), we get

14 - y + 4 =0 $\Rightarrow$ y = 18

Hence, required fraction =$\frac{7}{18}$.

41. It is given that angles of a cyclic quadrilateral ABCD are given by:

∠A = (4x + 20)°,

∠B = (3x - 5)°,

∠C = (4y)°

and ∠D = (7y + 5)°.

We know that the opposite angles of a cyclic quadrilateral are supplementary.

∠A + ∠C = 180°

4x + 20° + 4y = 180°

4x + 4y – 160° = 0 … (1)

And ∠B + ∠D = 180°

3x – 5 + 7y + 5 = 180°

3x + 7y - 180° = 0… (2)

By elimination method,

Step 1: Multiply equation (1) by 3 and equation (2) by 4 to make the coefficients of x equal.

Then, we get the equations as:

12x + 12y = 480 … (3)

12x + 16y = 540 … (4)

Step 2: Subtract equation (4) from equation (3),

(12x – 12x) + (16y - 12y) = 540 – 480

⇒ 4y = 60

y = 15

Step 3: Substitute value of y in (1),

4x + 4(15) – 160 = 0

⇒ x = 25

Hence, the angles of ABCD are

∠A = 120°, ∠B = 70°,

∠C = 60° and ∠D = 110°.

42. Let us suppose that the digit at unit place be x

Suppose the digit at tens place be y.

Thus, the number is 10y + x.

According to question it is given that the number is 4 times the sum of the two digits.

Therefore, we have

10y + x = 4(x + y)

⇒ 10y + x = 4x + 4y

⇒ 4x + 4y - 10y - x = 0

⇒ 3x - 6y = 0

⇒ 3(x - 2y) = 0

⇒ x - 2y = 0

After interchanging the digits, the number becomes 10x + y.

Again according to question If 18 is added to the number, the digits are reversed.

Thus, we have

(10y + x) + 18 = 10x + y

⇒ 10x + y - 10y - x = 18

⇒ 9x - 9y = 18

⇒ 9(x - y) = 18

⇒ $x - y = \frac{18}{9}$

⇒ x - y = 2

Therefore, we have the following systems of equations

x - 2y = 0 ....(1)

x - y = 2...............(2)

Here x and y are unknowns. Now let us solve the above systems of equations for x and y.

Subtracting the equation (1) from the (2), we get

(x - y) - (x - 2y) = 2 - 0

⇒ x - y - x + 2y = 2

⇒ y = 2

Now, substitute the value of y in equation (1), we get

x - 2 × 2 = 0

⇒ x - 4 = 0

⇒ x = 4

Therefore the number is 10 × 2 + 4 = 24

Thus the number is 24

43. $\frac{x+1}{2} + \frac{y-1}{3} = 9$

or, 3(x + 1) + 2(y -1) = 54

or, 3x + 3 + 2 y - 2 = 54

or, 3x + 2y +1 = 54

or, 3x + 2y = 53..........(i)

and $\frac{x-1}{3} + \frac{y+1}{2} = 8$

or, 2(x -1) + 3 (y + 1) = 48

or, 2x - 2 + 3y + 3 = 48

or, 2x + 3y +1 = 48

or, 2x +3y = 47........(ii)

Multiply eqn.(i) by 3, multiply eqn.(ii) by 2 and subtracting both eqn

Multiply eqn. (i) by 3, $9x + 6y = 159$
Multiply eqn. (ii) by 2, $4x + 6y = 94$

$$\begin{array}{r} 9x + 6y = 159 \\ 4x + 6y = 94 \\ - \quad - \quad - \\ \hline 5x = 65 \end{array}$$

On subtracting

$$\begin{array}{r} 9x + 6y = 159 \\ 4x + 6y = 94 \\ - \quad - \quad - \\ \hline 5x = 65 \end{array}$$

$\therefore x = \frac{65}{5} = 13$

Substitute the value of x in eqn. (ii),

2(13) + 3y = 47

or, 26 + 3y = 47

3y = 47-26 = 21

$\therefore y = \frac{21}{3} = 7$

Hence x = 13, y = 7.

44. Let the two numbers be x and y (x > y) then, according to the question,
the pair of linear equations formed is:

x - y = 26.........(1)

x = 3y.............(2)

Substitute the value of x from equation (2) in equation (1), we get

3y - y = 26

$\Rightarrow \quad 2y = 26$

$\Rightarrow \quad y = \frac{26}{2}$

$\Rightarrow \quad y = 13$

Substituting this value of y in equation (2), we get

x = 3(13) = 39

Hence, the required numbers are 39 and 13.

verification: Substituting x = 39 and y = 13, we find that both
the equation (1) and (2) are satisfied as shown below:

x - y = 39 - 13 = 26

3y = 3(13) = 39 = x.

This verifies the solution.

45. Let the one's digit be 'a' and ten's digit be 'b'.

Given, two digit number is 4 times the sum of its digits and twice the product of the digits.

$\Rightarrow$ 10b + a = 4(a + b)

$\Rightarrow$ a = 2b

Also, 10b + a = 2ab

Substituting value of a.

$\Rightarrow$ 10b + 2b = 2 $\times$ 2b $\times$ b

$\Rightarrow$ b = 3

Thus, a = 6

Thus, the number is 36.

Section B

46. x + y + 2 = 15

x + y = 13 ...(i)

Area of bedroom + Area of kitchen = 95

5 $\times$ x + 5 $\times$ x + 5 $\times$ y = 95

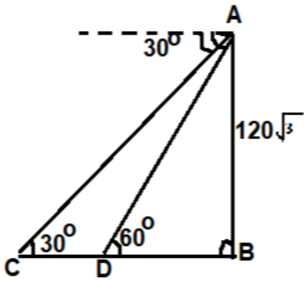
2x + y = 19 ...(ii)

In $\triangle$ABD

tan 60^o = $\frac{120\sqrt{3}}{BD}$

BD = $\frac{120\sqrt{3}}{\sqrt{3}}$

BD = 120 m



In $\triangle ABC$

$\tan 30^o = \frac{AB}{BC}$

$\frac{1}{\sqrt{3}} = \frac{120\sqrt{3}}{BC}$

BC = 360 m

$\therefore$ CD = BC - BD

= 360 - 120

= 240 m

47. Length of outer boundary

= 12 + 15 + 12 + 15

= 54 m

48.
$$\begin{array}{r} x + y = 13 \\ 2x + y = 19 \\ - \quad - \quad\quad - \\ \hline -x \quad\quad = -6 \end{array}$$

x = 6

Area of bedroom 1 = $5 \times x$

= 5×6 = 30 m^2

49. Area of living room = $(5 \times 2) + (9 \times 7)$

= 10 + 63

= 73 m^2

50. $S = a + bt^2$

At t = 1 sec

180 = a + b ...(i)

At t = 2 sec

132 = a + 4b ...(ii)

from (i) and (ii)

180 - 132 = -3b

48 = -3b

b = -16

Put b = -16, in equation (i)

180 = a + (-16)

a = 196

51. At t = 0

s = a + b(0)

s = a

s = 196

i.e., The height of Tower of Pisa = 196 feet

52. $s = a + bt^2$

$0 = 196 - 16t^2$

$-196 = -16t^2$

$196 \div 16 = t$

$t = \frac{14}{4}$

t = 3.5 sec

53. s = a + bt^2

s = 196 + (-16) $(2)^2$

s = 196 - 64

s = 132 feet

Section C

54. Let the digits of number be x and y

$\therefore$ number = 10x + y

According to the question,

10x + y = 8(x+y) - 5

10x +y = 8x + 8y -5

10x - 8x + y - 8y + 5 = 0

or, 2x - 7y + 5 = 0 ....(i)

also 16(x - y) + 3 = 10x + y

or, 16x - 16y + 3 = 10x + y

or, 16x - 16y + 3 - 10x -y = 0

$\Rightarrow$ 6x - 17y + 3 = 0 ....(ii)

On comparing the equation with ax + by + c = 0 we get

$a_1 = 2, b_1 = -7, c_1 = 5$

$a_2 = 6, b_2 = -17, c_2 = 3$

$\frac{x}{b_2c_1 - b_1c_2} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{c_1b_2 - a_2b_1}$

$\frac{x}{(-17)(5)-(-7)(3)} = \frac{y}{(5)(6)-(3)(2)}$

$= \frac{1}{(5)(-17)-(6)(-7)}$

or, $\frac{x}{85-21} = \frac{y}{30-6} = \frac{1}{-34+42}$

or, $\frac{x}{64} = \frac{y}{24} = \frac{1}{8}$

or, $\frac{x}{8} = \frac{y}{3} = 1$

Hence, x=8, y=3

So required number = 10 $\times$ 8 + 3 = 83.

55. Let father's age (in years) be x and that of son's be y.

$\Rightarrow x + 2y = 70$ (by first condition)

$2x + y = 95$ (by second condition)

This system of equations may be written as

$x + 2y - 70 = 0$

$2x + y - 95 = 0$

By cross-multiplication, we get

$\frac{x}{2\times -95-(-70)} = \frac{-y}{1\times -95 - 2\times -70} = \frac{1}{1\times 1 - 2\times 2}$

$\Rightarrow \frac{x}{-190+70} = \frac{-y}{-95+140} = \frac{1}{-3}$

$\Rightarrow \frac{x}{-120} = \frac{y}{-45} = \frac{1}{-3} \Rightarrow x = \frac{-120}{-3} = 40$ and $y = \frac{-45}{-3} = 15$

$\therefore$ father's age is 40 years and the son's age is 15 years.

56. Let the numerator and the denominator be x & y respectively. Hence, the fraction is $\frac{x}{y}$.

Now, according to the question, we have

$\frac{x-2}{y+3} = \frac{1}{4}$

$\Rightarrow 4(x-2) = y + 3$

$\Rightarrow 4x - 8 = y + 3$

$\Rightarrow 4x - y = 3 + 8$

$\Rightarrow 4x - y = 11 \ldots (i)$

Also according to the question, $\frac{x+6}{3y} = \frac{2}{3}$

$\Rightarrow \frac{3(x+6)}{3y} = 2$

$\Rightarrow x + 6 = 2y$

$\Rightarrow x - 2y = -6$ ...(ii)

Multiplying equation (i) by 2 & then subtracting equation (ii) from it, we get

$\Rightarrow$ 8x - x = 22 + 6

$\Rightarrow$ 7x = 28

$\Rightarrow x = \frac{28}{7} = 4$

Putting x = 4 in equation (ii), we get

$\Rightarrow$4 - 2y = -6

$\Rightarrow$ -2y = -6 - 4

$\Rightarrow$ -2y = -10

$\Rightarrow y = \frac{-10}{-2} = 5$

Hence, the fraction is $\frac{4}{5}$.

57. The given system of equations is

$\frac{x}{7} + \frac{y}{3} = 5$ ....(i)

$\frac{x}{2} - \frac{y}{9} = 6$ ....(ii)

From (i), we get

3x + 7y = 5(21)

$\Rightarrow$ 3x + 7y = 105

$\Rightarrow$ 3x = 105 - 7y

$x = \frac{105-7y}{3}$ .... (iii)

From (ii), we get

$\frac{9x-2y}{18} = 6$

$\Rightarrow$ 9x - 2y = 18(6)

$\Rightarrow$ 9x - 2y = 108 ...(iv)

Substituting (iii) in (iv), we get

$9\left(\frac{105-7y}{3}\right) - 2y = 108$

$\Rightarrow \frac{945-63y}{3} - 2y = 108$

$\Rightarrow 945 - 63y - 6y = 108 \times 3$

$\Rightarrow 945 - 69y = 324$

$\Rightarrow 945 - 324 = 69y$

$\Rightarrow 69y = 621$

$\Rightarrow y = \frac{621}{69} = 9$

Putting y = 9 in (iii), we get

$x = \frac{105-7\times9}{3}$

$= \frac{105-63}{3}$

$\Rightarrow x = \frac{42}{3}$

$\therefore x = 14$

Hence, the solution of the given system of equations is x = 14, y = 9.

58. Suppose the numerator and denominator of the fraction be x and y respectively.

Then the fraction is $\frac{x}{y}$.

If 1 is added to the numerator and 1 is subtracted from the denominator, the fraction becomes 1.

Thus, we have $\frac{x+1}{y-1} = 1$

$\Rightarrow (x+1) = (y-1)$

$\Rightarrow x + 1 - y + 1 = 0$

$\Rightarrow x - y + 2 = 0$

If 1 is added to the denominator, the fraction becomes $\frac{1}{2}$.

Thus, we have $\frac{x}{y+1} = \frac{1}{2}$

$\Rightarrow 2x = (y+1)$

$\Rightarrow 2x - y - 1 = 0$

We have two equations

$x - y + 2 = 0$

$2x - y - 1 = 0$

By using cross-multiplication, we have

$\frac{x}{(-1)\times(-1)-(-1)\times2} = \frac{-y}{1\times(-1)-2\times2} = \frac{1}{1\times(-1)-2\times(-1)}$

$\Rightarrow \frac{x}{1+2} = \frac{-y}{-1-4} = \frac{1}{-1+2}$
$\Rightarrow \frac{x}{3} = \frac{-y}{-5} = \frac{1}{1}$
$\Rightarrow \frac{x}{3} = \frac{y}{5} = 1$
So, $x = 3 \; and \; y = 5$.
The fraction is $\frac{3}{5}$.

59. Let the numerator and the denominator of the fraction be x and y respectively.
Hence, the fraction is $\frac{x}{y}$
Given, the numerator of the fraction is 4 less than the denominator.
So, x = y - 4
$\Rightarrow$ x - y = -4........... (i)
Also given, If the numerator is decreased by 2 and the denominator is increased by 1, then the denominator becomes 8 times of the numerator.
So, $y + 1 = 8(x - 2)$
$\Rightarrow y + 1 = 8x - 16$
$\Rightarrow 8x - y = 1 + 16$
$\Rightarrow 8x - y = 17$ .... (ii)
So, we have formed two linear equations in x & y as following:-
x - y = -4
8x - y = 17
Here x and y are unknowns.
We have to solve the above equations for x and y.
Now subtracting equation (ii) from equation (i), we get:
(x - y) - (8x - y) = -4 - 17
$\Rightarrow$ x - y - 8x + y = -21
$\Rightarrow$ -7x = -21
$\Rightarrow x = \frac{-21}{-7}$
$\Rightarrow$ x = 3
Substituting x = 3 in equation (i),
3 - y = -4
$\Rightarrow$ -y = -3 - 4
$\Rightarrow$ -y = -7
$\Rightarrow$ y = 7
So, we get x = 3 & y = 7
Hence the fraction is $\frac{x}{y} = \frac{3}{7}$

60. Suppose the numerator of the fraction be x
Denominator of the fraction be y
$\therefore$ the fraction is $\frac{x}{y}$
According to the question,
The sum of the numerator and denominator of the fraction is 12.
$\Rightarrow x + y = 12$
$\Rightarrow x + y - 12 = 0$
If the denominator is increased by 3, the fraction becomes $\frac{1}{2}$.
$\Rightarrow \frac{x}{y+3} = \frac{1}{2}$
$\Rightarrow 2x = (y + 3)$
$\Rightarrow 2x - y - 3 = 0$
So, we have two equations
$x + y - 12 = 0$
$2x - y - 3 = 0$
Here x and y are unknowns.
We have to solve the above equations for x and y.
By using cross-multiplication,
$\Rightarrow \frac{x}{(1)\times(-3)-(-1)\times-12} = \frac{-y}{1\times(-3)-2\times-12} = \frac{1}{1\times(-1)-2\times(1)}$

$\Rightarrow \frac{x}{-3-12} = \frac{-y}{-3+24} = \frac{1}{-1-2}$
$\Rightarrow \frac{x}{-15} = \frac{-y}{21} = \frac{1}{-3}$
$\Rightarrow \frac{x}{15} = \frac{y}{21} = \frac{1}{3}$
$\Rightarrow x = \frac{15}{3},\ y = \frac{21}{3}$
$\Rightarrow x = 5, y = 7$
The fraction is $\frac{5}{7}$

61. The given systems of equations is
$x + 2y = \frac{3}{2}$ ...(i)
$2x + y = \frac{3}{2}$ ...(ii)
Multiplying (i) by 1 and (ii) by 2, we get
$x + 2y = \frac{3}{2}$ ...(iii)
4x + 2y = 3 ...(iv)
Subtracting (iii) from (iv), we get
$4x - x + 2y - 2y = 3 - \frac{3}{2}$
$\Rightarrow 3x = \frac{6-3}{2}$
$\Rightarrow 3x = \frac{3}{2}$
$\Rightarrow x = \frac{3}{2\times 3}$
$\Rightarrow x = \frac{1}{2}$
Putting $x = \frac{1}{2}$, in equation (iv),we get
$4 \times \frac{1}{2} + 2y = 3$
2 + 2y = 3
2y = 3 - 2
$\Rightarrow y = \frac{1}{2}$
Hence, Solution of the given of equation is $x = \frac{1}{2}, y = \frac{1}{2}$.

62. Let the digits at the units and at the tens place of the given number be x and y respectively.
Thus, the number is 10y + x.
Given, the sum of the digits of the number is 5.
Hence, x + y = 5...........(1)
After interchanging the digits, the number becomes 10x + y.
Also given, the number obtained by interchanging the digits is greater by 9 from the original number.
Hence, 10x + y = (10y + x) +9
⇒ 10x + y - 10y - x= 9
⇒ 9x - 9y =9
⇒ 9(x - y) = 9
⇒ x - y = 1.................(2)
Adding equation (1) & equation (2), we get ;
(x + y) + (x - y) = 5 + 1
⇒ x + y + x - y = 5 +1
⇒ 2x = 6
$\Rightarrow x = \frac{6}{2}$
⇒ x = 3
Substituting the value of x in the equation (1), we get
3 + y = 5
⇒ y = 5 - 3
⇒ y = 2
Hence, the number is 10y + x = 10 × 2 + 3 = 23

63. $\frac{3x}{2} - \frac{5y}{3} = -2; \frac{x}{3} + \frac{y}{2} = \frac{13}{6}$
The given system of linear equation is
$\frac{3x}{2} - \frac{5y}{3} = -2 \quad \ldots\ldots\ldots\ldots(1)$
$\frac{x}{3} + \frac{y}{2} = \frac{13}{6}$ .... (2)
$\Rightarrow \quad 9x - 10y = -12$ .... (3)

2x + 3y = 13 ...... (4)

From equation (3)

9x - 10y = -12

9x = 10y - 12

$x = \frac{10y-12}{9}$

Substituting the value of y in equation (4), we get

$2\left(\frac{10y-12}{9}\right) + 3y = 13$

$20y - 24 + 27y = 117$

$47y = 117 + 24$

$y = \frac{141}{47}$

y = 3

Substituting the value of y in equation (4), we get

$2x + 3 \times 3 = 13$

$2x + 9 = 13$

$2x = 13 - 9$

$x = \frac{4}{2} = 2$

Therefore, the solution is

x = 2, y = 3

Verification, Substituting x = 2 and y = 3, we find that both the equations (1) and (2) are satisfied as shown below:

$\frac{3}{2}x - \frac{5y}{3} = \frac{3}{2}(2) - \frac{5}{3}(3) = 3 - 5 = -2$

$\frac{x}{3} + \frac{y}{2} = \frac{2}{3} + \frac{3}{2} = \frac{13}{6}$

This verifies the solution.

64. Let the numerator and the denominator of the fraction be x and y respectively.

Then the fraction is $\frac{x}{y}$.

Given, The sum of the numerator and the denominator of the fraction is 3 less than the twice of the denominator.

Thus, we have

$x + y = 2y - 3$

$\Rightarrow x + y - 2y + 3 = 0$

$\Rightarrow x - y + 3 = 0$ ....(1)

Also given, If the numerator and the denominator both are decreased by 1, the numerator becomes half the denominator. Thus, we have

$x - 1 = \frac{1}{2}(y - 1)$

$\Rightarrow 2(x - 1) = (y - 1)$

⇒ 2x - 2 = (y - 1)

⇒ 2x - y - 1 = 0.......................(2)

So, we have formed two linear equations in x & y as following:-

x - y + 3 = 0

2x - y - 1 = 0

Here x and y are unknowns.

We have to solve the above equations for x and y.

By using cross-multiplication method , we have

$\frac{x}{(-1)\times(-1)-(-1)\times 3} = \frac{-y}{1\times(-1)-2\times 3} = \frac{1}{1\times(-1)-2\times(-1)}$

$\Rightarrow \frac{x}{1+3} = \frac{-y}{-1-6} = \frac{1}{-1+2}$

$\Rightarrow \frac{x}{4} = \frac{-y}{-7} = \frac{1}{1}$

$\Rightarrow \frac{x}{4} = \frac{y}{7} = 1$

Using Part I & III , we get x = 4

& From part II & III, we get y = 7

⇒ x = 4, y = 7

Hence, The fraction is $\frac{x}{y}$ = $\frac{4}{7}$

65. Let the ten's and unit's digits of the required number be x and y respectively.

Then, xy = 14.

Required number = (10x + y).

Number obtained on reversing its digits = (10y + x).

$\therefore$ (10x + y) + 45 = (10y + x)

$\Rightarrow$ 9(y - x) = 45 $\Rightarrow$ y - x = 5 ...(i)

Now, $(y + x)^2 - (y - x)^2 = 4xy$

$\Rightarrow$ 9(y - x) = $\sqrt{(y-x)^2+4xy} = \sqrt{25+4\times 14} = \sqrt{81}$

$\Rightarrow$ y + x = 9 ...(ii) [$\because$ digits are never negative]

On adding (i) and (ii), we get

2y = 14 $\Rightarrow$ y = 7

Putting y = 7 in (ii), we get

7 + x = 9 $\Rightarrow$ x = 9 - 7 = 2

$\therefore$ x = 2 and y = 7

Hence, the required number is 27

66. Suppose that the digits at units and tens place of the given number be x and y respectively.

Thus, the number is $10y + x$.

The product of the two digits of the number is 20.

Thus, we have $xy = 20$

After interchanging the digits, the number becomes $10x + y$

If 9 is added to the number, the digits interchange their places.

Thus, we have

$(10y + x) + 9 = 10x + y$

$\Rightarrow 10y + x + 9 = 10x + y$

$\Rightarrow 10x + y - 10y - x = 9$

$9x - 9y = 9$

$\Rightarrow 9(x - y) = 9$

$\Rightarrow x - y = \frac{9}{9}$

$\Rightarrow x - y = 1$

So, we have the systems of equations

$xy = 20$ ....(i)

$x - y = 1$ ....(ii)

Here x and y are unknowns.

We have to solve the above systems of equations for x and y.

Substituting $x = 1 + y$ from the second equation to the first equation, we get $(1 + y)y = 20$

$\Rightarrow y + y^2 = 20$

$\Rightarrow y^2 + y - 20 = 0$

$\Rightarrow y^2 + 5y - 4y - 20 = 0$

$\Rightarrow y(y + 5) - 4(y + 5) = 0$

$\Rightarrow (y + 5)(y - 4) = 0$

$\Rightarrow y = -5 \; or \; y = 4$

Substituting the value of y in the second equation, we have

x	-4	5
y	-5	4

Note that in the first pair of solution the values of x and y are both negative. But the digits of the number can't be negative. So, we must remove this pair.

Hence, the number is 10 $\times$ 4 + 5 = 45

67. 2x - y + 3 = 0

3x - 5y + 1 = 0

$2x - y = -3 \ldots\ldots\ldots (1)$

$3x - 5y = -1 \ldots\ldots (2)$

Multiplying eqn. (i) by 3 and (ii) by 2, and subtracting (ii)

$6x\ -3\ y\ =-\ 9$

$6x\ -\ 10y\ =\ -2$

$(-)\ \ (+)\ \ \ \ \ \ (+)$

$7y\ =\ 11$

$\Rightarrow y\ =\ \frac{11}{7}$

Substituting the value of y in ,eqn. (i)

2x - y = 3

2 x = y + 3

$2x\ =\ 3\ +\frac{11}{7}$

$2x\ =\ \frac{21\ +\ 11}{7}\ \Rightarrow\ 2x\ =\ 32/7$

$\Rightarrow x\ =\ \frac{32}{14}$

$or\ x\ =\ \frac{16}{7}$

- 4 - y + 3 = 0

or, - y - 1 = 0

y = -1

Hence, x=-2

68. Suppose the numerator and denominator of the fraction be x and y respectively.

Then the fraction is $\frac{x}{y}$.

If 1 is subtracted from both numerator and the denominator, the fraction becomes $\frac{1}{3}$.

Thus, we have $\frac{x-1}{y-1}=\frac{1}{3}$

$\Rightarrow 3(x-1)=(y-1)$

$\Rightarrow 3x-3=y-1$

$\Rightarrow 3x-y-2=0$

If 1 is added to both numerator and the denominator, the fraction becomes $\frac{1}{2}$.

Thus, we have $\frac{x+1}{y+1}=\frac{1}{2}$

$\Rightarrow 2(x+1)=(y+1)$

$\Rightarrow 2x+2=y+1$

$\Rightarrow 2x-y+1=0$

We have two equations

$3x-y-2=0$

$2x-y+1=0$

By using cross-multiplication, we have

$\frac{x}{(-1)\times 1-(-1)\times(-2)}=\frac{-y}{3\times 1-2\times(-2)}=\frac{1}{3\times(-1)-2\times(-1)}$

$\Rightarrow \frac{x}{-1-2}=\frac{-y}{3+4}=\frac{1}{-3+2}$

$\Rightarrow \frac{x}{-3}=\frac{-y}{7}=\frac{1}{-1}$

$\Rightarrow \frac{x}{3}=\frac{y}{7}=1$

$\Rightarrow$ x = 3, y = 7

The fraction is $\frac{3}{7}$.

69. The given system of equation is

2x - 7y = 1 .... (1)

4x + 3 y = 15 ...... (2)

From equation (1),

7y = 2x - 1

$\Rightarrow\quad y=\frac{2x-1}{7}$ .... (3)

Substitute this value of y in equation (2), we get

$4x+3\left(\frac{2x-1}{7}\right)=15$

$\Rightarrow\quad 28x+6x-3=105$

$\Rightarrow\quad 34x-3=105$

$\Rightarrow\quad 34x=105+3$

$\Rightarrow \quad x=\frac{108}{34}$

$\Rightarrow \quad x=\frac{54}{17}$

Substituting this value of x in equation (3), we get

$\mathrm{y}=\frac{2\left(\frac{54}{17}\right)-1}{7}=\frac{\frac{108}{17}-1}{7}=\frac{108-17}{119}$

$=\frac{91}{119}=\frac{13}{17}$

Therefore the solution is $x=\frac{54}{17}, y=\frac{13}{17}$.

Verification, Substituting $x=\frac{54}{17}$ and, $y=\frac{13}{17}$ we find that both

the equation (1) and (2) are satisfied as shown below:

$2x-7y=2\left(\frac{54}{17}\right)-7\left(\frac{13}{17}\right)=\frac{108}{17}-\frac{21}{17}$

$=\frac{108-91}{17}=\frac{17}{17}=1$

$4x+3y=4\left(\frac{54}{17}\right)+3\left(\frac{13}{17}\right)=\frac{216}{17}+\frac{39}{17}$

$=\frac{216+39}{17}=\frac{255}{17}=15$ This verifies the solution.

70. Let the larger and smaller of two supplementary angles be x^{o} and y^{o} respectively.

Then, according to the question.

The pair of linear equations formed is

$x^{\circ}=y^{\circ}+18^{\circ}$ ...(i)

$x^{0}+y^{0}=180^{\circ}$ ...(ii)

∵ The two angles and supplementary

Substitute the value of x^{o} from equation (1) in equation (2), we get

$y^{\circ}+18^{\circ}+y^{\circ}=180^{\circ}$

$\Rightarrow \quad 2y^{0}+18^{\circ}=180^{\circ}$

$\Rightarrow \quad 2y^{\circ}=180^{\circ}-18^{\circ}$

$\Rightarrow \quad 2y=162^{\circ}$

$\Rightarrow \quad y^{\circ}=\frac{162^{\circ}}{2}=81^{\circ}$

Substituting this value of y^{o} in equation (1), we get

$x^{0}=81^{\circ}+18^{\circ}=99^{\circ}$

Hence, the larger and smaller of the two supplementary angles are 99^{o} and 81^{o} respectively.

Verification, Substituting x^{o} = 99^{o} and y^{o} = 81^{o}, we find that both the equations (1) and (2) are satisfied as shown below:

$y^{\circ}+18^{\circ}=81^{\circ}+18=99^{\circ}=x^{\circ}$

$x^{0}+y^{\circ}=99^{\circ}+81^{\circ}=180^{\circ}$

This verifies the solution.

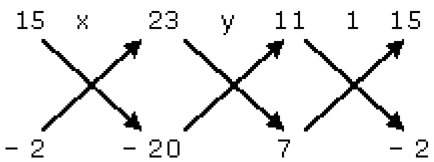
71. The given system of equations is

11x + 15y + 23 = 0 ....(1)

7x - 2y - 20 = 0 ....(2)

To solve the equations (1) and (2) by cross multiplication method,

we draw the diagram below:



Then,

$\Rightarrow \frac{x}{(15)(-20)-(-2)(23)}=\frac{y}{(23)(7)-(-20)(11)}=\frac{1}{(11)(-2)-(7)(15)}$

$\Rightarrow \frac{x}{-300+46}=\frac{y}{161+220}=\frac{1}{-22-105}$

$\Rightarrow \frac{x}{-254}=\frac{y}{381}=\frac{1}{-127}$

$\Rightarrow x=\frac{-254}{-127}=2$ and $y=\frac{381}{-127}=-3$

Hence, the required solution of the given pair of equations is

x = 2, y = -3

Verification: Substituting x = 2, y = -3,

We find that both the equations (1) and (2) are satisfied as shown below:

$11x + 15y + 23 = 11(2) + 15(-3) + 23$
$= 22 - 45 + 23 = 0$
$7x - 2y - 20 = 7(2) - 2(-3) - 20$
$= 14 + 6 - 20 = 0$
Hence, the solution we have got is correct.

72. Let x and y be the numerator and the denominator of the fraction.
According to the question,
$x + y = 8$...(1)
Also, we have,
$\frac{x+3}{y+3} = \frac{3}{4}$
$\Rightarrow 4(x + 3) = 3(y + 3)$
$\Rightarrow 4x + 12 = 3y + 9$
$\Rightarrow 4x - 3y = - 3$...(2)
Multiplying eq (1) by 4, we get,
$4x + 4y = 32$ ...(3)
Subtracting eq (2) from eq (3), we get,
$7y = 35$
Thus, $y = 5$
Substitute the value of y in eq (1), we get, $x = 3$.
Thus, we have $x = 3$ and $y = 5$.
Hence, the required fraction is $\frac{3}{5}$.

73. Suppose the digits at units and tens place of the given number be x and y respectively.
$\therefore$ the number is $10y + x$.
The number is 4 more than 6 times the sum of the two digits.
$\therefore 10y + x = 6(x + y) + 4$
$\Rightarrow 10y + x = 6x + 6y + 4$
$\Rightarrow 6x + 6y - 10y - x = -4$
$\Rightarrow 5x - 4y = -4$ ....(i)
After interchanging the digits, the number becomes $10x + y$.
If 18 is subtracted from the number, the digits are reversed. Thus, we have
$(10y + x) - 18 = 10x + y$
$\Rightarrow 10x + y - 10y - x = -18$
$\Rightarrow 9x - 9y = -18$
$\Rightarrow 9(x - y) = -18$
$\Rightarrow x - y = \frac{-18}{9}$
$\Rightarrow x - y = -2$ ....(ii)
So, we have the systems of equations
$5x - 4y = -4,$
$x - y = -2$
Here x and y are unknowns. We have to solve the above systems of equations for x and y.
Multiplying the second equation by 5 and then subtracting from the first, we have
$(5x - 4y) - (5x - 5y) = -4 - (-2 \times 5)$
$\Rightarrow 5x - 4y - 5x + 5y = -4 + 10$
$\Rightarrow y = 6$
Substituting the value of y in the second equation, we have
$x - 6 = -2$
$\Rightarrow x = 6 - 2$
$\Rightarrow x = 4$
Hence, the number is $10 \times 6 + 4 = 64$

74. Let the numerator be x and denominator be y
if 2 is added to both numerator and denominator, the fraction becomes $\frac{9}{11}$
$\frac{x+2}{y+2} = \frac{9}{11}$
$11(x + 2) = 9(y + 2)$

= 11x + 22 = 9y + 18

or, 11x + 22 - 9y - 18 = 0

or, 11 x - 9 y + 4 = 0 ....(i)

If 3 is added to both numerator and denominator the fraction becomes $\frac{5}{6}$

and $\frac{x+3}{y+3} = \frac{5}{6}$

6(x+3) = 5(y+3)

6x +18 = 5y + 15

or, 6x + 18 - 5y - 15 = 0

or, 6x -5y + 3 = 0 ...(ii)

On comparing with ax + by + c = 0

we get $a_1 = 11, b_1 = -9, c_1 = 4$

$a_2 = 6, b_2 = -5$ and $c_2 = 3$

Now, $\frac{x}{b_1c_2 - b_2c_1} = \frac{y}{c_1a_2 - c_2a_1} = \frac{1}{a_1b_2 - a_2b_1}$

$\frac{x}{(-9)(3)-(-5)(4)} = \frac{y}{(4)(6)-(3)(11)} = \frac{1}{(11)(-5)-(6)-(9)}$

$\frac{x}{-27+20} = \frac{y}{24-33} = \frac{1}{-55+54}$

$\Rightarrow \quad \frac{x}{-7} = \frac{y}{-9} = \frac{1}{-1}$

$\Rightarrow \quad \frac{x}{-7} = -1$

or, x = 7

Hence, x=7, y=9

$\therefore$ Fraction =$\frac{7}{9}$

75. Let the digits at units and tens place of the given number be x and y respectively.

Thus, the number is $10y + x$.

The sum of the digits of the number is 13.

Thus, we have $x + y = 13$

According to the question,

After interchanging the digits, the number becomes $10x + y$.

The difference between the number obtained by interchanging the digits and the original number is 45.

Thus, we have $(10x + y) - (10y + x) = 45$

$\Rightarrow 10x + y - 10y - x = 45$

$\Rightarrow 9x - 9y = 45$

$\Rightarrow 9(x - y) = 45$

$\Rightarrow x - y = 5$

So, we have two equations

$x + y = 13$

$x - y = 5$

Here x and y are unknowns.

We have to solve the above equations for x and y.

Adding the two equations, we have

$(x + y) + (x - y) = 13 + 5$

$\Rightarrow x + y + x - y = 18$

$\Rightarrow 2x = 18$

$\Rightarrow x = 9$

Substituting the value of x in the first equation,

$\Rightarrow 9 + y = 13$

$\Rightarrow y = 13 - 9$

$\Rightarrow y = 4$

Hence, the number is 10 $\times$ 4 + 9 = 49