

Solution

QUADRATIC EQUATIONS WS 4

Class 10 - Mathematics

1. Given, $3\sqrt{2}x^2 - 5x - \sqrt{2} = 0$

By splitting the middle term, we have

$$3\sqrt{2}x^2 - 6x + x - \sqrt{2} = 0$$

$$\Rightarrow 3\sqrt{2}x(x - \sqrt{2}) + 1(x - \sqrt{2}) = 0$$

$$\Rightarrow (3\sqrt{2}x + 1)(x - \sqrt{2}) = 0$$

$$\therefore 3\sqrt{2}x + 1 = 0 \text{ or } x - \sqrt{2} = 0$$

$$\therefore x = -\frac{1}{3\sqrt{2}} \text{ or } x = \sqrt{2}$$

2. $x^2 - 3x - 10 = 0$

$$\Rightarrow x^2 - 5x + 2x - 10 = 0$$

$$\Rightarrow x(x-5) + 2(x-5) = 0$$

$$\Rightarrow (x-5)(x+2) = 0$$

$$\Rightarrow x = 5, -2$$

3. Here we have, $ax^2 + (4a^2 - 3b)x - 12ab = 0$

$$\Rightarrow ax^2 + 4a^2x - 3bx - 12ab = 0$$

$$\Rightarrow ax(x + 4a) - 3b(x + 4a) = 0$$

$$\Rightarrow (ax - 3b)(x + 4a) = 0$$

$$\Rightarrow x = \frac{3b}{a} \text{ or } -4a \text{ are two roots of the equation.}$$

4. $6x^2 - x - 2 = 0$

$$\text{or, } 6x^2 + 3x - 4x - 2 = 0$$

$$\text{or, } 3x(2x + 1) - 2(2x + 1) = 0$$

$$\text{or, } (2x + 1)(3x - 2) = 0$$

$$\Rightarrow \text{either } 3x - 2 = 0 \text{ or } 2x + 1 = 0$$

$$\therefore x = \frac{2}{3} \text{ or } x = -\frac{1}{2}$$

Therefore, Roots of equation are $\frac{2}{3}$ and $-\frac{1}{2}$.

5. Given, $\frac{x+1}{x-1} + \frac{x-2}{x+2} = 4 - \frac{2x+3}{x-2}$

$$\Rightarrow \frac{x^2 + 3x + 2 + x^2 - 3x + 2}{x^2 + x - 2} = \frac{4x - 8 - 2x - 3}{x - 2}$$

$$\Rightarrow (2x^2 + 4)(x - 2) = (2x - 11)(x^2 + x - 2)$$

$$\text{or, } 5x^2 + 19x - 30 = 0$$

$$\Rightarrow 5x^2 + 25x - 6x - 30 = 0$$

$$\Rightarrow (x+5)(5x-6) = 0$$

$$\Rightarrow x = -5, \frac{6}{5}$$

6. We have, $16x - \frac{10}{x} = 27$

$$\Rightarrow \frac{16x^2 - 10}{x} = 27$$

$$\Rightarrow 16x^2 - 10 = 27x$$

$$\Rightarrow 16x^2 - 27x - 10 = 0$$

$$\Rightarrow 16x^2 - 32x + 5x - 10 = 0$$

$$\Rightarrow 16x(x - 2) + 5(x - 2) = 0$$

$$\Rightarrow (16x + 5)(x - 2) = 0$$

$$\Rightarrow (16x + 5) = 0 \text{ or } (x - 2) = 0$$

$$\Rightarrow x = -\frac{5}{16} \text{ or, } x = 2$$

Hence, the roots of given quadratic equation are 2 and $-\frac{5}{16}$

7. We have, $3\sqrt{7}x^2 + 4x - \sqrt{7} = 0$

Factorise the equation,

$$\Rightarrow 3\sqrt{7}x^2 + 7x - 3x - \sqrt{7} = 0$$

$$\Rightarrow \sqrt{7}x(3x + \sqrt{7}) - 1(3x + \sqrt{7}) = 0$$

$$\Rightarrow (3x + \sqrt{7})(\sqrt{7}x - 1) = 0$$

$$\Rightarrow (3x + \sqrt{7}) = 0 \text{ or } (\sqrt{7}x - 1) = 0$$

$$3x = -\sqrt{7} \text{ or } x = \frac{1}{\sqrt{7}}$$

$$x = \frac{-\sqrt{7}}{3} \text{ or } x = \frac{1 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{\sqrt{7}}{7}$$

8. Consider,

$$6 - x - x^2 = 0$$

This can be written as,

$$x^2 + x - 6 = 0$$

$$x^2 + 3x - 2x - 6 = 0$$

$$x(x + 3) - 2(x + 3) = 0$$

$$(x - 2)(x + 3) = 0$$

$$x - 2 = 0 \text{ (or) } x + 3 = 0$$

$$x = 2 \text{ (or) } x = -3$$

\therefore Solution set is {2, -3}

9. Ram and Bhagat together do the work in 4 days

\therefore Ram and Bhagat will do in one days = $\frac{1}{4}$ work

Let Bhagat alone does the same work in x days.

\therefore Ram will take = (x - 6) days

$$\therefore \frac{1}{x} + \frac{1}{x-6} = \frac{1}{4}$$

$$\Rightarrow \frac{x-6+x}{x(x-6)} = \frac{1}{4}$$

$$4x - 24 + 4x = x^2 - 6x$$

$$\Rightarrow x^2 - 6x - 8x + 24 = 0$$

$$\Rightarrow x^2 - 14x + 24 = 0$$

$$\Rightarrow x^2 - (12 + 2)x + 24 = 0$$

$$\Rightarrow x^2 - 12x - 2x + 24 = 0$$

$$\Rightarrow x^2 - 12x - 2x + 24 = 0$$

$$\Rightarrow x(x - 12) - 2(x - 12) = 0$$

$$\Rightarrow x = 12, x = 2$$

\therefore If Bhagat complete the work in 2 days

Ram will take = 2 - 6 = -4 days (impossible)

Hence, Bhagat can finish in 12 days

10. We have,

$$x^2 + \left(a + \frac{1}{a}\right)x + 1 = 0$$

$$\Rightarrow x^2 + ax + \frac{1}{a}x + 1 = 0$$

$$\Rightarrow x(x + a) + \frac{1}{a}(x + a) = 0$$

$$\Rightarrow (x + a) \left(x + \frac{1}{a} \right) = 0$$

$$\Rightarrow \text{either } x + a = 0 \text{ or, } x + \frac{1}{a} = 0$$

$$\Rightarrow x = -a \text{ or } x = -\frac{1}{a}$$

Hence, the roots of given quadratic equation are $-a$ and $-\frac{1}{a}$

11. We have, $abx^2 + (b^2 - ac)x - bc = 0$

$$\Rightarrow abx^2 + b^2x - acx - bc = 0$$

$$\Rightarrow bx(ax + b) - c(ax + b) = 0$$

$$\Rightarrow (ax + b)(bx - c) = 0$$

Either $ax + b = 0$ or $bx - c = 0$

$$\Rightarrow x = -\frac{b}{a}, \frac{c}{b}$$

Hence, $x = -\frac{b}{a}, \frac{c}{b}$ are the required solutions.

12. We have, $4x^2 - 4ax + (a^2 - b^2) = 0$

$$\Rightarrow 4x^2 - [2(a+b)x + 2(a-b)x] + (a^2 - b^2) = 0$$

$$\Rightarrow 4x^2 - 2(a+b)x - 2(a-b)x + (a^2 - b^2) = 0$$

$$\Rightarrow 2x(2x - a - b) - (a-b)(2x - a - b) = 0$$

$$\Rightarrow (2x - a - b)(2x - a + b) = 0$$

Either $2x = a + b$ or $2x = a - b$

$$\Rightarrow x = \frac{a+b}{2}, \frac{a-b}{2}$$

Hence, $x = \frac{a+b}{2}, \frac{a-b}{2}$ are the required solutions.

13. $x^2 + 3x = 9$

Subtract 9 from both sides of equation

$$x^2 + 3x - 9 = 0$$

Use the quadratic formula to find the solution

$$\frac{-b \pm \sqrt{b^2 - 4(ac)}}{2a}$$

Substitute the values $a = 1, b = 1, c = -9$ into the quadratic formula and solve for x .

$$\frac{-3 \pm \sqrt{3^2 - 4 \cdot (1 \cdot -9)}}{2 \cdot 1}$$

Simplify

$$x = \frac{-3 \pm 3\sqrt{5}}{2}$$

The result can be shown in multiple forms.

Exact Form:

$$x = \frac{-3 \pm 3\sqrt{5}}{2}$$

Decimal Form:

$$x = 1.85410196..., -4.85410196$$

14. We have the following equation,

$$100x^2 - 20x + 1 = 0$$

$$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$$

$$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$$

$$\Rightarrow (10x - 1)(10x - 1) = 0$$

$$\Rightarrow (10x - 1)^2 = 0$$

$$\Rightarrow 10x - 1 = 0$$

$$\Rightarrow x = \frac{1}{10}$$

15. Here, area of a right-angled triangle is 96 sq metres.

Let the altitude of a triangle be the x meter.

Then, base = $3x$ meter.

$$\begin{aligned} \text{Therefore, Area of triangle} &= \frac{1}{2} \times (3x \times x) \text{ cm}^2 \\ &= \frac{1}{2} \times 3x^2 = 96 \Rightarrow x^2 = \frac{96 \times 2}{3} \end{aligned}$$

$$\Rightarrow x^2 = 64$$

$$\Rightarrow x = \sqrt{64} = x = \pm 8$$

Therefore, $x = 8$ [length of altitude can never be negative]

Hence, the altitude of the triangle is 8 cm and base of triangle = $3x = 3(8)$ cm = 24cm

$$16. y^2 - 3 = 0$$

$$\Rightarrow y^2 - (\sqrt{3})^2 = 0$$

$$\Rightarrow (y + \sqrt{3})(y - \sqrt{3}) = 0 \text{ (Using Identity } a^2 - b^2 = (a + b)(a - b) \text{)}$$

Either $y + \sqrt{3} = 0$ or $y - \sqrt{3} = 0$

$$\therefore y = -\sqrt{3}, \sqrt{3}$$

Thus, $y = -\sqrt{3}$ and $\sqrt{3}$ are the roots of the given equation.

17. In factorization, we write the middle term of the quadratic equation either as a sum of two numbers or difference of two numbers such that the equation can be factorized.

Here we have, $x^2 - (\sqrt{3} + 1)x + \sqrt{3} = 0$

$$x^2 - \sqrt{3}x - x + \sqrt{3} = 0$$

$$x(x - \sqrt{3}) - 1(x - \sqrt{3}) = 0$$

$$(x - \sqrt{3})(x - 1) = 0$$

$$\therefore x - \sqrt{3} = 0 \text{ or } x - 1 = 0$$

$$x = \sqrt{3} \text{ or } x = 1.$$

$$18. 3x^2 - 8x - 1 = 0$$

$$\Rightarrow x = \frac{-(-8) \pm \sqrt{(-8)^2 - 4 \times 3 \times -1}}{2 \times 3}$$

$$= \frac{8 \pm \sqrt{64 + 12}}{6}$$

$$= \frac{8 \pm \sqrt{76}}{6}$$

$$= \frac{8 \pm 2\sqrt{19}}{6}$$

$$= \frac{4 \pm \sqrt{19}}{3}$$

$$19. \frac{2}{5}x^2 - x - \frac{3}{5} = 0$$

$$\text{or, } \frac{2x^2 - 5x - 3}{5} = 0$$

$$\text{or, } 2x^2 - 5x - 3 = 0$$

$$\text{or, } 2x^2 - 6x + x - 3 = 0$$

$$\text{or, } 2x(x - 3) + 1(x - 3) = 0$$

$$\text{or, } (2x + 1)(x - 3) = 0$$

$$\therefore x = -\frac{1}{2}, 3$$

20. We have, $2x^2 + 3x + k = 0$.

$a = 2$, $b = 3$ and, $c = k$

The given equation will have real roots, if

$$D \geq 0$$

$$\therefore D = b^2 - 4ac \geq 0$$

$$\Rightarrow 9 - 4 \times 2 \times k \geq 0$$

$$\Rightarrow 9 - 8k \geq 0$$

$$\Rightarrow k \leq \frac{9}{8}$$

21. We have the following equation,

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

Now,

$$4x^2 + 4bx - (a^2 - b^2) = 0$$

$$\Rightarrow 4x^2 - 2(a - b)x + 2(a + b)x - (a^2 - b^2) = 0$$

$$\Rightarrow 2x[2x - (a - b)] + (a + b)[2x - (a - b)] = 0$$

$$\Rightarrow [2x - (a - b)][2x + (a + b)] = 0$$

$$\Rightarrow 2x - (a - b) = 0 \text{ or } 2x + (a + b) = 0$$

$$\Rightarrow 2x = a - b \text{ or } 2x = -a - b$$

$$\Rightarrow x = \frac{a-b}{2} \text{ or } x = \frac{-a-b}{2}$$

22. We have,

$$\sqrt{3}x^2 + 11x + 6\sqrt{3} = 0$$

Factorise the equation,

$$\Rightarrow \sqrt{3}x^2 + 9x + 2x + 6\sqrt{3} = 0$$

$$\Rightarrow \sqrt{3}x(x + 3\sqrt{3}) + 2(x + 3\sqrt{3}) = 0$$

$$\Rightarrow (\sqrt{3}x + 2)(x + 3\sqrt{3}) = 0$$

$$\Rightarrow \sqrt{3}x + 2 = 0 \text{ or } x + 3\sqrt{3} = 0$$

$$x = \frac{-2}{\sqrt{3}} \text{ or } x = -3\sqrt{3}$$

$$x = \frac{-2 \times \sqrt{3}}{\sqrt{3} \times \sqrt{3}} \text{ or } x = -3\sqrt{3}$$

$$x = \frac{-2\sqrt{3}}{3} \text{ or } x = -3\sqrt{3}$$

23. Given: $kx^2 + 6x + 1 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$$a = k, b = 6 \text{ and } c = 1$$

For real and distinct roots: $D > 0$

$$\text{Discriminant, } D = b^2 - 4ac > 0$$

$$6^2 - 4k > 0$$

$$36 - 4k > 0$$

$$4k < 36$$

$$k < 9$$

24. According to the question,

$$\frac{m}{n}x^2 + \frac{n}{m} = 1 - 2x$$

$$\Rightarrow \frac{m}{n}x^2 + 2x + \frac{n}{m} - 1 = 0$$

$$\Rightarrow x^2 + \frac{2nx}{m} + \frac{n^2}{m^2} - \frac{n}{m} = 0 \text{ [multiplying both sides by 'n' and dividing both sides by 'm']}$$

$$\Rightarrow x^2 + \frac{2nx}{m} + \frac{n^2 - mn}{m^2} = 0$$

To factorize $x^2 + \frac{2nx}{m} + \frac{n^2 - mn}{m^2}$, we have to find two numbers 'a' and 'b' such that.

$$a + b = \frac{2n}{m} \text{ and } ab = \frac{n^2 - mn}{m^2}$$

$$\text{Clearly, } \frac{n + \sqrt{mn}}{m} + \frac{n - \sqrt{mn}}{m} = \frac{2n}{m} \text{ and } \frac{(n + \sqrt{mn})}{m} \times \frac{(n - \sqrt{mn})}{m} = \frac{n^2 - mn}{m^2} \text{ (} \therefore a = \frac{n + \sqrt{mn}}{m} \text{ and } b = \frac{n - \sqrt{mn}}{m} \text{)}$$

$$\Rightarrow x^2 + \frac{2nx}{m} + \frac{n^2 - mn}{m^2} = 0$$

$$\Rightarrow x^2 + \frac{(n+\sqrt{mn})}{m}x + \frac{(n-\sqrt{mn})}{m}x + \frac{n^2-mn}{m^2} = 0$$

$$\Rightarrow x \left[x + \frac{n+\sqrt{mn}}{m} \right] + \frac{n-\sqrt{mn}}{m} \left[x + \frac{n+\sqrt{mn}}{m} \right] = 0$$

$$\Rightarrow \left(x + \frac{n-\sqrt{mn}}{m} \right) \left(x + \frac{n+\sqrt{mn}}{m} \right) = 0$$

$$\Rightarrow x + \frac{n-\sqrt{mn}}{m} = 0 \text{ or } x + \frac{n+\sqrt{mn}}{m} = 0$$

$$\Rightarrow x = \frac{-n-\sqrt{mn}}{m} \text{ or } x = \frac{-n+\sqrt{mn}}{m}$$

25. Given;

$$a^2b^2x^2 + b^2x - a^2x - 1 = 0$$

$$\Rightarrow b^2x(a^2x + 1) - 1(a^2x + 1) = 0$$

$$\Rightarrow (a^2x + 1)(b^2x - 1) = 0$$

$$\Rightarrow x = \frac{-1}{a^2} \text{ or } x = \frac{1}{b^2}$$

26. Let the successive multiples of 3 be $3x$ and $3x + 3$. Then,

$$3x(3x + 3) = 270$$

$$\Rightarrow 9x^2 + 9x = 270$$

$$\Rightarrow 9x^2 + 9x - 270 = 0$$

$$\Rightarrow 9(x^2 + x - 30) = 0$$

$$\Rightarrow x^2 + x - 30 = 0$$

$$\Rightarrow x^2 + 6x - 5x - 30 = 0$$

$$\Rightarrow x(x + 6) - 5(x + 6) = 0$$

$$\Rightarrow (x + 6)(x - 5) = 0$$

$$\Rightarrow x - 5 = 0 [\because x + 6 \neq 0]$$

$$\therefore 3x = 3 \times 5 = 15$$

$$\text{And, } 3x + 3 = 3 \times 5 + 3 = 18$$

Hence, required multiples of 3 are 15 and 18.

27. We have, $100x^2 - 20x + 1 = 0$

$$\Rightarrow 100x^2 - 10x - 10x + 1 = 0$$

$$\Rightarrow 10x(10x - 1) - 1(10x - 1) = 0$$

$$\Rightarrow (10x - 1)(10x - 1) = 0$$

Either $10x - 1 = 0$ or $10x - 1 = 0$

$$\Rightarrow x = \frac{1}{10}, \frac{1}{10}$$

$$\therefore x = \frac{1}{10} \text{ are the repeated roots.}$$

28. We have

$$3x^2 + 5\sqrt{5}x - 10 = 0$$

$$\Rightarrow 3x^2 + 6\sqrt{5}x - \sqrt{5}x - 10 = 0$$

$$\Rightarrow 3x(x + 2\sqrt{5}) - \sqrt{5}(x + 2\sqrt{5}) = 0$$

$$\Rightarrow (x + 2\sqrt{5})(3x - \sqrt{5}) = 0$$

$$\Rightarrow x + 2\sqrt{5} = 0 \text{ or } 3x - \sqrt{5} = 0$$

$$\Rightarrow x = -2\sqrt{5} \text{ or } x = \frac{\sqrt{5}}{3}$$

29. Given, $\frac{2}{x^2} - \frac{5}{x} + 2 = 0$

$$\Rightarrow \frac{2 - 5x + 2x^2}{x^2} = 0$$

$$\Rightarrow 2 - 5x + 2x^2 = 0$$

$$\Rightarrow 2x^2 - 5x + 2 = 0$$

$$\Rightarrow 2x^2 - 4x - x + 2 = 0$$

$$\Rightarrow 2x(x - 2) - 1(x - 2) = 0$$

$$\Rightarrow (2x - 1)(x - 2) = 0$$

$$\Rightarrow (2x - 1) = 0 \text{ or } (x - 2) = 0$$

$$2x = 1 \text{ or } x = 2$$

The value of x will be

$$\Rightarrow x = \frac{1}{2} \text{ or } x = 2$$

Therefore roots of the equation will be $\frac{1}{2}$ and 2 .

$$30. x^2 - 45x + 324 = 0$$

$$\Rightarrow x^2 - 36x - 9x + 324 = 0 \Rightarrow x(x - 36) - 9(x - 36) = 0$$

$$\Rightarrow (x - 9)(x - 36) \Rightarrow x = 9, 36$$

31. Here, the hypotenuse of a right-angled triangle is 1 metre less than twice the shortest side.

Let the shorter side of a triangle be x meter.

Then, its hypotenuse = (2x - 1) meter

and let the altitude = x + 1 meter

$$\text{Then, } (2x - 1)^2 = x^2 + (x + 1)^2$$

$$\Rightarrow 4x^2 + 1 - 4x = x^2 + x^2 + 1 + 2x$$

$$\implies 4x^2 - 2x^2 - 4x - 2x = 1 - 1$$

$$\Rightarrow 2x^2 - 6x = 0$$

$$\Rightarrow 2x(x - 3) = 0$$

$$\Rightarrow x - 3 = 0 \text{ or } 2x = 0, \text{ as } 2 \text{ can not be zero.}$$

Therefore, x = 3 or x = 0

So, x = 3 [base cannot be zero]

Hence, base = 3m.

hypotenuse = (2(3) - 1)m = 5m and

altitude = (3 + 1)m = 4m

$$32. \frac{5}{2}x^2 + \frac{2}{5} = 1 - 2x$$

$$\Rightarrow \frac{5}{2}x^2 + 2x + \frac{2}{5} + 1 = 0$$

$$\Rightarrow 25x^2 + 20x + 14 = 0$$

$$\Rightarrow x = \frac{-b \pm \sqrt{D}}{2a}$$

$$= \frac{-20 \pm \sqrt{400 - 4 \times 25 \times 14}}{2 \times 25}$$

$$= \frac{-20 \pm \sqrt{400 - 1400}}{50}$$

$$= \frac{-20 \pm \sqrt{-1000}}{50}$$

$$= \frac{-20 \div 10 \pm \sqrt{-10}}{50}$$

$$\frac{-20 + 10\sqrt{-10}}{50} \text{ or } \frac{-20 - 10\sqrt{-10}}{50}$$

$$\frac{-20 + 10\sqrt{10}i}{50} \text{ or } \frac{-20 - 10\sqrt{10}i}{50}$$

$$33. \text{ We have, } 7x^2 + kx - 3 = 0$$

Since $x = \frac{2}{3}$ is the solution of the given equation

$\therefore x = \frac{2}{3}$ satisfies the given equation

$$7\left(\frac{2}{3}\right)^2 + k\left(\frac{2}{3}\right) - 3 = 0$$

$$\Rightarrow \frac{28}{9} + \frac{2k}{3} - 3 = 0$$

$$\Rightarrow \frac{1}{9} + \frac{2k}{3} = 0$$

$$\Rightarrow \frac{2k}{3} = -\frac{1}{9} \Rightarrow k = -\frac{3}{18}$$

$$\Rightarrow k = -\frac{1}{6}$$

34. We have, $kx^2 + \sqrt{2}x - 4 = 0$; $x = \sqrt{2}$

Since $x = \sqrt{2}$ is the solution of the given equation

Substituting $x = \sqrt{2}$ in the given equation

$$k(\sqrt{2})^2 + \sqrt{2} \times \sqrt{2} - 4 = 0$$

$$\Rightarrow 2k + 2 - 4 = 0$$

$$\Rightarrow 2k = 2$$

$$\Rightarrow k = 1$$

35. Let the base of the right triangle be x cm.

Then, altitude = $(x - 7)$ cm

From pythagoras theorem,

$$B^2 + P^2 = H^2$$

$$\therefore x^2 + (x - 7)^2 = 13^2$$

$$\Rightarrow x^2 + x^2 + 49 - 14x = 169$$

$$\Rightarrow 2x^2 - 14x - 120 = 0$$

$$\Rightarrow x^2 - 7x - 60 = 0$$

Factorise the equation,

$$\Rightarrow x^2 - 12x + 5x - 60 = 0$$

$$\Rightarrow x(x - 12) + 5(x - 12) = 0$$

$$\Rightarrow (x - 12)(x + 5) = 0$$

$$\Rightarrow x - 12 = 0 \text{ or } x + 5 = 0,$$

$$\Rightarrow x = 12 \text{ or } x = -5$$

Since sides are positive, $x = 12$.

Therefore, the base = 12 cm and the altitude = $(12 - 7)$ cm = 5 cm.

36. The given equation can be rewritten as

$$\frac{a(x-a) + b(x-b)}{(x-b)(x-a)} = 2$$

$$a(x-a) + b(x-b) = 2[x^2 - (a+b)x + ab]$$

$$ax - a^2 + bx - b^2 = 2x^2 - 2(a+b)x + 2ab$$

$$2x^2 - 3(a+b)x + (a+b)^2 = 0$$

$$2x^2 - 2(a+b)x - (a+b)x + (a+b)^2 = 0$$

$$2x[x - (a+b)] - (a+b)[x - (a+b)]$$

$$[2x - (a+b)][x - (a+b)] = 0$$

$$x = a + b, \frac{a+b}{2}$$

37. We have,

$$3x^2 - 14x - 5 = 0$$

$$\text{So, } 3x^2 - 14x - 5 = 0$$

$$\Rightarrow 3x^2 - 15x + 1x - 5 = 0$$

$$\Rightarrow 3x(x - 5) + 1(x - 5) = 0$$

$$\Rightarrow (x - 5)(3x + 1) = 0$$

$$\Rightarrow x - 5 = 0 \text{ or } 3x + 1 = 0$$

$$\Rightarrow x = 5 \text{ or } x = -\frac{1}{3}. \text{ Hence the roots are } 5 \text{ and } -\frac{1}{3}$$

$$\begin{aligned}
38. \quad & \frac{1}{x+4} - \frac{1}{x+7} = \frac{11}{30} \\
& \Rightarrow \frac{(x+7) - (x+4)}{(x+4)(x+7)} = \frac{11}{30} \\
& \Rightarrow \frac{3}{x^2+4x+7x+28} = \frac{11}{30} \\
& \Rightarrow \frac{3}{x^2+11x+28} = \frac{11}{30} \\
& \Rightarrow 11x^2 + 121x + 308 = 90 \\
& \Rightarrow 11x^2 + 121x + 218 = 0 \\
& \Rightarrow 11(x^2 + 11x + 18) = 0 \\
& \Rightarrow x^2 + 11x + 18 = 0 \\
& \Rightarrow x^2 + 9x + 2x + 18 = 0 \\
& \Rightarrow x^2(x+9) + 2(x+9) = 0 \\
& \Rightarrow (x+2)(x+9) = 0 \\
& \Rightarrow x = -2, -9
\end{aligned}$$

39. Let the whole number be x.

According to question,

$$\begin{aligned}
(x-20) &= 69\left(\frac{1}{x}\right) \\
&\Rightarrow x^2 - 20x - 69 = 0 \\
&\Rightarrow x^2 - 23x + 3x - 69 = 0 \\
&\Rightarrow (x-23)(x+3) = 0 \\
&\Rightarrow x = 23, -3. \text{ Rejecting } -3 \text{ as } -3 \text{ is not a whole number.} \\
&\Rightarrow x = 23
\end{aligned}$$

40. We have,

$$\begin{aligned}
x^2 - 4ax + 4a^2 - b^2 &= 0 \\
&\Rightarrow (x-2a)^2 - b^2 = 0 \\
&\Rightarrow (x-2a+b)(x-2a-b) = 0 \\
&\Rightarrow x-2a+b = 0 \text{ or } x-2a-b = 0 \\
&\Rightarrow x = 2a-b \text{ or } 2a+b
\end{aligned}$$

$$41. 15x^2 - 10\sqrt{6}x + 10 = 0.$$

$$\begin{aligned}
3x^2 - 2\sqrt{6}x + 2 &= 0 \\
3x^2 - \sqrt{6}x - \sqrt{6}x + 2 &= 0 \\
\sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) &= 0 \\
(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) &= 0 \\
\therefore x &= \frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}
\end{aligned}$$

42. We have the following equation,

$$\frac{1}{x} - \frac{1}{x-2} = 3$$

Taking LCM

$$\begin{aligned}
&\Rightarrow \frac{x-2-x}{x(x-2)} = 3 \\
&\Rightarrow -2 = 3x(x-2) \\
&\Rightarrow 3x(x-2) + 2 = 0 \\
&\Rightarrow 3x^2 - 6x + 2 = 0
\end{aligned}$$

Factorise the equation

$$\begin{aligned}
&\Rightarrow 3x^2 - [(3 + \sqrt{3})x + (3 - \sqrt{3})x] + 2 = 0 \\
&\Rightarrow 3x^2 - (3x + \sqrt{3})x - (3 - \sqrt{3})x + 2 = 0
\end{aligned}$$

$$\Rightarrow 3x^2 - \sqrt{3}(\sqrt{3} + 1)x - \sqrt{3}(\sqrt{3} - 1)x + 1(\sqrt{3} + 1)(\sqrt{3} - 1) = 0$$

$$\Rightarrow \sqrt{3}x(\sqrt{3}x - (\sqrt{3} + 1)) - (\sqrt{3} - 1)(\sqrt{3} - (\sqrt{3} + 1)) = 0$$

$$\Rightarrow [\sqrt{3}x - (\sqrt{3} - 1)][\sqrt{3}x - (\sqrt{3} + 1)] = 0$$

$$x = \frac{\sqrt{3}-1}{\sqrt{3}} \text{ or } \frac{\sqrt{3}+1}{\sqrt{3}}$$

$$x = \frac{3-\sqrt{3}}{3} \text{ or } \frac{3+\sqrt{3}}{3}$$

43. Here we have, $6x^2 - \sqrt{2}x - 2 = 6x^2 - 3\sqrt{2}x + 2\sqrt{2}x - 2$

$$= 3x(2x - \sqrt{2}) + \sqrt{2}(2x - \sqrt{2})$$

$$= (3x + \sqrt{2})(2x - \sqrt{2})$$

Now, $6x^2 - \sqrt{2}x - 2 = 0$ gives $(3x + \sqrt{2})(2x - \sqrt{2}) = 0$ i.e., $3x + \sqrt{2} = 0$ or $2x - \sqrt{2} = 0$

So, the roots are $-\frac{\sqrt{2}}{3}$ and $\frac{\sqrt{2}}{2}$.

44. Let the required number be x.

Then, square of a number will be x^2 .

According to given information, we have

$$x^2 - 84 = 3(x + 8)$$

$$\Rightarrow x^2 - 84 = 3x + 24$$

$$\Rightarrow x^2 - 3x - 108 = 0$$

$$\Rightarrow x^2 - 12x + 9x - 108 = 0$$

$$\Rightarrow x(x - 12) + 9(x - 12) = 0$$

$$\Rightarrow (x - 12)(x + 9) = 0$$

$$\Rightarrow x - 12 = 0 \text{ or } x + 9 = 0$$

$$\Rightarrow x = 12 \text{ or } x = -9$$

Since x is a natural number, $x \neq -9$.

$$\Rightarrow x = 12$$

Hence, required number is 12.

45. For the given equation, $2x^2 - 2\sqrt{2}x + 1 = 0$

Now discriminant $D = b^2 - 4ac$

$$D = (2\sqrt{2})^2 - 4 \times 2 \times 1 = 8 - 8 = 0$$

\therefore The given equation has real roots.

So the roots are given by, $x_1 = \frac{-b + \sqrt{D}}{2a}$ and $x_2 = \frac{-b - \sqrt{D}}{2a}$

$$x_1 = \frac{-(-2\sqrt{2}) + 0}{2 \times 2}, x_2 = \frac{-(-2\sqrt{2}) - 0}{2 \times 2}$$

$$x_1 = \frac{2\sqrt{2}}{4}, x_2 = \frac{2\sqrt{2}}{4}$$

$$\therefore x_1 = \frac{1}{\sqrt{2}} \text{ and } x_2 = \frac{1}{\sqrt{2}}$$

46. $2x^2 + x - 6 = 0$

$$\Rightarrow 2x^2 + 4x - 3x - 6 = 0 \Rightarrow 2x(x + 2) - 3(x + 2) = 0$$

$$\Rightarrow (2x - 3)(x + 2) = 0 \Rightarrow x = \frac{3}{2}, -2$$

47. Let the required natural numbers be x and x - 3. Then,

$$x^2 + (x - 3)^2 = 117$$

$$\Rightarrow x^2 + (x^2 + 9 - 6x) = 117 \quad [\because (a - b)^2 = a^2 + b^2 - 2ab]$$

$$\Rightarrow 2x^2 - 6x + 9 - 117 = 0$$

$$\Rightarrow 2x^2 - 6x - 108 = 0$$

$$\Rightarrow 2(x^2 - 3x - 54) = 0$$

$$\Rightarrow x^2 - 3x - 54 = 0$$

$$\begin{aligned} &\Rightarrow x^2 - 9x + 6x - 54 = 0 \\ &\Rightarrow x(x - 9) + 6(x - 9) = 0 \\ &\Rightarrow (x - 9)(x + 6) \quad [\because x \text{ is natural number } \therefore x + 6 \neq 0] \\ &\Rightarrow x = 9 \\ &\Rightarrow x - 3 = 9 - 3 = 6 \end{aligned}$$

Hence, required numbers are 6 and 9.

48. We have, $\sqrt{7}x^2 - 6x - 13\sqrt{7} = 0 \Rightarrow \sqrt{7}x^2 - 13x + 7x - 13\sqrt{7} = 0$

$$\Rightarrow x(\sqrt{7}x - 13) + \sqrt{7}(\sqrt{7}x - 13) = 0$$

$$\Rightarrow (x + \sqrt{7})(\sqrt{7}x - 13) = 0$$

$$\Rightarrow x + \sqrt{7} = 0 \text{ or } \sqrt{7}x - 13 = 0$$

$$x = -\sqrt{7} \text{ or } x = \frac{13}{\sqrt{7}} = \frac{13 \times \sqrt{7}}{\sqrt{7} \times \sqrt{7}} = \frac{13\sqrt{7}}{7}.$$

49. Let the number be x and its reciprocal be $\frac{1}{x}$

According to the question

$$x + \frac{1}{x} = 2\frac{1}{30}$$

$$\Rightarrow \frac{x^2 + 1}{x} = \frac{61}{30}$$

$$\Rightarrow 30x^2 - 61x + 30 = 0$$

$$\Rightarrow 30x^2 - 36x - 25x + 30 = 0$$

$$\Rightarrow 6x(5x - 6) - 5(5x - 6) = 0$$

$$\Rightarrow (6x - 5)(5x - 6) = 0$$

Either $6x - 5 = 0$ or $5x - 6 = 0$

$$\Rightarrow x = \frac{5}{6}, \frac{6}{5}$$

Hence, the required number is $\frac{5}{6}$ or $\frac{6}{5}$

50. Let the first natural number be x

$$\therefore \text{Second consecutive natural number} = x + 1$$

According to the question,

$$x^2 + (x + 1)^2 = 421$$

$$\text{or, } x^2 + x^2 + 2x + 1 = 421$$

$$2x^2 + 2x - 420 = 0$$

$$\text{or, } x^2 + x - 210 = 0 \text{ [dividing by 2 b/s]}$$

$$\text{or, } x^2 + 15x - 14x - 210 = 0$$

$$\text{or, } x(x + 15) - 14(x + 15) = 0$$

$$\text{or, } (x + 15)(x - 14) = 0$$

$$\text{or, } x + 15 = 0 \text{ or } x - 14 = 0$$

$$\therefore x = -15 \text{ or } x = 14$$

Rejecting negative value

First number = 14

and consecutive number = 15

51. Given, $x^2 + 3\sqrt{3}x - 30 = 0$

$$\Rightarrow x^2 + 5\sqrt{3}x - 2\sqrt{3}x - 30 = 0$$

$$\Rightarrow x(x + 5\sqrt{3}) - 2\sqrt{3}(x + 5\sqrt{3}) = 0$$

$$\Rightarrow (x + 5\sqrt{3})(x - 2\sqrt{3}) = 0$$

$$\Rightarrow x + 5\sqrt{3} = 0 \text{ or } x - 2\sqrt{3} = 0$$

$$\Rightarrow x = -5\sqrt{3} \text{ or } x = 2\sqrt{3}$$

52. We have

$$2x^2 - x + \frac{1}{8} = 0$$

$$\Rightarrow 16x^2 - 8x + 1 = 0$$

$$\begin{aligned} &\Rightarrow 16x^2 - 4x - 4x + 1 = 0 \\ &\Rightarrow 4x(4x - 1) - 1(4x - 1) = 0 \\ &\Rightarrow (4x - 1)(4x - 1) = 0 \\ &\Rightarrow (4x - 1)^2 = 0 \\ &\Rightarrow x = \frac{1}{4}. \text{ Hence, } x = \frac{1}{4} \text{ is repeated root of the polynomial.} \end{aligned}$$

53. Let the tens digit be x .

$$\text{Then, the units digit} = \frac{16}{x}.$$

$$\therefore \text{Number} = 10x + \frac{16}{x}$$

$$\text{And, number obtained by interchanging the digits} = 10 \times \frac{16}{x} + x = \frac{160}{x} + x.$$

$$\therefore 10x + \frac{16}{x} - 54 = \frac{160}{x} + x$$

$$\Rightarrow 10x - x + \frac{16}{x} - \frac{160}{x} - 54 = 0$$

$$\Rightarrow 9x + \frac{16-160}{x} - 54 = 0$$

$$\Rightarrow 9x - \frac{144}{x} - 54 = 0$$

$$\Rightarrow 9x^2 - 144 - 54x = 0 \text{ [Multiplying both sides of equation by } x]$$

$$\Rightarrow 9x^2 - 54x - 144 = 0$$

$$\Rightarrow 9(x^2 - 6x - 16) = 0$$

$$\Rightarrow x^2 - 6x - 16 = 0$$

$$\Rightarrow x^2 - 8x + 2x - 16 = 0$$

$$\Rightarrow x(x - 8) + 2(x - 8) = 0$$

$$\Rightarrow (x - 8)(x + 2) = 0$$

$$\Rightarrow x - 8 = 0 \text{ or } x + 2 = 0$$

$$\Rightarrow x = 8 \text{ or } x = -2$$

But, a digit can never be negative.

So, $x = 8$

$$\text{Hence, the required number} = 10 \times 8 + \frac{16}{8} = 82.$$

54. We have, $x^2 - x(a+b) + k = 0$

Since $x = a$ is the solution of the given equation.

$\therefore x = a$ satisfies the given equation.

$$a^2 - a(a+b) + k = 0$$

$$\implies a^2 - a^2 - ab + k = 0$$

$$\implies k - ab = 0 \text{ or } k = ab$$

55. Given,

$$5^{(x+1)} + 5^{(2-x)} = 5^3 + 1$$

$$\Rightarrow 5^x \cdot 5 + 5^2 \cdot 5^{-x} = 125 + 1$$

$$\Rightarrow 5^x \cdot 5 + \frac{25}{5^x} = 126$$

Let $y = 5^x$

$$\Rightarrow 5y + \frac{25}{y} = 126$$

$$\Rightarrow 5y^2 + 25 = 126y$$

$$\Rightarrow 5y^2 - 126y + 25 = 0$$

Factorise the equation,

$$\Rightarrow 5y^2 - 125y - y + 25 = 0$$

$$\Rightarrow 5y(y - 25) - 1(y - 25) = 0$$

$$\Rightarrow (y - 25)(5y - 1) = 0$$

$$\Rightarrow y - 25 = 0 \text{ or } 5y - 1 = 0$$

$$\Rightarrow y = 25 \text{ or } y = 1/5$$

Now,

$$\Rightarrow 5^x = 25 \text{ or } 5^x = 1/5$$

$$\Rightarrow 5^x = 5^2 \text{ or } 5^x = 5^{-1}$$

$$\Rightarrow x = 2 \text{ or } x = -1.$$

56. We have $\frac{16}{x} - 1 = \frac{15}{x+1}$

$$\Rightarrow \frac{16-x}{x} = \frac{15}{x+1}$$

Cross multiply,

$$\Rightarrow (16-x)(x+1) = 15x$$

$$\Rightarrow 16x + 16 - x^2 - x = 15x$$

$$\Rightarrow 15x - 16x - 16 + x^2 + x = 0$$

$$\Rightarrow x^2 - 16 = 0$$

$$\Rightarrow x^2 = 16$$

$$\therefore x = \pm 4$$

57. We have,

$$5x^2 - 3x - 2 = 0$$

In order to factorize $5x^2 - 3x - 2$, we have to find two numbers 'a' and 'b' such that

$$a + b = -3 \text{ and } ab = 5 \times -2 = -10$$

Clearly, $-5 + 2 = -3$ and $-5 \times 2 = -10$

$$\Rightarrow a = -5 \text{ and } b = 2$$

Now,

$$5x^2 - 3x - 2 = 0$$

$$\Rightarrow 5x^2 - 5x + 2x - 2 = 0$$

$$\Rightarrow 5x(x-1) + 2(x-1) = 0$$

$$\Rightarrow (x-1)(5x+2) = 0$$

$$\Rightarrow x-1 = 0 \text{ or } 5x+2 = 0$$

$$\Rightarrow x = 1 \text{ or } x = -\frac{2}{5}$$

Hence, the roots of given equation are 1 and $-\frac{2}{5}$

58. $y^2 + \frac{3\sqrt{5}}{2}y - 5 = 0$

$$\frac{2y^2 + 3\sqrt{5}y - 10}{2} = 0$$

$$\Rightarrow 2y^2 + 3\sqrt{5}y - 10 = 0$$

$$\Rightarrow 2y^2 + 4\sqrt{5}y - \sqrt{5}y - 10 = 0$$

$$\Rightarrow 2y(y + 2\sqrt{5}) - \sqrt{5}(y - 2\sqrt{5})$$

$$\Rightarrow (2y - \sqrt{5})(y + 2\sqrt{5}) = 0$$

$$\therefore y = \frac{\sqrt{5}}{2}, -2\sqrt{5}$$

59. Given, $2x^2 + \frac{5}{3}x - 2 = 0$

$$\Rightarrow 6x^2 + 5x - 6 = 0$$

By splitting the middle term, we have

$$6x^2 + 9x - 4x - 6 = 0$$

$$\Rightarrow 3x(2x+3) - 2(2x+3) = 0$$

$$\Rightarrow (2x+3)(3x-2) = 0$$

$$\therefore 2x+3 = 0 \text{ or } 3x-2 = 0$$

$$\therefore x = -\frac{3}{2} \text{ or } x = \frac{2}{3}$$

60. $2x^2 - ax - a^2 = 0$

$$\implies 2x^2 - 2ax + ax - a^2 = 0$$

$$\implies 2x(x-a) + a(x-a) = 0$$

$$\implies (2x+a)(x-a)=0$$

$$\implies \text{either } 2x+a=0 \text{ or } x-a=0$$

$$\implies x = -\frac{a}{2} \text{ or } x = a$$

$$\implies x = -\frac{a}{2}, a$$

Thus, $x = -\frac{a}{2}$ and $x = a$ are two roots of the given quadratic equation.

61. Given that,

$$x^2 - (1 + \sqrt{2})x + \sqrt{2} = 0$$

$$\Rightarrow x^2 - x - \sqrt{2}x + \sqrt{2} = 0$$

Factorise the equation, we get

$$\Rightarrow x(x-1) - \sqrt{2}(x-1) = 0$$

$$\Rightarrow (x-1)(x-\sqrt{2}) = 0$$

$$\Rightarrow x-1=0 \text{ or } x-\sqrt{2}=0$$

Therefore, $x = 1$ or $x = \sqrt{2}$

62. The given equation may be written as

$$\frac{(x-2)^{-x}}{x(x-2)} = 3$$

$$\Rightarrow 3x(x-2) = -2$$

$$\Rightarrow 3x^2 - 6x + 2 = 0 \dots(i)$$

This equation is of the form $ax^2 + bx + c = 0$, where $a = 3$, $b = -6$ and $c = 2$.

$$\therefore D = (b^2 - 4ac) = (-6)^2 - 4 \times 3 \times 2 = 36 - 24 = 12 > 0.$$

So, the given equation has real roots.

$$\text{Now, } \sqrt{D} = \sqrt{12} = 2\sqrt{3}$$

$$\therefore \alpha = \frac{-b + \sqrt{D}}{2a} = \frac{6 + 2\sqrt{3}}{2 \times 3} = \frac{6 + 2\sqrt{3}}{6} = \frac{3 + \sqrt{3}}{3}$$

$$\beta = \frac{-b - \sqrt{D}}{2a} = \frac{6 - 2\sqrt{3}}{2 \times 3} = \frac{6 - 2\sqrt{3}}{6} = \frac{3 - \sqrt{3}}{3}$$

Hence, the required values of x are $\frac{(3+\sqrt{3})}{3}$ and $\frac{(3-\sqrt{3})}{3}$

63. Let the required number be x and $x + 4$. Then,

$$x \times (x + 4) = 192$$

$$\Rightarrow x^2 + 4x = 192$$

$$\Rightarrow x^2 + 4x - 192 = 0$$

$$\Rightarrow x^2 + 16x - 12x - 192 = 0$$

$$\Rightarrow x(x + 16) - 12(x + 16) = 0$$

$$\Rightarrow (x + 16)(x - 12) = 0$$

$$\Rightarrow x + 16 = 0 \text{ or } x - 12 = 0$$

$$\Rightarrow x = -16 \text{ or } x = 12$$

Case I: When $x = -16$

$$\therefore x + 4 = -16 + 4 = -12$$

Case II: When $x = 12$

$$\therefore x + 4 = 12 + 4 = 16$$

Hence, the numbers are $-16, -12$ or $12, 16$.

64. We have,

$$3x^2 - 2\sqrt{6}x + 2 = 0$$

Factorise the equation,

$$\Rightarrow 3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$\Rightarrow \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow (\sqrt{3}x - \sqrt{2})^2 = 0$$

$$\Rightarrow \sqrt{3}x - \sqrt{2} = 0$$

$$\Rightarrow x = \frac{\sqrt{2}}{\sqrt{3}}$$

65. Let the natural number be x

Since the difference between the two natural numbers is 3, therefore, the other number is x + 3.

Given that the difference of the reciprocal

of the natural numbers is $\frac{3}{28}$.

$$\Rightarrow \frac{1}{x} - \frac{1}{x+3} = \frac{3}{28}$$

$$\Rightarrow \frac{x+3-x}{x(x+3)} = \frac{3}{28}$$

$$\Rightarrow \frac{3}{x(x+3)} = \frac{3}{28}$$

$$\Rightarrow \frac{1}{x^2+3x} = \frac{1}{28}$$

$$\Rightarrow x^2 + 3x = 28$$

$$\Rightarrow x^2 + 3x - 28 = 0$$

$$\Rightarrow x^2 + 7x - 4x - 28 = 0$$

$$\Rightarrow x(x+7) - 4(x+7) = 0$$

$$\Rightarrow (x-4)(x+7) = 0$$

$$\Rightarrow x-4 = 0 \text{ or } x+7 = 0$$

$$\Rightarrow x = 4 \text{ or } x = -7$$

As a natural number cannot be negative, the two natural numbers are 4 and 7.

66. The given equation is:

$$a^2x^2 - 3abx + 2b^2 = 0$$

$$a^2x^2 - abx - 2abx + 2b^2 = 0$$

$$ax(ax-b) - 2b(ax-b) = 0$$

$$(ax-b)(ax-2b) = 0$$

$$ax-b = 0; x = \frac{b}{a}$$

$$ax-2b = 0; x = \frac{2b}{a}$$

So zeroes are $\frac{b}{a}$ and $\frac{2b}{a}$

67. Given that the quadratic equation $2x^2 + px + 8 = 0$ has real roots

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

$$a = 2, b = p \text{ and } c = 8$$

$$\text{Thus, } D = 0$$

$$\text{Discriminant, } D = b^2 - 4ac \geq 0$$

$$(p)^2 - 4(2)(8) \geq 0$$

$$(p)^2 - 64 \geq 0$$

$$p^2 \geq 64$$

Taking square root on both sides, we get

$$p \geq 8 \text{ or } p \leq -8$$

The roots of equation are real for $p \geq 8$ or $p \leq -8$.

68. Let the two consecutive natural numbers be x and x+1.

According to the question

$$x^2 + (x+1)^2 = 313$$

$$\Rightarrow x^2 + x^2 + 2x + 1 = 313$$

$$\Rightarrow 2x^2 + 2x - 312 = 0$$

$$\Rightarrow x^2 + x - 156 = 0$$

$$\Rightarrow x^2 + 13x - 12x - 156 = 0$$

$$\Rightarrow x(x+13) - 12(x+13) = 0$$

$$\Rightarrow (x+13)(x-12) = 0$$

Either $x+13 = 0$ or $x-12 = 0$

$$\Rightarrow x = -13, 12$$

Since x being a natural number. It cannot be negative.

$$\therefore x = 12$$

Hence, two consecutive numbers are 12, and 13.

69. Given quadric equation is $x^2 - kx + 4 = 0$, and roots are equal.

we have to find the value of k .

Here, $a = 1$, $b = -k$ and, $c = 4$

As we know that $D = b^2 - 4ac$

Putting the value of $a = 1$, $b = -k$ and, $c = 4$

$$= (-k)^2 - 4 \times 1 \times 4$$

$$= k^2 - 16$$

The given equation will have equal roots, if $D = 0$

$$k^2 - 16 = 0$$

$$k^2 = 16$$

$$k = \sqrt{16}$$

$$k = \pm 4$$

Therefore, the value of $k = \pm 4$

70. Let the numbers be x and $15 - x$.

According to question,

$$\frac{1}{x} + \frac{1}{15-x} = \frac{3}{10}$$

$$\Rightarrow \frac{15-x+x}{x(15-x)} = \frac{3}{10}$$

$$\Rightarrow \frac{15}{x(15-x)} = \frac{3}{10}$$

$$\Rightarrow 150 = 3x(15-x)$$

$$\Rightarrow 150 = 45x - 3x^2$$

$$\Rightarrow 150 = 3(15x - x^2)$$

$$\Rightarrow x^2 - 15x + 50 = 0$$

$$\Rightarrow x^2 - 10x - 5x + 50 = 0$$

$$\Rightarrow x(x-10) - 5(x-10) = 0$$

$$\Rightarrow (x-10)(x-5) = 0$$

$$\Rightarrow x-10 = 0 \text{ or, } x-5 = 0$$

$$\Rightarrow x = 10 \text{ or, } x = 5$$

Therefore, the two numbers are 10 and 5

71. The given equation is: $48x^2 - 13x - 1 = 0$

$$\Rightarrow 48x^2 - 16x + 3x - 1 = 0$$

$$\Rightarrow 16x(3x-1) + 1(3x-1) = 0$$

$$\Rightarrow 16x+1 = 0 \text{ or } (3x-1) = 0$$

$$\Rightarrow x = \frac{-1}{16} \text{ or } x = \frac{1}{3}$$

72. We have, $2x^2 - x + \frac{1}{8} = 0$

$$\Rightarrow 2x^2 - \frac{1}{2}x - \frac{1}{2}x + \frac{1}{8} = 0$$

$$\Rightarrow x(2x - \frac{1}{2}) - \frac{1}{4}(2x - \frac{1}{2}) = 0$$

$$\Rightarrow (2x - \frac{1}{2})(x - \frac{1}{4}) = 0$$

Either $(2x - \frac{1}{2}) = 0$ or $(x - \frac{1}{4}) = 0$

$$\Rightarrow x = \frac{1}{4}, \frac{1}{4}$$

So, this root is repeated root.

\therefore both the roots are $\frac{1}{4}$.

73. $3x^2 - 2\sqrt{6}x + 2 = 0$

$$3x^2 - \sqrt{6}x - \sqrt{6}x + 2 = 0$$

$$= \sqrt{3}x(\sqrt{3}x - \sqrt{2}) - \sqrt{2}(\sqrt{3}x - \sqrt{2})$$

$$= (\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2})$$

$$(\sqrt{3}x - \sqrt{2})(\sqrt{3}x - \sqrt{2}) = 0$$

$$\Rightarrow \sqrt{3}x - \sqrt{2} = 0 \text{ or } x = \sqrt{\frac{2}{3}}$$

\therefore the roots are $\sqrt{\frac{2}{3}}, \sqrt{\frac{2}{3}}$

74. We have, $x^2 - \frac{11}{4}x + \frac{15}{8} = 0$

Multiplying the given equation by 8, we get

$$8x^2 - 22x + 15 = 0$$

$$\Rightarrow 8x^2 - 12x - 10x + 15 = 0$$

$$\Rightarrow 4x(2x - 3) - 5(2x - 3) = 0$$

$$\Rightarrow (2x - 3)(4x - 5) = 0$$

Either $2x - 3 = 0$ or $4x - 5 = 0$

$$\Rightarrow x = \frac{3}{2}, \frac{5}{4}$$

Hence, $x = \frac{3}{2}, \frac{5}{4}$ are the required solution.

75. We have the following equation,

$$10x - \frac{1}{x} = 3$$

$$\Rightarrow 10x^2 - 1 = 3x$$

$$\Rightarrow 10x^2 - 3x - 1 = 0$$

Factorise the equation,

$$\Rightarrow 10x^2 - 5x + 2x - 1 = 0$$

$$\Rightarrow 5x(2x - 1) + 1(2x - 1) = 0$$

$$\Rightarrow (5x + 1)(2x - 1) = 0$$

$$\Rightarrow 5x + 1 = 0 \text{ or } 2x - 1 = 0$$

Therefore, $x = \frac{-1}{5}$ or $x = \frac{1}{2}$