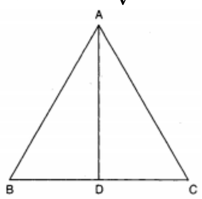
Solution

QUADRATIC EQUATIONS WS 3

Class 10 - Mathematics

1. According to the question,
   quadratic equation is $2x^2 - 6x + a = 0$
   $\therefore \alpha, \beta$ are roots of the equation
   $\therefore \alpha + \beta = \frac{-b}{a}$
   $\Rightarrow \alpha + \beta = -\frac{-6}{2}$
   $\Rightarrow \alpha + \beta = 3$
   $\Rightarrow \alpha = 3 - \beta$ ...(i)
   Also $2\alpha + 5\beta = 12$
   $\Rightarrow 2(3 - \beta) + 5\beta = 12$ [using equation (i)]
   $\Rightarrow 6 - 2\beta + 5\beta = 12$
   $\Rightarrow 3\beta = 6$
   $\Rightarrow \beta = 2$
   where β = 2, eq. (i) becomes $\alpha = 3 - 2 = 1$
   Now, product of roots = $\frac{c}{a}$
   $\Rightarrow \alpha \cdot \beta = \frac{a}{2} \Rightarrow 1 \times 2 = \frac{a}{2} \Rightarrow a = 4.$
2. Let ABC be the isosceles triangle in which AB = AC = 13 cm.
   Draw AD perpendicular from A on BC. Let BC = 2x cm. Then, BD = DC = x cm.
   In $\triangle ABD$,
   $AB^2 = AD^2 + BD^2$
   $\Rightarrow 13^2 = AD^2 + x^2$
   $\Rightarrow AD = \sqrt{13^2 - x^2} = \sqrt{169 - x^2}$
   

   Now, Area = 60 cm^2
   $\Rightarrow \frac{1}{2}(BC \times AD) = 60$
   $\Rightarrow \frac{1}{2}\left\{\left(2x \times \sqrt{169 - x^2}\right\} = 60$
   $\Rightarrow x\sqrt{169 - x^2} = 60$
   Squaring both sides,
   $\Rightarrow x^2(169 - x^2) = 3600$
   $\Rightarrow x^4 - 169x^2 + 3600 = 0$
   $\Rightarrow (x^2 - 144)(x^2 - 25) = 0$
   $\Rightarrow x^2 = 144$ or, $x^2 = 25$
   $\Rightarrow$ x = 12 or, x = 5
   If x = 12 $\Rightarrow$Base = 2x = 24cm
   If x = 5 $\Rightarrow$Base = 2x = 10cm
   Therefore, Base = 24 cm or 10 cm
3. Let the denominator be y, then numerators = y - 3
   So the fraction be $\frac{y-3}{y}$
   By the given condition, new fraction = $\frac{y-3+2}{y+2}$
   $= \frac{y-1}{y+2}$
   $\frac{y-3}{y} + \frac{y-1}{y+2} = \frac{29}{20}$
   $\frac{(y-3)(y+2)+y(y-1)}{y(y+2)} = \frac{29}{20}$

$20[(y-3)(y+2)+y(y-1)]=29\left(y^2+2y\right)$
$20[(y^2-3y+2y-6)+(y^2-y)]=29\left(y^2+2y\right)$
$20\left(y^2-y-6+y^2-y\right)=29y^2+58y$
$20\left(2y^2-2y-6\right)=29y^2+58y$
$11y^2-98y-120=0$
$11y^2-110y+12y-120=0$
$(11y+12)(y-10)=0$
$\therefore y=10$
$\therefore$ The fraction is $\frac{7}{10}$

4. The given equation is:
   $\frac{2}{x^2}-\frac{5}{x}+2=0$
   $\Rightarrow$ 2 - 5x + $2x^2$ = 0
   $\Rightarrow$ $2x^2$ - 5x + 2 = 0
   On multiplying each term by 2, we obtain:
   $4x^2$ - 10x + 4 = 0
   $\Rightarrow$ $4x^2$ - 10x = -4
   Adding $\left(\frac{5}{2}\right)^2$ on both sides, we have:
   $(2x)^2$ - 2 $\times$ 2x $\times\frac{5}{2}+\left(\frac{5}{2}\right)^2=-4+\left(\frac{5}{2}\right)^2$
   $\Rightarrow\left(2x-\frac{5}{2}\right)^2=-4+\frac{25}{4}$
   $\Rightarrow\left(2x-\frac{5}{2}\right)^2=\frac{9}{4}$
   $\Rightarrow$ 2x - $\frac{5}{2}=\pm\frac{3}{2}$
   Therefore, either 2x = $=\frac{5}{2}+\frac{3}{2}$ or 2x = $\frac{5}{2}-\frac{3}{2}$
   $\Rightarrow$ 2x = 4 or 2x = 1
   $\Rightarrow$ x = 2 or x = $\frac{1}{2}$
   Hence, the roots of given equation are 2 and $\frac{1}{2}$.
5. Let digit at unit's place be x and digit at ten's place be y
   Therefore, Number = 10y + x
   According to given situation we have,
   10y + x = 7(x + y)
   $\Rightarrow$10y + x = 7x + 7y
   $\Rightarrow$6x = 3y
   $\Rightarrow$y =2x ....(i)
   Also 10y + x = 3xy - 12
   $\Rightarrow$10 $\times$ 2x + x = 3x$\cdot$ 2x - 12
   $\Rightarrow$$6x^2$ - 21x -12 = 0 $\Rightarrow$ $2x^2$ - 7x - 4 = 0
   $\Rightarrow$$2x^2$ - 8x + x - 4 = 0
   Factorize above quadratic equation we get,
   2x(x - 4) + 1(x - 4) = 0
   $\Rightarrow$(x - 4)(2x + 1) = 0
   $\Rightarrow$x =4 or x = -$\frac{1}{2}$(rejected)
   When x = 4, y = 2 $\times$4 = 8
   $\therefore$Number is 10$\times$8 + 4 = 84.
6. Let the consecutive multiples of 7 be x and (x + 7).
   According to the question ;
   x^2 + $(x + 7)^2$ = 1225
   $\Rightarrow$ x^2 + x^2 + 14x + 49 = 1225
   $\Rightarrow$ $2x^2$ + 14x - 1176 = 0
   $\Rightarrow$ x^2 + 7x - 588 = 0 (dividing both sides by 2)

$\Rightarrow$ $x^2 + 28x - 21x - 588 = 0$
$\Rightarrow$ x(x + 28) - 21(x + 28) = 0
$\Rightarrow$ x + 28 = 0 or x - 21 = 0
$\Rightarrow$ x = -28 or x = 21 (both values are accepted as both are multiples of 7)
When x = -28
x + 7 = -28 + 7 = -21
When x = 21
x + 7 = 21 + 7 = 28
Hence, the required numbers are 21, 28 or -21, -28

7. Let the natural number be x.
Then, its positive square root will be $\sqrt{x}$
According to the question,
x + $\sqrt{x}$ = 132 ....(1) (sum of the number & it's square root is 132)
Let $\sqrt{x}$ =y $\Rightarrow x = y^2$
Hence, from (1), we have :-
$\Rightarrow$ $y^2 + y = 132$
$\Rightarrow$ $y^2 + y - 132 = 0$
$\Rightarrow$ $y^2 + 12y - 11y - 132 = 0$
$\Rightarrow$ y(y + 12) - 11(y + 12) = 0
$\Rightarrow$ (y + 12)(y - 11) = 0
$\Rightarrow$ y + 12 = 0 or y - 11 = 0
$\Rightarrow$ y = -12 or y = 11
Square root of a number cannot be negative, y $\neq$ -12 (as y =$\sqrt{x}$)
Hence, y = 11
$\Rightarrow$ $\sqrt{x}$ = 11 $\Rightarrow x = 11^2$
$\Rightarrow$ x = 121
Hence, the required natural number is x = 121.

8. $\because$ one root of eqn is $\frac{5}{2}$
So, $2x^2 - 8x - k = 0$
Putting x = $\frac{5}{2}$
$2 \times \left(\frac{5}{2}\right)^2 - 8 \times \frac{5}{2} - k = 0$
$\frac{25}{2} - 4 \times 5 - k = 0$
$\frac{25}{2} - 20 = k$
$k = \frac{-15}{2}$
Now for second root
$\because \alpha + \beta = \frac{-b}{a}$
$\frac{5}{2} + \beta = \frac{-(-8)}{2}$
$\beta = 4 - \frac{5}{2} = \frac{8-5}{2} = \frac{3}{2}$

9. The given quadratic equation is:
$(k + 1)x^2 + 2(k + 3)x + (k + 8) = 0$
Here, a = k + 1, b = 2 (k + 3) and c = k + 8
We know that $D = b^2 - 4ac$
Therefore,
$D = [2(k + 3)]^2 - 4 \times (k + 1) \times (k + 8)$
$= 4[k + 3]^2 - 4[k^2 + 8k + k + 8]$
$= 4[k^2 + 9 + 6k] - 4[k^2 + 9k + 8]$
$= 4k^2 + 36 + 24k - 4k^2 - 36k - 32$
= -12k + 4
Hence, D = -12k + 4
It is given that the given equation has real and equal roots, therefore Discrimnanat is equal to zero i.e.,

D = 0

⇒ -12k + 4 = 0

⇒ 12k = 4

⇒ $k = \frac{4}{12} = \frac{1}{3}$

$\therefore k = \frac{1}{3}$

10. Let the required numbers be x and (x - 5).

Then, according to question we have:

$\frac{1}{x-5} - \frac{1}{x} = \frac{5}{14}$

⇒ $\frac{x-x+5}{x(x-5)} = \frac{5}{14}$

⇒ 70 = $5x^2$ - 25x

⇒ $5x^2$ - 25x - 70 = 0

⇒ x^2 - 5x - 14 = 0

By factorization method, we have:

x^2 - 7x + 2x - 14 = 0

⇒ x(x - 7) + 2(x - 7) = 0

⇒ (x + 2) (x - 7) = 0

Therefore, either (x + 2) = 0 or (x - 7) = 0

⇒ x = - 2 or x = 7

Since x is a natural number, x $\neq$ -2.

⇒ x = 7 and x - 5 = 7 - 5 = 2

Hence, the required numbers are 7 and 2.

11. We have,

$\frac{1}{x-2} + \frac{2}{x-1} = \frac{6}{x}$

⇒ $\frac{(x-1)+2(x-2)}{(x-2)(x-1)} = \frac{6}{x}$

⇒ $\frac{3x-5}{x^2-3x+2} = \frac{6}{x}$

⇒ $3x^2$ - 5x = $6x^2$ - 18x + 12

⇒ $3x^2$ - 13x + 12 = 0

⇒ $3x^2$ - 9x - 4x + 12 = 0

⇒ 3x(x - 3) - 4(x - 3) = 0

⇒ (x - 3)(3x - 4) = 0

⇒ x - 3 = 0 or 3x - 4 = 0

⇒ x = 3 or $x = \frac{4}{3}$

Hence , the roots of given quadratic equation are 3 and $\frac{4}{3}$.

12. Given;

$\frac{3}{x+1} + \frac{4}{x-1} = \frac{29}{4x-1}$

⇒ $\frac{3(x-1)+4(x+1)}{(x+1)(x-1)} = \frac{29}{4x-1}$

⇒ $\frac{3x-3+4x+4}{x^2-1} = \frac{29}{4x-1}$

⇒ $\frac{7x+1}{x^2-1} = \frac{29}{4x-1}$

⇒ $(7x+1)(4x-1) = 29\left(x^2-1\right)$

⇒ $28x^2$ - 7x + 4x - 1 = $29x^2$ - 29

⇒ $29x^2$ - $28x^2$ + 3x - 28 = 0

⇒ x^2 + 3x - 28 = 0

⇒ x^2 + 7x - 4x - 28 = 0

⇒ x(x + 7) - 4(x + 7) = 0

⇒ (x - 4) (x + 7) = 0

⇒ x - 4 = 0 or x + 7 = 0

⇒ x = 4 or x = -7

Hence, the factors are 4 and -7.

13. The given equation is:

$\frac{x-2}{x-3}+\frac{x-4}{x-5}=\frac{10}{3}$

By cross multiplication method we have,

$\frac{(x-2)(x-5)+(x-4)(x-3)}{(x-3)(x-5)}=\frac{10}{3}$

$\Rightarrow \frac{(x^2-7x+10)+(x^2-7x+12)}{(x^2-8x+15)}=\frac{10}{3}$

$\Rightarrow \frac{(2x^2-14x+22)}{(x^2-8x+15)}=\frac{10}{3}$

$\Rightarrow$ 3(2x^2 - 14x + 22) = 10(x^2 - 8x + 15) [by cross multiplication method]

$\Rightarrow$ 6x^2 - 42x + 66 = 10x^2 - 80x +150

$\Rightarrow$ 4x^2 - 38x + 84 = 0 $\Rightarrow$ 2x^2 - 19x + 42 = 0

$\Rightarrow$ 2x^2 - 12x - 7x + 42 = 0 $\Rightarrow$ 2x(x -6) - 7(x - 6) = 0

$\Rightarrow$ (x - 6)(2x - 7) = 0 $\Rightarrow$ x - 6 = 0 or 2x - 7 = 0

Either x = 6 or x = $\frac{7}{2}$.

Hence, the roots of given equation are 6 and $\frac{7}{2}$.

14. The given equation is:

$\frac{x}{x-1}+\frac{x-1}{x}=4\frac{1}{4}$

$\Rightarrow \frac{x}{x-1}+\frac{x-1}{x}=\frac{17}{4}$

put $\frac{x}{x-1}$ = y , we obtain

$y+\frac{1}{y}=\frac{17}{4}$

$\Rightarrow \frac{y^2+1}{y}=\frac{17}{4}$

$\Rightarrow$ 4y^2 + 4 = 17y

$\Rightarrow$ 4y^2 - 17y + 4 = 0

$\Rightarrow$ 4y^2 - 16y - y + 4 = 0

$\Rightarrow$ 4y(y - 4) -1(y - 4) = 0

$\Rightarrow$ (y - 4) (4y - 1) = 0

$\Rightarrow$ y - 4 = 0 or 4y - 1 = 0

Therefore, either y = 4 or y = $\frac{1}{4}$.

Now $\frac{x}{x-1}$ = y

$\Rightarrow \frac{x}{x-1}=4$ or $\frac{x}{x-1}=\frac{1}{4}$

$\Rightarrow$ x = 4x - 4 or 4x = x - 1

$\Rightarrow$ 3x = 4 or 3x = -1

Hence the values of x are $x=\frac{4}{3}$ and $x=\frac{-1}{3}$.

15. The given equation is:

$\frac{4}{x}$ - 3 = $\frac{5}{2x+3}$, x $\neq$0, -$\frac{3}{2}$

$\Rightarrow \frac{4-3x}{x}$ = $\frac{5}{2x+3}$

$\Rightarrow$(4 - 3x) (2x + 3) = 5x

$\Rightarrow$ 8x + 12 - 6x^2 - 9x = 5x

$\Rightarrow$ - 6x^2 - 6x + 12 = 0

$\Rightarrow$ - 6 (x^2 + x - 2) = 0

$\Rightarrow$ x^2 + x - 2 = 0

Factorize the above quadratic equation we get

x^2 + 2x - x - 2 = 0

$\Rightarrow$x(x + 2) - 1(x + 2) = 0

$\Rightarrow$(x + 2)(x - 1) = 0

Therefore, Either x + 2 = 0 or x - 1 = 0

$\Rightarrow$x = -2, 1.

16. Put $\frac{2x-1}{x+3}$ = y in the given equation we have,

2y - $\frac{3}{y}$ = 5

$\Rightarrow 2y^2 - 3 = 5y$

$\Rightarrow 2y^2 - 5y - 3 = 0$

By factorization method we have,

$2y^2 - 6y + y - 3 = 0$

$\Rightarrow$ 2y(y - 3) + (y - 3) = 0

$\Rightarrow$ (y - 3)(2y + l) = 0

Either y - 3 = 0 or 2y + 1 = 0

$\Rightarrow$ y = 3 or y = $-\frac{1}{2}$.

Now substitute y i.e.,

Case I: when y = 3 $\Rightarrow$ $\frac{2x-1}{x+3}$ = 3

$\Rightarrow$ 2x - 1 = 3(x + 3) [by cross multiplication]

$\Rightarrow$ 2x - 1 = 3x + 9

$\Rightarrow$ x = -10

Case II : when y = $\frac{-1}{2}$ $\Rightarrow$ $\frac{2x-1}{x+3} = \frac{-1}{2}$

$\Rightarrow$ 2(2x - 1) = -(x + 3)

$\Rightarrow$ 5x = -1

$\Rightarrow$ x = $\frac{-1}{5}$

Therefore, the roots of the given equation are -10 and $\frac{-1}{5}$.

17. Let the positive integer be x.

Then, the consecutive odd integer = x + 2.

Square of the positive integer = x^2

Given that the sum of the squares of the consecutive odd positive numbers is 970.

$\Rightarrow x^2 + (x+2)^2 = 970$

$\Rightarrow x^2 + x^2 + 4 + 4x = 970$

$\Rightarrow 2x^2 + 4x - 970 + 4 = 0$

$\Rightarrow 2x^2 + 4x - 966 = 0$

$\Rightarrow x^2 + 2x - 483 = 0$

$\Rightarrow x^2 + 23x - 21x - 483 = 0$

$\Rightarrow x(x+23) - 21(x+23) = 0$

$\Rightarrow (x-21)(x+23) = 0$

$\Rightarrow$ x - 21 = 0 or x + 23 = 0

$\Rightarrow$ x = 21 or x = -23

As x cannot be negative, therefore consecutive odd positive integers are 21 and 23.

18. Let the required numbers be x and 48 - x. Then,

$x(48 - x) = 432$

$\Rightarrow 48x - x^2 = 432$

$\Rightarrow x^2 - 48x + 432 = 0$

$\Rightarrow x^2 - 36x - 12x + 432 = 0$

$\Rightarrow$ x(x - 36) - 12(x - 36) = 0

$\Rightarrow$ (x - 36)(x - 12) = 0

$\Rightarrow$ x = 36 or x = 12

Hence, the required numbers are 12 and 36.

19. Let the breadth be x cm.

Length = (x + 8) cm.

According to question,

$\Rightarrow$ length $\times$ breadth = Area

$\Rightarrow$ length $\times$ breadth = 240

$\Rightarrow$ (x + 8)x = 240

$\Rightarrow x^2 + 8x - 240 = 0$

$\Rightarrow x^2 + 20x - 12x - 240 = 0$

$\Rightarrow$ x(x + 20) - 12(x + 20) = 0

$\Rightarrow$ (x + 20) (x - 12) = 0
$\Rightarrow$ x = 12 or, x = -20
Breadth cannot be negative. So, x = 12.
Therefore, L = x + 8 = 12 + 8 = 20 cm and B = 12 cm.

20. The given equation is $9x^2 - 24x + k = 0$.
Here, $a = 9, b = -24\ and\ c = k$.
$\therefore D = b^2 - 4ac$
$= (-24)^2 - (4)(9)(k)$
$= 576 - 36k$
The given equation will have real and equal roots, if
$D = 0$
$\Rightarrow 576 - 36k = 0$
$\Rightarrow$ k = 16
Putting k = 16 in the given equation, we get
$9x^2 - 24x + 16 = 0$
$\Rightarrow (3x - 4)^2 = 0$
$\Rightarrow$ 3x - 4 = 0
$\Rightarrow x = \frac{4}{3}$
Hence, both the roots of the given equation are equal to $\frac{4}{3}$.

21. Given,
$\frac{2}{(x+1)} + \frac{3}{2(x-2)} = \frac{23}{5x}$
Taking LCM, we get
$\Rightarrow \frac{4(x-2)+3(x+1)}{2(x+1)(x-2)} = \frac{23}{5x} \Rightarrow \frac{7x-5}{2(x^2-x-2)} = \frac{23}{5x}$
By cross multiplication
$\Rightarrow 5x(7x - 5) = 46(x^2 - x - 2)$
$\Rightarrow 35x^2 - 25x = 46x^2 - 46x - 92$
$\Rightarrow 46x^2 - 35x^2 - 46x + 25x - 92 = 0$
$\Rightarrow 11x^2 - 21x - 92 = 0$
$\Rightarrow 11x^2 - 44x + 23x - 92 = 0$
$\Rightarrow$ 11x(x - 4) + 23(x - 4) = 0
$\Rightarrow$ (x - 4)(11x + 23) = 0
$\Rightarrow$ x - 4 = 0 or 11x + 23 = 0
$\Rightarrow$ x = 4 or x = $\frac{-23}{11}$
Therefore, 4 or $\frac{-23}{11}$ are the roots of the given equation.

22. Let the base of the right triangle be x cm.
Then altitude = (x - 7) cm
Hypotenuse = 13 cm
By Pythagoras theorem
$(\text{Base})^2 + (\text{Altitude})^2 = (\text{Hypotenuse})^2$
$\implies x^2 + (x-7)^2 = (13)^2$
$\implies x^2 + x^2 - 14x + 49 = 169$
$\implies 2x^2 - 14x - 120 = 0$
$\implies 2(x^2 - 7x - 60) = 0$ or $x^2 - 7x - 60 = 0$
$\implies x^2 - 12x + 5x - 60 = 0$
$\implies$ x (x - 12) +5 (x - 12) = 0
$\implies$ (x + 5) (x - 12) = 0
Either x + 5 = 0 or x -12 = 0
$\implies$ x = -5, 12
Since side of the triangle cannot be negative. So, x =12 cm and x = - 5 is rejected.
Hence, length of the other two sides are 12cm, (12 - 7) = 5cm.

23. Let ABC be the given right angled triangle such that base = BC = x cm and hypotenuse AC = 25 cm.

Now, Perimeter = 60 cm

$\Rightarrow$ AB + BC + AC = 60

$\Rightarrow$ AB + x + 25 = 60

$\Rightarrow$ AB = 35 - x

By Pythagoras theorem, we have

$AB^2 + BC^2 = AC^2$

$\Rightarrow (35 - x)^2 + x^2 = 252$

$\Rightarrow 2x^2 - 70x + 600 = 0$

$\Rightarrow x^2 - 35x + 300 = 0$

$\Rightarrow x^2 - 20x - 15x + 300 = 0$

$\Rightarrow$ (x - 20) (x - 15) = 0 $\Rightarrow$ x = 20 or, x = 15

If x = 20, then AB = 35 - x = 15 and BC = x = 20.

$\therefore$ Area $= \frac{1}{2}(BC \times AB) = \frac{1}{2}(20 \times 15) = 150\text{cm}^2$

If x = 15, then AB = 35 - x = 20 and BC = x = 15

$\therefore$ Area $= \frac{1}{2}(BC \times AB) = \frac{1}{2}(15 \times 20) = 150\text{cm}^2$

Hence, Area = 150 cm^2.

24. We have,

$2x^2 - x + 9 = x^2 + 4x + 3$

or $2x^2 - x + 9 - x^2 - 4x - 3 = 0$

or $x^2 - 5x + 6 = 0$

Now on Substituting x = 2,

LHS = $x^2 - 5x + 6$

= $(2)^2 - 5(2) + 6$

= 10 - 10

= 0 = RHS

Again on Substituting x = 3, we have

LHS = $x^2 - 5x + 6$

= $(3)^2 - 5(3) + 6$

= 9 - 15 + 6

= 15 - 15

= 0 = RHS

Hence, x = 2 and x = 3 are the solutions of the given equation.

25. We have

$\sqrt{3}x^2 + 10x - 8\sqrt{3} = 0$

$\sqrt{3}x^2 + 12x - 2x - 8\sqrt{3} = 0$

$\sqrt{3}x(x + 4\sqrt{3}) - 2(x + 4\sqrt{3}) = 0$

$(\sqrt{3}x - 2)(x + 4\sqrt{3}) = 0$

$(\sqrt{3}x - 2) = 0$ or $(x + 4\sqrt{3}) = 0$

x = $-4\sqrt{3}$ or x = $\frac{2}{\sqrt{3}}$

Hence roots of equation are x = $-4\sqrt{3}$ or x = $\frac{2}{\sqrt{3}}$

26. The given equation is:

$(3k + 1)x^2 + 2(k + 1)x + 1 = 0$

This is of the form $ax^2 + bx + c = 0$, where

a = 3k + 1, b = 2(k + 1) = 2k + 2 and c = 1

As it is given that the given equation has real and equal roots, i.e., D = $b^2 - 4ac = 0$.

$\Rightarrow (2k + 2)^2 - 4(3k + 1)(1) = 0$

$\Rightarrow$ 4k^2 + 8k + 4 - 12k - 4 = 0

$\Rightarrow$ 4k^2 - 4k = 0

$\Rightarrow$ 4k(k - 1) = 0

Therefore, either 4k = 0 or k - 1 = 0

$\Rightarrow$ k = 0 or k = 1

Hence, the roots of given equation are 1 and 0.

27. Assume denominator = x then, numerator = x - 1

$\therefore$ Fraction = $\frac{x-1}{x}$

According to given situation, we have

$\frac{x-1+3}{x+3} = \frac{x-1}{x} + \frac{3}{28}$

$\Rightarrow \frac{x+2}{x+3} - \frac{x-1}{x} = \frac{3}{28}$

$\Rightarrow \frac{(x+2)x-(x-1)(x+3)}{(x+3)x} = \frac{3}{28}$

$\Rightarrow \frac{x^2+2x-\left(x^2+2x-3\right)}{x^2+3x} = \frac{3}{28}$

$\Rightarrow 3\times 28 = 3(x^2 + 3x)$

$\Rightarrow x^2 + 3x - 28 = 0$

Factorize the above quadratic equation, we get

$\Rightarrow$(x + 7)(x - 4) = 0 $\Rightarrow$x = -7 or x = 4

Rejecting x = -7 $\therefore$x = 4

$\therefore$ Fraction is $\frac{4-1}{4} = \frac{3}{4}$

28. Let the width of the path be x meters,

We know that, if length of rectangle = l, width of rectangle =b & width of path around it x then,

Area of path = $lb \ - (l-2x)(b-2x)$

$\Rightarrow$16 $\times$ 10 - (16 - 2x)(10 - 2x) = 120

$\Rightarrow$ 16 $\times$ 10 - {160 - 32x - 20x + 4x^2} = 120

$\Rightarrow$ 160 - 160 + 32x + 20x - 4x^2 = 120

$\Rightarrow$ -4x^2 + 52x - 120 = 0

$\Rightarrow$ 2x^2 - 26x + 60 = 0

$\Rightarrow$ x^2 - 13x + 30 = 0

$\Rightarrow$ x^2 - 10x - 3x + 30 = 0 $\Rightarrow$ x(x - 10) - 3(x - 10) = 0

$\Rightarrow$ (x - 10) (x - 3) = 0

$\Rightarrow$ x - 10 = 0 or x - 3 = 0

But x $\neq$10 ,as width of path can't be greater than width of rectangle. So, x = 3m

Hence, the width of the path = 3 m

29. The given equation is:

$\frac{3x-4}{7} + \frac{7}{3x-4} = \frac{5}{2}$

put $\frac{3x-4}{7}$ = y, we obtain

$y + \frac{1}{y} = \frac{5}{2}$

$\Rightarrow \frac{y^2+1}{y} = \frac{5}{2}$

$\Rightarrow$ 2y^2 + 2 = 5y

$\Rightarrow$ 2y^2 - 5y + 2 = 0

By Factorisation we have:

2y^2 - 4y - y + 2 = 0

$\Rightarrow$ 2y(y - 2) - 1(y - 2) = 0

$\Rightarrow$ (y - 2) (2y - 1) = 0

$\Rightarrow$ y - 2 = 0 or 2y - 1 = 0

Therefore, either y = 2 or y = $\frac{1}{2}$

Now, $y = \frac{3x-4}{7}$

$\Rightarrow \frac{3x-4}{7} = 2$ or $\frac{3x-4}{7} = \frac{1}{2}$

$\Rightarrow$ 3x - 4 = 14 or 6x - 8 = 7

$\Rightarrow$ 3x = 18 or 6x = 15

Therefore, $x = 6 \; or \; \frac{5}{2}$

30. $3^{x+2} + 3^{-x} = 10$

$3^x.3^2 + 3^{-x} = 10$

$\Rightarrow$ 9y + $\frac{1}{y}$ = 10 where $3^x = y$

$\Rightarrow 9y^2 - 9y - y + 1 = 0$

$\Rightarrow 9y(y-1) - 1(y-1) = 0$

$\Rightarrow (9y-1) = 0 \; or \; y - 1 = 0$

$\Rightarrow$ y = $\frac{1}{9}$ or y = 1

If $3^x = \frac{1}{9} \Rightarrow 3^x = (3)^{-2} \Rightarrow x = -2$

If $3^x = 1 = 3^0 \Rightarrow x = 0$

Hence , -2, 0 are the roots of given equation.

31. The given equation is $(k + 1)x^2 - 6(k + 1)x + 3(k + 9) = 0, k \neq -1$

For equal roots, discriminant vanishes. So,

$36(k+1)^2 - 4(k + 1) \times 3(k + 1) = 0$

$\Rightarrow 36(k^2 + 1 + 2k) - 12(k^2 + 10k + 9) = 0$

$\Rightarrow 36k^2 + 72k + 36 - 12x^2 - 120k - 10 = 0$

$\Rightarrow 24k^2 - 48k - 72 = 0$

$\Rightarrow k^2 - 2k - 3 = 0$

$\Rightarrow k = \frac{2 \pm \sqrt{4+12}}{2}$

$\Rightarrow k = \frac{2 \pm 4}{2}$

$\Rightarrow k = \frac{6}{2}, \frac{-2}{2}$

$\Rightarrow$ k = 3, -1

32. We have,

$\frac{x-1}{2x+1} + \frac{2x+1}{x-1} = \frac{5}{2}, x \neq -\frac{1}{2}, 1$

$\Rightarrow (x - 1)(x - 1) + (2x + 1)(2x + 1) = \frac{5}{2} (2x + 1)(x - 1)$ [Multiplying both sides by (2x + 1)(x - 1)]

$\Rightarrow x^2 + 1 - 2x + (2x)^2 + 2x + 2x + 1 = \frac{5}{2} [2x(x - 1) + 1(x - 1)]$

$\Rightarrow x^2 + 1 - 2x + 4x^2 + 4x + 1 = \frac{5}{2} [2x^2 - 2x + x - 1]$

$\Rightarrow 5x^2 + 2x + 2 = \frac{5}{2}[2x^2 - x - 1]$

$\Rightarrow 10x^2 + 4x + 4 = 5[2x^2 - x - 1]$

$\Rightarrow$ 4x + 5x + 4 + 5 = 0

$\Rightarrow$ 9x + 9 = 0

$\Rightarrow$ 9(x + 1) = 0

So, either x + 1 = 0 or, 9 = 0 which is not possible.

Thus, x = -1.

Hence, the given quadratic equation has repeated (equal) roots and are given by -1 and -1

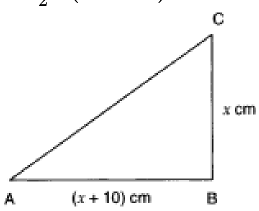
33. Let the altitude be x cm.

Therefore, Base = (x + 10) cm.

$\therefore$ Area $= \frac{1}{2}($ Base $\times$ Height $)$

$\Rightarrow$ Area $= \frac{1}{2}(x+10)x\text{cm}^2$

$\Rightarrow \frac{1}{2}x(x+10) = 600$



$\Rightarrow x^2 + 10x = 1200$

$\Rightarrow x^2 + 10x - 1200 = 0$

$\Rightarrow x^2 + 40x - 30x - 1200 = 0$

$\Rightarrow x(x + 40) - 30(x + 40) = 0$

$\Rightarrow (x + 40)(x - 30) = 0$

$\Rightarrow x = 30, -40$ (altitude can't be negative)

$\Rightarrow x = 30$

Hence, Base = (30 +10) cm = 40 cm and, Altitude = 30 cm.

34. According to the question,the given two equations are,

$ax^2 + bx + c = 0$ ....(i)

And $-ax^2 + bx + c = 0$ ....(ii)

The discriminant of equation (i) is given by

$D_1 = b^2 - 4ac$

Now, the discriminant of equation (ii) is given by

$D_2 = b^2 - 4(-a)c$

$\Rightarrow D_2 = b^2 + 4ac$

Now, $D_1 + D_2 = b^2 - 4ac + b^2 + 4ac$

$\Rightarrow D_1 + D_2 = 2b^2 \geqslant 0$ [since, b is real]

$\Rightarrow$ At least one of D_1 and D_2 is greater than or equal to zero.

Therefore, at least one of the two equations has real roots.

35. Let the successive multiples of 5 be 5x and 5x + 5. Then according to question we have,

$5x \times (5x + 5) = 300$

$\Rightarrow 25x^2 + 25x = 300$

$\Rightarrow 25x^2 + 25x - 300 = 0$

$\Rightarrow 25(x^2 + x - 12) = 0$

$\Rightarrow x^2 + x - 12 = 0$

Solve by factorization method we have,

$x^2 + 4x - 3x - 12 = 0$

$\Rightarrow x(x + 4) - 3(x + 4) = 0$

$\Rightarrow (x + 4)(x - 3) = 0$

Therefore, either x = -4 or x = 3

If x = -4, we have $5x = 5 \times (-4) = -20$ and $5x + 5 = 5 \times (-4) + 5 = -15$

Aslo If x = 3, we have $5x = 5 \times 3 = 15$ and $5x + 5 = 5 \times 3 + 5 = 20$

Hence, the required multiples of 5 are 15, 20. or -20, -15

36. Let the required natural number be x and (8 - x) and their product is 15

$x (8 - x) = 15$

$8x - x^2 = 15$

$x^2 - 8x + 15 = 0$

$x^2 - 5x - 3x + 15 = 0$

$x(x - 5) - 3(x - 5) = 0$

$(x - 5)(x - 3) = 0$

x = 5 or x = 3

Hence the required natural numbers are 5 and 3.

37. We have the following equation,

$25x (x + 1) = - 4$

$\Rightarrow 25x^2 + 25x = - 4$

$\Rightarrow 25x^2 + 25x + 4 = 0$

Factorise the equation,

$\Rightarrow 25x^2 + 20x + 5x + 4 = 0$

$\Rightarrow$ 5x(5x + 4) + 1(5x + 4) = 0

$\Rightarrow$ (5x + 4)(5x + 1) = 0

$\Rightarrow$ 5x + 4 = 0 or 5x + 1 = 0

$\Rightarrow x = -\frac{4}{5}$

or

$x = -\frac{1}{5}$

38. Given,

$\frac{x+3}{x-2} - \frac{1-x}{x} = \frac{17}{4}$

Taking LCM, we get

$\Rightarrow \quad \frac{x(x+3)-(1-x)(x-2)}{x(x-2)} = \frac{17}{4}$

$\Rightarrow \quad \frac{x^2+3x-(x-2-x^2+2x)}{x^2-2x} = \frac{17}{4}$

$\Rightarrow \quad \frac{2x^2+2}{x^2-2x} = \frac{17}{4}$

After cross multiplication,we get

$\Rightarrow$ $8x^2 + 8 = 17x^2 - 34x$

$\Rightarrow$ $9x^2 - 34x - 8 = 0$

$\Rightarrow$ $9x^2 - 36x + 2x - 8 = 0$

$\Rightarrow 9x(x-4)+2(x-4)=0$

$\Rightarrow$ (x - 4)(9x + 2) = 0 $\Rightarrow$ x - 4 = 0 or, 9x + 2 = 0 $\Rightarrow$ $x = 4$ or, $x = -\frac{2}{9}$

39. Let the length of the shortest side be x metres.

As per given condition

The hypotenuse of a grassy land in the shape of a right triangle is 1 metre more than twice the shortest side.

So, Hypotenuse = (2x + 1) metres

And if the third side is 7 metres more than the shortest side

So, third side = (x + 7) metres.

By Pythagoras theorem, we have

$(\text{Hypotenuse})^2$ = Sum of the squares of the remaining two sides

$\Rightarrow$ $(2x + 1)^2 = x^2 + (x + 7)^2$

$\Rightarrow$ $4x^2 + 4x + 1 = x^2 + x^2 + 14x + 49$

$\Rightarrow$ $4x^2 + 4x + 1 = 2x^2 + 14x + 49$

$\Rightarrow$ $2x^2 - 10x - 48 = 0$

$\Rightarrow$ $x^2 - 5x - 24 = 0$

$\Rightarrow$ $x^2 - 8x + 3x - 24 = 0$

$\Rightarrow x(x-8)+3(x-8)=0$

$\Rightarrow$ (x - 8) (x + 3) = 0

$\Rightarrow$ x = 8 , - 3

$\Rightarrow$x = 8 [$\because$ x = -3 is not possible]

Hence, the lengths of the sides of the grassy land are 8 metres, 17 metres and 15 metres.

40. $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$

$\Rightarrow \left(\frac{x+1}{x-1} - 1\right) + \left(\frac{x-2}{x+2} - 1\right) + \left(\frac{2x+3}{x-2} - 2\right) = 0$

$\Rightarrow \left(\frac{x+1-x+1}{x-1}\right) + \left(\frac{x-2-x-2}{x+2}\right) + \left(\frac{2x+3-2x+4}{x-2}\right) = 0$

$\Rightarrow \left(\frac{2}{x-1}\right) + \left(\frac{-4}{x+2}\right) + \left(\frac{7}{x-2}\right) = 0$

$\Rightarrow \frac{2}{x-1} + \frac{-4x+8+7x+14}{x^2-4} = 0$

$\Rightarrow \frac{2}{x-1} + \frac{3x+22}{x^2-4} = 0$

$\Rightarrow \frac{2(x^2-4)+(x-1)(3x+22)}{(x-1)(x^2-4)} = 0$

$\Rightarrow 2x^2 - 8 + 3x^2 + 22x - 3x - 22 = 0$

$\Rightarrow 5x^2 + 19x - 30 = 0$

$\Rightarrow 5x^2 + 25x - 6x - 30 = 0$

$\Rightarrow 5x(x+5)-6(x+5)=0$
$\Rightarrow (x+5)(5x-6)=0$
$\Rightarrow$ x + 5 = 0 or 5x - 6 = 0
$\Rightarrow$ x = -5 or $x=\frac{6}{5}$

41. According to the question,
$(x-5)(x-6)=\frac{25}{(24)^2}$
$\Rightarrow x(x-6)-5(x-6)=\frac{25}{(24)^2}$
$\Rightarrow x^2-6x-5x+30-\frac{25}{(24)^2}=0$
$\Rightarrow x^2-11x+30-\frac{25}{(24)^2}=0$
$\Rightarrow x^2-11x+\frac{30\times 24^2-25}{(24)^2}=0$
$\Rightarrow x^2-11x+\frac{30\times 576-25}{(24)^2}=0$
$\Rightarrow x^2-11x+\frac{17280-25}{(24)^2}=0$
$\Rightarrow x^2-\frac{264x}{24}+\frac{145}{24}\times\frac{119}{24}=0$
$\Rightarrow x^2-\left(\frac{145}{24}+\frac{119}{24}\right)x+\frac{145}{24}\times\frac{119}{24}=0$
$\Rightarrow x^2-\frac{145}{24}x-\frac{119}{24}x+\frac{145}{24}\times\frac{119}{24}=0$
$\Rightarrow x\left(x-\frac{145}{24}\right)-\frac{119}{24}\left(x-\frac{145}{24}\right)=0$
$\Rightarrow \left(x-\frac{145}{24}\right)\left(x-\frac{119}{24}\right)=0$
$\Rightarrow x-\frac{145}{24}=0$ or $x-\frac{119}{24}=0$
$\Rightarrow x=\frac{145}{24}$ or $x=\frac{119}{24}$

42. Let the consecutive positive even numbers be x & (x + 2).
According to the question ;
$x^2 + (x + 2)^2 = 452$ (sum of squares of the numbers is 452)
$\Rightarrow x^2 + x^2 + 4x + 4 = 452$
$\Rightarrow 2x^2 + 4x - 448 = 0$
$\Rightarrow x^2 + 2x - 224 = 0$ (dividing both sides by 2)
$\Rightarrow x^2 - 14x + 16x - 224= 0$
$\Rightarrow$ x(x - 14) + 16(x - 14) = 0
$\Rightarrow$ x- 14 = 0 or x + 16 = 0
$\Rightarrow$ x = 14 or x = -16
Since x is a positive number, x $\neq$ -16
$\Rightarrow$ x = 14
$\Rightarrow$ x + 2 = 14 + 2 = 16
Hence, the required positive even numbers are x & (x+2) i.e. 14 and 16.

43. Given,
$x=\frac{1}{2-\frac{1}{2-\frac{1}{2-x}}}$
Now, $2-\frac{1}{2-x}\Rightarrow\frac{2(2-x)-1}{(2-x)}\Rightarrow\frac{4-2x-1}{2-x}\Rightarrow\frac{3-2x}{2-x}$
and $2-\frac{1}{2-\frac{1}{2-x}}\Rightarrow 2-\frac{2-x}{3-2x}\Rightarrow\frac{2(3-2x)-(2-x)}{3-2x}\Rightarrow\frac{4-3x}{3-2x}$
$Hence,\ x=\frac{3-2x}{4-3x}$
Cross multiplication,
$\Rightarrow$ x(4 - 3x) = (3 - 2x)
$\Rightarrow 4x - 3x^2 = 3 - 2x$
$\Rightarrow 3x^2 - 6x + 3 = 0 \Rightarrow x^2 - 2x + 1 = 0 \Rightarrow (x - 1)^2 =0 \Rightarrow x = 1, 1$

44. Let us first split the middle term – 5x as –2x, –3x [because (–2x) $\times$ (–3x) = $6x^2 = (2x^2) \times 3$]
So, $2x^2 – 5x + 3 = 2x^2 – 2x – 3x + 3$ = 2x(x – 1) –3(x – 1) = (2x – 3)(x – 1)

Now, $2x^2 - 5x + 3 = 0$ can be rewritten as $(2x - 3)(x - 1) = 0$

So, the values of x for which $2x^2 - 5x + 3 = 0$ are the same for which $(2x - 3)(x - 1) = 0$

i.e., either $2x - 3 = 0$ or $x - 1 = 0$.

Now, $2x - 3 = 0$ gives $x = \frac{3}{2}$ and $x - 1 = 0$ gives x = 1.

So, $x = \frac{3}{2}$ and x = 1 are the roots of the given quadratic equation.

45. Given, $x^2 + 5x - (a^2 + a - 6) = 0$

splitting $a^2 + a - 6$

$\Rightarrow x^2 + 5x - (a^2 + 3a - 2a - 6) = 0$

$\Rightarrow x^2 + 5x - [a(a + 3) - 2 (a + 3)] = 0$

$\Rightarrow x^2 + 5x - (a + 3)(a - 2) = 0$

Now splitting the middle term

$\Rightarrow x^2 + (a + 3)x -(a - 2)x - (a + 3) (a - 2) = 0$

$\Rightarrow x [x + (a + 3)] - (a - 2)[x + (a + 3)] = 0$

$\Rightarrow [x + (a + 3)] [x - (a - 2)] = 0$

$\Rightarrow x + (a + 3) = 0$ or $x - (a - 2) = 0$

Therefore, $x = -(a + 3)\ or\ (a - 2)$

46. According to the question, the sum of the squares of two consecutive odd numbers is 394.

Let the odd no. be 2x + 1

$\therefore$ Consecutive odd number = 2x + 1 + 2 = 2x + 3

Now, according to question

$(2x + 1)^2 + (2x + 3)^2 = 394$

$\Rightarrow 4x^2 + 4x + 1 + 4x^2 + 12x + 9 = 394$

$\Rightarrow 8x^2 + 16x - 384 = 0$

$\Rightarrow x^2 + 2x - 48 = 0$

$\Rightarrow x^2 + 8x - 6x - 48 = 0$

$\Rightarrow x(x + 8) - 6(x + 8) = 0$

$\Rightarrow (x - 6)(x + 8)$

$\Rightarrow x = 6, x = -8$

Rejecting -8, hence

$\therefore$ 1st number = $2 \times 6 + 1$ = 13

and second odd number = 15

47. We have,

$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} = \sqrt{4x^2 - 14x + 16}$

$\Rightarrow \sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} - \sqrt{4x^2 - 14x + 16} = 0$

Substituting x = 3 in the given equation, we get

$\sqrt{(3)^2 - 4(3) + 3} + \sqrt{(3)^2 - 9} - \sqrt{4(3)^2 - 14 \times 3 + 16}$

$= \sqrt{9 - 12 + 3} + \sqrt{9 - 9} - \sqrt{36 - 42 + 16}$

$\sqrt{12 - 12} + \sqrt{0} - \sqrt{52 - 42}$

$= 0 + 0 - \sqrt{10}$

$= -\sqrt{10} \neq 0$

For x = 3,

$\sqrt{x^2 - 4x + 3} + \sqrt{x^2 - 9} - \sqrt{4x^2 - 14x + 16} \neq 0$

Therefore, x = 3 is not a solution or root of the given equation.

48. Let the base of the right-angled triangle be 'x' m.

Hence, Altitude = (x + 7) m [$\because$altitude is 7m. greater than base]

$\because$ Area = $\frac{1}{2}$ (Base $\times$ Altitude)

$\Rightarrow$ Area = $\frac{1}{2}$ (x)(x + 7) . m^2

$\Rightarrow \frac{1}{2}$ x (x + 7) = 165[$\because$ Given, Area = 165 m^2]

$\Rightarrow x(x + 7) = 165 \times 2$

$\Rightarrow x^2 + 7x = 330$

$\Rightarrow x^2 + 7x - 330 = 0$

$\Rightarrow x^2 + 22x - 15x - 330 = 0$

$\Rightarrow x(x + 22) - 15(x + 22) = 0$

$\Rightarrow (x + 22)(x - 15) = 0$

$\Rightarrow x - 15 = 0$ [$\because$ Length can-not be negative, $\therefore x + 22 \neq 0$]

$\Rightarrow x = 15$

Hence, Base of the right-angled triangle = x=15 m

and Altitude of the right-angled triangle= x + 7 = 22 m

49. We have,

$\frac{1}{x-3} + \frac{2}{x-2} = \frac{8}{x}$

$\Rightarrow \frac{1(x-2)+2(x-3)}{(x-3)(x-2)} = \frac{8}{x}$

$\Rightarrow \frac{x-2+2x-6}{(x-3)(x-2)} = \frac{8}{x}$

$\Rightarrow \frac{3x-8}{x^2-2x-3x+6} = \frac{8}{x}$

$\Rightarrow \frac{3x-8}{x^2-5x+6} = \frac{8}{x}$

Cross multiply,

$\Rightarrow x(3x-8) = 8(x^2-5x+6)$

$\Rightarrow 3x^2 - 8x = 8x^2 - 40x + 48$

$\Rightarrow 8x^2 - 40x + 48 - 3x^2 + 8x = 0$

$\Rightarrow 5x^2 - 32x + 48 = 0$

Factorise the equation,

$\Rightarrow 5x^2 - 20x - 12x + 48 = 0$

$\Rightarrow 5x(x - 4) - 12(x - 4) = 0$

$\Rightarrow (5x - 12)(x - 4) = 0$

$\Rightarrow (5x - 12) = 0$ or $(x - 4) = 0$

$\Rightarrow x = \frac{12}{5}$ or x = 4

50. The given equation is; $\frac{x}{x-1} + \frac{x-1}{x} = 4\frac{1}{4}, x \neq 0, 1$

$\Rightarrow \frac{x^2+(x-1)^2}{x(x-1)} = \frac{17}{4}$

$\Rightarrow \frac{x^2+x^2-2x+1}{x^2-x} = \frac{17}{4}$

$\Rightarrow \frac{2x^2-2x+1}{x^2-x} = \frac{17}{4}$

$\Rightarrow 8x^2 - 8x + 4 = 17x^2 - 17x$

$\Rightarrow 9x^2 - 9x - 4 = 0$

$\Rightarrow 9x^2 - 12x + 3x - 4 = 0$

$\Rightarrow 3x(3x - 4) + 1(3x - 4) = 0$

$\Rightarrow (3x - 4)(3x + 1) = 0$

$\Rightarrow 3x - 4 = 0$ or $3x + 1 = 0$

$\Rightarrow x = \frac{4}{3}$ or $x = -\frac{1}{3}$

Hence, $\frac{4}{3}$ and $-\frac{1}{3}$ are the roots of the given equation.

51. Given,

$(x-3)(x-4) = \frac{34}{33^2}$

$\Rightarrow x^2 - 4x - 3x + 12 = \frac{34}{33^2}$

$\Rightarrow \quad x^2 - 7x + 12 - \frac{34}{33^2} = 0$

$\Rightarrow \quad x^2 - 7x + \frac{13034}{33^2} = 0$

$\Rightarrow \quad x^2 - 7x + \frac{98}{33} \times \frac{133}{33} = 0$

$\Rightarrow \quad x^2 - \frac{231}{33}x + \frac{98}{33} \times \frac{133}{33} = 0$

$\Rightarrow \quad x^2 - \left(\frac{98}{33} + \frac{133}{33}\right)x + \frac{98}{33} \times \frac{133}{33} = 0$

$\Rightarrow \quad x^2 - \frac{98}{33}x - \frac{133}{33}x + \frac{98}{33} \times \frac{133}{33} = 0$

$\Rightarrow \quad \left(x^2 - \frac{98}{33}x\right) - \left(\frac{133}{33}x - \frac{98}{33} \times \frac{133}{33}\right) = 0$

$\Rightarrow \quad x\left(x-\frac{98}{33}\right)-\frac{133}{33}\left(x-\frac{98}{33}\right)=0$

$\Rightarrow \quad \left(x-\frac{98}{33}\right)\left(x-\frac{133}{33}\right)=0 \Rightarrow x=\frac{98}{33}$ or , $x=\frac{133}{33}$

52. The given equation is:

$\frac{1}{2x-3}+\frac{1}{x-5}=\frac{10}{9}$

$\Rightarrow \frac{x-5+2x-3}{(2x-3)(x-5)}=\frac{10}{9}$

$\Rightarrow \frac{3x-8}{(2x-3)(x-5)}=\frac{10}{9}$

$\Rightarrow$ 27x - 72 = 10[(2x - 3) (x - 5)] (By cross multiplication method)

$\Rightarrow$ 27x - 72 = 10[2x^2 - 10x - 3x + 15]

$\Rightarrow$ 27x - 72 = 10[2x^2 - 13x + 15]

$\Rightarrow$ 27x - 72 = 20x^2 - 130x + 150

$\Rightarrow$ 20x^2 - 157x + 222 = 0

Here , a = 20, b = -157, c = 222

Therefore, by quadratic formula we have:

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$\Rightarrow x=\frac{157\pm\sqrt{(-157)^2-4(20)(222)}}{40}$

$\Rightarrow x=\frac{157\pm\sqrt{24649-17760}}{40}$

$\Rightarrow x=\frac{157\pm\sqrt{6889}}{40}$

$\Rightarrow x=\frac{157\pm 83}{40}$

$\Rightarrow x=\frac{157+83}{40}$ or $x=\frac{157-83}{40}$

$\Rightarrow$ x = 6 or x = $\frac{37}{20}$

53. Let S_1 and S_2 be two squares. Let the side of the square S_2 be x cm in length. Then,

the side of square S_1 is (x + 4) cm.

Therefore, area of square S_1 = $(x + 4)^2$ [Because, Area = $(\text{side})^2$]

and, Area of square S_2 = x^2

It is given that

Area of square S_1 + Area of square S_2 =400 cm^2

$\Rightarrow$ $(x + 4)^2 + x^2$ = 400

$\Rightarrow$ (x^2 + 8x + 16) + x^2 = 400

$\Rightarrow$ 2x^2 + 8x - 384 = 0

$\Rightarrow$ x^2 + 4x -192 = 0

$\Rightarrow$ x^2 + 16x - 12x -192 = 0

$\Rightarrow$ x(x + 16) - 12(x + 16) = 0

$\Rightarrow$ (x + 16) (x - 12) = 0

$\Rightarrow$ x = 12 or, x = -16

As the length of the side of a square cannot be negative. Therefore, x = 12.

Therefore, side of square S_1 = x + 4 = 12 + 4 = 16 cm and, Side of square S_2 = 12 cm.Hence the side of square S_1 and S_2 are 16 cm and 12 cm respectively.

54. Let the smaller number be x. Then,

Larger number = 2x - 5.

According to given question we have,

$(2x - 5)^2 - x^2$ = 88

$\Rightarrow$ 4x^2 + 25 - 20x - x^2 = 88 [$\because$ $(a - b)^2 = a^2 + b^2 - 2ab$]

$\Rightarrow$ 3x^2 - 20x + 25 - 88 = 0 (by factorization method)

$\Rightarrow$ 3x^2 - 20x - 63 = 0

$\Rightarrow 3x^2 - 27x + 7x - 63 = 0$
$\Rightarrow 3x(x - 9) + 7(x - 9) = 0$
$\Rightarrow (x - 9)(3x + 7) = 0$
$\Rightarrow x - 9 = 0$ [$\because 3x + 7 \neq 0$]
$\Rightarrow x = 9$
Therefore, smaller number = 9 and accordingly the larger number is:
$2x - 5 = 2 \times 9 - 5 = 13$
Hence, required numbers are 9 and 13.

55. Let the 1st natural number is x.
Since, sum of the two natural numbers is 28. Hence, 2nd number will be (28 - x).
According to the question;
$x(28 - x) = 192$
$\Rightarrow 28x - x^2 = 192$
$\Rightarrow x^2 - 28x + 192 = 0$
$\Rightarrow x^2 - 16x - 12x + 192 = 0$
$\Rightarrow x(x - 16) - 12(x - 16) = 0$
$\Rightarrow x - 16 = 0$ or $x - 12 = 0$
$\Rightarrow x = 16$ or $x = 12$
Hence, the required numbers are 16 and 12.

56. Given,
$\frac{1}{(x+3)} + \frac{1}{(2x-1)} = \frac{11}{(7x+9)}$
Taking L.C.M, we get
$\Rightarrow \frac{(2x-1)+(x+3)}{(x+3)(2x-1)} = \frac{11}{(7x+9)} \Rightarrow \frac{(3x+2)}{2x^2+5x-3} = \frac{11}{(7x+9)}$
Now cross multiply
$\Rightarrow (3x + 2)(7x + 9) = 11(2x^2 + 5x - 3)$
$\Rightarrow 21x^2 + 41x + 18 = 22x^2 + 55x - 33$
$\Rightarrow x^2 + 14x - 51 = 0$
$\Rightarrow x^2 + 17x - 3x - 51 = 0$
$\Rightarrow x(x + 17) - 3(x + 17) = 0$
$\Rightarrow (x + 17)(x - 3) = 0$
$\Rightarrow x + 17 = 0$ or $x - 3 = 0$
$\Rightarrow x = -17$ or $x = 3$.
Therefore, -17 and 3 are the roots of the given equation.

57. We have, $p^2x^2 + (p^2 - q^2)x - q^2 = 0$
Comparing this equation with $ax^2 + bx + c = 0$, we have
$a = p^2, b = p^2 - q^2 \; and \; c = -q^2$
$\therefore D = b^2 - 4ac = (p^2 - q^2)^2 - 4p^2(-q^2)$
$= (p^2 - q^2)^2 + 4p^2q^2$
$= (p^2 + q^2)^2 > 0$
So, the given equation has real roots.
Let the roots of given equation are $\alpha \; and \; \beta$
$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(p^2-q^2)+(p^2+q^2)}{2p^2} = \frac{q^2}{p^2}$
and, $\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-(p^2-q^2)-(p^2+q^2)}{2p^2} = -1$
$\therefore x = -1$ or $x = \frac{q^2}{p^2}$

58. Given equation is: $2x^2 + px + 30 = 0$
$\Rightarrow 2 \times (3)^2 + p \times 3 + 30 = 0$
$\Rightarrow 2 \times 9 + 3p + 30 = 0$
$\Rightarrow 18 + 3p + 30 = 0$

$\Rightarrow$ 3p + 48 = 0
$\Rightarrow$ 3p = -48
$\Rightarrow$ p = -16

59. Let the breadth of the given hall be x m. Then, length = (x + 5)m

Now, Area = 84 m^2
$\Rightarrow$ Length $\times$ Breadth = 84
$\Rightarrow$ (x + 5) $\times$ x = 84
$\Rightarrow$ x^2 + 5x = 84
$\Rightarrow$ x^2 + 5x - 84 = 0
$\Rightarrow$ x^2 + 12x - 7x - 84 = 0
$\Rightarrow$ x(x + 12) - 7(x + 12) = 0
$\Rightarrow$ (x + 12)(x - 7) = 0
$\Rightarrow$ x - 7 = 0 [$\because$ Length can not be negative, $\therefore$ x + 12 $\neq$ 0]
$\Rightarrow$ x = 7
$\therefore$ Breadth of the hall = 7 m
Hence, length of the hall = x + 5
= 7 + 5
= 12 m

60. Let the denominator of the fraction be x.

Since it is given that the numerator of the fraction is 3 less than the denominator, hence numerator = (x - 3).
$\Rightarrow$ Fraction = $\frac{x-3}{x}$
If denominator is increased by 1 keeping numerator unchanged ,new fraction = $\frac{x-3}{x}$
According to the question, we have :-
$\frac{x-3}{x+1} = \frac{x-3}{x} - \frac{1}{15}$
$\Rightarrow \frac{x-3}{x} - \frac{x-3}{x+1} = \frac{1}{15}$
$\Rightarrow \frac{(x+1)(x-3)-x(x-3)}{x(x+1)} = \frac{1}{15}$
$\Rightarrow \frac{\left(x^2-2x-3\right)-\left(x^2-3x\right)}{x^2+x} = \frac{1}{15}$
$\Rightarrow \frac{x-3}{x^2+x} = \frac{1}{15}$
$\Rightarrow$ 15x - 45 = x^2 + x
$\Rightarrow$ x^2 - 14x + 45 = 0
$\Rightarrow$ x^2 - 9x - 5x + 45 = 0
$\Rightarrow$ x(x - 9) - 5(x - 9) = 0
$\Rightarrow$ x - 9 = 0 or x - 5 = 0
$\Rightarrow$ x = 9 or x = 5
When x = 9,
x - 3 = 9 - 3 = 6
$\Rightarrow$ fraction = $\frac{6}{9} = \frac{2}{3}$
When x = 5
x - 3 = 5 - 3 = 2
$\Rightarrow$ fraction = $\frac{2}{5}$
Since numerator is 3 less than the denominator, required fraction is $\frac{2}{5}$.

61. We have $3x^2 - 2\sqrt{6}x + 2$ it can be factorise as:

$3x^2 - 2\sqrt{6}x + 2 = 3x^2 - \sqrt{6}x - \sqrt{6}x + 2$
$= \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2})$
$= (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2})$
So, the roots of the equation are the values of x for which
$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$
Now, $\sqrt{3}x - \sqrt{2} = 0$ for $x = \sqrt{\frac{2}{3}}$

So, this root is repeated twice, one for each repeated factor $\sqrt{3}x-\sqrt{2}$

Therefore, the roots of $3x^2-2\sqrt{6}x+2=0$ are $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

62. Given,

$\frac{1}{(a+b+x)}=\frac{1}{a}+\frac{1}{b}+\frac{1}{x}$

$\Rightarrow \quad \frac{1}{(a+b+x)}-\frac{1}{x}=\frac{1}{a}+\frac{1}{b} \Rightarrow \frac{x-(a+b+x)}{x(a+b+x)}=\frac{b+a}{ab}$

$\Rightarrow \quad \frac{-(a+b)}{x(a+b+x)}=\frac{(a+b)}{ab}$

On dividing both sides by (a+b)

$\Rightarrow \quad \frac{-1}{x(a+b+x)}=\frac{1}{ab}$

Now cross multiply

⇒ x(a + b + x) = -ab

⇒ x^2 + ax + bx + ab = 0

⇒ x(x +a) + b(x +a) = 0

⇒ (x + a) (x + b) = 0

⇒ x + a = 0 or x + b = 0

⇒ x = -a or x = -b.

Therefore, -a and -b are the roots of the equation.

63. Given,

$\frac{4}{x}-3=\frac{5}{2x+3}$

Taking LCM, we get

$\Rightarrow \quad \frac{4-3x}{x}=\frac{5}{2x+3}$

After cross multiplication,

⇒ (4 - 3x) (2x + 3) = 5x

⇒ 8x + 12 - $6x^2$ - 9x = 5x

⇒ 12 - x - $6x^2$ = 5x

⇒ $6x^2$ + 6x - 12 = 0

⇒ x^2 + x - 2 = 0

⇒ x^2 + 2x - x - 2 = 0

⇒ x (x + 2) - 1(x+ 2) = 0

⇒ (x + 2)(x - 1) = 0 ⇒ x + 2 = 0 or, x - l = 0 ⇒ x = - 2 or, x = 1

64. Given that,

$\frac{x+1}{x-1}-\frac{x-1}{x+1}=\frac{5}{6}$

$\Rightarrow \frac{(x+1)^2-(x-1)^2}{(x-1)(x+1)}=\frac{5}{6}$

⇒ (x^2 + 1 + 2x) - (x^2 + 1 - 2x) = $\frac{5}{6}$(x^2 - 1^2)

⇒ x^2 + 1 + 2x - x^2 - 1 + 2x = $\frac{5}{6}$(x^2 - 1)

⇒ 4x = $\frac{5}{6}$(x^2 - 1)

⇒ 24x = 5(x^2 - 1)

⇒ 24x = $5x^2$ - 5

⇒ $5x^2$ - 24x - 5 = 0

⇒ $5x^2$ - 25x + 1x - 5 = 0

⇒ 5x(x - 5) + 1(x - 5) = 0

⇒ (x - 5)(5x + 1) = 0

⇒ x - 5 = 0 or 5x + 1 = 0

⇒ x = 5 or $x=-\frac{1}{5}$

65. Consider $\frac{1}{2a+b+2x}=\frac{1}{2a}+\frac{1}{b}+\frac{1}{2x}$

$\Rightarrow \frac{1}{2a+b+2x}-\frac{1}{2x}=\frac{1}{2a}+\frac{1}{b}$

⇒ 2ab (2x - 2a – b – 2x) = (2a + b)2x (2a + b + 2x)

⇒ 2ab (-2a - b) = 2(2a+ b) (2ax + bx + $2x^2$)

⇒ -ab = 2ax + bx + $2x^2$

⇒ $2x^2$ + 2ax + bx + ab = 0

⇒ 2x (x + a) + b (x + a) = 0

⇒ (2x + b) (x + a) = 0

⇒ $x = -a, -\frac{b}{2}$

Hence the roots are $-a, -\frac{b}{2}$.

66. Given: $\frac{x+3}{x-2} - \frac{1-x}{x} = 4\frac{1}{4}$

⇒ $\frac{(x+3)}{(x-2)} - \frac{(1-x)}{x} = \frac{17}{4}$

⇒ $\frac{x(x+3)-(1-x)(x-2)}{(x-2)x} = \frac{17}{4}$

⇒ $\frac{x^2+3x-\left(x-2-x^2+2x\right)}{x^2-2x} = \frac{17}{4}$

⇒ $\frac{x^2+3x+x^2-3x+2}{x^2-2x} = \frac{17}{4}$

⇒ $\frac{2x^2+2}{x^2-2x} = \frac{17}{4}$

⇒ $8x^2 + 8 = 17x^2 - 34x$ [On cross multiplying]

⇒ $- 9x^2 + 34x + 8 = 0$

⇒ $9x^2 - 36x + 2x - 8 = 0$

⇒ 9x(x - 4) + 2(x - 4) = 0

⇒ (x - 4) (9x + 2) = 0

⇒ x - 4 = 0 or 9x + 2 = 0

⇒ x = 4 or x = $\frac{-2}{9}$

Hence, the roots of the equation are 4 and $\frac{-2}{9}$

67. Let the sides of the two squares be x and y.

Their areas will be:

x^2 and y^2 respectively.

Now their perimeters will be :

4x and 4y respectively.

We do this since the sides of a square are equal.

From the question we can do the substitution as follows:

Sum of the areas:

$x^2 + y^2 = 157$ ...(1)

4x + 4y = 68 ...(2)

Divide equation 2 all through by 4 to get :

x + y = 17

We need to solve for x and y.

By substitution we have :

x = 17 - y

Replace this in equation 1 as follows:

$(17 - y)^2 + y^2 = 157$

$289 - 34y + y^2 + y^2 = 157$

Collecting the like terms together we have :

$2y^2 - 34y + 132 = 0$

Solving the quadratic equation:

Divide through by 2 to get :

y2 - 17y + 66 = 0

We expand the equation as follows:

$y^2 - 11y - 6y + 66 = 0$

y(y - 11) - 6(y - 11) = 0

(y - 6)(y - 11) = 0

y = 6 or 11

When y is 11 x is (17 - 11) = 6

So the values can either be 6 or 11 for y or vice versa for x.

The sides are thus :

11 meters and 6 meters

68. We have,

$\frac{2x}{x-4} + \frac{2x-5}{x-3} = \frac{25}{3}$

$2x(x-3) + (2x-5)(x-4) = \frac{25}{3}(x-3)(x-4)$ [Multiplying both sides by (x -3)(x -4)]

$\Rightarrow 2x^2 - 6x + 2x(x-4) - 5(x-4) = \frac{25}{3}[x(x-4) - 3(x-4)]$

$\Rightarrow 2x^2 - 6x + 2x^2 - 8x - 5x + 20 = \frac{25}{3}[x^2 - 4x - 3x + 12]$

$\Rightarrow 4x^2 - 19x + 20 = \frac{25}{3}[x^2 - 7x + 12]$

$\Rightarrow 12x^2 - 57x + 60 = 25x^2 - 175x + 300$ [Multiplying both sides by 3]

$\Rightarrow 25x^2 - 12x^2 - 175x + 57x + 300 - 60 = 0$

$\Rightarrow 13x^2 - 118x + 240 = 0$

In order to factorize $13x^2$ - 118x + 240, we have to find two numbers 'a' and 'b' such that.

a + b = -118 and ab = 13 $\times$ 240 = 3120

CLearly, (-40) + (-78) = -118 and (-40) $\times$ (-78) = 3120

$\therefore$ a = -40 and b = -78

Now,

$13x^2$ - 118x + 240 = 0

$\Rightarrow$ $13x^2$ - 40x - 78x + 240 = 0

$\Rightarrow$ x(13x - 40) - 6(13x - 40) = 0

$\Rightarrow$ (13x - 40)(x - 6) = 0

$\Rightarrow$ 13x - 40 = 0 or x - 6 = 0

$\Rightarrow x = \frac{40}{13}$ or x = 6

69. Let the length of the sides of the right triangle be x cm and (x + 5) cm. Given, length of hypotenuse =25 cm.

According to pythagoras theorem;

$p^2 + b^2 = h^2$ (where, p,b & h are respectively perpendicular, base & hypotenuse of right angled triangle)

$\therefore$ $x^2 + (x + 5)^2 = (25)^2$ [Using pythagoras theorem]

$\Rightarrow$ $x^2 + x^2$ + 25 + 10x = 625

$\Rightarrow$ $2x^2$ + 10x + 25 - 625 = 0

$\Rightarrow$ $2x^2$ + 10x - 600 = 0

$\Rightarrow$ $2(x^2$ + 5x - 300) = 0

$\Rightarrow$ x^2 + 5x - 300 = 0

$\Rightarrow$ x^2 + 20x - 15x - 300 = 0

$\Rightarrow$ x(x + 20) - 15(x + 20) = 0

$\Rightarrow$ (x + 20)(x - 15) = 0

$\Rightarrow$ x - 15 = 0 [$\because$ Length can never be negative $\therefore$ x + 20 $\neq$ 0]

$\Rightarrow$ x = 15 cm

$\therefore$ x + 5 =15 + 5 = 20 cm

Hence, the lengths of required sides are 15 cm and 20 cm.

70. Assume digit at ten's place = x and digit at unit's place = y

Therefore number = 10x + y

Also xy = 15 $\Rightarrow$ x = $\frac{15}{y}$ ...(i)

According to given situation we have,

10x + y + 18 = 10y + x

$\Rightarrow$9x - 9y + 18 = 0

$\Rightarrow$x - y + 2 = 0

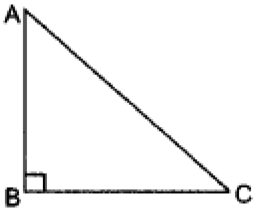
$\Rightarrow \frac{15}{y}$ - y + 2 = 0 (From (i))

$\Rightarrow$15 - y^2 + 2y = 0

$\Rightarrow$$y^2$ - 2y - 15 = 0

On factorizing the above quadratic equation we get
(y - 5) (y + 3) = 0
$\Rightarrow$y = 5, y = -3 [y = -3 is rejected]
Put the value of y = 5 in equation (i), we obtain
x = $\frac{15}{5}$ = 3
$\therefore$ Number = 3 $\times$ 10 + 5 = 35.

71. Let the consecutive positive odd numbers be x and (x + 2).
According to the question ;
$x^2 + (x + 2)^2 = 514$ (sum of squares of both numbers is 514)
$\Rightarrow x^2 + x^2 + 4x + 4 = 514$
$\Rightarrow 2x^2 + 4x - 510 = 0$
$\Rightarrow x^2 + 2x - 255 = 0$ (dividing both sides by 2)
$\Rightarrow x^2 - 15x + 17x - 255 = 0$
$\Rightarrow$ x(x - 15) + 17(x - 15) = 0
$\Rightarrow$ x - 15 = 0 or x + 17 = 0
$\Rightarrow$ x = 15 or x = -17
Since x is a positive number, x $\neq$ -17
$\Rightarrow$ x = 15
&, x + 2 = 15 + 2 = 17
Hence, the required positive odd numbers are x & (x+2) i.e. 15 and 17.

72. Let ABC is the right-angled triangle right angled at B.
Let BC = x units and AB = y units.
Also AC Hypotenuse = 29 units (given)


Perimeter = 70 units (given)
$\Rightarrow$x + y +29 = 70
$\Rightarrow$ x + y = 41
$\Rightarrow$y = 41 - x ....(i)
Also we know that $AC^2 = AB^2 + BC^2$
$\Rightarrow (29)^2 = (y)^2+x^2$
$\Rightarrow 841 = (41 - x)^2 + x^2$ (by using (i))
$\Rightarrow 841 = 1681 + x^2 - 82x + x^2$
$\Rightarrow 2x^2 - 82x + 840 = 0 \Rightarrow x^2 - 41x + 420 = 0$
By factorizing the above quadratic equation we have,
(x - 20) (x - 21) = 0 $\Rightarrow$ x = 20 or x = 21
Now When x = 20, y = 41 - x = 21 and when x = 21, y = 41 - 21 = 20
Therefore, length of other two sides are 20 units and 21 units.

73. $x^2 - 2ax - (4b^2 - a^2) = 0$
$\Rightarrow x^2 - 2ax + (a^2 - 4b^2) = 0$
$\Rightarrow x^2 - 2ax + (a - 2b)(a + 2b) = 0$
$\Rightarrow x^2 - (a - 2b)x - (a + 2b)x + (a - 2b)(a + 2b) = 0$ $[2ax = (a - 2b)x + (a + 2b)x]$
$\Rightarrow x[x - (a - 2b)] - (a + 2b)[x - (a - 2b)] = 0$
$\Rightarrow$ [x - (a - 2b)] [x - (a + 2b)] = 0
$\Rightarrow$ x - (a - 2b) = 0 or x - (a + 2b) = 0
$\Rightarrow$ x = a -2b or x = a + 2b

74. Let the smaller side of the right triangle be x cm and the larger side be y cm .
Then, Using Pythagoras Theorem, we get

$x^2 + y^2 = (3\sqrt{5})^2$

$\implies x^2 + y^2 = 9(5)$

$\Rightarrow x^2 + y^2 = 45$ ....(i)

And second condition is if the smaller side is tripled and the larger side be doubled, the new hypotenuse is 15 cm.

Therefore, $(3x)^2 + (2y)^2 = 15^2$

$\Rightarrow 9x^2 + 4y^2 = 225$ ....(ii).

From equation (i), we get $y^2 = 45 - x^2$

Putting $y^2 = 45 - x^2$ in equation (ii), we get

$9x^2 + 4(45 - x^2) = 225$

$\Rightarrow 5x^2 + 180 = 225$

$\Rightarrow 5x^2 = 45$

$\Rightarrow x^2 = 9$

$\Rightarrow x = \pm 3$.

But, length of a side cannot be negative. Therefore, x = 3.

Putting x = 3 in (i), we get

$9 + y^2 = 45$

$\Rightarrow y^2 = 36$

$\Rightarrow y = 6$

Hence, the length of the smaller side is 3 cm and the length of the larger side is 6 cm.

75. Given, the roots of the equation $(b - c)x^2 + (c - a)x + (a - b) = 0$ are equal.

Hence, $D = 0$

$\Rightarrow (c-a)^2 - 4(b-c)(a-b) = 0$

$\Rightarrow c^2 + a^2 + 4b^2 - 2ac - 4ab + 4ac - 4bc = 0$

$\Rightarrow (c + a - 2b)^2 = 0$

$\Rightarrow c + a - 2b = 0$

$\Rightarrow 2b = a + c$