Solution

POLYNOMIALS WS 2

Class 10 - Mathematics

Section A

1. (a) - (iv), (b) - (iii), (c) - (i), (d) - (ii)
2. (a) - (iii), (b) - (i), (c) - (iv), (d) - (ii)
3. (a) - (iii), (b) - (i), (c) - (iv), (d) - (ii)
4. (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)
5. (a) - (iii), (b) - (iv), (c) - (i), (d) - (ii)

Section B

6. $7y^2 - \frac{11}{3}y - \frac{2}{3}$

$= \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$

$= \frac{1}{3}(3y - 2)(7y + 1)$

$\Rightarrow y = \frac{2}{3}, \frac{-1}{7}$ are zeroes of the polynomial.

If Given polynimoal is $7y^2 - \frac{11}{3}y - \frac{2}{3}$

Then a = 7 , b = $-\frac{11}{3}$ and c = $-\frac{2}{3}$

Sum of zeroes = $\frac{2}{3} + \frac{-1}{7} = \frac{14-3}{21} = \frac{11}{21}$ .... (i)

Also, $\frac{-b}{a} = \frac{-\left(\frac{-11}{3}\right)}{7} = \frac{11}{21}$ .... (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

Now, product of zeroes = $\frac{2}{3} \times \frac{-1}{7} = \frac{-2}{21}$ .... (iii)

Also, $\frac{c}{a} = \frac{\frac{-2}{3}}{7} = \frac{-2}{21}$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

7. Given polynomial is $f(x) = x^2 - 2x + 3$

Compare with $ax^2 + bx + c$, we get

a = 1 , b = -2 and c = 3

Sum of the zeroes $= \alpha + \beta = -\frac{b}{a} = -\frac{-2}{1} = 2$

Product of the zeroes $= \alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

Sum of the zeroes of new polynomial = $(\alpha + 2) + (\beta + 2)$

$= \alpha + \beta + 4$

$= 2 + 4 = 6$

Product of the zeroes of new polynomial = $(\alpha + 2)(\beta + 2)$

$= \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha + \beta) + 4$

$= 3 + 2(2) + 4$

$= 11$

So, quadratic polynomial is: x^2 - (sum of the zeroes)x + (product of the zeroes)

$= x^2 - 6x + 11$

Hence, the required quadratic polynomial is $f(x) = (x^2 - 6x + 11)$

8. Sum of zeroes = $\frac{-q}{p} < 0$ [as zeroes are negative means sum of zeroes is negative]

   So that $\frac{q}{p} > 0$

   $\Rightarrow\ q > 0,\ p > 0\ or\ q < 0,\ \ p < 0$…….. (i)

   Product of zeros = $\frac{r}{p} > 0$ [as zeroes are negative means product of zeroes is positive]

   $\Rightarrow\ r > 0,\ p > 0\ or\ r < 0,\ p < 0$ ………. (ii)

   ∴ From (i) and (ii), p, q and r will have same signs i.e.

   Either $p > 0,\ q > 0,\ \ r > 0$

   Or $p < 0,\ q < 0,\ r < 0$.

9. Let the given polynomial is $p(x) = x^2 + 7x + 7$

   Here, a = 1, b = 7, c = 7

   $\therefore\ \alpha, \beta$ are both zeroes of p(x)

   $\therefore\ \ \alpha + \beta = \frac{-b}{a}$ = -7.........(i)

   $\alpha\beta = \frac{c}{a}$ = 7...........(ii)

   Now,

   $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta = \frac{\beta+\alpha}{\alpha\beta} - 2\alpha\beta$

   $= \frac{-7}{7} - 2 \times 7$

   = -1 - 14

   = -15

   Hence the value of $\frac{1}{\alpha} + \frac{1}{\beta} - 2\alpha\beta$ is - 15.

10. Consider general quadratic polynomial $p(x) = ax^2 + bx + c,\ a \neq 0$

    b = 0 (given)

    Let α, β be the zeroes of p(x)

    ∴ Sum of zeroes = $\alpha + \beta = -\frac{b}{a} = \frac{0}{a} = 0$

    $\Rightarrow \alpha + \beta = 0$

    $\Rightarrow \alpha = -\beta$

    In other words $\beta = -\alpha$

    ∴ The zeroes are α, - α.

    Hence, the zeroes are equal in magnitude but opposite in sign.

11. $\because\ \alpha$ and β are zeroes of given polynomial

    So, $x^2 + 9x + 20 = 0$

    $x^2 + 4x + 5x + 20 = 0$

    $x(x + 4) + 5(x + 4) = 0$

    $(x + 5)(x + 4) = 0$

    x = -5 and x = -4

    ∴ α = -5 and β = -4

    Now, $\alpha + 1 = -4$ and $\beta + 1 = -3$

    So, product of zeroes= (-4) × (-3) = 12

    Sum of zeroes = -7

    Now polynomial = x^2 - (sum of zeroes)x + (product of zeroes)

    Polynomial = $x^2 + 7x + 12$

12. The given quadratic polynomial is:

    $v^2 + 4\sqrt{3}v - 15$

    By factorizing it we have

    $v^2 + 4\sqrt{3}v - 15 = v^2 + 5\sqrt{3}v - \sqrt{3}v - 15$

$= v(v + 5\sqrt{3}) - \sqrt{3}(v + 5\sqrt{3})$

$= (v - \sqrt{3})(v + 5\sqrt{3})$

For zeroes, put the factors equal to zero i.e.,

$(v - \sqrt{3})(v + 5\sqrt{3}) = 0$

$\Rightarrow v = \sqrt{3}, \ -5\sqrt{3}$ are zeroes of the polynomial.

Verification: In the given polynomial a = 1, b = $4\sqrt{3}$ and c = - 15

Now Sum of the zeroes = $\sqrt{3} + (-5\sqrt{3}) = -4\sqrt{3}$

Also sum of zeroes = $\frac{-b}{a}, \frac{-b}{a} = \frac{-4\sqrt{3}}{1} = -4\sqrt{3}$

And product of zeroes = $\sqrt{3} \times -5\sqrt{3}$ = -15

Also, product of zeroes = $\frac{c}{a} = \frac{-15}{1} = -15$

13. Given polynomial is

$f(x) = x^2 - 2x + 3$

Compare with $ax^2 + bx + c$, we get

a = 1 , b = -2 and c = 3

Sum of the zeroes =$\alpha + \beta = -\frac{b}{a} = -\frac{-2}{1} = 2$

Product of the zeroes $= \alpha\beta = \frac{c}{a} = \frac{3}{1} = 3$

i. Sum of the zeroes of new polynomial = $(\alpha + 2) + (\beta + 2)$

$= \alpha + \beta + 4$

$= 2 + 4 = 6$

Product of the zeroes of new polynomial = $(\alpha + 2)(\beta + 2)$

$= \alpha\beta + 2\alpha + 2\beta + 4$

$= \alpha\beta + 2(\alpha + \beta) + 4$

$= 3 + 2(2) + 4$

$= 11$

So, quadratic polynomial is: x^2 - (sum of the zeroes)x + (product of the zeroes)

$= x^2 - 6x + 11$

Hence, the required quadratic polynomial is $f(x) = (x^2 - 6x + 11)$

ii. Sum of the zeroes of new polynomial = $\frac{\alpha-1}{\alpha+1} + \frac{\beta-1}{\beta+1}$

$= \frac{(\alpha-1)(\beta+1) + (\beta-1)(\alpha+1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta+\alpha-\beta-1+\alpha\beta+\beta-\alpha-1}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta-1+\alpha\beta-1}{\alpha\beta+\alpha+\beta+1}$

$= \frac{3-1+3-1}{3+1+2}$

$= \frac{4}{6}$

$= \frac{2}{3}$

Product of the zeroes of new polynomial $= \frac{\alpha-1}{\alpha+1} \times \frac{\beta-1}{\beta+1}$

$= \frac{(\alpha-1)(\beta-1)}{(\alpha+1)(\beta+1)}$

$= \frac{\alpha\beta - \alpha-\beta+1}{\alpha\beta + \alpha+\beta+1}$

$= \frac{\alpha\beta - (\alpha+\beta)+1}{\alpha\beta + (\alpha+\beta)+1}$

$= \frac{3 - 2+1}{3+2+1}$

$= \frac{2}{6} = \frac{1}{3}$

So, the quadratic polynomial is, x^2 - (sum of the zeroes)x + (product of the zeroes)

$= x^2 - \frac{2}{3}x + \frac{1}{3}$

Thus, the required quadratic polynomial is $f(x) = k\left(x^2 - \frac{2}{3}x + \frac{1}{3}\right)$.

14. $x^2 - 6$

Let $p(x) = x^2 - 6$

For zeroes of p(x), p(x) = 0

$\Rightarrow x^2 - 6 = 0 \Rightarrow (x)^2 - (\sqrt{6})^2 = 0$

$\Rightarrow (x - \sqrt{6})(x + \sqrt{6}) = 0$

Using the identity $a^2 - b^2 = (a - b)(a + b)$

$\Rightarrow x - \sqrt{6} = 0$ or $x + \sqrt{6} = 0$

$\Rightarrow x = \sqrt{6}$ or $x = -\sqrt{6} \Rightarrow x = \sqrt{6}, -\sqrt{6}$

So, the zeroes of $x^2 - 6$ are $\sqrt{6}$ and $-\sqrt{6}$

Sum of zeroes

$= (\sqrt{6}) + (-\sqrt{6}) = 0 = \frac{-0}{1} = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^2}$

Product of zeroes

$= (\sqrt{6}) \times (-\sqrt{6}) = -6 = \frac{-6}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence the relation between zeroes and coefficient is verified.

15. Let $p(x) = x^2 - 2x - (7p + 3)$

Since -1 is a zero of p(x). Therefore,

p(-1) = 0

$(-1)^2 - 2(-1) - (7p + 3) = 0$

1 + 2 - 7p - 3 = 0

3 - 7p - 3 = 0

7p = 0

p = 0

Thus, $p(x) = x^2 - 2x - 3$

For finding zeros of p(x), we put,

p(x) = 0

$x^2 - 2x - 3 = 0$

$x^2 - 3x - x - 3 = 0$

x(x-3) + 1(x-3) = 0

(x-3)(x+1) = 0

Put x - 3 = 0 and x + 1 = 0, we get,

Thus, x = 3, -1

Thus, the other zero is 3.

16. Let the given polynomial is $p(x) = 4x^2 + 4x + 1$

Since, α, β are zeroes of p(x),

$\therefore \alpha + \beta$= sum of zeroes = $\frac{-4}{4}$

Also, $\alpha.\beta$= Product of zeroes = $\alpha.\beta = \frac{1}{4}$

Now a quadratic polynomial whose zeroes are 2α and 2β

x^2 - (sum of zeroes)x + Product of zeroes

$= x^2 - (2\alpha + 2\beta)x + 2\alpha \times 2\beta$

$= x^2 - 2(\alpha + \beta)x + 4(\alpha\beta)$

$= x^2 - 2 \times (-1)x + 4 \times \frac{1}{4}$

$= x^2 + 2x + 1$

The quadratic polynomial whose zeroes are 2α and 2β is $x^2 + 2x + 1$

17. Let $p(x) = x^2 - 2x - 8$

By the method of splitting the middle term,

$x^2 - 2x - 8 = x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4) = (x - 4)(x + 2)$

For zeroes of p(x),

$p(x) = 0$

$\Rightarrow (x - 4)(x + 2) = 0$

$\Rightarrow x - 4 = 0$ or $x + 2 = 0$

$\Rightarrow x = 4$ or $x = -2$

$\Rightarrow x = 4, -2$

So, the zeroes of p(x) are 4 and -2.

We observe that, Sum of its zeroes

$= 4 + (-2) = 2$

$= \frac{-(-2)}{1} = \frac{-\text{(Coefficient of } x)}{\text{Coefficient of } x^2}$

Product of its zeroes

$= 4x(-2) = -8 = \frac{-8}{1} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

Hence, relation between zeroes and coefficients is verified.

18. $f(x) = x^2 - 2x - 8$

$= x^2 - 4x + 2x - 8$

$= x(x - 4) + 2(x - 4)$

$= (x + 2)(x - 4)$

$f(x) = 0$ if $x+2 = 0$ or $x-4 = 0$

$x = -2$ or 4

So the zeroes of the polynomials are -2 and 4.

For the Polynomial $f(x)=x^2 - 2x - 8$

a=1,b=-2, c=-8

Sum of the zeroes = $- 2 + 4 = 2 = -\frac{b}{a}$

Product of zeros =$(-2)(4) = -8 = \frac{c}{a}$

Hence, the relationship between the zeros and coefficients is verified.

19. We know that, if x = a is a zero of a polynomial then x - a is a factor of quadratic polynomials.

Since $\frac{-1}{4}$ and 1 are zeros of polynomial.

Therefore $\left(x + \frac{1}{4}\right)(x - 1)$

$= x^2 + \frac{1}{4}x - x - \frac{1}{4}$

$= x^2 + \frac{1}{4}x - \frac{4}{4}x - \frac{1}{4}$

$= x^2 + \frac{1-4}{4}x - \frac{1}{4}$

$= x^2 - \frac{3}{4}x - \frac{1}{4}$

Hence, the family of quadratic polynomials is $f(x) = k\left(x^2 - \frac{3}{4}x - \frac{1}{4}\right)$, where k is any non-zero real number.

20. Let $f(x) = 6x^2 + x - 2$

a = 6, b = 1 and c = -2

And α and β are the zeros of polynomial,

$\alpha + \beta = -\frac{b}{a} = -\frac{1}{6}$

$\alpha\beta = \frac{c}{a} = \frac{-2}{6} = \frac{-1}{3}$

$\therefore \frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{(\alpha+\beta)^2 - 2\alpha\beta}{\alpha\beta}$

$= \frac{\left(-\frac{1}{6}\right)^2 - 2\left(-\frac{1}{3}\right)}{\left(-\frac{1}{3}\right)}$

$= -\frac{\frac{1}{36} + \frac{2}{3}}{\frac{1}{3}}$

$= -\frac{\frac{25}{36}}{\frac{1}{3}}$

$= -\frac{25}{36} \times \frac{3}{1}$

$= -\frac{25}{12}$

21. The given polynomial p(x) = $x^2 + 2\sqrt{2}$ x - 6

$= x^2 + 3\sqrt{2}$ x - $\sqrt{2}$ x - 6

$= x(x + 3\sqrt{2}) - \sqrt{2}(x + 3\sqrt{2})$

$= (x + 3\sqrt{2})(x - \sqrt{2})$

p(x) = 0 if $x+3\sqrt{2} = 0$ or x = $\sqrt{2}$

Zeroes of the polynomials are $\sqrt{2}$ and $-3\sqrt{2}$

For p(x) = $x^2 + 2\sqrt{2}$ x - 6

a = 1, b = $2\sqrt{2}$, c = -6

Sum of the zeroes $\sqrt{2} - 3\sqrt{2} = -2\sqrt{2} = -\frac{2\sqrt{2}}{1} = -\frac{b}{a}$

Product of the zeroes $=\sqrt{2} \times -3\sqrt{2} = \frac{-6}{1} = \frac{c}{a}$

Hence, the relationship is verified.

22. Let the polynomial be $ax^2 + bx + c$.

and its zeroes be α and β.

Then, $\alpha + \beta = \sqrt{2} = -\frac{b}{a}$ and $\alpha\beta = \frac{1}{3} = \frac{c}{a}$

If a = 3, then $b = -3\sqrt{2}$ and c = 1.

So, one quadratic polynomial which fits the given conditions is $3x^2 - 3\sqrt{2}x + 1$.

It is given that $\alpha + \beta = \sqrt{2}$ and $\alpha\beta = \frac{1}{3}$

Now, standard form of quadratic polynomial is given by $x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \sqrt{2}x + \frac{1}{3}$

$= \frac{1}{3}\left(3x^2 - 3\sqrt{2}x + 1\right)$

Hence the required quadratic polynomial is $3x^2 - 3\sqrt{2}x + 1$

23. $y^2 + \frac{3}{2}\sqrt{5}y - 5 = \frac{1}{2}\left(2y^2 + 3\sqrt{5}y - 10\right)$

$= \frac{1}{2}\left(2y^2 + 4\sqrt{5}y - \sqrt{5}y - 10\right)$

$= \frac{1}{2}[2y(y + 2\sqrt{5}) - \sqrt{5}(y + 2\sqrt{5})]$

$= \frac{1}{2}(y + 2\sqrt{5})(2y - \sqrt{5})$

$\Rightarrow \quad y = -2\sqrt{5}, \frac{\sqrt{5}}{2}$ are zeroes of the polynomial.

If given polynomial is $y^2 + \frac{3}{2}\sqrt{5}y$ - 5 then a = 1, b=$\frac{3}{2}\sqrt{5}$ and c = -5

Sum of zeroes = $-2\sqrt{5} + \frac{\sqrt{5}}{2} = \frac{-3\sqrt{5}}{2}$ .... (i)

Also, $\frac{-b}{a} = \frac{-3\sqrt{5}}{2}$ ------ (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

Product of zeroes = $-2\sqrt{5} \times \frac{\sqrt{5}}{2} = -5$ .... (iii)

Also, $\frac{c}{a} = \frac{-5}{1} = -5$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

24. The given polynomial is:

p(y) = $y^2 + \frac{3}{2}\sqrt{5}y - 5$

For zeroes of f(y), put f(y) = 0

$\Rightarrow y^2 + \frac{3}{2}\sqrt{5}y - 5 = 0$

$\Rightarrow 2y^2 + 3 \cdot \sqrt{5}y - 10 = 0$

$\Rightarrow 2y^2 + 4\sqrt{5}y - 1\sqrt{5}y - 10 = 0$

$\Rightarrow 2y(y + 2\sqrt{5}) - \sqrt{5}[y + 2\sqrt{5}] = 0$

$\Rightarrow (y + 2\sqrt{5})(2y - \sqrt{5}) = 0$

Therefore, either $y + 2\sqrt{5} = 0$ or $2y - \sqrt{5} = 0$

$\Rightarrow y = -2\sqrt{5}$ or $y = \frac{\sqrt{5}}{2}$

Now Verification of the relations between α, β and a, b, c

We have, $\alpha = -2\sqrt{5}, \ \beta = \frac{\sqrt{5}}{2}$, a = 1, $b = \frac{3}{2}\sqrt{5}$ and c = -5

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow -2\sqrt{5} + \frac{\sqrt{5}}{2} = \frac{\frac{-3}{2}\sqrt{5}}{1}$

$\Rightarrow \frac{-4\sqrt{5} + \sqrt{5}}{2} = \frac{-3}{2}\sqrt{5}$

$\Rightarrow \frac{-3\sqrt{5}}{2} = \frac{-3}{2}\sqrt{5}$

$\Rightarrow$ LHS = RHS

Hence, verified.

Also we know that $\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow (-2\sqrt{5})\left(\frac{\sqrt{5}}{2}\right) = \frac{-5}{1}$

$\Rightarrow$ -5 = -5

$\Rightarrow$ LHS = RHS

Hence, verified.

25. Here, $\alpha + \beta = -2\sqrt{3}$ and $\alpha\beta = -9$

f(x) = $x^2 - (\alpha + \beta)x + \alpha\beta$ [Formula]

$= x^2 - (-2\sqrt{3})x + (-9)$

$\Rightarrow f(x) = x^2 + 2\sqrt{3}x - 9$

For zeroes of polynomial f(x), f(x) = 0

$\Rightarrow x^2 + 2\sqrt{3}x - 9 = 0$

$\Rightarrow x^2 + 3\sqrt{3}x - 1\sqrt{3}x - 9 = 0$

$\Rightarrow x(x + 3\sqrt{3}) - \sqrt{3}(x + 3\sqrt{3}) = 0$

$\Rightarrow (x + 3\sqrt{3})(x - \sqrt{3}) = 0$

$\Rightarrow x + 3\sqrt{3} = 0$ or $(x - \sqrt{3}) = 0$

$\Rightarrow x = -3\sqrt{3}$ or $x = \sqrt{3}$

$\therefore \alpha = -3\sqrt{3}$ and $\beta = \sqrt{3}$

Hence the polynomial is $x^2 + 2\sqrt{3}$ x - 9 and its zeros are $-3\sqrt{3}$ and $\sqrt{3}$.

26. $2s^2 + (1 + 2\sqrt{2})s + \sqrt{2}$

$= 2s^2 + s + 2\sqrt{2}s + \sqrt{2}$

$= s(2s + 1) + \sqrt{2}(2s + 1)$

$= (2s + 1)(s + \sqrt{2})$

$\Rightarrow s = -\frac{1}{2}, -\sqrt{2}$ are zeroes of the polynomial.

Sum of zeroes = $-[\frac{1}{2} + \sqrt{2}] = -\frac{1+2\sqrt{2}}{2}$

Also, $\frac{-b}{a} = -\frac{1+2\sqrt{2}}{2}$

$\Rightarrow$ Sum of zeroes = $\frac{-b}{a}$

Product of zeroes = $\frac{-1}{2} \times -\sqrt{2} = \frac{\sqrt{2}}{2}$

and $\frac{c}{a} = \frac{\sqrt{2}}{2}$

$\Rightarrow$ Product of zeroes = $\frac{c}{a}$

27. Since,

Sum of the zeroes of polynomial = $\alpha + \beta = \frac{-b}{a}$

and product of zeroes of polynomial = $\alpha\beta = \frac{c}{a}$

Simplify the given expression and substitutie the values, we obtain

$$\frac{\beta}{a\alpha+b} + \frac{\alpha}{a\beta+b} = \frac{\beta(a\beta+b)+\alpha(a\alpha+b)}{(a\alpha+b)(a\beta+b)}$$

$$= \frac{\alpha\beta^2+b\beta+\alpha^2a+b\alpha}{a^2\alpha\beta+ab\alpha+ab\beta+b^2}$$

$$= \frac{a\alpha^2+b\beta^2+b\alpha+b\beta}{a^2\times\frac{c}{a}+ab(\alpha+\beta)+b^2}$$

$$= \frac{a\left[(\alpha+\beta)^2+b(\alpha+\beta)\right]}{ac}$$

$$= \frac{a\left[(a+\beta)^2-2\alpha\beta\right]-\frac{b^2}{a}}{ac}$$

$$= \frac{a\left[\frac{b^2}{a}-\frac{2c}{a}\right]-\frac{b^2}{a}}{ac}$$

$$= \frac{a\left[\frac{b^2-ac}{a}\right]-\frac{b^2}{a}}{ac}$$

$$= \frac{a\left[\frac{b^2-ac-b^2}{a}\right]}{ac}$$

$= \frac{b^2 - 2c - b^2}{ac}$

$= \frac{-2c}{ac} = \frac{-2}{a}$

28. Let the required polynomial be $ax^2 + bx + c$

and let its zeroes be α and β

Then, $\alpha + \beta = \frac{1}{4} = -\frac{b}{a}$ and $\alpha\beta = -1 = \frac{c}{a}$

If a = 4, then b = -1 and c = -4

So, one quadratic polynomial which satisfies the given conditions is $4x^2 - x - 4$

Or

If α and β zeroes of the polynomials then standard quadratic polynomial is given by

$x^2 - (\alpha + \beta)x + \alpha\beta$, where $\alpha + \beta = \frac{1}{4}$ *and* $\alpha\beta = -1$ [Given]

Now, we have,

$x^2 - (\alpha + \beta)x + \alpha\beta$

$= x^2 - \left(\frac{1}{4}\right)x + (-1)$

$= \frac{1}{4}\left(4x^2 - x - 4\right)$

Required polynomial is $4x^2 - x - 4$

29. We know, quadratic polynomial = x^2 - (Sum of zeroes)x + Product of zeroes

Given, Sum of zeroes = $-\frac{21}{8}$ and Product of zeroes = $\frac{5}{16}$

$\therefore$ Quadratic Polynomial = $x^2 + \frac{21}{8}x + \frac{5}{16}$

$= \frac{1}{16}(16x^2 + 42x + 5)$

$\Rightarrow$ Quadratic polynomial is $16x^2 + 42x + 5$

Now, we rewrite the polynomial as $16x^2 + 2x + 40x + 5$

$= 2x \cdot (8x + 1) + 5 \cdot (8x + 1)$

$= (2x + 5) \cdot (8x + 1)$

Now, for Zeros, $(8x + 1) \cdot (2x + 5) = 0$

$\Rightarrow x = \frac{-1}{8}, \frac{-5}{2}$

30. We have to find the zeroes of the quadratic polynomial $4y^2 – 15$ and verify the relationship between the zeroes and coefficient of polynomial.

Let $f(y) = 4y^2 - 15$

Compare it with the quadratic $ay^2 + by + c$.

Here, coefficient of $y^2 = 4$, coefficient of y = 0 and constant term = - 15.

Now $4y^2 - 15 = (2y)^2 - (\sqrt{15})^2$

$= (2y + \sqrt{15})(2y - \sqrt{15})$

The zeroes of f(y) are given by $f(y) = 0$

$\Rightarrow (2y) + \sqrt{15})(2y - \sqrt{15}) = 0$

$\Rightarrow (2y) + \sqrt{15}) = 0$ or $(2y - \sqrt{15}) = 0$

$\Rightarrow 2y = -\sqrt{15}$ or $2y = \sqrt{15}$

$\Rightarrow y = -\frac{\sqrt{15}}{2}$ or $y = \frac{\sqrt{15}}{2}$

Hence, the zeroes of the given quadratic polynomial are $-\frac{\sqrt{15}}{2}, \frac{\sqrt{15}}{2}$

Verification of relationship between zeroes and coefficients

Sum of the zeroes = $-\frac{\sqrt{15}}{2} + \frac{\sqrt{15}}{2} = \frac{-\sqrt{15} + \sqrt{15}}{2} = \frac{0}{2} = 0 = \frac{0}{4}$

$= \frac{coefficient\ of\ y}{coefficient\ of\ y^2}$

Product of zeroes = $-\frac{\sqrt{15}}{2} \times \frac{\sqrt{15}}{2} = -\frac{15}{4} = \frac{constant\ term}{coefficient\ of\ y^2}$.

31. The quadratic polynomial $ax^2 + bx + c = f(x)$

α and β are the zeroes of an equation.

$\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$

$\frac{1}{\alpha} - \frac{1}{\beta} = \frac{\beta - \alpha}{\alpha\beta} = \frac{-(\alpha - \beta)}{\alpha\beta}$ ...... (i)

consider,

$(\alpha - \beta)^2 = (\alpha + \beta)^2 + 4\alpha\beta$

$(\alpha - \beta)^2 = \left(\frac{-b}{a}\right)^2 + \frac{4c}{a}$

$\alpha - \beta = \sqrt{\frac{b^2}{a^2} + \frac{4c}{a}} = \sqrt{\frac{b^2 + 4ac}{a^2}} = \frac{\sqrt{b^2 + 4ac}}{a}$

$\frac{1}{\alpha} - \frac{1}{\beta} = \frac{-(\alpha - \beta)}{\alpha\beta} = \frac{\frac{\sqrt{b^2 + 4ac}}{a}}{\frac{c}{a}} = \frac{\sqrt{b^2 + 4ac}}{c}$

32. Here, $p(x) = 3x^2 - 2$.

Now p(x) = 0

$\Rightarrow \quad 3x^2 - 2 = 0$

$\Rightarrow 3x^2 = 2$

$\Rightarrow \quad x^2 = \frac{2}{3}$

$\Rightarrow x = \pm\sqrt{\frac{2}{3}}$

Therefore, zeroes are $\sqrt{\frac{2}{3}}$ and $-\sqrt{\frac{2}{3}}$.

If $p(x) = 3x^2 - 2$, then a = 3, b = 0 and c = -2

Now, sum of zeroes = $\sqrt{\frac{2}{3}} + \left(-\sqrt{\frac{2}{3}}\right) = 0$ .... (i)

Also, $\frac{-b}{a} = \frac{-0}{3} = 0$ .... (ii)

From (i) and (ii)

Sum of zeroes = $\frac{-b}{a}$

and product of zeroes = $\sqrt{\frac{2}{3}} \times -\sqrt{\frac{2}{3}} = \frac{-2}{3}$ .... (iii)

Also, $\frac{c}{a} = \frac{-2}{3}$ .... (iv)

From (iii) and (iv)

Product of zeroes = $\frac{c}{a}$

33. $p(x) = x^2 + 3x + 2$

α, β are its zeroes

$\therefore \ \alpha + \beta = -3, \alpha\beta = 2$

Now,

$(\alpha + 1) + (\beta + 1) = \alpha + \beta + 2 = -3 + 2 = -1$

$(\alpha + 1)(\beta + 1) = \alpha\beta + (\alpha + \beta) + 1 = +2 - 3 + 1 = 0$

$\therefore$ Required Polynomial is $k(x^2 + x)$ or $x^2 + x$

34. Sum of the zeroes: $(2 + \beta) = (-1)$

Product of the zeroes : $2\beta = -20$

So, required Quadratic polynomial

$= \left[x^2 + (\alpha + \beta)x + 2\beta\right]$

$= \left[x^2 + (-1)x + (-20)\right]$

$= x^2 - x - 20$

$\Rightarrow x^2 - x - 20 = 0$ is the polynomial

35. $4x^2 + 5\sqrt{2}x - 3$

$= 4x^2 + 6\sqrt{2}x - \sqrt{2}x - 3$

$= 2\sqrt{2}x(\sqrt{2}x + 3) - 1(\sqrt{2}x + 3)$

$= (2\sqrt{2}x - 1)(\sqrt{2}x + 3)$

$\Rightarrow x = \frac{1}{2\sqrt{2}}$ and $x = -\frac{3}{\sqrt{2}}$ are zeroes of the polynomial

If given polynomial is $4x^2 + 5\sqrt{2}x - 3$, then $a = 4$, $b = 5\sqrt{2}$ and $c = -3$

Now, Sum of zeroes $= \frac{1}{2\sqrt{2}} + \frac{-3}{\sqrt{2}} = \frac{1-6}{2\sqrt{2}} = \frac{-5}{2\sqrt{2}}$ .... (i)

Also, $\frac{-b}{a} = \frac{-5\sqrt{2}}{4} = \frac{-5}{2\sqrt{2}}$ .... (ii)

From (i) and (ii)

Sum of zeroes $= \frac{-b}{a}$

Product of zeroes $= \frac{1}{2\sqrt{2}} \times \frac{-3}{\sqrt{2}} = \frac{-3}{4}$ .... (iii)

Also, $\frac{c}{a} = \frac{-3}{4}$ .... (iv)

From (iii) and (iv)

Product of zeroes $= \frac{c}{a}$

36. The given quadratic polynomial is $p(x) = 2x^2 - 3x + p$

Since, 3 is a root (zero) of p(x)

$\Rightarrow 2(3)^2 - 3 \times 3 + p = 0$

$\Rightarrow 18 - 9 + p = 0$

$\Rightarrow 9 + p = 0$

$\Rightarrow p = -9$

Now $p(x) = 2x^2 - 3x - 9$

$= 2x^2 - 6x + 3x - 9$

$= 2x(x - 3) + 3(x - 3)$

$= (x - 3)(2x + 3)$

For roots of polynomial, $p(x) = 0$

$\Rightarrow (x - 3)(2x + 3) = 0$

$\Rightarrow x = 3$ or $x = -\frac{3}{2}$

Hence the other root is $-\frac{3}{2}$.

37. Let $p(x) = 6x^2 - 3 - 7x$

For zeroes of p(x),

$p(x) = 0$

$\Rightarrow 6x^2 - 3 - 7x = 0$

$\Rightarrow 6x^2 - 7x - 3 = 0$

$\Rightarrow 6x^2 - 9x + 2x - 3 = 0$

$\Rightarrow 3x(2x - 3) + (2x - 3) = 0$

$\Rightarrow$ (2x - 3) (3x + 1) = 0

$\Rightarrow$ 2x - 3 = 0 or 3x + 1 = 0

$\Rightarrow x = \frac{3}{2}$ or $x = -\frac{1}{3} \Rightarrow x = \frac{3}{2}, -\frac{1}{3}$

So, the zeroes of p(x) are $\frac{3}{2}$ and $-\frac{1}{3}$

We observe that Sum of its zeroes

$= \frac{3}{2} + \left(-\frac{1}{3}\right) = \frac{3}{2} - \frac{1}{3}$

$= \frac{9-2}{6} = \frac{7}{6} = \frac{-(-7)}{6} = -\frac{\text{Coefficient of } x}{\text{Coefficient of } x^2}$

Product of its zeroes = $\left(\frac{3}{2}\right) \times \left(-\frac{1}{3}\right)$

$= -\frac{1}{2} = -\frac{3}{6} = \frac{\text{Constant term}}{\text{Coefficient of } x^2}$

38. The given polynomial is

p(x) = $6x^2$ -7x -3

Factorize the above quadratic polynomial, we have

$6x^2$ -7x -3 = $6x^2$ -9x + 2x - 3

= 3x(2x - 3) + 1(2x - 3)

= (3x + 1)(2x - 3)

For p(x) = 0, either 3x + 1 = 0 or 2x - 3 = 0

$\Rightarrow \quad x = \frac{-1}{3}$ or $x = \frac{3}{2}$

Verification:we have a = 6, b = -7, c = -3

Sum of zeroes = $\frac{-1}{3} + \frac{3}{2} = \frac{7}{6}$

Also, $\frac{-b}{a} = \frac{-(-7)}{6} = \frac{7}{6}$

Now, product of zeroes= $\left(-\frac{1}{3}\right) \times \frac{3}{2} = \frac{-1}{2}$

Also, $\frac{c}{a} = \frac{-3}{6} = \frac{-1}{2}$

39. P (x) = $2x^2 - 4x + 5$

Here, a = 2 , b = – 4, c = 5

Let zeroes be α, β

Sum of zeroes $\alpha + \beta = \frac{-b}{a} = \frac{-(-4)}{2} = 2$

Product of zeroes $\alpha \times \beta = \frac{c}{a} = \frac{5}{2}$

i. $\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

$= (2)^2 - 2(\frac{5}{2})$

= 4 – 5

$\Rightarrow \alpha^2 + \beta^2 = -1$

ii. $(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$= (2)^2 - 4(\frac{5}{2})$

= 4 – 2(5)

= 4 – 10

= - 6

$(\alpha - \beta)^2 = -6$

40. Let $P(x) = 2x^2 + 3x + \lambda$

Its one zero is $\frac{1}{2}$ so $P(\frac{1}{2}) = 0$

$P(\frac{1}{2}) = 2 \times (\frac{1}{2})^2 + 3(\frac{1}{2}) + \lambda = 0$

$\Rightarrow 2 \times \frac{1}{4} + 3/2 + \lambda = 0$

$\Rightarrow \frac{1}{2} + \frac{3}{2} + \lambda = 0$

$\Rightarrow \frac{4}{2} + \lambda = 0$

$\Rightarrow 2 + \lambda = 0$

$\Rightarrow \lambda = -2$

Let the other zero be α

Then $\alpha + \frac{1}{2} = -\frac{3}{2}$

$\Rightarrow \alpha = -\frac{3}{2} - \frac{1}{2} = -\frac{4}{2} = -2$

41. Since α, β are the zeros of the polynomial f(x) = x^2 - 5x + k.

Compare f(x) = x^2 - 5x + k with ax^2 + bx + c.

So, a = 1 , b = -5 and c = k

$\alpha + \beta = -\frac{(-5)}{1} = 5$

$\alpha\beta = \frac{k}{1} = k$

Given, $\alpha - \beta = 1$

Now, $(\alpha + \beta)^2 = (\alpha - \beta)^2 + 4\alpha\beta$

$\Rightarrow (5)^2 = (1)^2 + 4k$

$\Rightarrow 25 = 1 + 4k$

$\Rightarrow 4k = 24$

$\Rightarrow k = 6$

Hence the value of k is 6.

42. Here, $\alpha + \beta = \frac{-3}{2\sqrt{5}}$ and $\alpha \cdot \beta = -\frac{1}{2}$ [Given]

f(x) = $x^2 - (\alpha + \beta)x + \alpha\beta$ [Formula]

$= x^2 - \left(\frac{-3}{2\sqrt{5}}\right)x + \left(-\frac{1}{2}\right)$

$\Rightarrow f(x) = x^2 + \frac{3}{2\sqrt{5}}x - \frac{1}{2}$

$\Rightarrow f(x) = 2\sqrt{5}x^2 + 3x - \sqrt{5}$

For zeroes of polynomial f(x), f(x) = 0

$\Rightarrow 2\sqrt{5}x^2 + 3x - \sqrt{5} = 0$

$\Rightarrow 2\sqrt{5}x^2 + 5x - 2x - \sqrt{5} = 0$

$\Rightarrow \sqrt{5}x(2x + \sqrt{5}) - 1(2x + \sqrt{5}) = 0$

$\Rightarrow (2x + \sqrt{5})(\sqrt{5}x - 1) = 0$

$\Rightarrow (2x + \sqrt{5}) = 0$ or $\sqrt{5}x - 1 = 0$

$\Rightarrow x = \frac{-\sqrt{5}}{2}$ or $x = \frac{1}{\sqrt{5}}$

$\therefore \alpha = \frac{-\sqrt{5}}{2}$ and $\beta = \frac{1}{\sqrt{5}}$.

43. $p(y) = 7y^2 - \frac{11}{3}y - \frac{2}{3} = \frac{1}{3}(21y^2 - 11y - 2)$

$= \frac{1}{3}(21y^2 - 14y + 3y - 2)$

$= \frac{1}{3}[7y(3y - 2) + 1(3y - 2)]$

$= \frac{1}{3}[(7y + 1)(3y - 2)]$

$\therefore$ Zeroes are $\frac{2}{3}, -\frac{1}{7}$

Sum of Zeroes = $\frac{2}{3} - \frac{1}{7} = \frac{11}{21}$

$\frac{-b}{a} = \frac{11}{21}$

$\therefore$ sum of zeroes = $\frac{-b}{a}$

Product of Zeroes = $(\frac{2}{3})(-\frac{-1}{7}) = -\frac{2}{21}$

$\frac{c}{a} = -\frac{2}{3}(\frac{1}{7}) = -\frac{2}{21}$

$\therefore$ Product = $\frac{c}{a}$

Section C

44. a is a non zero real number and b and c are any real numbers.
45. D = 0
46. $2x^2 - x + 8k$

$\alpha \times \frac{1}{\alpha} = \frac{8k}{2}$

1 = 4k

$k = \frac{1}{4}$

47. $\alpha + \beta = \frac{-b}{a}$ i.e., $\left(\frac{-\text{coefficient of x}}{\text{coefficient of } x^2}\right)$

$\alpha\beta = \frac{c}{a}$ i.e., $\left(\frac{\text{constant term}}{\text{coeff of } x^2}\right)$

48. For Zero of the polynomial p(t) we have

p(t) = 0

$\Rightarrow 20t - 16t^2 = 0$

$\Rightarrow 16t^2 - 20t = 0$

$\Rightarrow 4t(4t - 5) = 0$

Now,

4t = 0 gives

$t = \frac{0}{4}$

$\Rightarrow t = 0$

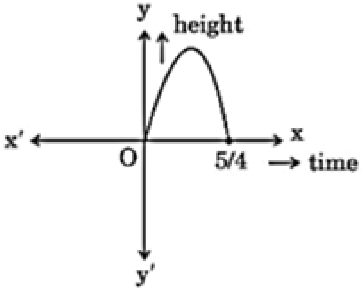
Again, 4t - 5 = 0 gives

4t = 5

$\Rightarrow t = \frac{5}{4}$

Hence zeros of the polynomial $p(t) = 20t - 16t^2$ are 0 and $\frac{5}{4}$

49. **(c)**



Explanation: Since the leading coefficient of the given quadratic is negative the graph will be downward parabola. And will intersect x axis where the polynomial is zero. i.e 0 and $\frac{5}{4}$.

50. $h = 20t - 16t^2$

$= (20 \times \frac{3}{2}) - 16 \times (\frac{3}{2})^2$

$= 30 - (16 - \frac{9}{4})$

$= 30 - 36 = -6$

The results interprets that dolphin is swimming at a depth of 6m below the surface of water.

51. Since, the Dolphin hits the water level again at $t = \frac{5}{4}$

Distance covered by Dolphin = Velocity $\times$ time

$= 20 \times \frac{5}{4}$

= 25 m

Hence, Dolphin covers a distance of 25m before hitting the surface of water again.

52. Zeroes are -2 and 8

$\alpha + \beta = -2 + 8 = 6$

$\alpha\beta = -2 \times 8 = -16$

expression of polynomial

$x^2 - (\alpha + \beta)x + \alpha\beta$

$x^2 - 6x - 16$

53. $P(x) = x^2 - 6x - 16$

$P(4) = 4^2 - 6(4) - 16$

$= 16 - 24 - 16$

$= -24$

54. $P(x) = -x^2 + 3x - 2$

$\alpha + \beta = \frac{-3}{-1}$

$\alpha + \beta = 3$ ...(i)

$\alpha\beta = \frac{-2}{-1}$

$\alpha\beta = 2$ ...(ii)

$(\alpha - \beta)^2 = (\alpha + \beta)^2 - 4\alpha\beta$

$(\alpha - \beta)^2 = (3)^2 - 4(2)$

$(\alpha - \beta)^2 = 9 - 8$

$\alpha - \beta = \pm\sqrt{1}$

$\alpha - \beta = \pm 1$

Taking

$\alpha - \beta = 1$

$\alpha + \beta = 3$

$2\alpha = 4$

$\alpha = 2$

Put $\alpha = 2$ in, $\alpha - \beta = 1$

$2 - \beta = 1$

$\beta = 1$

55. $\alpha + \beta = \frac{-3}{-1} = 3$

$\alpha\beta = \frac{-2}{-1} = 2$

56. Graph of y = f(x) intersects X-axis at two distinct points. So we can say that no of zeros of y = f(x) is 2.

57. There will not be any zero if graph of f(x) does not intersect x- axis.

58. $x^2 + (a + 1)x + b$ is the quadratic polynomial.

2 and -3 are the zeros of the quadratic polynomial.

Thus, $2 + (-3) = \frac{-(a+1)}{1}$

$\Rightarrow \frac{(a+1)}{1} = 1$

$\Rightarrow a + 1 = 1$

$\Rightarrow a = 0$

Also, 2 × (-3) = b

⇒ b = -6

59. If -4 is zero of given polynomial then,

$(-4)^2$ - 2(-4) - (7p + 3) = 0

⇒ 16 + 8 - 7p - 3 = 0

⇒ 7p = 21

⇒ p = 3

60. 2

61. 81.2 m

62. quadratic polynomial

63. (x - 3) and (x - 2)

Section D

64. Since α and β are the zeros of the quadratic polynomial f(x) = ax^2 + bx + c

$$\alpha + \beta = \frac{-\text{ Coefficient of } x}{\text{Coefficient of } x^2} = \frac{-b}{a}$$

$$\alpha\beta = \frac{\text{Constant term}}{\text{Coefficient of } x^2} = \frac{c}{a}$$

We have,

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\alpha^3+\beta^3}{\alpha\beta}\right) + b\left(\frac{\alpha^2+\beta^2}{\alpha\beta}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{(\alpha+\beta)^3-3\alpha\beta(\alpha+\beta)}{\alpha\beta}\right) + \left(\frac{(\alpha+\beta)^2-2\alpha\beta}{\alpha\beta}\right)$$

By substituting $\alpha + \beta = \frac{-b}{a}$ and $\alpha\beta = \frac{c}{a}$ we get,

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\left(\frac{-b}{a}\right)^3 - 3\times\frac{c}{a}\left(\frac{-b}{a}\right)}{\frac{c}{a}}\right) + b\left(\frac{\left(\frac{-b}{a}\right)^2 - 2\times\frac{c}{a}}{\frac{c}{a}}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{\frac{-b^3}{a^3} + \frac{3bc}{a^2}}{\frac{c}{a}}\right) + b\left(\frac{\frac{b^2}{a^2} - \frac{2c}{a}}{\frac{c}{a}}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{-b^3+3bca}{a^3} \times \frac{a}{c}\right) + b\left(\frac{b^2-2ca}{a^2} \times \frac{a}{c}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = a\left(\frac{-b^3+3abc}{a^2c}\right) + b\left(\frac{b^2-2ca}{ac}\right)$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{-b^3+3abc}{ac} + \frac{b^3-2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta} + \frac{\beta^2}{\alpha}\right) + b\left(\frac{\alpha}{\beta} + \frac{\beta}{\alpha}\right) = \frac{-b^3+3abc+b^3-2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=\frac{3abc-2abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=\frac{abc}{ac}$$

$$a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)=b$$

Hence, the value of $a\left(\frac{\alpha^2}{\beta}+\frac{\beta^2}{\alpha}\right)+b\left(\frac{\alpha}{\beta}+\frac{\beta}{\alpha}\right)$ is b.

65. According to the question, α and β are zeroes of p(x) = $6x^2 - 5x + k$

So, Sum of zeroes = $\alpha+\beta = -\left(\frac{-5}{6}\right)=\frac{5}{6}$.......(i)

$\alpha-\beta=\frac{1}{6}$(Given) ....(ii)

Adding equations (i) and (ii) , we get

$2\alpha = 1$

or, $\alpha=\frac{1}{2}$

On putting the value of α in equation (ii), we get

$\frac{1}{2}-\beta=\frac{1}{6}$

$\beta=\frac{1}{2}-\frac{1}{6}$

$\beta=\frac{2}{6}=\frac{1}{3}$

$\therefore \quad \alpha\beta=\frac{k}{6}=\frac{1}{2}\times\frac{1}{3}=\frac{1}{6}$

Hence, k = 1

66. Here, $f(v)=v^2+4\sqrt{3}v-15$

For zeroes of f(v), put f(v) = 0

$\Rightarrow v^2+4\sqrt{3}v-15=0$

$\Rightarrow v^2+5\sqrt{3}v-1\sqrt{3}v-15=0$ (By splitting the middle term)

$\Rightarrow v(v+5\sqrt{3})-\sqrt{3}(v+5\sqrt{3})=0$

$\Rightarrow (v+5\sqrt{3})(v-\sqrt{3})=0$

$\Rightarrow (v+5\sqrt{3})=0$ or $(v-\sqrt{3})=0$

Therefore, either $v=-5\sqrt{3}$ or $v=\sqrt{3}$

Verification of relations between α, β, a, b, c:

we have, $\alpha=-5\sqrt{3},\ \beta=\sqrt{3},$ a = 1, $b=4\sqrt{3},$ and c = -15

$\alpha+\beta=\frac{-b}{a}$

$\Rightarrow -5\sqrt{3}+\sqrt{3}=\frac{-4\sqrt{3}}{1}$

$\Rightarrow -4\sqrt{3}=-4\sqrt{3}$

$\Rightarrow$ LHS = RHS

Hence, verified.

Also we know that

$\alpha\cdot\beta=\frac{c}{a}$

$\Rightarrow (-5\sqrt{3})(\sqrt{3})=\frac{-15}{1}$

$\Rightarrow$ -5 × 3 = -15

$\Rightarrow$ -15 = -15

$\Rightarrow$ LHS = RHS

Hence, verified.

67. Given quadratic polynomial is

f(y) = $7y^2 - \frac{11}{3}y - \frac{2}{3}$

For zeroes of f(y), put f(y) = 0

$\Rightarrow 7y^2 - \frac{11}{3}y - \frac{2}{3} = 0$

$\Rightarrow$ $21y^2 - 11y - 2 = 0$

$\Rightarrow$ $21y^2 - 14y + 3y - 2 = 0$ (by splitting the middle term method)

$\Rightarrow$ 7y(3y - 2) + 1(3y - 2) = 0

$\Rightarrow$ (3y - 2)(7y + 1) = 0

Therefore, either 3y - 2 = 0 or 7y + 1 = 0

$\Rightarrow y = \frac{2}{3}$ or $y = \frac{-1}{7}$

Now Verification of the relations between α, β ,a, b, and c:

We have $\alpha = \frac{2}{3},\ \beta = \frac{-1}{7},$ a = 7, $b = -\frac{11}{3},\ c = \frac{-2}{3}$

$\Rightarrow \alpha + \beta = \frac{-b}{a}$

$\Rightarrow \left(\frac{2}{3}\right) - \frac{1}{7} = \frac{+\frac{11}{3}}{7}$

$\Rightarrow \frac{14-3}{21} = \frac{11}{3} \times \frac{1}{7}$

$\Rightarrow \frac{11}{21} = \frac{11}{21}$

$\Rightarrow$ LHS = RHS

Hence verified.

Also, we know that $\alpha \cdot \beta = \frac{c}{a}$

$\Rightarrow \left(\frac{2}{3}\right) \times \left(\frac{-1}{7}\right) = \frac{\frac{-2}{3}}{7}$

$\Rightarrow \frac{-2}{21} = \frac{-2}{3} \times \frac{1}{7}$

$\Rightarrow \frac{-2}{21} = \frac{-2}{21}$

$\Rightarrow$ LHS = RHS

Hence verified.

68. Zeroes are -2, -3

factors are (x + 2), (x + 3)

g(x) = (x + 2) (x + 3) = $x^2 + 5x + 6$

$\frac{x^4 + 2x^3 - 7x^2 - 8x + 12}{x^2 + 5x + 6}$ = $x^2 - 3x + 2$

$x^2 - 3x + 2 = (x - 2)(x - 1)$

Other zeroes are 2, 1

69. Sum of zeroes of a quadratic polynomial is $\frac{-b}{a}$ and the product is $\frac{c}{a}$

So a + b = $\frac{-2}{5}$ and ab = $\frac{-3}{5}$

According to question

Sum of zeroes of the polynomial is $\frac{1}{a} + \frac{1}{b}$

$= \frac{a+b}{ab}$

$= \frac{\frac{-2}{5}}{\frac{-3}{5}}$

$= \frac{2}{3}$

Product of zeroes of the polynomial is $\frac{1}{ab}$

$= \frac{1}{\frac{-3}{5}}$

$= \frac{-5}{3}$

We know that a quadratic equation is of the form $ax^2 + bx + c$

$= x^2 - \frac{2}{3}x - \frac{5}{3}$

70. Since α and β are the zeroes of the quadratic polynomial $x^2 + 4x + 3$

So, $\alpha + \beta = -4$

and $\alpha\beta = 3$

Sum of zeroes of new polynomial $= 1 + \frac{\beta}{\alpha} + 1 + \frac{\alpha}{\beta}$

$= \frac{\alpha\beta + \beta^2 + \alpha\beta + \alpha^2}{\alpha\beta}$

$= \frac{\alpha^2 + \beta^2 + 2\alpha\beta}{\alpha\beta}$

$= \frac{(\alpha + \beta)^2}{\alpha\beta} = \frac{(-4)^2}{3} = \frac{16}{3}$

Product of zeroes $= \left(1 + \frac{\beta}{\alpha}\right)\left(1 + \frac{\alpha}{\beta}\right)$

$= \left(\frac{\alpha + \beta}{\alpha}\right)\left(\frac{\beta + \alpha}{\beta}\right)$

$= \frac{(\alpha + \beta)^2}{\alpha\beta}$

$= \frac{(-4)^2}{3} = \frac{16}{3}$

So required polynomial = x^2 - (Sum of the zeroes)x + Product of the zeroes

$= x^2 - \left(\frac{16}{3}\right)x + \frac{16}{3}$

$= \left(x^2 - \frac{16}{3}x + \frac{16}{3}\right)$

$= \frac{1}{3}\left(3x^2 - 16x + 16\right)$

71. Given, β and $\frac{1}{\beta}$ are zeroes of the polynomial $(\alpha^2 + \alpha)x^2 + 61x + 6\alpha$.

$\therefore \quad \beta + \frac{1}{\beta} = -\frac{61}{\alpha^2 + \alpha}$

or, $\frac{\beta^2 + 1}{\beta} = \frac{-61}{\alpha^2 + \alpha}$.......(i)

and $\beta \cdot \frac{1}{\beta} = \frac{6\alpha}{\alpha^2 + \alpha}$

or, $1 = \frac{6}{\alpha + 1}$

$\alpha + 1 = 6$

$\alpha = 5$

Substituting this value of α in (i), we get

$\frac{\beta^2 + 1}{\beta} = \frac{-61}{5^2 + 5} = -\frac{61}{30}$

$30\beta^2 + 30 = -61\beta$

$30\beta^2 + 61\beta + 30 = 0$

or, $\frac{-61 \pm \sqrt{(-61)^2 \times 4 \times 30 \times 30}}{2 \times 30}$

$= \frac{-61 \pm \sqrt{3721 - 3600}}{60} = \frac{-61 \pm 11}{60}$

$\beta = \frac{-5}{6}$ or $\frac{-6}{5}$

Hence, $\alpha = 5, \beta = \frac{-5}{6}, \frac{-6}{5}$

72. Since α and β are the zeroes of polynomial $3x^2 + 2x + 1$.

Hence, $\alpha + \beta = -\frac{2}{3}$

and $\alpha\beta = \frac{1}{3}$

Now, for the new polynomial,

Sum of zeroes = $\frac{1-\alpha}{1+\alpha} + \frac{1-\beta}{1+\beta}$

$= \frac{(1-\alpha+\beta-\alpha\beta) + (1+\alpha-\beta-\alpha\beta)}{(1+\alpha)(1+\beta)}$

$= \frac{2-2\alpha\beta}{1+\alpha+\beta+\alpha\beta} = \frac{2-\frac{2}{3}}{1-\frac{2}{3}+\frac{1}{3}}$

$\therefore$ Sum of zeroes = $\frac{4/3}{2/3} = 2$

Product of zeroes = $\left[\frac{1-\alpha}{1+\alpha}\right]\left[\frac{1-\beta}{1+\beta}\right]$

$= \frac{(1-\alpha)(1-\beta)}{(1+\alpha)(1+\beta)}$

$= \frac{1-(\alpha+\beta)+\alpha\beta}{1+(\alpha+\beta)+\alpha\beta}$

$= \frac{1+\frac{2}{3}+\frac{1}{3}}{1-\frac{2}{3}+\frac{1}{3}} = \frac{\frac{6}{3}}{\frac{3}{3}} = 3$

Hence, Required polynomial = x^2 - (Sum of zeroes)x + Product of zeroes

$= x^2 - 2x + 3$

73. Here the given polynomial is

$f(s) = 2s^2 - (1 + 2\sqrt{2})s + \sqrt{2}$

$= s(2s - 1) - \sqrt{2}(2s - 1)$

$= (2s - 1)(s - \sqrt{2})$

Hence f(s) = 0 if 2s - 1 = 0 or $s - \sqrt{2} = 0$

$s = \frac{1}{2}$ or $s = \sqrt{2}$

Verification of the relation between α, β, a, b and c

$\alpha = \frac{1}{2}, \beta = \sqrt{2}$,, a = 2, $b = -(1 + 2\sqrt{2})$, $c = \sqrt{2}$

$\alpha + \beta = \frac{-b}{a}$

$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{+(1+2\sqrt{2})}{2}$

$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \frac{2\sqrt{2}}{2}$

$\Rightarrow \frac{1}{2} + \sqrt{2} = \frac{1}{2} + \sqrt{2}$

$\Rightarrow$ LHS = RHS

Now, $\alpha \times \beta = \frac{c}{a}$

$\Rightarrow \left(\frac{1}{2}\right)(\sqrt{2}) = \frac{\sqrt{2}}{2}$

$\Rightarrow \frac{\sqrt{2}}{2} = \frac{\sqrt{2}}{2}$

$\Rightarrow$ LHS = RHS

Hence verified.

74. $2x^2 + 3x - 14 = 2x^2 + 7x - 4x - 14$

$= (x - 2)(2x + 7)$

$x = 2, -\frac{7}{2}$

Sum of zeroes = $2 + \left(-\frac{7}{2}\right) = -\frac{3}{2}$

Product of zeroes = $2 \times -\frac{7}{2} = -7$

$-\frac{b}{a} = -\frac{3}{2}$

$\frac{c}{a} = -\frac{14}{2} = -7$

$\Rightarrow$ \Hence, sum of zeroes = $-\frac{b}{a}$

Product of zeroes = $\frac{c}{a}$

75. According to the question, $\alpha$and $\beta$are zeroes of $p(x) = 6x^2 - 5x + k$

So, Sum of zeroes =$\alpha + \beta = -\left(\frac{-5}{6}\right) = \frac{5}{6}$ ....(i)

$\alpha - \beta = \frac{1}{6}$(Given).......(ii)

Adding equations (i) and (ii), we get

2α= 1

or, $\alpha = \frac{1}{2}$

On putting the value of α in equation (ii), we get

$\frac{1}{2} - \beta = \frac{1}{6}$

$\beta = \frac{1}{2} - \frac{1}{6}$

$\beta = \frac{2}{6} = \frac{1}{3}$

$\therefore \quad \alpha\beta = \frac{k}{6} = \frac{1}{2} \times \frac{1}{3}$

Hence, k = 1