Solution

QUADRATIC EQUATIONS WS 2

Class 10 - Mathematics

Section A

1. **(a)** 45

Explanation: Let the digits at units and tens place of the given number be x and y respectively. Thus, the number is 10y + x.
The product of the two digits of the number is 20.
Thus, we have xy = 20.
After interchanging the digits, the number becomes 10x + y.
If 9 is added to the number, the digits interchange their places. Thus, we have
(10y + x) + 9 = 10x + y
$\Rightarrow$ 10y + x + 9 = 10x + y
$\Rightarrow$ 10x + y - 10y - x = 9
$\Rightarrow$ 9x - 9y = 9
$\Rightarrow$ 9(x - y) = 9
$\Rightarrow x - y = \frac{9}{9}$
$\Rightarrow$ x - y = 1
So, we have the systems of equations
xy = 20
x - y = 1
Here x and y are unknown. We have to solve the above systems of equations for x and y.
Substituting x = 1 + y from the second equation to the first equation, we get
(1 + y)y = 20
$\Rightarrow$ y + y^2 = 20
$\Rightarrow$ y^2 + y = 20 = 0
$\Rightarrow$ y^2 + 5y - 4y - 20 = 0
$\Rightarrow$ y(y + 5) - 4(4 + 5) = 0
$\Rightarrow$ (y + 5) (y - 4) = 0
$\Rightarrow$ y = -5 or y = 4
Substituting the value of y in the second equation, we have

y	-5	4
x	-4	5

Hence, the number is 10 $\times$ 4 + 5 = 45
Note that in the first pair of solution the values of x and y are both negative. But the digits of the number can't be negative. So we must remove this pair.

2.

(b) x^2 - 3x - 10 = 0

Explanation: Sum of the roots = 5 + (-2) = 3, product of roots = 5 $\times$ (-2) = -10.
$\therefore$ x^2 - (sum of roots) x + product of roots = 0.
Hence, x^2 - 3x - 10 = 0.

3.

(c) 5, 3

Explanation: Let the two numbers be x and y.
According to the given conditions,
x + y = 8 ...(i)
and $\frac{1}{x}+\frac{1}{y}=\frac{8}{15}$ ...(ii)
Putting value of x = 8 - y in (ii), we get
$\frac{1}{8-y}+\frac{1}{y}=\frac{8}{15} \Rightarrow \frac{y+8-y}{y(8-y)}=\frac{8}{15}$

$\Rightarrow y^2 - 8y + 15 = 0 \Rightarrow y^2 - 5y - 3y + 15 = 0$
$\Rightarrow (y - 5)(y - 3) = 0 \Rightarrow y = 5$ or $y = 3$
From (i), x = 3 or x = 5
Thus, the numbers are 5 and 3.

4. **(a)** p = 1, q = - 2
Explanation: Given sum of roots, S = p + q = – p and product pq = q
$\Rightarrow$ q(p – 1) = 0 i.e. q = 0 or p = 1
Now If q = 0 then p = 0, this implies p = q
If p = 1, then p + q = – p
q = – 2p
q = – 2(1)
q = – 2

5.
(c) 2
Explanation: Dividing the equation by the coefficient of x^2 i.e., 2 we got
$x^2 + \frac{kx}{2} - 2 = 0$
$\left(x + \frac{k}{4}\right)^2 - \frac{k^2}{16} - 2 = 0$
$\left(x + \frac{k}{4}\right)^2 = \frac{k^2+32}{16}$
Hence for rational roots, $\frac{k^2+32}{16}$ has to be a perfect square.
We get a perfect square at k = ± 2 for $\left(\frac{k^2+32}{16}\right)$ i.e., $\frac{36}{16}$ which becomes $\frac{6}{4}$ upon removing the square
We get a perfect square at k = ± 7 for $\left(\frac{k^2+32}{16}\right)$ i.e., $\frac{81}{16}$ which becomes $\frac{9}{4}$ upon removing the square
Hence the least positive value of k is 2.

6.
(b) 2, -5
Explanation: Given; $x^2 + 3x - 10 = 0$
$x^2 + 5x - 2x - 10 = 0$
x(x + 5) - 2(x + 5) = 0
(x - 2) (x + 5) = 0
x = 2, -5
hence, roots are 2, -5

7.
(b) 3
Explanation: In the equation $ax^2 + ax + 3 = 0$ and $x^2 + x + b = 0$
Substituting the value of x = 1, then in $ax^2 + ax + 3 = 0$
$a(1)^2 + a(1) + 3 = 0 \Rightarrow a + a + 3 = 0$
$\Rightarrow 2a + 3 = 0 \Rightarrow 2a = -3 \Rightarrow a = \frac{-3}{2}$
and in $x^2 + x + b = 0$
$(1)^2 + 1 + b = 0 \Rightarrow 1 + 1 + b = 0 \Rightarrow b = -2$
$\therefore$ ab = $\frac{-3}{2} \times (-2) = 3$

8.
(b) $\frac{b^2}{4a}$
Explanation: Since the roots are equal, we have D = 0
$\therefore b^2 - 4ac = 0 \Rightarrow 4ac = b^2 \Rightarrow c = \frac{b^2}{4a}$.

9. **(a)** 3
Explanation: We are given that quadratic equations have real roots and the quadratic equation does not change by squaring their roots. We have to find the number of quadratic equations.
The possible roots (1,1),(1,0),(0,0)

The general formula of quadratic equation is;

x^2- (sum of roots)x + product of roots

So, we have;

Case-I: When roots are 1 and 1

$x^2 - (1+1)x + 1 = 0$

$x^2 - 2x + 1 = 0$

Case-II: When roots are 1 and 0

$x^2 - x = 0$

Case-III: When roots are 0 and 0

Then, $x^2 = 0$

Therefore, 3 possible quadratic equation.

10.

(d) $\frac{25cd}{9ab}$ and $\frac{-cd}{ab}$

Explanation: Using factorisation method

$9a^2b^2x^2 - 16abcdx - 25c^2d^2 = 0$

$\Rightarrow 9a^2b^2x^2 - 25abcdx + 9abcdx - 25c^2d^2 = 0$

⇒ abx(9abx - 25cd) + cd(9abx - 25cd) = 0

⇒ (abx + cd)(9abx - 25cd) = 0

⇒ abx + cd = 0 and 9abx - 25cd = 0

⇒ x = $\frac{-cd}{ab}$ and x = $\frac{25cd}{9ab}$

11.

(c) (a + 1), -(a + 2)

Explanation: Given equation is $x^2 + x - (a + 1)(a + 2) = 0$

$\Rightarrow x^2 + (a + 2)x - (a + 1)x - (a + 1)(a + 2) = 0$

⇒x(x + (a + 2)) - (a + 1)(x + (a + 2)) = 0

⇒(x - (a + 1))(x + (a + 2)) = 0

⇒x = (a + 1) or x = -(a + 2)

12.

(b) -12

Explanation: $4x^2 - 6x + 3 = 0$

a = 4, b = -6, c = 3

$D = b^2 - 4ac$

$=(-6)^2 - 4(4)(3)$

= 36 - 48

= -12

13.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: As one root is $\frac{2}{3}$, x = $\frac{2}{3}$

$6 \times \left(\frac{2}{3}\right)^2 - \frac{2}{3} - k = 0$

$6 \times \frac{4}{9} - \frac{2}{3} = k$

$k = \frac{8}{3} - \frac{2}{3} = \frac{6}{3} = 2$

k = 2

So, both A and R are true but R is not the correct explanation of A.

14.

(b) Both A and R are true but R is not the correct explanation of A.

Explanation: Both A and R are true and R is the correct explanation of A.

Assertion $(2x - 1)^2 - 4x^2 + 5 = 0$

-4x + 6 = 0

Reason $2x^2 - 6x = 0$

2x(x - 3) = 0

x = 0

and x = 3

15. $3x^2-2x-1=0 \Rightarrow 3x^2-3x+x-1=0$

$\Rightarrow 3x(x-1)+1(x-1)=0$

$\Rightarrow (3x+1)(x-1)=0$

$\Rightarrow$ 3x + 1 = 0 or x - 1 = 0

x = $\frac{-1}{3}$ or x = 1

16. Since, $\frac{1}{3}$ is the root of quadratic equation then it satisfies the equation.

So, put x = $\frac{1}{3}$ in the quadratic equation

$2\left(\frac{1}{3}\right)^2+2\left(\frac{1}{3}\right)+k=0$

$\frac{2}{9}+\frac{2}{3}+k=0$

$\frac{6+18}{9}+k=0$

$k=\frac{-8}{3}$

Therefore, the value of k is $\frac{-8}{3}$

17. $15x^2-28=x \Rightarrow 15x^2-x-28=0$

$\Rightarrow 15x^2-21x+20x-28=0$

$\Rightarrow 3x(5x-7)+4(5x-7)=0$

$\Rightarrow$ (5x - 7) (3x + 4) = 0

$\Rightarrow$ 5x - 7 = 0 or 3x + 4 = 0

$\Rightarrow$ x = $\frac{7}{5}$ or x = $\frac{-4}{3}$

18. $\sqrt{3}x^2$ + 15x - x - $5\sqrt{3}$ = 0

[x + $5\sqrt{3}$] [$\sqrt{3}$x - 1] = 0

x = $-5\sqrt{3}$ or x = $\frac{1}{\sqrt{3}}$

19. Given, $3x^2$ - 243 = 0

$\Rightarrow$ 3(x^2 - 81) = 0

$\Rightarrow x^2-81=0$

$\Rightarrow$ x^2 = 81

$\Rightarrow$ x = $\pm\sqrt{81}=\pm 9$

$\Rightarrow$ x = 9, -9

20. Substitute the value of x in equation,

$\Rightarrow 3x^2+13x+14=0$

$\Rightarrow 3(-2)^2+13(-2)+14=0$

$\Rightarrow 3(4)-26+14=0$

$\Rightarrow$ -26+26=0

Since it gives the value 0, it is the solution the given equation.

21. The given equation is $2x^2$ - 5x - 3 = 0

On substituting x = 4 in the given equation, we get

LHS = 2 $\times$ 4^2 - 5 $\times$ 4 - 3 = (32 - 20 - 3) = 9 $\neq$ 0

Thus, LHS $\neq$ RHS

$\therefore$ x = 4 is not a solution of $2x^2$ - 5x - 3 = 0

22. $\sqrt{3}x^2-2x-\sqrt{3}=0$

or, $\sqrt{3}x^2-3x+x-\sqrt{3}=0$

or, $\sqrt{3}x(x-\sqrt{3})+1(x-\sqrt{3})=0$

or, $(x-\sqrt{3})(\sqrt{3}x+1)=0$

$\therefore \quad x=\sqrt{3}, \frac{-1}{\sqrt{3}}$

23. $x^2-55x+750=0$

$\Rightarrow x^2-25x-30x+750=0 \Rightarrow x(x-25)-30(x-25)=0$

$\Rightarrow (x-30)(x-25) \Rightarrow x=30, 25$

24. Given equation is $5x^2 - kx + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

a = 5, b = - k and c = 1

For real and distinct roots: D > 0

Discriminant $D = b^2 - 4ac > 0$

$(-k)^2 - 4.5.1 = k^2 - 20 > 0$

$k^2 > 20$

This gives;

$k > 2\sqrt{5}$ or $k < -2\sqrt{5}$

25. Given equation is, (2x - 3)(3x + 1) = 0

$\Rightarrow$ 2x - 3 = 0 or 3x + 1 = 0

$\Rightarrow$ 2x = 3 or 3x = -1

Therefore, $x = \frac{3}{2}\ or\ -\frac{1}{3}$

Section B

26. Let the length of field be x m and breadth be y m

Area of field = xy

$\therefore$ xy = 1500 ...(i)

Now, A/c to question

x + 2y = 110

from (i)

$\frac{1500}{y}$ + 2y = 110

2y = 110 - $\frac{1500}{y}$ ...(ii)

Perimeter of field = 2x + 2y

= 2x + 110 - $\frac{1500}{y}$

27. Let the length of field be x m and breadth be y m

A/c to question

x + 2y = 110

$\frac{1500}{y}$ + 2y = 110 ...form (i)

$\frac{1500+2y^2}{y}$ = 110

$1500 + 2y^2 = 110y$

$2y^2 - 110y + 1500 = 0$

$y^2 - 55y + 750 = 0$

y = $\frac{-(-55)\pm\sqrt{(-55)^2-4(1)(750)}}{2\times1}$

y = $\frac{-55\pm\sqrt{3025-3000}}{2}$

y = $\frac{55\pm\sqrt{25}}{2}$

y = $\frac{55\pm5}{2}$

y = $\frac{55+5}{2}$ = 30

or y = $\frac{55-5}{2}$ = 25

if y = 30 m, x = 110 - 2y

= 110 - 2 $\times$ 30

= 50 m

if y = 25 m, x = 110 - 2 $\times$ 25

= 60 m

hence, the dimension of field will be either (50m, 30 m) or (60 m, 25 m)

28. 8

Explanation:

Let D_1 and D_2 be the discriminant of the first and second given equations respectively.

Since equation has real roots,

$D_1 = 0$ and $D_2 = 0$

$D = b^2 - 4ac$

$\Rightarrow (4p^2 - 4 \Rightarrow 64) = 0$ and $(64 - 8p) = 0$

$\Rightarrow p^2 - 64 = 0$ and $64 - 8p = 0$

$\Rightarrow p^2 = 64$ and $8p = 64$

$\Rightarrow p = 8$.

29. 6

Explanation:

According to question,

3 + 5 + 7 + 9 +... to n terms = 48

$\Rightarrow \frac{n}{2}[2 \times 3 + (n - 1)2] = 48$ [Using: $S_n = \frac{n}{2}\{2a + (n - 1)d\}$ where a = 3 and d = 2]

$\Rightarrow n(6 + 2n - 2) = 96$

$\Rightarrow n(4 + 2n) = 96$

$\Rightarrow 2n^2 + 4n = 96$

$\Rightarrow n^2 + 2n - 48 = 0$

Factorise the equation,

$\Rightarrow n^2 + 8n - 6n - 48 = 0$

$\Rightarrow n(n + 8) - 6(n + 8) = 0$

$\Rightarrow (n + 8)(n - 6) = 0$

$\Rightarrow n = -8$ or, $n = 6$

Therefore, n = 6 [$\because n > 0$]

30. 20

Explanation:

We have,

2 + 4 + 6 + 8 + ... to n terms = 420

$\Rightarrow \frac{n}{2}[2 \times 2 + (n - 1) \times 2] = 420$

$\Rightarrow n(2 + n - 1) = 420$

$\Rightarrow n(n + 1) = 420$

$\Rightarrow n^2 + n - 420 = 0$

$\Rightarrow n^2 + 21n - 20n - 420 = 0$

$\Rightarrow n(n + 21) - 20(n + 21) = 0$

$\Rightarrow (n + 21)(n - 20) = 0$

$\Rightarrow n = 20, -21$

$\Rightarrow n = 20$ [$\because$ n is a natural number, $\therefore n > 0$]

31. 3

Explanation:

Let α and 6α be the roots of equation.

We have, $px^2 - 14x + 8 = 0$ where a = p, b = -14, c = 8

Sum of zeroes $= -\frac{b}{a} = -\frac{-14}{p}$

$\alpha + 6\alpha = \frac{14}{p}$

$7\alpha = \frac{14}{p}$

$\alpha = \frac{2}{p}$ ..... (i)

Also, Product of the zeroes $= \frac{8}{p} = \frac{c}{a}$

$\alpha \times 6\alpha = \frac{8}{p}$

$6\alpha^2 = \frac{8}{p}$

From (i)

$6(\frac{2}{p})^2 = \frac{8}{p}$

$6 \times \frac{4}{p^2} = \frac{8}{p}$

$\frac{6}{p^2} = \frac{2}{p}$

$\frac{6}{2} = \frac{p^2}{p}$

Hence, p = 3

32. 23

Explanation:

Let the whole number be x.

According to question,

(x - 20) = 69 $\left(\frac{1}{x}\right)$

$\Rightarrow$ x^2 - 20x - 69 = 0

$\Rightarrow$ x^2 - 23x + 3x - 69 = 0

$\Rightarrow$ (x - 23) (x + 3) = 0

$\Rightarrow$ x = 23, -3. Rejecting -3 as -3 is not a whole number.

$\Rightarrow$ x = 23

33. 11

Explanation:

we have to find the numerical difference of the roots of quadratic equation: x^2 - 7x - 18 = 0

$\Rightarrow$ x^2 - 9x + 2x - 18 = 0

$\Rightarrow$ x(x - 9) + 2(x - 9) = 0

$\Rightarrow$ (x + 2)(x - 9) = 0

$\Rightarrow$ x = -2 or x = 9

Now, difference of roots = 9 - (-2) = 11

34. -1

Explanation:

$\frac{2x}{x-3} + \frac{1}{2x+3} + \frac{3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow \frac{2x(2x+3)+x-3+3x+9}{(x-3)(2x+3)} = 0$

$\Rightarrow$ $4x^2$ + 6x + x - 3 + 3x + 9 = 0

$\Rightarrow$ $4x^2$ + 10x + 6 = 0

$\Rightarrow$ $2x^2$ + 5x + 3 = 0

$\Rightarrow$ $2x^2$ + 2x + 3x + 3 = 0

$\Rightarrow$ 2x(x + 1) + 3(x + 1) = 0

$\Rightarrow$ (x + 1)(2x + 3) = 0

$\Rightarrow$ x = -1 or x = $\frac{-3}{2}$

When x = $\frac{-3}{2}$, given equation is not defined,

$\therefore$ x = -1.

35. 0

Explanation:

(k + 1)x^2 + 2(k + 1)x + 1 = 0

has equal roots if D = 0

i.e b^2 = 4ac

Here, a = (k + 1), b = 2(k + 1), c = 1

or, $4(k + 1)^2$ = 4(k + 1)

k^2 + 2k + 1 = k + 1

k^2 + 2k + 1 - k - 1 = 0

k^2 + k = 0

k(k + 1) = 0

k = 0, -1

Since k = -1 does not satisfy the equation

$\Rightarrow$ k = 0

36. 12

Explanation:

Let the natural number be x.

Then, its square will be x^2

According to given information, we have

$x + x^2 = 156$

$\Rightarrow x^2 + x - 156 = 0$

$\Rightarrow x^2 + 13x - 12x - 156 = 0$

$\Rightarrow x(x + 13) - 12(x + 13) = 0$

$\Rightarrow x + 13 = 0$ or $x - 12 = 0$

$\Rightarrow x = -13$ or $x = 12$

Since x is a natural number, $x \neq -13$.

Hence, the required natural number is 12.

Section C

37. $h = 5t^2 + 14t + 3$

When ball touches ground h = 0

$= -5t^2 + 14t + 3 = 0$

$= 5t^2 - 14t - 3 = 0$

$= 5t^2 - 15t + t - 3 = 0$

$5t(t - 3) + 1(t - 3) = 0$

$(5t + 1)(t - 3) = 0$

$t = \frac{-1}{5}$ Not possible

t = 3s

Hence the ball touches the ground after 3s.

38. Ball is at maximum height at $t = \frac{3}{2}$ sec

∵ time of ascent = time of decent

$h_{max} = -5t^2 + 14t + 3$

$= -5\left(\frac{3}{2}\right)^2 + 14 \times \left(\frac{3}{2}\right) + 3$

$= -5 \times \frac{9}{4} + 14 \times \frac{3}{2} + 3$

$= \frac{-45}{4} + 24$

$\frac{-45+96}{4}$

$= \frac{51}{4}$

= 12.75 m

39. At h_{max}, vertical component of velocity is always zero.

40. $h = -5t^2 + 14t + 3$

(h) $t = 18 = -5(1)^2 + 14(1) + 3$

= -5 + 14 + 3

h = 12 m

i.e., ball is at a height of 12 m from the ground.

41. State True or False:

(i) **(a)** True

Explanation: True

(ii) **(b)** False

Explanation: False

(iii) **(a)** True

Explanation: True, since in this case discriminant is always negative, so it has no real roots i.e., if b = 0, then $b^2 - 4ac \Rightarrow -4ac < 0$ and $ac > 0$.

(iv) **(a)** True

Explanation: True

Section D

42. Given equation, $\frac{y+3}{y-2}-\frac{1-y}{y}=\frac{17}{4}$

$\Rightarrow \quad \frac{y(y+3)-(1-y)(y-2)}{y(y-2)}=\frac{17}{4}$

$\Rightarrow \frac{(y^2+3y)-(-y^2+3y-2)}{y^2-2y}=\frac{17}{4}$

$\Rightarrow \frac{y^2+3y+y^2-3y+2}{y^2-2y}=\frac{17}{4}$

$\Rightarrow \quad \frac{2y^2+2}{y^2-2y}=\frac{17}{4}$

$\Rightarrow \quad 4(2y^2+2)=17(y^2-2y)$

$\Rightarrow \quad 8y^2+8=17y^2-34y$

$\Rightarrow \quad 9y^2-34y-8=0$

$\Rightarrow \quad 9y^2-36y+2y-8=0$

$\Rightarrow \quad 9y(y-4)+2(y-4)=0$

$\Rightarrow \quad (y-4)(9y+2)=0$

$\Rightarrow \quad y-4=0 \quad or \quad 9y+2=0$

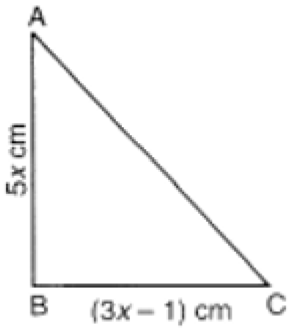
$\Rightarrow \quad y=4 \quad or \quad y=-\frac{2}{9}$

$\therefore \quad y=4,-\frac{2}{9}$

43. Area of triangle = $\frac{1}{2}\times$ base $\times$ h$eight$

$=\frac{1}{2}\times 5x\times(3x-1)$

According to the question,



$15x^2-5x=120$

or, $3x^2-x-24=0$

or, $3x^2-9x+8x-24=0$

or, 3x (x - 3) + 8 (x - 3) = 0

or, (x - 3) (3x + 8) = 0

$\therefore \quad x=3, x=-\frac{8}{3}$

Length can't be negative, so x = 3

AB = 5 × 3 = 15 cm, BC = 3x - 1 = 9 - 1

= 8 cm

$AC=\sqrt{15^2+8^2}$

$=\sqrt{225+64}$

$=\sqrt{289}=17\text{cm}$

Hence hypotenuse = 17 cm

44. Given, $\frac{1}{(x-1)(x-2)}+\frac{1}{(x-2)(x-3)}=\frac{2}{3}$

$\frac{(x-3)+(x-1)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\frac{x-3+x-1}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\frac{2x-4}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\frac{2(x-2)}{(x-1)(x-2)(x-3)}=\frac{2}{3}$

$\frac{2}{(x-1)(x-3)}=\frac{2}{3}$

(x - 1) (x - 3) = 3

x^2 - 4x + 3 = 3

x^2 -4x = 0
x(x - 4) = 0
x = 0 , x - 4= 0
x = 0, x = 4

45. Let the larger number be x. Then,
Square of the smaller number = 4x
Also, Square of the larger number = x^2
It is given that the difference of the squares of the numbers is 45.
$\therefore$ x^2 - 4x = 45
$\Rightarrow$$x^2$ - 4x - 45 = 0
$\Rightarrow$ x^2 - 9x + 5x - 45 = 0
$\Rightarrow$ x (x - 9) + 5 (x - 9) = 0
$\Rightarrow$ x - 9 = 0 or, x + 5 = 0 $\Rightarrow$ x = 9, -5
Case I When x = 9: In this case, we have
Square of the smaller number = 4x = 36
$\therefore$ Smaller number = $\pm$6.
Thus, the numbers are 9, 6 or 9, - 6
CASE II When x = -5: In this case, we have
Square of the smaller number = 4x = - 20. But, square of a number is always positive. Therefore, x = - 5 is not possible.
Hence, the numbers are 9, 6 or 9, - 6.

46. Let the width of the path be x m
Length of the field including the path = (20 + 2x) m
Breadth of the field including the path = (14 + 2x) m.
Area of rectangle = L $\times$ B
Area of the field including the path = (20 + 2x) (14 + 2x) m^2 .
Area of the field excluding the path = (20 $\times$ 14) m^2= 280 m^2.
$\therefore$ Area of the path = (20 + 2x)(14 + 2x) - 280
(20 + 2x) (14 + 2x) - 280 = 111
$\Rightarrow 4x^2 + 68x - 111 = 0$
Factorise the equation,
$\Rightarrow$ $4x^2 + 74x - 6x - 111 = 0$
$\Rightarrow$ 2x(2x + 37) - 3(2x + 37) = 0
$\Rightarrow$ $(2x + 37)(2x - 3) = 0$
$\Rightarrow$ x = $-\frac{37}{2}$ or x = $\frac{3}{2}$
As width can't be negative.
$\Rightarrow$ x = $\frac{3}{2}$ = 1.5
Therefore, the width of the path is 1.5 m.

47. Let the first number be x
$\therefore$ Second number = x + 5
Now according to the question
$\frac{1}{x} - \frac{1}{x+5} = \frac{1}{10}$
$\Rightarrow \quad \frac{x+5-x}{x(x+5)} = \frac{1}{10}$
$\Rightarrow \quad 50 = x^2 + 5x$
$\Rightarrow \quad x^2 + 5x - 50 = 0$
$\Rightarrow \quad x^2 + 10x - 5x - 50 = 0$
$\Rightarrow x(x + 10) - 5(x + 10) = 0$
$\Rightarrow \quad (x + 10)(x - 5) = 0$
x = 5 ,- 10 rejected
The numbers = 5 and 10.

48. Let the altitude of the triangle = x cm
base = (2x+8) cm

area = 480 sq cm
$\frac{1}{2}$ × base × altitude = 480
⇒ x(2x+8)= 2 ×480
⇒ $2x^2 + 8x - 960=0$
⇒ $x^2 + 4x - 480 = 0$
⇒ $x^2 + 24x - 20x - 480 = 0$
⇒ x(x + 24) -20(x + 24) = 0
⇒ (x + 24)(x - 20) = 0
⇒ x + 24 = 0 or x - 20 = 0
⇒ x = -24 or x = 20
Length never negative so value of x = 20
base = 2x + 8= 2(20) + 8 = 48 cm
By Pythagoras theorem
$\text{hypotenuse}^2 = \text{base}^2 + \text{altitude}^2$
$= (48)^2 + (20)^2$
= 2304 + 400
= 2704
hypotenuse = 52
Three sides of a triangle are 48cm,20cm and 52cm

49. Let the two consecutive positive integers be x and (x + 1)
According to given condition,
x(x + 1) = 306
$x^2 + x - 306 = 0$
$x^2 + 18x - 17x - 306 = 0$
x(x + 18) - 17(x + 18) = 0
(x + 18) (x – 17) = 0
(x + 18) = 0 or (x – 17) = 0
x = - 18 or x = 17
but x = 17 (x is a positive integers)
x + 1 = 17 + 1 = 18
Thus the two consecutive positive integers are 17 and 18.

50. According to the question, let the consecutive multiples of 7 be 7x and 7x +7
$(7x)^2 + (7x+7)^2 = 637$
or, $49x^2 + 49x^2 + 49 + 98x = 637$
or, $98x^2 + 98x - 588 = 0$
or, $x^2 + x - 6 = 0$
or, (x + 3)(x - 2) = 0
or, x = -3 ,2
Rejecting the value, x=2
Thus,the required multiples are, 14 and 21.

51. Let the lengths of the radii of the smaller and larger circles be r cm and R cm respectively.
It is given that,R - r = 7.....(i).
It is also given that the difference between the areas of two circles is 1078 cm^2
$\therefore \quad \pi R^2 - \pi r^2 = 1078$
$\Rightarrow \quad \pi\left(R^2 - r^2\right) = 1078$
$\Rightarrow \quad \frac{22}{7}(R+r)(R-r) = 1078$
$\Rightarrow \quad \frac{22}{7}(R+r) \times 7 = 1078$
⇒ R + r = 49 ....(ii)
Subtracting (i) from (ii), we get
2r = 42⇒r = 21
Hence, the radius of the smaller circle is of length 21 cm.

52. Let breadth of the rectangular park = x m

Then, length of the rectangular park = (x + 3) m

Now, area of the rectangular park is = x(x + 3) = (x^2 + 3x)m^2 [∵ area = length × breadth]

Given, base of the triangular park = Breadth of the rectangular park

Therefore, base of triangular park is = x m

altitude of triangular park is = 12 m

Therefore, area of the triangular park will be = $\frac{1}{2} \times x \times$ 12 = 6x m^2 [∵ area = $\frac{1}{2} \times$ base × height]

As per the question area of rectangular park is = 4 + Area of triangular park

⇒ x^2 + 3x = 4 + 6x

⇒ x^2 + 3x - 6x - 4 = 0

⇒ x^2 - 3x - 4 = 0

⇒ x^2 - 4x + x - 4 = 0[by factorization]

⇒ x(x - 4) + 1(x - 4) = 0

⇒ (x - 4)(x + 1) = 0

⇒ x - 4 = 0 or x + 1 = 0

⇒ x = 4 or x = - 1

Since, breadth cannot be negative , so we will neglect x = -1 and choose x = 4

Hence, breadth of the rectangular park will be = 4 m

and length of the rectangular park will be = x + 3 = 4 + 3 = 7m

53. $\frac{1}{a+b+x} = \frac{1}{a} + \frac{1}{b} + \frac{1}{x}$

$\Rightarrow \frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\Rightarrow \quad \frac{-(a+b)}{x^2+(a+b)x} = \frac{b+a}{ab}$

$\Rightarrow \quad x^2 + (a+b)x + ab = 0$

$\Rightarrow \quad (x+a)(x+b) = 0$

$\Rightarrow \quad x = -a, x = -b$

Hence ,x = - a, -b.

54. Suppose, the smaller side of the right triangle be x cm and the larger side be y cm. Then,

$\therefore x^2 + y^2 = \left(3\sqrt{10}\right)^2$ [Using pythagoras theorem]

$\Rightarrow x^2 + y^2 = 90$ ....(i)

If the smaller side is tripled and the larger side be doubled, the new hypotenuse is $9\sqrt{5}$ cm.

$\therefore (3x)^2 + (2y)^2 = \left(9\sqrt{5}\right)^2$ [Using pythagoras theorem]

$\Rightarrow 9x^2 + 4y^2 = 405$ ....(ii)

Putting $y^2 = 90 - x^2$ in equation (ii), we get

$9x^2 + 4(90 - x^2) = 405$

$\Rightarrow 9x^2 + 360 - 4x^2 = 405$

$\Rightarrow 5x^2 = 405 - 360$

$\Rightarrow 5x^2 = 45$

$\Rightarrow x^2 = 9$

⇒ x = ±3

But, length of a side can not be negative. Therefore, $x = 3$

Putting $x = 3$ in (i), we get

$(3)^2 + y^2 = 90$

$\Rightarrow y^2 = 90 - 9$

$\Rightarrow y^2 = 81$

⇒ y = ±9

But, length of a side can not be negative. Therefore, $y = 9$

Hence, the length of the smaller side is 3 cm and the length of the larger side is 9 cm.

55. The given equation is: $\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

$\Rightarrow \quad \frac{x-1+2x-4}{(x-2)(x-1)} = \frac{6}{x}$

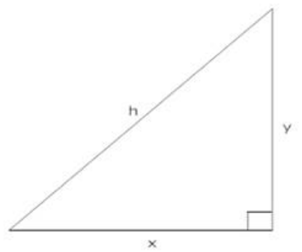
$\Rightarrow \quad 3x^2 - 5x = 6x^2 - 18x + 12$

$\Rightarrow \quad 3x^2 - 13x + 12 = 0$

$\Rightarrow 3x^2 - 4x - 9x + 12 = 0$
$\Rightarrow x(3x - 4) - 3(3x - 4) = 0$
⇒ (3x -4)(x -3) = 0
$\Rightarrow x = \frac{4}{3}$ and 3
Hence, $x = 3, \frac{4}{3}$

56. We have,
$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$
Here, $a = \sqrt{3}$, $b = 10$ and $c = -8\sqrt{3}$
$\therefore D = b^2 - 4ac$
$= (10)^2 - 4 \times (\sqrt{3}) \times (-8\sqrt{3})$
= $100 + 96 = 196 > 0$
$\therefore D > 0$
So, the given equation has real roots, given by
$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-10+\sqrt{196}}{2\times\sqrt{3}}$
$= \frac{-10+14}{2\sqrt{3}} = \frac{4}{2\sqrt{3}} = \frac{2}{\sqrt{3}}$
And, $\beta = \frac{-b-\sqrt{D}}{2a}$
$= \frac{-10-\sqrt{196}}{2\sqrt{3}}$
$= \frac{-10-14}{2\sqrt{3}} = -\frac{24}{2\sqrt{3}} = -\frac{12}{\sqrt{3}}$
$= -\frac{4\times\sqrt{3}\times\sqrt{3}}{\sqrt{3}} = -4\sqrt{3}$
$\therefore\ x = \frac{2}{\sqrt{3}}, -4\sqrt{3}$

57. Let base = x
Altitude = y
Hypotenuse = h



According to question,
h = x + 2
h = 2y + 1
⇒ x + 2 = 2y + 1
⇒ x + 2 - 1 = 2y
⇒ x - 1 = 2y
$\Rightarrow \frac{x-1}{2} = y$
And $x^2 + y^2 = h^2$
$\Rightarrow x^2 + \left(\frac{x-1}{2}\right)^2 = (x+2)^2$
⇒ x^2 - 15x + x - 15 = 0
⇒ x^2 - 15x + x - 15 = 0
⇒ (x - 15)(x + 1) = 0
⇒ x = 15 or x = -1
Base = 15 cm
Altitude = $\frac{x+1}{2} = 8\text{cm}$
Height, $h = 2 \times 8 + 1 = 17\text{cm}$

58. We have given,
$\left(\frac{2x}{x-5}\right)^2 + 5\left(\frac{2x}{x-5}\right) - 24 = 0$
Let $\frac{2x}{(x-5)}$ be y

$\therefore \quad y^2+5y-24=0$

Now factorise,

$y^2+8y-3y-24=0$

$y(y+8)-3(y+8)=0$

$(y+8)(y-3)=0$

$y=3,-8$

Putting y=3

$\frac{2x}{x-5}=3$

2x = 3x - 15

x = 15

Putting y = -8

$\frac{2x}{x-5}=-8$

2x = -8x + 40

10x = 40

x = 4

Hence, x is 15 , 4

59. Let the boy and his sister's ages be 'x' years and 'y' years, respectively

According to the question,

x + y = 25 ...(i)

and xy = 150

or, y = $\frac{150}{x}$ ...(ii)

Using equation (ii) in equation (i), we get

$x+\frac{150}{x}$ = 25

⇒ x^2 - 25x + 150 = 0

⇒ x^2 - 15x - 10x + 150 = 0

⇒ x(x - 15) -10(x - 15) = 0

⇒ x - 15 = 0 or x - 10 = 0

⇒ x = 15 or x = 10

When x = 15 i.e., boy's age is 15 years.

Then, sister's age, y = $\frac{150}{15}$ = 10 years

When x = 10 i.e, boy's age is 10 years

Then, sister's age, y = $\frac{150}{10}$ = 15 years

60. Let the time taken by smaller diameter tap be x hrs.

Time taken by larger diameter tap is (x - 2) hrs.

Therefore $\frac{1}{x-2}$ + $\frac{1}{x}$ = $\frac{8}{15}$

⇒ 15(2x - 2) = 8x(x - 2)

⇒ $8x^2$ - 46x + 30 = 0

⇒ $4x^2$ - 23x + 15 = 0

⇒(4x - 3)(x - 5) = 0

⇒ x = $\frac{3}{4}$, x = 5

61. Perimeter = 82 m

⇒ 2 (l +b) = 82 m

or, l + b = 41 m

Area = 400 m^2

⇒ l × b = 400 m^2

Let length be x m. Then,

breadth = (41- x) m

Now, $x(41-x)=400$

$41x-x^2=400$

$x^2-41x+400=0$

(x - 16)(x - 25) = 0

x = 16 or x = 25
Hence, if length = 16 m, then breadth = 25 m
or, if length = 25 m, then breadth = 16 m

62. $\frac{1}{a+b+x} - \frac{1}{x} = \frac{1}{a} + \frac{1}{b}$

$\frac{x-a-b-x}{x(a+b+x)} = \frac{b+a}{ab}$

$-ab = x^2 + (a + b)x$

$x^2 + (a + b)x + ab = 0$

(x + a) (x + b) = 0

x = -a, x = -b

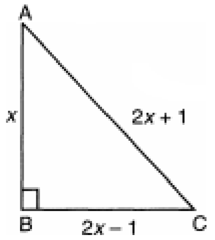
63. Then, according to given condition, we have
x + y = 34 ...(i)
and (x - 3) (y + 2) = 260 ...(ii)
On substituting the value of y from equation (i) in equation (ii), we get
(x - 3)(34 - x + 2) = 260 ...[∵ y = 34 - x]
⇒ (x - 3)(36 - x) = 260
⇒ $36x - x^2 - 108 + 3x = 260$
⇒ $x^2 - 39x + 368 = 0$
⇒ $x^2 - (16x + 23x) + 368 = 0$
⇒ $x^2 - 16x - 23x + 368 = 0$
⇒ x(x - 16) - 23(x - 16) = 0
⇒ (x - 16)(x - 23) = 0
⇒ x = 16 or x = 23
When x = 16, then y = 34 - 16 = 18
When x = 23, then y = 34 - 23 = 11
So, the numbers are 16, 18 or 23, 11

64. Let sides of two squares be a cm and b cm
Sum of areas of squares = $a^2 + b^2$
Sum of Perimater = 4a + 4b
A.T.Q $a^2 + b^2 = 544$
4a - 4b = 32
or, a - b = 8
a = b + 8
⇒ $a^2 + b^2 = 544$
$(b + 8)^2 + b^2 = 544$
⇒ $b^2 + 64 + 16b + b^2 = 544$
⇒ $2b^2 + 16b + 64 = 544$
⇒ $b^2 + 8b + 32 = 272$
⇒ $b^2 + 8b - 240 = 0$
⇒ $b^2 + 20b - 12b - 240 = 0$
⇒ b(b + 20) - 12(b + 20) = 0
b = 12 or b = -20
Sides cant be -ve
b = 12
a = 20
Therefore, sides of two squares are 20 cm and 12 cm respectively

65. Let the present age of father be x years.
Son's present age = (45 - x) years.
Five years ago:
Father's age = (x - 5) years
Son's age = (45 - x - 5) years = (40 - x) years.

According to question,
∴ (x - 5) (40 - x) = 124
⇒ 40x - x^2 - 200 + 5x = 124
⇒ x^2 - 45x + 324 = 0
Spilting the middle term,
⇒ x^2 - 36x - 9 x + 324 = 0
⇒ x(x - 36) - 9(x - 36) = 0
⇒ (x - 9)(x - 36) = 0
⇒ x = 9, or 36
We can't take father age as 9 years
So, x = 36, we have
Father's present age = 36 years
Son's present age = 9 years
Hence, Father's present age = 36 years and Son's present age = 9 years.

66. Let the ten's digit be x and the one's digit be y.
The number will be 10x + y
Given, a product of digits is 24
∴ xy = 24
or, y = $\frac{24}{x}$ ...(i)
Given that when 18 is subtracted from the number, the digits interchange their places.
∴ 10x + y - 18 = 10y + x
or, 9x - 9y = 18
Substituting y from equation (i) in equation (ii), we get
$9x - 9\left(\frac{24}{x}\right)$ = 18
or, $x - \frac{24}{x}$ = 2
or, x^2 - 24 - 2x = 0
or, x^2 - 2x - 24 = 0
or, x^2 - 6x + 4x - 24 = 0
or, x(x - 6) + 4(x - 6) = 0
or, (x - 6)(x + 4) = 0
or, x - 6 = 0 and x + 4 = 0
or, x = 6 and x = -4
Since, the digit cannot be negative, so, x = 6
Substituting x = 6 in equation (i), we get
y = $\frac{24}{6}$ = 4
∴ The number = 10(6) + 4 = 60 + 4 = 64

67. Let altitude of triangle be x.
∴ hypotenuse of triangle = 2x + 1 and base of triangle = 2x - 1.
Using Pythagoras theorem,
$(2x+1)^2 = x^2 + (2x-1)^2$
or, $4x^2 + 1 + 4x = x^2 + 4x^2 + 1 - 4x$
or, $x^2 - 8x = 0$
or, x(x - 8) = 0


either x = 0 or x - 8 = 0

Rejecting x = 0,∴ x = 8

hypotenuse of triangle 2 ×8 + 1 = 17 cm and base of triangle 2 × 8 - 1 = 15 cm

68. We have,

$\frac{2}{x+1}+\frac{3}{2(x-2)}=\frac{23}{5x}$

$\frac{2x}{x+1}+\frac{3x}{2(x-2)}=\frac{23}{5}$

Taking LCM

$\frac{2\times 2x(x-2)+3x(x+1)}{2(x+1)(x-2)}=\frac{23}{5}$

$\frac{4x^2-8x+3x^2+3x}{2\left(x^2-x-2\right)}=\frac{23}{5}$

$\frac{7x^2-5x}{2\left(x^2-x-2\right)}=\frac{23}{5}$

By cross multiplication,

$35x^2-25x=46x^2-46x-92$

$11x^2-21x-92=0$

$\therefore \quad x=\frac{21\pm\sqrt{(-21)^2-4(11)(-92)}}{2\times 11}$

$=\frac{21\pm\sqrt{441+4048}}{22}$

$=\frac{21\pm\sqrt{4489}}{22}$

$=\frac{21\pm 67}{22}$

$x=\frac{21+67}{22}$ or $x=\frac{21-67}{22}$

$\therefore \quad x=4,\frac{-23}{11}$

69. Let two numbers of x and y

According to the question,

x + y = 45

⇒ y = 45 - x ...(i)

And (x - 5)(y - 5) = 124

(x - 5)[(45 - x) - 5] = 124 [From equation (i)]

(x - 5)(40 - x) = 124

40x - 200 - x^2 + 5x = 124

x^2 - 45x + 324 = 0

x^2 - 36x - 9x + 324 = 0

x(x - 36) -9(x - 36) = 0

(x - 36)(x - 9) = 0

x - 36 = 0 or x - 9 = 0

x = 36 or x = 9

Now from equation (i),

When x = 36, then y = 45 - 36 = 9

When x = 9, then y = 45 - 9 = 36

Hence, the numbers are 9 and 36

70. The given equation is:

$\frac{3}{x+1}-\frac{1}{2}=\frac{2}{3x-1}$

$\Rightarrow \frac{3}{x+1}-\frac{2}{3x-1}=\frac{1}{2}$

$\Rightarrow \frac{3(3x-1)-2(x+1)}{(x+1)(3x-1)}=\frac{1}{2}$ (By cross multiplication method)

$\Rightarrow \frac{9x-3-2x-2}{3x^2-x+3x-1}=\frac{1}{2}$

$\Rightarrow \frac{7x-5}{3x^2+2x-1}=\frac{1}{2}$

⇒ 14x - 10 = $3x^2$ + 2x - 1

⇒ $3x^2$ + 2x - 1 - 14x +10 = 0

⇒ $3x^2$ - 12x + 9 = 0

⇒ x^2 - 4x + 3 = 0

Now by factorization method we have,

$x^2 - 3x - x + 3 = 0$

$\Rightarrow x(x - 3) -1(x - 3) = 0$

$\Rightarrow (x - 3)(x - 1) = 0$

$\Rightarrow x - 3 = 0$ or $x - 1 = 0$

Therefore either x = 3 or x = 1

71. There is an error in the question, so full marks to be awarded to the Candidates, who attempted.

72. Given

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = 0$

Let $\frac{x-1}{2x+1}$ be y so $\frac{2x+1}{x-1} = \frac{1}{y}$

$\therefore$ Substituting this value

$y + \frac{1}{y} = 2$ or $\frac{y^2+1}{y} = 2$

or $y^2 + 1 = 2y$

or $y^2 - 2y + 1 = 0$

or $(y - 1)^2 = 0$

Putting $y = \frac{x-1}{2x+1}$,

$\frac{x-1}{2x+1} = 1$ or $x - 1 = 2x + 1$

or x = -2