Solution

QUADRATIC EQUATIONS WS 5

Class 10 - Mathematics

Section A

1. Since x = 2 is a root of the equation $3x^2 - 2kx + 5 = 0$
   Therefore it satisfies the equation
   $i.e.\ 3(2)^2 - 2k(2) + 5 = 0$
   $\implies 12 - 4k + 5 = 0$
   $\implies 4k = 17$
   $\implies k = \frac{17}{4}$
2. We have,
   $3x^2 - 5x + 2k = 0$
   Here, a = 3, b = -5 and c = 2k
   $\therefore D = b^2 - 4ac$
   $= (-5)^2 - 4 \times 3 \times 2k$
   = 25 - 24k
   $\Rightarrow$ D = 25 - 24k
   Now, it is given that given equation will has real and equal roots,
   So, discriminant D = 0
   $\Rightarrow$ 25 - 24k = 0
   $\Rightarrow$ 24k = 25
   $\Rightarrow k = \frac{25}{24}$
   Hence, $k = \frac{25}{24}$ so that given equation has real and equal roots.
3. We have the following equation,
   $x^2 + px - q^2 = 0$
   a = 1, b = p and $c = -q^2$
   $\therefore D = b^2 - 4ac$
   $= (p)^2 - 4(1)(-q^2)$
   $= p^2 + 4q^2 > 0$
   Therefore, the roots of the given equation are real for all real value of p and q.
4. The given quadric equation is 3x2 - 5x - 2k = 0, and roots are real and equal
   Then find the value of k.
   Here, a = 3, b = -5 and c = -2k
   As we know that $D = b^2 - 4ac$
   Putting the value of a = 3, b = -5 and c = -2k
   $= (-5)^2 - 4 \times (3) \times (-2k)$
   = 25 + 24k
   The given equation will have real and equal roots, if D = 0
   Thus,
   25 + 24 k = 0
   24 k = -25
   $k = \frac{-25}{24}$
   Therefore, the value of $k = \frac{-25}{24}$
5. Let D_1 and D_2 be the discriminant of equations $ax^2 + 2bx + c = 0$ and bx^2 - 2ac−−√2acx + b = 0
   First equation, $ax^2 + 2bx + c = 0$
   $D_1 = 0$ [For equal roots]
   $b^2 - 4ac = 0 \Rightarrow 4b^2 - 4ac = 0$ ...(iii)
   Second equation, $bx^2 - 2\sqrt{ca}x + b = 0$

$D_2 = 0$

$b^2 - 4ac = 0 \Rightarrow (-2\sqrt{ac})^2 - 4b^2 = 0$ ...(iv)

Equating Eqn's (i) & (ii), to get the required condition,

$4b^2 - 4ac = 4ac - 4b^2 \Rightarrow 8ac = 8b^2$

$ac = b^2$

Hence, $ac = b^2$ is the required condition for the two equations to have equal roots.

6. The given quadratic equation is

   $2x^2 + kx + 3 = 0$

   Here, a = 2, b = k, c = 3

   Therefore, discriminant = $b^2 - 4ac$

   $= (k)^2 - 4(2)(3) = k^2 - 24$

   If the given quadratic equation has two equal real roots, then

   $b^2 - 4ac = 0$

   $\Rightarrow \mathrm{k}^2 - 24 = 0 \Rightarrow \mathrm{k}^2 = 24$

   $\Rightarrow \mathrm{k} = \pm\sqrt{24} \Rightarrow \mathrm{k} \pm 2\sqrt{6}$

   Hence, the required values of k are $\pm 2\sqrt{6}$.

   i.e., $2\sqrt{6}$ and $-2\sqrt{6}$

7. $(k + 1)x^2 + 2(k + 1)x + 1 = 0$

   has equal roots if D = 0

   i.e $b^2 = 4ac$

   Here, a = (k + 1), b = 2(k + 1), c = 1

   or, $4(k + 1)^2 = 4(k + 1)$

   $k^2 + 2k + 1 = k + 1$

   $k^2 + 2k + 1 - k - 1 = 0$

   $k^2 + k = 0$

   k(k + 1) = 0

   k = 0, -1

   Since k = -1 does not satisfy the equation

   $\Rightarrow$ k = 0

8. To check whether the quadratic equation has real roots or not, we need to check the discriminant value i.e.,

   $D = b^2 - 4ac$

   Given, $5x^2 - 2x - 10 = 0$

   $\therefore D = (-2)^2 - 4(5)(-10)$

   $\Rightarrow$ D = 4 + 200 > 0

   Hence, the roots are real and distinct.

   To find the roots, use the formula,

   $\mathrm{x} = \frac{-\mathrm{b} \pm \sqrt{\mathrm{b}^2 - 4\mathrm{ac}}}{2\mathrm{a}}$

   $\therefore \mathrm{x} = \frac{2 \pm \sqrt{204}}{2(5)}$

   $= \frac{2 \pm 2\sqrt{51}}{10} = \frac{1+\sqrt{51}}{5}, \frac{1-\sqrt{51}}{5}$

9. Given quadratic equation is: $4x^2 - 5x - 1 = 0$

   Comparing the above equation with ax2 + bx + c = 0

   We get a = 4, b = -5 and c = - 1

   Now, $b^2 - 4ac = (-5)^2 - 4(4)(-1)$

   = 25 + 16

   = 41 > 0

   Hence, roots of the quadratic equation are real and distinct.

10. The given equation is $9x^2 + 8kx + 16 = 0$

    Comparing it with, $ax^2 + bc + c = 0$, we have

    $a = 9,\ b = 8k\ and\ c = 16$

Now, for equal roots, we have

Discriminant, D = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (8k)^2 - 4(9)(16) = 0$

$\Rightarrow 64k^2 - 576 = 0$

$\Rightarrow 64k^2 = 576$

$\Rightarrow k^2 = \frac{576}{64}$

$\Rightarrow k^2 = 9$

$\Rightarrow k = \pm 3$

$\Rightarrow$ k = 3 or k = -3

Hence, the required value of k is 3 ,-3.

11. The given equation is

$2x^2 - 3x + 5 = 0$

Here, a = 2, b = -3, c = 5

Therefore, discriminant = $b^2 - 4ac$

$= (-3)^2 - 4(2)(5)$

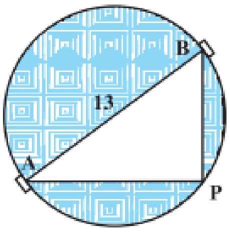
$= 9 - 40 = -31 < 0$

So, the given equation has no real roots.

12. Let P be the position of the pole and A & B be the opposite fixed gates. Let, BP = x metres.

$\therefore$ AP = x + 7

In right triangle APB,



$AP^2 + BP^2 = AB^2$

$\Rightarrow (x + 7)^2 + x^2 = 13^2$

$\Rightarrow x^2 + 49 + 14x + x^2 = 169$

$\Rightarrow 2x^2 + 14x + 49 - 169 = 0$

$\Rightarrow 2x^2 + 14x - 120 = 0$

$\Rightarrow 2(x^2 + 7x - 60) = 0$

$\Rightarrow x^2 + 7x - 60 = 0$

$\Rightarrow x^2 + 12x - 5x - 60 = 0$

$\Rightarrow x(x + 12) - 5(x + 12) = 0$

$\Rightarrow (x + 12)(x - 5) = 0$

Either x+12 = 0, then x = -12 which is not possible being negative or x - 5 = 0, then x = 5.

Thus P is at a distance of 5m from B and 5+7 = 12m from A.

13. Let first root of given quadratic equation be α

According to question second root = 3α

We know that

Sum of roots = $-\frac{b}{a}$

$\alpha + 3\alpha = \frac{-b}{a}$

$\Rightarrow 4\alpha = \frac{-12}{1}$

$\Rightarrow \alpha = -3$

Product of roots = $\frac{c}{a}$

$\alpha \times 3\alpha = \frac{-k}{1}$

$3\alpha^2 = -k$

Put the value of α

$3 \times (-3)^2 = -k$

$k = -27$

14. The given equation is:

    $u^2 + 5ku + 16 = 0$

    If we compare the given equation with the general form, i.e

    $ax^2 + bx + c = 0$

    we get,

    $a = 1, b = 5k, c = 16$

    Now,

    Discriminant, $D = b^2 - 4ac$

    For, the equation to have real and equal roots, D should be equal to zero.

    $b^2 - 4ac = 0$

    $(5k)^2 - 4 \times 1 \times 16 = 0$

    $(5k)^2 = 4 \times 16$

    $25k^2 = 4 \times 16$

    $k^2 = \frac{4\times16}{25}$

    $k = \pm\sqrt{\frac{4\times16}{25}}$

    $k = \pm\frac{2\times4}{5}$

    $k = \pm\frac{8}{5}$

    $\therefore k = \frac{8}{5}$ or $k = \frac{-8}{5}$

15. The given equation is $kx(x-2\sqrt{5})+10=0$

    or, $kx^2-2\sqrt{5}kx+10=0$

    Here, $a=k,\ b=-2\sqrt{5}k,\ c=10$

    It is given that the roots are equal,

    $\therefore D=b^2-4ac=0$

    or, $(-2\sqrt{5}k)^2-4\times k\times10=0$

    or, $20k^2-40k=0$

    or, $20k(k - 2) = 0$

    or, $k (k -2) = 0$

    $k = 0$ or $k = 2$

16. Let the breadth of the rectangle be x metres

    and the length is 2x metres.

    So, area of rectangle is 800 sq.m.

    $(x)(2x) = 800$

    $2x^2 = 800$

    $x^2 = 400$

    $x = \pm 20$

    But breadth of rectangle cannot be negative, so x = 20

    and yes, it is possible to design it.

    So, Breadth is 20 m and length is 40 m.

17. Given that the quadratic equation $x^2 - 4kx + k = 0$ has equal roots

    Comparing with general equation $ax^2 + bx + c = 0$, for the given equation

    $a = 1, b = -4k$ and $c = k$

    Hence, $D = b^2 - 4ac = 0$

    $(-4k)^2 - 4 \times 1 \times k = 0$

    $16k^2 - 4k = 0$

    $4k(4k - 1) = 0$

    $4k = 0$ or $(4k - 1) = 0$

k = 0 or k = $\frac{1}{4}$

Hence 0 and $\frac{1}{4}$ are values of k which the equation has equal roots.

18. We have, (k- 12)x^2 + 2(k - 12)x + 2 = 0.

    a = k - 12,b = 2(k - 12) and c = 2.

    The given equation will have equal roots, if

    $\therefore$ D = b^2 - 4ac = 0

    $\Rightarrow$ 4$(k - 12)^2$ - 4 (k - 12)$\times$ 2 = 0

    $\Rightarrow$4(k- 12) [(k - 12) - 2] = 0

    $\Rightarrow$4(k - 12)(k - 14) = 0

    $\Rightarrow$4(k - 12) (k - 14) = 0 $\Rightarrow$ k - 12 = 0 or, k - 14 = 0 $\Rightarrow$ k = 12 or, k = 14
19. We have, 5x^2 - kx + 1 = 0.

    a = 5, b = -k and c = 1

    $\therefore$ D = b^2 - 4ac = $(- k)^2$ - 4 $\times$ 5 $\times$ 1 = k^2 - 20

    To have a real roots,

    $D \geq 0$

    $\Rightarrow \quad k^2 - 20 \geq 0$

    $\Rightarrow \quad k \leq -\sqrt{20}$ or , $k \geq \sqrt{20}$
20. we are given that 2 is a root of the equation x^2 + kx + 12 = 0 and the equation x^2 + kx + q = 0 has equal roots, find the value of q.

    If 2 is the root of $x^2 + kx + 12 = 0$,then

    $(2)^2 + 2k + 12 = 0$

    or, 2k + 16 = 0

    k = - 8

    Put k = -8, in $x^2 + kx + q = 0$,we get

    $x^2 - 8x + q = 0$

    For equal roots

    $(-8)^2 - 4(1)q = 0$

    64 -4q = 0

    4q = 64

    q = 16
21. We have the following equation,

    5x(x - 2) + 6 = 0

    $\Rightarrow$ 5x^2 -10x + 6 = 0

    a = 5, b = -10 and c = 6

    $\therefore$ D = b^2 - 4ac

    = $(-10)^2$ - 4(5)(6)

    = 100 - 120

    = -20 < 0

    Therefore, the given equation has no real roots.
22. For real roots of quadratic equation, b^2 – 4ac > 0

    We have, -2x^2+ 3x + 2 = 0

    Now, b^2 – 4ac > 0

    ⇒ $(3)^2$ – 4(–2)(2)>0 ($\because$a = –2, b = 3, c = 2)

    ⇒9 + 16 > 0

    ⇒ 25 > 0

    Now, $\sqrt{D} = 5$

    And, $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm 5}{2(-2)} = \frac{-3 \pm 5}{-4}$

    $\Rightarrow x = \frac{-3+5}{-4}$ and $x = \frac{-3-5}{-4}$

    $\Rightarrow x = \frac{2}{-4}$ and $x = \frac{-8}{-4}$

    $\Rightarrow x = \frac{-1}{2}$ and 2

    Therefore, the roots of the given equation are 2 and $\frac{-1}{2}$.

23. The given equation is: $9x^2 + 3kx + 4 = 0$

Comparing with standard quadratic equation $ax^2 + bx + c = 0$

a = 9, b = 3k and c = 4

For real and distinct roots: D > 0

Discriminant, $D = b^2 - 4ac > 0$

$(3k)^2 - 4(4)(9) = 9k^2 - 144 > 0$

$9k^2 > 144$

$k^2 > 16$

Taking square root on both sides, we get

k > 4 or k < - 4

24. We have, $(k+1)x^2 - 2(k - 1)x+1 = 0$.

a = k + 1, b = -2(k - 1), c = 1.

$D = b^2 - 4ac = 4(k-1)^2 - 4(k + 1) = 4(k^2 - 3k)$

The given equation will have real and equal roots, if

$D = 0 \Rightarrow 4 (k^2 - 3k) = 0 \Rightarrow k^2 - 3k = 0 \Rightarrow k (k - 3) = 0 \Rightarrow k = 0, 3$

25. The roots are -7, 9

Sum of roots = -7 + 9

= 2

Product of roots

$= -7 \times 9$

= -63

Required quadratic equation

x^2 - (sum of roots)x + Product of roots = 0

$x^2 - 2x - 63 = 0$

26. As the roots of the equation are given to be equal. So,

D = 0

$\Rightarrow 4(1 - p^2) = 0$

$\because 4(1 - p^2) = 0$

$\therefore (1 - p^2) = 0$

$\Rightarrow p^2 = 1$

$\Rightarrow p = \pm 1$

So, p = 1 or -1

Therefore, the given equation will have equal roots for p = 1 or -1.

27. The given quadratic equation is

$3x^2 - 4\sqrt{3}x + 4 = 0$

Here, a = 3, $b = -4\sqrt{3}$, c = 4

$\therefore$ discriminant = $b^2 - 4ac$

$= (-4\sqrt{3})^2 - 4(3)(4)$

= 48 - 48 = 0

Hence, the given quadratic equation has two equal real roots.

The roots are $= -\frac{b}{2a}, -\frac{b}{2a}$

$\text{i.e. } -\frac{(-4\sqrt{3})}{2\times 3}, -\frac{(-4\sqrt{3})}{2\times 3}, \text{ i.e. } \frac{2}{\sqrt{3}}, \frac{2}{\sqrt{3}}$

28. In the given equation a = 1, b = -5 and c = 9

substitute these values in the formula of discriminant, we get

$(-5)^2 - 4(1)(9)$

25 - 36

-11

It is obtained that by substituting these values in the formula, we get the final answer as -11.

If the discriminant value is negative, the roots are said to be imaginary.

The root of this equation is $\pm 11i$ where i represents the imaginary roots.
Therefore, the nature of the root is obtained as imaginary.

29. $a = 13\sqrt{3}, b = 10, c = \sqrt{3}$
$b^2 - 4ac = (10)^2 - 4(13\sqrt{3})(\sqrt{3})$
= 100 - 156
= - 56
As d < 0
So, the equation has not real roots.

30. We have the following equation,
$4x^2 + px + 3 = 0$
This is the form of $ax^2 + bx + c = 0$, where
a = 4, b = p and c = 3
For real and equal roots, we must have
D = 0
$\Rightarrow b^2 - 4ac = 0$
$\Rightarrow p^2 - 4(4)(3) = 0$
$\Rightarrow p^2 - 48 = 0$
$\Rightarrow p^2 = 48$
$\Rightarrow p = \pm 4\sqrt{3}$

31. For equation $x^2 + kx + 64 = 0$
$b^2 - 4ac = 0$
or, $k^2 - 4 \times 1 \times 64 = 0$
or, $k^2 - 256 = 0$
or, $k = \pm 16$ ..(i)
and for equation $x^2 - 8x + k = 0$
$b^2 - 4ac = 0$
or, $(-8)^2 - 4 \times 1 \times k = 0$
or, 64 = 4k
or, $k = \frac{64}{4} = 16 \ldots (ii)$
From (i) and (ii), we get k=16
For k= 16, given equations have equal roots.

32. kx(x − 2) + 6 = 0
$\Rightarrow kx^2 - 2kx + 6 = 0$
Comparing quadratic equation $kx^2 - 2kx + 6 = 0$ with general form $ax^2 + bx + c = 0$, we get a = k, b = -2k and c = 6
Discriminant = $b^2 - 4ac = (-2k)^2 - 4(k)(6) = 4k^2 - 24k$
We know that two roots of quadratic equation are equal only if discriminant is equal to zero.
Putting discriminant equal to zero
$4k^2 - 24k = 0$
$\Rightarrow 4k(k - 6) \Rightarrow k = 0, 6$
The basic definition of quadratic equation says that quadratic equation is the equation of the form $ax^2 + bx + c = 0$, where $a \neq 0$
Therefore, in equation $kx^2 - 2kx + 6 = 0$, we cannot have k = 0.
Therefore, we discard k = 0.
Hence the answer is k = 6.

33. Here, a = 9, b = - 3 k, c = k
Since roots of the equation are equal
So, $b^2 - 4ac = 0$
$(-3k)^2 - (4 \times 9 \times k) = 0$
$9k^2 - 36k = 0$
$k^2 - 4k = 0$
k(k - 4) = 0

k = 0 or k = 4

But given k is non zero hence k = 4 for which roots of the quadratic equation are real and equal.

34. $\because$ The given equation has equal roots

$\therefore$ D = 0

D = b^2 - 4ac

0 = $(2ab)^2 - 4(1 + a^2)(b^2 - c^2)$

0 = $4a^2b^2 - 4(b^2 - c^2 + a^2b^2 - a^2c^2)$

0 = $4a^2b^2 - 4b^2 + 4c^2 - 4a^2b^2 + 4a^2c^2$

0 = $-4b^2 + 4c^2 + 4a^2c^2$

$4b^2 = 4(c^2 + a^2c^2)$

$b^2 = c^2(1+ a^2)$

35. We have, $x^2 + px + 1 = 0$

Here a = 1, b = p, c = 1

$\implies D = b^2 - 4ac = p^2 - 4(1)(1) = p^2 - 4$

For real roots, $D \geq 0$

$\implies p^2 - 4 \geq 0 \implies (p + 2)(p - 2) \geq 0$

$\implies p \leq -2 \, or \, p \geq 2$

36. We have the following equation,

$12x^2 - 4\sqrt{15}x + 5 = 0$

a = 12, b = $-4\sqrt{15}$ and c = 5

$\therefore$ D = b^2 - 4ac

= $(-4\sqrt{15})^2 - 4(12)(5)$

= 240 - 240

= 0

D = 0

Therefore, the roots of the given equation are real and equal.

37. We have, $25x^2 + 30x + 7 = 0$

a = 25, b = 30 and c = 7

$\therefore$ D = $b^2 - 4ac = (30)^2 - 4(25)(7) = 900 - 700 = 200 > 0$

So, the given equation has real roots, given by

$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-30+\sqrt{200}}{2\times25} = \frac{-30+10\sqrt{2}}{50} = \frac{10(-3+\sqrt{2})}{50} = \frac{(-3+\sqrt{2})}{5}$

$\beta = \frac{-b-\sqrt{D}}{2a} = \frac{-30-\sqrt{200}}{2\times25} = \frac{-30-10\sqrt{2}}{50} = \frac{10(-3-\sqrt{2})}{50} = \frac{-3-\sqrt{2}}{5}$

Therefore, $\frac{(-3+\sqrt{2})}{5}$ and $\frac{(-3-\sqrt{2})}{5}$ are the roots of the given equation.

38. We have, $kx^2 + 2x + 1 = 0$

here, a = k, b=2, c=1

$\therefore D = b^2 - 4ac = (2)^2 - 4(k)(1) = 4 - 4k$

The given equation will have real and distinct roots,

$D > 0$

$\Rightarrow 4 - 4k > 0 \Rightarrow k < 1$

39. The given equation is $x^2 - 4x + k = 0$.

a = 1, b = -4 and c = k

$\therefore$ D = $b^2 - 4ac = (-4)^2 - 4 \times 1 \times k = 16 - 4k$

The given equation will have real and distinct roots, if

D > 0

$\Rightarrow$ 16 - 4k > 0

$\Rightarrow$ 16 > 4k

$\Rightarrow$ 4k < 16

$\Rightarrow k < \frac{16}{4} = 4$

$\Rightarrow$ k < 4

40. We have,

$kx^2 - 2\sqrt{5}x + 4 = 0$

Here, a = k, $b = -2\sqrt{5}$ and c = 4

$\therefore D = b^2 - 4ac$

$= \left(-2\sqrt{5}\right)^2 - 4 \times k \times 4$

= $20 - 16k$

$\Rightarrow D = 20 - 16k$

The given equation will have real and equal roots, if

$D = 0$

$\Rightarrow 20 - 16k = 0$

$\Rightarrow 16k = 20$

$\Rightarrow k = \frac{20}{16} = \frac{5}{4}$

$\therefore k = \frac{5}{4}$

41. The given equation is $x^2 + k(2x + k - 1) + 2 = 0$

$\Rightarrow x^2 + 2kx + k(k-1) + 2 = 0$

So, a = 1, b = 2 k, c = k(k - 1) + 2

We know $D = b^2 - 4ac$

$\Rightarrow D = (2k)^2 - 4 \times 1 \times [k(k-1) + 2]$

$\Rightarrow D = 4k^2 - 4[k^2 - k + 2]$

$\Rightarrow D = 4k^2 - 4k^2 + 4k - 8$

$\Rightarrow D = 4k - 8 = 4(k-2)$

For equal roots, D = 0

Thus, 4(k - 2) = 0

So, k = 2.

42. We have the following equation,

$mx^2 + 2x + m = 0$

Here, a = m, b = 2 and c = m

Now, D = 0

$\Rightarrow b^2 - 4ac = 0$

$\Rightarrow (2)^2 - 4(m)(m) = 0$

$\Rightarrow 4 - 4m^2 = 0$

$\Rightarrow 4m^2 = 4$

$\Rightarrow m^2 = \frac{4}{4}$

$\Rightarrow m^2 = 1$

$\Rightarrow m = \sqrt{1} \Rightarrow m = \pm 1$

Therefore, m = -1 and m = 1

43. We have, $4x^2 - 3kx + 9 = 0$

Here, a=4, b=-3k, c=9

$\therefore D = b^2 - 4ac = (-3k)^2 - 4(4)(9) = 9k^2 - 144$

For the equation having real and distinct roots.

$\therefore D > 0 \implies 9k^2 - 144 > 0 \implies k^2 > \frac{144}{9} \implies k^2 > 16$

$\implies k > 4 \, or \, k < -4$

44. **Given:** $9x^2 + 8kx + 16 = 0$

Here,

a = 9, b = 8k and c = 16

It is given that the roots of the equation are real and equal; therefore, we have: D = 0

$\Rightarrow (b^2 - 4ac) = 0$

$\Rightarrow (8k)^2 - 4 \times 9 \times 16 = 0$

$\Rightarrow 64k^2 - 576 = 0$

$\Rightarrow 64k = 576$

$\Rightarrow$ k^2 = 9

k = ± 3

So,

k=-3 or k=3

Hence, for k=-3,3 the equation 9x^2 + 8kx + 16 = 0 has equal roots.

45. The given quadratic equation is $3\sqrt{3}x^2 + 10x + \sqrt{3} = 0$

    On comparing with standard form of quadratic equation i.e, $ax^2 + bx + c = 0$, we get a = $3\sqrt{3}$, b = 10 , c = $\sqrt{3}$

    Now, $D = b^2 - 4ac = (10)^2 - 4 \times 3\sqrt{3} \times \sqrt{3}$

    = 100 - 36 = 64

    Therefore, D = 64

    We know that , When D > 0 , then equation has real and distinct roots.

    Therefore , Given equation has real and distinct roots.

46. -4 is root of x^2 + px - 4 = 0

    $\therefore$ (-4)^2 + p(-4) - 4 = 0

    $\Rightarrow$ 16 - 4p - 4 = 0

    $\Rightarrow$ -4p = -12

    $\Rightarrow$ p = 3

    x^2 - px + k = 0 (Given)

    x^2 + 3x + k = 0

    D = b2 - 4ac

    a = 1, b = 3, c = 1

    $\Rightarrow 0 = (3)^2 - 4 \times 1 \times k$ [For equal roots D = 0]

    $\Rightarrow$ 4k = 9

    $\Rightarrow k = \frac{9}{4}$

47. Here we have, $4x^2 + kx + 3 = 0$

    Comparing with $ax^2 + bx + c = 0$, we get

    $a = 4,\ b = k,\ c = 3$

    Now, Discriminant i,e,. D $\geq$ 0

    $\Rightarrow$ k^2 - 4 $\times$ 4 $\times$ 3 $\geq$ 0

    $\Rightarrow k^2 - 48 \geq 0$

    $k^2 \geq 48$

    $k \geq \pm\sqrt{48}$

48. We have,

    $x^2 + kx + 4 = 0$

    Here, $a = 1,\ b = k\, and\, c = 4$

    $\therefore D = b^2 - 4ac$

    = (k)^2 - 4 $\times$ 1 $\times$ 4

    = $k^2 - 16$

    $\Rightarrow D = k^2 - 16$

    The given equation will have real roots, if

    $D \geq 0$

    $\Rightarrow k^2 - 16 \geq 0$

    $\Rightarrow k^2 - (4)^2 \geq 0$

    $\Rightarrow k \leq -4$ or $k \geq 4$ $\left[\because x^2 - a^2 \geq 0 \Rightarrow x \leq -a \text{ or, } x \geq a\right]$

    $\because k \geq 4$

    $\therefore$ k = 4 is the least positive value for which the given equation has real roots

49. For the quadratic equation, (m - 1)x^2 - 2(m - 1)x + 1 = 0

    Discriminant = (−2(m - 1))^2 - 4 $\times$ (m - 1) = 0.... Since roots are real equal roots

    4m^2 - 8m + 4 - 4m + 4 = 0

    4m^2 - 12m+8=0

    m^2 - 3m + 2 = 0

$\therefore$ m = 1, 2

Neglect m = 1 as the equation will not be quadratic anymore.

Hence, m = 2

50. We have, $2x^2 + x - 1 = 0$

Here a = 2, b = 1, c = -1

$\therefore D = b^2 - 4ac = (1)^2 - 4(2)(-1)$

= 1 + 8 = 9 > 0

Hence, the given equation has real roots.

51. The given quadratic equation is

$2x^2 - 6x + 3 = 0$

Here, a = 2, b = -6, c = 3

Therefore, discriminant = $b^2 - 4ac$

$= (-6)^2 - 4(2)(3) = 36 - 24$

= 12 > 0

So, the given quadratic equation has two distinct real roots

Solving the quadratic equation $2x^2 - 6x + 3 = 0$, by the quadratic formula, $x = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$

we get $= \frac{-(-6)\pm\sqrt{12}}{2(2)} = \frac{6\pm2\sqrt{3}}{4} = \frac{3\pm\sqrt{3}}{2}$

Therefore, the roots are $\frac{3\pm\sqrt{3}}{2}$, i.e. $\frac{3+\sqrt{3}}{2}$ and $\frac{3-\sqrt{3}}{2}$

52. We have the following equation,

$3x^2 - 2kx + 27 = 0$

a = 3, b = -2k and c = 27

$\therefore D = b^2 - 4ac$

$= (-2k)^2 - 4(3)(27)$

$= 4k^2 - 324$

Roots are real and equal if D = 0

$\Rightarrow 4k^2 - 324 = 0$

$k^2 = \frac{324}{4} = 81$

$\therefore k = \pm 9$

53. Since the roots are equal, then

D = 0

$D = b^2 - 4ac = 0$

or, $(-2k)^2 - 4(k)(6) = 0$

or, $4k^2 - 24k = 0$

or, 4k(k - 6) = 0

or, k = 0,6

But $k \neq 0$ as coefficient of x^2 can not be zero.

k= 6

Section B

54. The given equation is; (x - a)(x - b)+(x - b)(x - c)+(x - c)(x - a) =0

$\Rightarrow (x^2 - ax - bx + ab) + (x^2 - bx - cx + bc) + (x^2 - cx - ax + ac) = 0$

$\Rightarrow 3x^2 - 2x(a + b + c) + (ab + bc + ca) = 0$.....(1).

Discriminant 'D' of quadratic equation (1) is given by;

$\therefore D = 4(a + b + c)^2 - 12(ab + bc + ca)$

$= 4[(a + b + c)^2 - 3(ab + bc + ca)]$

$= 4(a^2 + b^2 + c^2 - ab - bc - ca)$

$= 2(2a^2 + 2b^2 + 2c^2 - 2ab - 2bc - 2ca)$

$= 2[(a - b)^2 + (b - c)^2 + (c - a)^2] \geq 0$

$[\because (a - b)^2 \geq 0, (b - c)^2 \geq 0$ and $(c - a)^2 \geq 0]$.

This shows that both the roots of the given equation are real.

For equal roots, we must have D = 0.

Now, D = 0 ⇒ $(a - b)^2 + (b - c)^2 + (c - a)^2 = 0$

⇒ (a - b) = 0, (b - c) = 0 and (c - a) = 0 (sum of squares can be zero only if they all are equal to 0)

⇒ a = b = c.

Hence, the roots are equal only when a = b = c

55. Given quadratic equations are;

$x^2 + px + q = 0$ —(i)

and, $x^2 + rx + s = 0$ ......(ii)

Also given ; pr = 2(q + s)........(iii)

Let D_1 and D_2 be the discriminant of quadratic equations (i) and (ii) respectively. Then,

$D_1 = p^2 - 4q$ and $D_2 = r^2 - 4s$

⇒ $D_1 + D_2 = p^2 - 4q + r^2 - 4s = (p^2 + r^2) - 4(q + s)$

$\Rightarrow D_1 + D_2 = p^2 + r^2 - 4\left(\frac{pr}{2}\right)$ ([from equation (iii)]

$\Rightarrow D_1 + D_2 = p^2 + r^2 - 2pr = (p - r)^2 \geq 0 \quad \left[\because (p - r)^2 \geq 0 \text{ for all real } p, r\right]$

Now, Since sum of both D_2 & D_1 is greater than or equal to 0. Hence, both can't be negative.

⇒ At least one of D_1 and D_2 is greater than or equal to zero

Case 1. If $D_1 \geq 0$, equation (i) has real roots.

Case 2.If $D_2 \geq 0$, equation (ii) has real roots.

Case 3. If D_1 & D_2 both ≥ 0, then equation (i) & (ii) both have equal roots.

Clearly, from case 1,2 & 3 at least one of the two given quadratic equations has real roots.

56. Given, $x^2 + k(4x + k - 1) + 2 = 0$

$x^2 + 4kx + k^2 - k + 2 = 0.$

Here, a =1, b = 4k, c = k^2-k + 2 = 0

Now equation has real roots, D = 0

i,e. $b^2 - 4ac = 0$

$(4k)^2 - 4 \times 1 \times (k^2-k+2) = 0$

$16k^2 - 4k^2 + 4k - 8 = 0$

$12k^2 + 4k - 8 = 0$

$4(3k^2 + k - 2) = 0$

$3k^2 + 3k - 2k - 2 = 0$

3k(k+1)-2(k+1) = 0

(3k-2)(k+1) = 0

k = $\frac{2}{3}$ or k = -1

57. Let the length and breadth of the park be l and b.

Perimeter = $2(l + b) = 80$

$l + b = 40 \; or, \; b = 40 - l$

Area = $l \times b = l(40 - l)$

$= 40l - l^2 = 400$ Given

$l^2 - 40l + 400 = 0$

Comparing this equation with $al^2 + bl + c = 0$, we obtain

a = 1, b = -40, c = 400

Discriminant D = $b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$

As $b^2 - 4ac = 0$

Therefore, this equation has equal real roots and hence, this situation is possible.

Root of this equation,

$l = -\frac{b}{2a}$

$l=-\frac{(-40)}{2(1)}=\frac{40}{2}=20$

Therefore, length of park, l = 20m

And breadth of park, b = 40 - l = 40 - 20 = 20m

58.

Let D_1 and D_2 be the discriminant of given two equations respectively. Therefore,

$D_1 = (2c)^2 - 4 \times 1 \times ab = 4c^2 - 4ab = 4(c^2 - ab)$

and, $D_2 = \{-2(a + b)\}^2 - 4 \times 1 \times (a^2 + b^2 + 2c^2)$

$\Rightarrow D_2 = 4(a + b)^2 - 4(a^2 + b^2 + 2c^2)$

$\Rightarrow D_2 = 4\{a^2 + b^2 + 2ab - a^2 - b^2 - 2c^2\}$

$\Rightarrow D_2 = 4(2ab - 2c^2) = -8(c^2 - ab)$

According to the question, we are given that the roots of first equation are real and unequal.

Therefore,

$D_1 > 0$

$\Rightarrow 4(c^2 - ab) > 0$

$\Rightarrow c^2 - ab > 0$

$\Rightarrow -8(c^2 - ab) < 0$

$\Rightarrow D_2=-8(c^2-ab) < 0$

$\Rightarrow$ Roots of equation second are not real.

59. Let the present age of one friend be x years.

Also, sum of ages of both friends = 20 years

hence age of 2nd friend will be (20 - x) years.

4 years ago, age of 1st friend = (x - 4) years.

age of 2nd friend= (20-x)- 4 = (16-x) years.

According to the question;

(x - 4)(1 6 - x) = 48

$\Rightarrow x^2 - 20x + 112 = 0$

Let D be the discriminant of this quadratic. Then,

D = b^2-4ac = 400 - 448 = -48 < 0. (here, a = 1, b = -20, c = 112)

So, above equation does not have real roots. Hence, the given situation is not possible.

60. Given equation- $abx^2 + (b^2 - ac)^2 - bc = 0$

Using quadratic formula

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$

$x=\frac{-(b^2-ac)\pm\sqrt{\left(b^2-ac\right)^2-4\times ab\times(-bc)}}{2ab}$

$x=\frac{-b^2+ac\pm\sqrt{b^4+a^2c^2-2b^2ac+4ab^2c}}{2ab}$

$x=\frac{-b^2+ac\pm\sqrt{b^4+a^2c^2+2ab^2c}}{2ab}$

$x=\frac{-b^2+ac\pm\sqrt{\left(b^2+ac\right)^2}}{2ab}$

$x=\frac{-b^2+ac\pm\left(b^2+ac\right)}{2ab}$

Taking positive

$x=\frac{c}{b}$

Taking negative

$x=\frac{-b}{a}$

61. Given quadratic equation is

$p(x - 4)(x - 2) + (x - 1)^2 = 0$

$\Rightarrow p(x^2 - 4x - 2x + 8) + (x^2 + 1 - 2x) = 0$

$\Rightarrow$ $px^2 - 6px + 8p + x^2 + 1 - 2x = 0$

$\Rightarrow$ $x^2(p + 1) - 2x(3p + 1) + (8p + 1) = 0$

Comparing the above equation with $ax^2 + bx + c = 0$, we get

a = p + 1, b = -2(3p + 1) and c = 8p + 1

For real and equal roots

D = 0 i.e., $b^2 - 4ac = 0$

$\therefore$ $[-2(3p + 1)]^2 - 4(p + 1)(8p + 1) = 0$

$\Rightarrow$ $4(3p + 1)^2 - 4(8p^2 + 9p + 1) = 0$

$\Rightarrow$ $4(9p^2 + 1 + 6p) -32p^2 - 36p - 4 = 0$

$\Rightarrow$ $36p^2 + 4 + 24p - 32p^2 - 36p - 4 = 0$

$\Rightarrow$ $4p^2 - 12p = 0$

$\Rightarrow$ 4p(p - 3) = 0

$\Rightarrow$ p = 0 or p = 3

Hence, for p = 0 or p = 3, the given quadratic equation has real and equal roots.

62. We have,

$kx^2 + kx + 1 = -4x^2 - x$

$\Rightarrow kx^2 + 4x^2 + kx + x + 1 = 0$

$\Rightarrow (k + 4)x^2 + (k + 1)x + 1 = 0$

Here, a = k + 4, b = k + 1 and c = 1

$\therefore D = b^2 - 4ac$

= $(k + 1)^2 - 4 \times (k + 4) \times (1)$

$= k^2 + 1 + 2k - 4k - 16$

$= k^2 - 2k - 15$

$\Rightarrow D = k^2 - 2k - 15$

The given equation will have real and equal roots, if

D = 0

$\Rightarrow k^2 - 2k - 15 = 0$

$\Rightarrow k^2 - 5k + 3k - 15 = 0$

$\Rightarrow$ k(k - 5) + 3(k - 5) = 0

$\Rightarrow$ (k - 5)(k + 3) = 0

$\Rightarrow$ k - 5 = 0 or k + 3 = 0

$\Rightarrow$ k = 5 or k = -3

63. Given, $x^2 - 2x(1 + 3k) + 7(3 + 2k) = 0$

Here, a = 1, b = -2(1 + 3k) and c = 7(3 + 2k)

The given equation will have equal roots, if

D = 0

$\Rightarrow$ $b^2 - 4ac = 0$

$\Rightarrow$ $4(1 + 3k)^2 - 4 \times 1 \times 7(3 + 2k) = 0$

$\Rightarrow$ $4(3k + 1)^2 - 4 \times 1 \times 7(3 + 2k) = 0$

$\Rightarrow$ $4(9k^2 + 1 + 6k) - 4(21 + 14k) = 0$

$\Rightarrow$ $4(9k^2 + 1 + 6k) - 84 - 56k = 0$

$\Rightarrow 36k^2 + 4 + 24k - 84 - 56k = 0$

$\Rightarrow$ $4(9k^2 - 8k - 20) = 0$

$\Rightarrow$ $9k^2 - 8k - 20 = 0$

$\Rightarrow$ $9k^2 - 18k + 10k - 20 = 0$

$\Rightarrow$ $(k - 2)(9k + 10) = 0 \Rightarrow k - 2 = 0$ or, $9k + 10 = 0 \Rightarrow k = 2$ or, $k = -\frac{10}{9}$

64. Since -2 is a root of the equation $3x^2 + 7x + p = 0$, we have

$3 \times (-2)^2 + 7 \times (-2) + p = 0$

$\Rightarrow$ 12 - 14 + p = 0

$\Rightarrow$ p = 2.

For p = 2, the other given equation becomes $x^2 + 4kx + k(k-1) + 2 = 0$.

This is of the form $ax^2 + bx + c = 0$, where

$a = 1, b = 4k \; and \; c = (k^2 - k + 2)$.

$D = (b^2 - 4ac) = 16k^2 - 4 \times 1 \times (k^2 - k + 2)\}$

$= (12k^2 + 4k - 8)$.

For equal roots

D = 0 $\Rightarrow 12k^2 + 4k - 8 = 0 \Rightarrow 4(3k^2 + k - 2) = 0$

$\Rightarrow 3k^2 - k - 2 = 0$

$\Rightarrow 3k^2 + 3k - 2k - 2 = 0$

$\Rightarrow$ 3k(k + 1) - 2(k + 1) =0

$\Rightarrow$ (k + 1)(3k - 2) = 0

$\Rightarrow$ k + 1 = 0 or 3k - 2 = 0

$\Rightarrow$ k = - 1 or k = $\frac{2}{3}$

Hence, the required value of k is -1 or $\frac{2}{3}$

65. According to the question,

$16x^2 = 24x + 1$

$\Rightarrow 16x^2 - 24x - 1 = 0$

Here, a = 16, b = -24 and c = -1

$\therefore D = b^2 - 4ac$

$= (-24)^2 - 4 \times 16 \times (-1)$

= 576 + 64

= 640 > 0

$\therefore$ D > 0

So, the given equation has real roots given by

$\alpha = \frac{-b+\sqrt{D}}{2a} = \frac{-(-24)+\sqrt{640}}{2\times16}$

$= \frac{24+\sqrt{640}}{32}$

$= \frac{24+8\sqrt{10}}{32}$

$= \frac{8[3+\sqrt{10}]}{32}$

$= \frac{3+\sqrt{10}}{4}$

$\therefore \; \alpha = \frac{3+\sqrt{10}}{4}$

And, $\beta = \frac{-b-\sqrt{D}}{2a}$

$= \frac{-(-24)-\sqrt{640}}{2\times16}$

$= \frac{24-8\sqrt{10}}{32}$

$= \frac{3-\sqrt{10}}{4}$

$\therefore \; x = \frac{3+\sqrt{10}}{4}, \frac{3-\sqrt{10}}{4}$

Section C

66. In equation $(m+1)y^2 - 6(m+1)y + 3(m+9) = 0$

A = m + 1, B = - 6(m + 1), C = 3(m + 9)

For equal roots, $D = B^2 - 4AC = 0$

$36(m+1)^2 - 4(m+1) \times 3(m+9) = 0$

$\Rightarrow 3\left(m^2 + 2m + 1\right) - (m+1)(m+9) = 0$

$\Rightarrow 2m^2 - 4m - 6 = 0$

$\Rightarrow m^2 - 2m - 3 = 0$

$\Rightarrow m^2 - 3m + m - 3 = 0$

$\Rightarrow m(m-3) + 1(m-3) = 0$

$\Rightarrow (m-3)(m+1) = 0$

$\therefore$ m = -1, 3

Neglecting $m \neq -1$

$\therefore m=3$
$\therefore$ the equation becomes $4y^2-24y+36=0$
$\Rightarrow y^2-6y+9=0$
$\Rightarrow (y-3)(y-3)=0$
$\Rightarrow (y-3)=0 \quad and \quad (y-3)=0$
$\therefore$ roots are y = 3,3

67. Given,
(x - a)(x - b) + (x - b)(x - c) + (x - c)(x - a) = 0
$\Rightarrow x^2-ax-bx+ab+x^2-bx-cx+bc+x^2-cx-ax+ac=0$
$\Rightarrow 3x^2-2ax-2bx-2cx+ab+bc+ca=0$
For equal roots $B^2-4AC=0$
or, $\{-2(a+b+c)\}^2=4\times 3(ab+bc+ca)$
or, $4(a+b+c)^2-12(ab+bc+ca)=0$
or, $a^2+b^2+c^2+2ab+2bc+2ac-3ab-3bc-3ac=0$
or, $\frac{1}{2}\left[2a^2+2b^2+2c^2-2ab-2ac-2bc\right]=0$
or, $\frac{1}{2}\left[\left(a^2+b^2-2ab\right)+\left(b^2+c^2-2bc\right)+\left(c^2+a^2-2ac\right)\right]=0$
or, $\frac{1}{2}\left[\left(a^2+b^2-2ab\right)+\left(b^2+c^2-2bc\right)+\left(c^2+a^2-2ac\right)\right]=0$
or, $(a-b)^2+(b-c)^2+(c-a)^2=0$ if $a\neq b\neq c$
Since $(a-b)^2>0,(b-c)^2>0(c-a)^2>0$
Hence, $(a-b)^2=0\Rightarrow a=b$
$(a-c)^2=0\Rightarrow b=c$
$(c-a)^2=0\Rightarrow c=a$
$\therefore a=b=c$ Hence Proved.

68. For equal roots k^2 - 64 = 0
$\Rightarrow$ k = ± 8
Equations are $2x^2$ + 8x + 8 = 0 and $2x^2$ - 8x + 8 = 0
$\Rightarrow$ $2(x + 2)^2$ = 0 or $2(x - 2)^2$ = 0
$\Rightarrow$ x = -2 or x = 2

69. x = - 4 is the root of the equation $x^2+2x+4p=0$
$(-4)^2+(2\times -4)+4p=0$
or, p = -2
Equation $x^2-2(1+3k)x+7(3+2k)=0$ has equal roots.
$\therefore 4(1+3x)^2-28(3+2k)=0$
or, $9k^2-8k-20=0$
or, (9k +10)(k-2) = 0
or, $k=\frac{-10}{9},2$
Hence, the value of k = $-\frac{10}{9},2$

70. Here roots are equal,
$\therefore D=B^2-4AC=0$
Here, $A=1+m^2,B=2mc,C=\left(c^2-a^2\right)$
$\therefore (2mc)^2-4\left(1+m^2\right)\left(c^2-a^2\right)=0$
or, $4m^2c^2-4\left(1+m^2\right)\left(c^2-a^2\right)=0$
or, $m^2c^2-\left(c^2-a^2+m^2c^2-m^2n^2\right)=0$
or, $m^2c^2-c^2+a^2-m^2c^2+m^2a^2=0$
or, $-c^2+a^2+m^2a^2=0$
or, $c^2=a^2\left(1+m^2\right)$
Hence Proved.

71. Since (-5) is a root of given quadratic equation $2x^2$ + px + 15 =0,then,
$2(-5)^2+p(-5)-15=0$
$50-5p-15=0$
$5p=35\Rightarrow p=7$
Now $p\left(x^2+x\right)+k=0$ has equal roots

$px^2+px+k=0$
So $(b)^2-4ac=0$
$(p)^2-4p\times k=0$
$(7)^2-4\times 7\times k=0$
28k = 49
$k=\frac{49}{28}=\frac{7}{4}$
hence p = 7 and $k=\frac{7}{4}$

72. We, $A=\left(c^2-ab\right)$, B$=-2(a^2-bc), C=b^2-ac$
For real equal roots, $D=B^2-4AC=0$
$\Rightarrow \quad \left[-2\left(a^2-bc\right)\right]^2-4\left(c^2-ab\right)\left(b^2-ac\right)=0$
$\Rightarrow 4\left(a^4+b^2c^2-2a^2bc\right)-4\left(b^2c^2-c^3a-ab^3-a^2bc\right)=0$
$\Rightarrow 4\left[a^4+b^2c^2-2x^2bc-b^2c^2+c^3a+ab^3-a^2bc\right)=0$
$\Rightarrow 4\left[a^4+ac^3+ab^3-3a^2bc\right]=0$
$\Rightarrow a\left(a^3+c^3+b^3-3abc\right)=0$
$\Rightarrow a=0$ or $a^3+b^3+c^3=3abc$

73. For equal roots of $x^2+2px+mn=0, 4p^2-4mn=0$
or, $p^2=mn$ ......(i)
For equal roots of
$x^2-2(m+n)x+\left(m^2+n^2+2p^2\right)=0$
$4(m+n)^2-4\left(m^2+n^2+2p^2\right)=0$
$m^2+n^2+2mn-m^2-n^2-2(mn)=0$ {From (i))
$\therefore$ If root of $x^2+2px+mn=0$ are equal then those of $x^2-2a(m+n)x+\left(m^2+n^2+2p^2\right)=0$ are equal.

74. Since the given equation has equal roots,
$D=b^2-4ac=0$
Here, a = (3k + 1), b = 2(k + 1), c = 1
$[2(k+1)]^2-4(3k+1)(1)=0$
or, $4\left(k^2+2k+1\right)-(12k+4)=0$
or, $4k^2+8k+4-12k-4=0$
$\therefore 4k^2-4k=0$
k = 0,1
Put k = 0, in the given equation,
$x^2+2x+1=0$
or, $(x+1)^2=0$
or, x = -1
Again put k = 1, in the given equation,
$4x^2+4x+1=0$
$(2x+1)^2=0$
or, $x=-\frac{1}{2}$
Hence, roots = $-1,-\frac{1}{2}$

75. Here x = - 2 is the root of the equation $3x^2+7x+p=0$
then, $3(-2)^2+7(-2)+p=0$
or, p=2
Roots of the equation $x^2+4kx+k^2-k+2=0$ are equal,then,
$16k^2-4\left(k^2-k+2\right)=0$
or, $16k^2-4k^2+4k-8=0$
or, $12k^2+4k-8=0$
or, $3k^2+k-2=0$
or, (3k-2)(k + 1) = 0
or, $k=\frac{2}{3},-1$
Hence, roots = $\frac{2}{3},-1$