

Solution
REAL NUMBERS WS 1
Class 10 - Mathematics

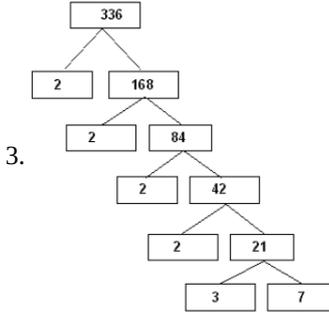
Section A

1. (c) prime, co-prime
Explanation: If their greatest common factor is 1, then one of the two numbers must be a prime or co-prime. Their least common multiple must be the product of the two numbers.

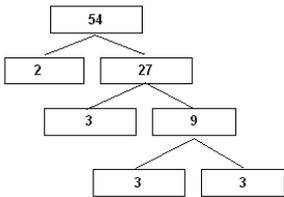
Section B

2. (a) - (iii), (b) - (iv), (c) - (ii), (d) - (i)

Section C



So, $336 = 2 \times 2 \times 2 \times 2 \times 3 \times 7 = 2^4 \times 3 \times 7$



So, $54 = 2 \times 3 \times 3 \times 3 = 2 \times 3^3$

Therefore,

$LCM(336, 54) = 2^4 \times 3^3 \times 7 = 3024$

$HCM(336, 54) = 2 \times 3 = 6$.

Verification:

$LCM \times HCF = 3024 \times 6 = 18144$ and $336 \times 54 = 18144$

i.e. $LCM \times HCF = \text{product of two numbers}$

4. We need to find the H.C.F. of 506 and 1155 and express it as a linear combination of 506 and 1155.

By applying Euclid's division lemma

$1155 = 506 \times 2 + 143$.

$506 = 143 \times 3 + 77$.

$143 = 77 \times 1 + 66$.

$77 = 66 \times 1 + 11$.

$66 = 11 \times 6 + 0$.

Therefore, H.C.F. = 11.

Now, $11 = 77 - 66 \times 1 = 77 - [143 - 77 \times 1] \times 1 \{ \because 143 = 77 \times 1 + 66 \}$

$= 77 - 143 \times 1 + 77 \times 1$

$= 77 \times 2 - 143 \times 1$

$= [506 - 143 \times 3] \times 2 - 143 \times 1 \{ \because 506 = 143 \times 3 + 77 \}$

$= 506 \times 2 - 143 \times 6 - 143 \times 1$

$= 506 \times 2 - 143 \times 7$

$= 506 \times 2 - [1155 - 506 \times 2] \times 7 \{ \because 1155 = 506 \times 2 + 143 \}$

$= 506 \times 2 - 1155 \times 7 + 506 \times 14$

$= 506 \times 16 - 1155 \times 7$

Hence obtained.

5. Using Euclid's Division Lemma,

$$963 = 657 \times 1 + 306$$

$$657 = 306 \times 2 + 45$$

$$306 = 45 \times 6 + 36$$

$$45 = 36 \times 1 + 9$$

$$36 = 9 \times 4 + 0$$

$$\therefore \text{HCF}(963, 657) = 9$$

Now it is given that

$$\text{HCF} = 657x + 963(-15)$$

$$\text{or } 9 = 657x + 963(-15)$$

$$657x = 9 + 14445 = 14454$$

$$x = \frac{14454}{657} = 22$$

6. Using Euclid's algorithm, the HCF(30, 72)

$$72 = 30 \times 2 + 12$$

$$30 = 12 \times 2 + 6.$$

$$12 = 6 \times 2 + 0.$$

$$\text{HCF}(30, 72) = 6$$

Now it given that

$$\text{HCF} = 30x + 72y$$

$$\text{So } 6 = 30x + 72y$$

$$\text{or } 1 = 5x + 12y$$

$$x = \frac{1-12y}{5} \dots\dots (1)$$

$$\text{if } y = -2 \text{ then } x = \frac{1+24}{5} = 5$$

$$\text{if } y = -12 \text{ then } x = \frac{1+144}{5} = 29$$

$$\text{if } y = -22 \text{ then } x = \frac{1+264}{5} = 53$$

So the value of (x, y) possible are (5, -2), (29, -12) (53, -22)

Hence infinite no. of solutions are possible.

7. LCM (84, 90, 120)

$$84 = 2^2 \times 3 \times 7$$

$$90 = 2 \times 3^2 \times 5$$

$$120 = 2^3 \times 3 \times 5$$

$$\text{LCM} = 2520 \text{ min or } 42 \text{ hours}$$

After 42 h all the bells will ring at same time.

8. Since, the three persons start walking together.

\therefore The minimum distance covered by each of them in complete steps = LCM of the measures of their steps

$$40 = 8 \times 5 = 2^3 \times 5$$

$$42 = 6 \times 7 = 2 \times 3 \times 7$$

$$45 = 9 \times 5 = 3^2 \times 5$$

Hence LCM (40, 42, 45)

$$= 2^3 \times 3^2 \times 5 \times 7 = 8 \times 9 \times 5 \times 7 = 2520$$

\therefore The minimum distance each should walk so that each can cover the same distance

$$= 2520 \text{ cm} = 25.20 \text{ meters.}$$

9. Prime factorisation of 404 and 96 is:

$$404 = 2 \times 2 \times 101$$

$$\text{or } 404 = 2^2 \times 101$$

$$96 = 2 \times 2 \times 2 \times 2 \times 2 \times 3$$

$$\text{or } 96 = 2^5 \times 3$$

$$\therefore \text{HCF}(404, 96) = 2^2 = 4$$

$$\text{LCM}(404, 96) = 101 \times 2^5 \times 3$$

$$\text{LCM}(404, 96) = 9696$$

Now we have to verify that,

$$\text{HCF}(404, 96) \times \text{LCM}(404, 96) = 404 \times 96$$

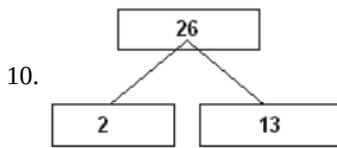
Hence, $LHS = HCF \times LCM = 4 \times 9696 = 38784$

$RHS = \text{Product of numbers} = 404 \times 96 = 38784$

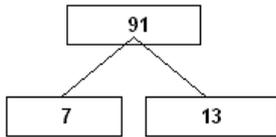
Since, $LHS = RHS$

$\therefore HCF \times LCM = \text{Product of 404 and 96.}$

Hence verified.



So, $26 = 2 \times 13$



So, $91 = 7 \times 13$

Therefore, $LCM(26, 91) = 2 \times 7 \times 13 = 182$

$HCF(26, 91) = 13$

Verification:

$LCM \times HCF = 182 \times 13 = 2366$

and $26 \times 91 = 2366$

i.e., $LCM \times HCF = \text{product of two numbers.}$

11. To distribute the fruits equally Renu has to take the H.C.F. of 45 and 20.

H.C.F. of 20 and 45 = 5

i.e. 5 fruits can be placed in 1 pack

$\therefore \text{Total no. of packs} = \frac{\text{Total available fruits}}{\text{no. of fruits in 1 packs}}$

$= \frac{45+20}{5}$

$= \frac{65}{5}$

$= 13$

Hence, maximum no. of packets required = 13

12. **HCF (highest common factor)** : The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.

We need to find H.C.F of 475 and 495.

By applying Euclid's Division lemma, we have

$495 = 475 \times 1 + 20.$

Since remainder $\neq 0$, apply division lemma on 475 and remainder 20

$475 = 20 \times 23 + 15.$

Since remainder $\neq 0$, apply division lemma on 20 and remainder 15

$20 = 15 \times 1 + 5.$

Since remainder $\neq 0$, apply division lemma on 15 and remainder 5

$15 = 5 \times 3 + 0.$

Therefore, H.C.F. of 475 and 495 = 5

13. Let length of park = x metres

We are given area of rectangular park = 400 m^2

Therefore, breadth of park = $\frac{400}{x}$ metres {Area of rectangle = length \times breadth}

Perimeter of rectangular park = $2(\text{length} + \text{breadth}) = 2\left(x + \frac{40}{x}\right)$ metres

We are given perimeter of rectangle = 80 metres

According to condition:

$2\left(x + \frac{400}{x}\right) = 80$

$\Rightarrow 2\left(\frac{x^2+400}{x}\right) = 80$

$\Rightarrow 2x^2 + 800 = 80x$

$\Rightarrow 2x^2 - 80x + 800 = 0$

$$\Rightarrow x^2 - 40x + 400 = 0 \text{ (Dividing whole equation by 2)}$$

Comparing equation, $x^2 - 40x + 400 = 0$ with general quadratic equation $ax^2 + bx + c = 0$, we get $a = 1$, $b = -40$ and $c = 400$

$$\text{Discriminant} = b^2 - 4ac = (-40)^2 - 4(1)(400) = 1600 - 1600 = 0$$

Discriminant is equal to 0.

Therefore, two roots of the equation are real and equal which means that it is possible to design a rectangular park of perimeter 80 metres and area 400 m^2 .

Using the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ to solve the equation,

$$x = \frac{40 \pm \sqrt{0}}{2} = \frac{40}{2} = 20$$

Here, both the roots are equal to 20.

Therefore, length of the rectangular park = 20 metres

$$\text{The breadth of rectangular park} = \frac{400}{x} = \frac{400}{20} = 20 \text{ m}$$

14. We have,

$$60 = 2^2 \times 3 \times 5$$

$$168 = 2^3 \times 3 \times 7$$

$$330 = 2 \times 3 \times 5 \times 11$$

$$\text{HCF} = 6$$

They can put 6 food items in 1 packet.

$$\text{so the number of packets required for 60 pieces of pastries} = \frac{60}{6} = 10$$

$$\text{the number of packets required for 168 pieces of cookies} = \frac{168}{6} = 28$$

$$\text{the number of packets required for 330 chocolate bars} = \frac{330}{6} = 55$$

$$\text{Total Packets required} = 10 + 28 + 55 = 93$$

15. Here, $x - 3$ is the HCF of

$$x^3 - 2x^2 + px + 6 \text{ and } x^2 - 5x + q$$

Since $x - 3$ is a common factor of given expression

$$f(x) = x^3 - 2x^2 + px + 6, \text{ then by factor theorem}$$

$$f(x) = 0$$

$$\Rightarrow 3^3 - 2 \times 3^2 + p \times 3 + 6 = 0$$

$$\Rightarrow 27 - 18 + 3p + 6 = 0 \Rightarrow 15 + 3p = 0$$

$$\Rightarrow 3p = -15 \Rightarrow p = \frac{-15}{3} = -5$$

Since $x - 3$ is a factor of $g(x) = x^2 - 5x + q$,

then by factor theorem, $g(3) = 0$

$$\Rightarrow 3^2 - 5 \times 3 + q = 0 \Rightarrow 9 - 15 + q = 0$$

$$\Rightarrow -6 + q = 0 \Rightarrow q = 6$$

$$\therefore 6p + 5q = 6 \times (-5) + 5 \times 6$$

$$= -30 + 30 = 0$$

$$\text{Hence } 6p + 5q = 0$$

16. For maximum number of students to put into each group

Mr patil sir should have to take H.C.F of 28, 42 and 56

so

maximum number of students Mr. Patil can put into each group is 14.

17. The greatest number of cartons is the HCF of 144 and 90

Now the prime factorization of 144 and 90 are

$$144 = 16 \times 9 = 2^4 \times 3^2$$

$$90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$$

$$\text{HCF}(144, 90) = 2 \times 3^2 = 18$$

\therefore The greatest number of cartons each stack would have = 18.

18. Given numbers are 156, 208 and 260.

Here, $260 > 208 > 156$

Let us find the HCF of 260 and 208,

By using Euclid's division lemma for 260 and 208,

we get

$$260 = (208 \times 1) + 52$$

Here, the remainder is 52, not zero.

On taking 208 as new dividend and 52 as new divisor and then apply Euclid's division lemma, we get

$$208 = (52 \times 4) + 0$$

Here, the remainder is zero and the divisor is 52.

So, HCF of 208 and 260 is 52.

Now, $156 > 52$

Let us find the HCF of 52 and 156. By using Euclid's division lemma, we get

$$156 = (52 \times 3) + 0$$

Here, the remainder is zero and the divisor is 52.

So, HCF of 52 and 156 is 52.

Thus, HCF of 156, 208 and 260 is 52.

Hence, the minimum number of buses

$$= \frac{156}{52} + \frac{208}{52} + \frac{260}{52} = \frac{156+208+260}{52} = \frac{624}{52} = 12$$

The minimum number of buses is 12.

19. For any finite set of primes $\{p_1, p_2, p_3, \dots, p_n\}$, Euclid considered the number

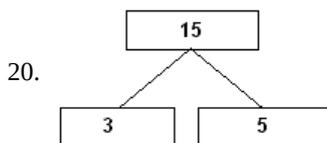
$$n = 1 + p_1 \times p_2 \times p_3 \times \dots \times p_n$$

n has a prime divisor p (every integer has at least one prime divisor). But p is not equal to any of the p_i . (If p were equal to any of the p_i , then p would have to divide 1, which is impossible).

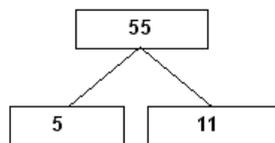
So for any finite set of prime numbers, it is possible to find another prime that is not in that set.

In other words, a finite set of primes cannot be the collection of all prime numbers.

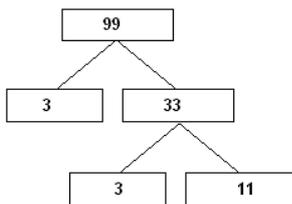
Hence, there are infinitely many positive primes.



$$\text{so, } 15 = 3 \times 5$$



$$\text{So, } 55 = 5 \times 11$$



$$\text{So, } 99 = 3 \times 3 \times 11 = 3^2 \times 11$$

Therefore,

$$\text{HCF} (15, 55, 99) = 1$$

$$\text{LCM} (15, 55, 99) = 3^2 \times 5 \times 11 = 495.$$

21. **HCF (highest common factor)** : The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.

We need to find H.C.F. of 75 and 243.

By applying Euclid's Division lemma

$$243 = 75 \times 3 + 18.$$

Since remainder $\neq 0$, apply division lemma on 75 and remainder 18

$$75 = 18 \times 4 + 3.$$

Since remainder $\neq 0$, apply division lemma on divisor 18 and remainder 3

$$18 = 3 \times 6 + 0.$$

Therefore, H.C.F. of 75 and 243 = 3

22. First find the HCF of 65 and 117 by Using Euclid's division algorithm,

$$117 = 65 \times 1 + 52$$

$$65 = 52 \times 1 + 13$$

$$52 = 13 \times 4 + 0$$

So, HCF of 117 and 65 = 13

$$\text{HCF} = 65m + 117n$$

For, $m = 2$ and $n = -1$,

$$\text{HCF} = 65 \times 2 + 117 \times (-1)$$

$$= 130 - 117$$

$$= 13$$

Hence, the integral values of m and n are 2 and -1 respectively and the HCF of 117 and 65 is 13.

23. Since, the three persons start walking together.

\therefore The minimum distance each should walk so that all can cover the same distance in complete steps will be equal to LCM of 80,85 and 90.

Prime factors of 80,85 and are as following

$$80 = 16 \times 5 = 2^4 \times 5$$

$$85 = 5 \times 17$$

$$90 = 2 \times 9 \times 5 = 2 \times 3^2 \times 5$$

$$\text{So LCM of 80,85 and 90} = 2^4 \times 3^2 \times 5 \times 17$$

$$= 16 \times 9 \times 5 \times 17 = 12240$$

\therefore Each person should walk the minimum distance

$$= 12240 \text{ cm} = 122 \text{ meter } 40 \text{ cm}$$

Value of morning walk :

“An early morning walk is a blessing for the whole day.”

24. Let $p(x) = 2x^2 - kx - 9$ and $q(x) = x^2 + x - 12$

Since $(x-k)$ is the HCF of both $p(x)$ and $q(x)$,

therefore $(x-k)$ divides both $p(x)$ and $q(x)$ exactly.

$\Rightarrow x - k$ is a factor of both $p(x)$ and $q(x)$

\therefore By factor theorem $p(k) = 0$

and also $q(k) = 0$

Now $p(k) = 0$

$$\Rightarrow 2k^2 - kk - 9 = 0 \Rightarrow k^2 - 9 = 0 \quad \text{Using Identity } a^2 - b^2 = (a + b)(a - b)$$

$$\Rightarrow (k - 3)(k + 3) = 0 \Rightarrow k = 3, -3 \dots\dots\dots(i)$$

Again $q(k) = 0$

$$\Rightarrow k^2 + k - 12 = 0 \Rightarrow k^2 + 4k - 3k - 12 = 0$$

$$\Rightarrow k(k + 4) - 3(k + 4) \Rightarrow (k + 4)(k - 3) = 0$$

$$\Rightarrow k = -4, 3 \dots\dots\dots(ii)$$

Hence from (i) and (ii), $k = 3$

25. This problem can be solved using Least Common Multiple because we are trying to figure out when the soonest (Least) time will be that as the event of exercising continues (Multiple), it will occur at the same time (Common).

L.C.M. of 12 and 8 is 24.

So,

They will exercise together again in 24 days.

26. The required greatest capacity is the HCF of 120, 180 and 240.

$$240 = 180 \times 1 + 60$$

$$180 = 60 \times 3 + 0$$

HCF is 60.

Now HCF of 60, 120

$$120 = 60 \times 2 + 0$$

∴ HCF of 120, 180 and 240 is 60.

∴ The required capacity is 60 litres.

27. We have to find Prime Factors of the following numbers

$$48 = 2^4 \times 3$$

$$72 = 2^3 \times 3^2$$

$$108 = 2^2 \times 3^3$$

so the LCM of 48, 72 and 108 is

$$LCM = 2^4 \times 3^3$$

$$LCM = 16 \times 27 = 432$$

$$432 \text{ seconds} = \frac{432}{60} \text{ mins}$$

$$432 \text{ seconds} = 7.2 \text{ mins}$$

So the time it will change together again is

$$= 8 : 07 : 12 \text{ am}$$

28. We have to find the greatest number that divides 445, 572 and 699 and leaves remainders of 4, 5 and 6 respectively. This means when the number divides 445, 572 and 699, it leaves remainders 4, 5 and 6. It means that

$$445 - 4 = 441,$$

$$572 - 5 = 567$$

$$\text{and } 699 - 6 = 693$$

are completely divisible by the required number.

For the highest number which divides the above numbers we need to calculate HCF of 441, 567 and 693 .

Therefore, the required number is the H.C.F. of 441, 567 and 693 Respectively.

First, consider 441 and 567.

By applying Euclid's division lemma, we get

$$567 = 441 \times 1 + 126$$

$$441 = 126 \times 3 + 63$$

$$126 = 63 \times 2 + 0.$$

Therefore, H.C.F. of 441 and 567 = 63

Now, consider 63 and 693

again we have to apply Euclid's division lemma, we get

$$693 = 63 \times 11 + 0.$$

Therefore, H.C.F. of 441, 567 and 693 is 63

Hence, the required number is 63. 63 is the highest number which divides 445, 572 and 699 will leave 4, 5 and 6 as remainder respectively.

29. Let us find HCF of 48 and 18

$$48 = 18 \times 2 + 12$$

$$18 = 12 \times 1 + 6$$

$$12 = 6 \times 2 + 0$$

Hence HCF (48, 18) = 6

$$\text{Now, } 6 = 18 - 12 \times 1$$

$$6 = 18 - (48 - 18 \times 2)$$

$$6 = 18 - 48 \times 1 + 18 \times 2$$

$$6 = 18 \times 3 - 48 \times 1$$

$$6 = 18 \times 3 + 48 \times (-1)$$

$$\text{i.e., } 6 = 18x + 48y \dots\dots (1)$$

$$\text{where } x = 3, y = -1$$

$$\therefore 6 = 18 \times 3 + 48 \times (-1)$$

$$= 18 \times 3 + 48 \times (-1) + 18 \times 48 - 18 \times 48$$

$$= 18(3 + 48) + 48(-1 - 18)$$

$$= 18 \times 51 + 48 \times (-19)$$

$$6 = 18x + 48y \dots\dots (2)$$

$$\text{where } x = 51, y = -19$$

Hence, x and y are not unique.

30. The condition of the question is, the number of orange forms taken must be equal to the number of green forms taken.

Let us assume that he takes 10 orange and 10 green forms.

10 green forms can be fit exactly on 2 pages at 5 forms/page. But, 10 orange forms can't be fit exactly on any number of pages.

Because, 3 orange forms can be fit exactly on a page. In 10 orange forms, 9 forms can be fit exactly on 3 pages and 1 form will be remaining.

To get the number of forms in orange and green which can be fit exactly on some number of pages, we have to find L.C.M of (3,5). That is 15.

15 orange forms can be fit exactly on 5 pages at 3 forms/page.

15 green forms can be fit exactly on 3 pages at 5 forms/page.

Hence, the smallest number of each form could be printed is 15.

31. $35 = 5 \times 7$

$$56 = 2^3 \times 7$$

$$91 = 13 \times 7$$

$$\text{L.C.M of } 35, 56 \text{ and } 91 = 2^3 \times 7 \times 5 \times 13 = 3640$$

The smallest number that when divided by 35, 56, 91 leaves a remainder 7 in each case = $3640 + 7 = 3647$.

Hence 3647 is the smallest number that, when divided by 35, 56 and 91 leaves a remainder of 7 in each case.

32. **Given:** Number of goats for trip = 105

Number of donkey for trip = 140

Number of cows for trip = 175

Therefore, The largest number of animals in one trip = HCF of 105, 140 and 175.

First consider 105 and 140

By applying Euclid's division lemma, we get

$$140 = 105 \times 1 + 35$$

$$105 = 35 \times 3 + 0$$

Therefore, HCF of 105 and 140 = 35

Now consider 35 and 175

Again applying Euclid's division lemma, we get

$$175 = 35 \times 5 + 0$$

HCF of 105, 140 and 175 is 35.

So 35 animals of same kind can go for trip in a single trip and number of trip is $105/35 + 140/35 + 175/35 = 12$

33. The largest positive integer that will divide 398, 436 and 542 leaving remainders 7, 11 and 15 respectively is the HCF of the numbers $(398 - 7)$, $(436 - 11)$ and $(542 - 15)$ i.e. 391, 425 and 527.

HCF of 391, 425 and 527:

HCF of 425 and 391:

$$425 = 391 \times 1 + 34$$

$$391 = 34 \times 11 + 17$$

$$34 = 17 \times 2 + 0$$

HCF of 425 and 391 = 17

$$527 = 17 \times 31$$

Similarly, HCF of 17 and 527 = 17

So, HCF of (391, 425, 527) = 17

\therefore Required number is 17.

34. Numbers are of two types - prime and composite.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

It can be observed that

$$7 \times 11 \times 13 + 13 = 13 \times (7 \times 11 + 1)$$

$$= 13 \times (77 + 1) = 13 \times 78 = 13 \times 13 \times 6$$

The given expression has 6 and 13 as its factors.

Therefore, it is a composite number.

$$7 \times 6 \times 5 \times 4 \times 3 \times 2 \times 1 + 5$$

$$= 5 \times (7 \times 6 \times 4 \times 3 \times 2 \times 1 + 1)$$

$$= 5 \times (1008 + 1) = 5 \times 1009$$

1009 cannot be factorized further

Therefore, the given expression has 5 and 1009 as its factors.

Hence, it is a composite number.

35. In order to arrange the books as required, we have to find the largest number that divides 96, 240 and 336 exactly.

Clearly, such a number is their HCF.

We have,

$$96 = 2^5 \times 3, 240 = 2^4 \times 3 \times 5 \text{ and } 336 = 2^4 \times 3 \times 7$$

$$\therefore \text{HCF of } 96, 240 \text{ and } 336 \text{ is } 2^4 \times 3 = 48$$

So, there must be 48 books in each stack.

$$\therefore \text{Number of stacks of English books} = \frac{96}{48} = 2$$

$$\text{Number of stacks of Hindi books} = \frac{240}{48} = 5$$

$$\text{Number of stacks of Mathematics books} = \frac{336}{48} = 7$$

36. To make all the groups identical and find the greatest number of groups, we have to find the greatest number which can divide by 8 and 12 exactly.

That is nothing but H.C.F. of 8 and 12.

$$\text{H.C.F of } (8, 12) = 4$$

That is, 8 U.S stamps can be displayed in 4 groups at 2 stamps/group.

And 12 international stamps can be displayed in 4 groups at 3 stamps/group.

In this way, each of the 4 groups would have 2 U.S. stamps and 3 international stamps. And all the 4 groups would be identical.

Hence, the greatest number of groups can be made is 4.

37. This problem can be solved using H.C.F. because we are cutting or “dividing” the strips of cloth into smaller pieces of 36 and 24 and we are looking for the widest possible strips .

So,

$$\text{H.C.F. of } 36 \text{ and } 24 \text{ is } 12$$

So we can say that

Maya should cut each piece to be 12 inches wide.

38. GIVEN: Their steps measure 80 cm, 85 cm and 90 cm.

We have to find the L.C.M of the measures of their steps i.e. 80 cm, 85 cm, and 90 cm, to calculate the required distance each should walk.

$$\text{L.C.M of } 80 \text{ cm, } 85 \text{ cm, and } 90 \text{ cm.}$$

$$80 = 2^4 \times 5$$

$$85 = 17 \times 5$$

$$90 = 2 \times 3 \times 3 \times 5$$

$$\text{L.C.M of } 80, 85 \text{ and } 90 = 2^4 \times 3 \times 3 \times 5 \times 17$$

$$\text{L.C.M of } 80, 85 \text{ and } 90 = 12240 \text{ cm}$$

Hence, the minimum distance each should walk so that all can cover the same distance in complete steps is 12240 cm.

39. As per question, the required number of books are to be distributed equally among the students of section A or B.

There are 30 students in section A and 28 students in section B.

So, the number of these books must be a multiple of 30 as well as that of 28.

Consequently, the required number is LCM(30, 28).

$$\text{Now, } 30 = 2 \times 3 \times 5$$

$$\text{and } 28 = 2^2 \times 7 .$$

$$\therefore \text{LCM}(30, 28) = \text{product of prime factors with highest power}$$

$$= 2^2 \times 3 \times 5 \times 7$$

$$= 4 \times 3 \times 5 \times 7$$

$$= 420$$

Hence, the required number of books = 420.

40. The greatest number of plants that can be planted in a row = HCF(81, 45,63)

$$81 = 3^4$$

$$45 = 3^2 \times 5$$

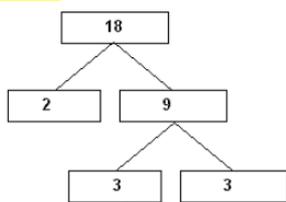
$$63 = 3^2 \times 7$$

$$\text{HCF} = 9$$

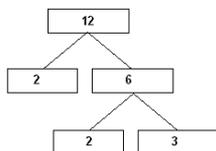
9 plants to be planted in a row

41. By taking LCM of time taken (in minutes) by Sonia and Ravi, We can get the actual number of minutes after which they meet again at the starting point after both start at the same point and at the same time, and go in the same direction.

$$18 = 2 \times 3 \times 3 = 2 \times 3^2$$



$$12 = 2 \times 2 \times 3 = 2^2 \times 3$$



$$\text{LCM}(18, 12) = 2^2 \times 3^2 = 36$$

Therefore, both Sonia and Ravi will meet again at the starting point after 36 minutes.

42. For the least possible number of casks of equal size, the size of each cask must be of the greatest volume.

To get the greatest volume of each cask, we have to find the largest number which exactly divides 403, 434 and 465. That is nothing but the H.C.F of (403, 434, 465)

The H.C.F of (403, 434, 465) = 31 liters

Each cask must be of the volume 31 liters.

Req. No. of casks is

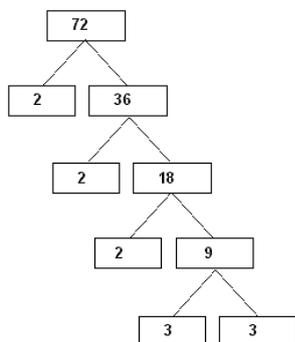
$$= \left(\frac{403}{31}\right) + \left(\frac{434}{31}\right) + \left(\frac{465}{31}\right)$$

$$= 13 + 14 + 15$$

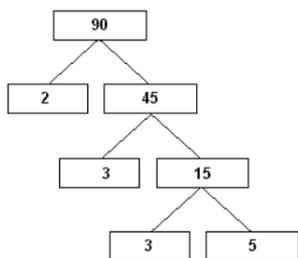
$$= 42$$

Hence, the least possible number of casks of equal size required is 42.

43.



$$\text{So, } 72 = 2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$$



$$\text{So, } 90 = 2 \times 3 \times 3 \times 5 = 2 \times 3^2 \times 5$$

Therefore,

$$\text{HCF}(72, 90) = 2 \times 3^2 = 18$$

$$\text{LCM}(72, 90) = 2^3 \times 3^2 \times 5 = 360$$

44. $P(x) = x(8x^3 + 27)$

$$= x(2x + 3)(4x^2 - 6x + 9) \text{ Using identity } a^3 + b^3 = (a + b)(a^2 + b^2 - ab)$$

$$Q(x) = 2x^2(2x^2 + 6x + 3x + 9)$$

$$= 2 \times x^2[2x(x + 3) + 3(x + 3)]$$

$$= 2 \times x^2 \times (x + 3)(2x + 3)$$

Common on factors: $x, (2x + 3)$

Uncommon on factors: $(4x^2 - 6x + 9)$ and $2, x, (x + 3)$

\therefore LCM of $P(x)$ and $Q(x) = 2 \times x^2(x + 3)(2x + 3)(4x^2 - 6x + 9)$

45. HCF of two or more positive integers is the largest positive integer that divides all the numbers exactly.

Here, the given numbers are 32 and 54

By applying division lemma we have

$$54 = 32 \times 1 + 22$$

Since remainder $\neq 0$, apply division lemma on 32 and remainder 22

$$32 = 22 \times 1 + 10$$

Since remainder $\neq 0$, apply division lemma on 22 and remainder 10

$$22 = 10 \times 2 + 2$$

Since remainder $\neq 0$, apply division lemma on 10 and 2

$$10 = 2 \times 5 + 0$$

Therefore, H.C.F. of 32 and 54 is 2.

46. **Given:** Size of bathroom = 10 ft by 8 ft.

$$= (10 \times 12) \text{ inch by } (8 \times 12) \text{ inch}$$

$$= 120 \text{ inch by } 96 \text{ inch}$$

Area of bathroom = 120 inch by 96 inch

To find the largest size of tile required, we find HCF of 120 and 96.

By applying Euclid's division lemma

$$120 = 96 \times 1 + 24$$

$$96 = 24 \times 4 + 0$$

Therefore, HCF = 24

Therefore, Largest size of tile required = 24 inches

$$\text{no. of tiles required} = \frac{\text{area of bathroom}}{\text{area of a tile}} = \frac{120 \times 96}{24 \times 24} = 5 \times 4 = 20 \text{ tiles}$$

Hence number of tiles required is 20 and size of tiles is 24 inches.

47. (i) The required number of minutes after which they start preparing a new card together = LCM of 10, 16 and 20 minutes

Prime factorisation of 10 = 2×5

and prime factorisation of 16 = $2 \times 2 \times 2 \times 2$

and prime factorisation of 20 = $2 \times 2 \times 5$

$$\text{Now, LCM}(10, 16, 20) = 2 \times 2 \times 2 \times 2 \times 5 = 80$$

Therefore, Number of minutes after which they start preparing a new card together = 80 minutes.

(ii) Recognition and care for elders removes the loneliness due to age related diseases. Moreover they feel happy to help young minds through their experience.

48. **HCF (highest common factor)** : The largest positive integer that divides given two positive integers is called the Highest Common Factor of these positive integers.

We need to find H.C.F. of 155 and 1385.

By applying Euclid's Division lemma, we have

$$1385 = 155 \times 8 + 145.$$

Since remainder $\neq 0$, apply division lemma on divisor 155 and remainder 145

$$155 = 145 \times 1 + 10.$$

Since remainder $\neq 0$ apply division lemma on divisor 145 and remainder 10

$$145 = 10 \times 14 + 5.$$

Since remainder $\neq 0$, apply division lemma on divisor 10 and remainder 5

$$10 = 5 \times 2 + 0.$$

Therefore, H.C.F. of 155 and 1385 = 5

49. In order to arrange the books as required, we have to find the largest number that divides 96, 240 and 336 exactly. Clearly, such a number is their HCF.

We have,

$$96 = 2^5 \times 3,$$

$$240 = 2^4 \times 3 \times 5$$

$$336 = 2^4 \times 3 \times 7$$

∴ HCF of 96,240 and 336 is $2^4 \times 3 = 48$

So, there must be 48 books in each stack.

∴ Number of stacks of Biography books = $\frac{96}{48} = 2$

Number of stacks of Mystery books = $\frac{240}{48} = 5$

Number of stacks of Self-help books = $\frac{336}{48} = 7$

50. $f(x) = (x + 3)(2x^2 - 3x + a)$

$$g(x) = (x - 2)(3x^2 + 10x - b)$$

since $(x + 3)(x - 2)$ is the HCF of $f(x)$ and $g(x)$

∴ $x - 2$ is a factor of $2x^2 - 3x + a \dots (i)$

and $x + 3$ is a factor of $3x^2 + 10x - b \dots (ii)$

From (i) follow that 2 is a zero of $2x^2 - 3x + a$

$$\Rightarrow 2 \times 2^2 - 3 \times 2 + a = 0$$

$$\Rightarrow 8 - 6 + a = 0$$

$$\Rightarrow 2 + a = 0 \Rightarrow a = -2$$

From (ii), it follows that

-3 is a zero of $3x^2 + 10x - b$

$$\Rightarrow 3x(-3)^2 + 10x(-3) - b = 0$$

$$\Rightarrow 27 - 30 - b = 0$$

$$\Rightarrow -3 - b = 0 \Rightarrow b = -3$$

The values of a and b are -2 and -3 respectively.

Section D

51. HCF (96, 240, 336) = 48

52. Number of stacks = $\frac{336}{48} = 7$

53. Total number of stacks = $\frac{96}{48} + \frac{240}{48} + \frac{336}{48}$
= 14

54. Height of each stack of History = $48 \times 1.8 = 86.4$ cm

Height of each stack of Science = $48 \times 2.2 = 105.6$ cm

Height of each stack of Mathematics = $48 \times 2.5 = 120$ cm

55. The number of students in Section A is 32, and the number of students in Section B is 36.

Step 1: Find the prime factors of each number:

$$32 = 2 \times 2 \times 2 \times 2 \times 2$$

$$36 = 2 \times 2 \times 3 \times 3$$

Step 2: Identify the common and uncommon prime factors. The common ones are 2×2 .

Step 3: Multiply the common and uncommon prime factors together to get the LCM:

$$\text{LCM} = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 288$$

So, the minimum number of books needed to be acquired for the class library is 288, so they can be distributed equally among students of Section A or Section B.

56. Step 1: Find the prime factors of each number:

$$32 = 2 \times 2 \times 2 \times 2$$

$$36 = 2 \times 2 \times 3 \times 3$$

Step 2: Identify the common prime factors and their minimum exponent:

The common prime factors are 2×2 .

Step 3: Calculate the HCF by multiplying the common prime factors:

$$\text{HCF} = 2 \times 2 = 4$$

So, the HCF of 32 and 36 is 4.

57. Given number $(7 \times 11 \times 13 \times 15 + 15)$

It can also be written as $15(7 \times 11 \times 13 + 1)$.

As it is a product of two composite numbers

hence it is a composite number.

58. Given:

$$p = ab^2$$

$$q = a^2b$$

Take the highest power of each prime factor:

$$\text{LCM} = a^2 \times b^2$$

So, the LCM of p and q is a^2b^2 .

59. Khushi can invite guests = $\text{HCF}(36, 60) = 12$

So, she can invite at most 12 guests.

60. Each guest gets bananas = $\frac{60}{12} = 5$ bananas

Each guest get apples = $\frac{36}{12} = 3$ apples

61. If Khushi adds 42 mangoes

She can invite guests = $\text{HCF}(36, 60, 42)$

$$= 6$$

62. Total amount spent = $5 \times (60) + 15 \times (36) + (42) \times (20)$

$$= 300 + 540 + 840$$

$$= ₹ 1680$$

63. The Number of room will be minimum if each room accomodates maximum number of participants. Therefore, the number of participants in each room must be the HCF of 60, 84 and 108. The prime factorisations of 60, 84 and 108 are as under.

$$60 = 2^2 \times 3 \times 5, 84 = 2^2 \times 3 \times 7 \text{ and } 108 = 2^2 \times 3^3$$

$$\therefore \text{HCF of } 60, 84 \text{ and } 108 \text{ is } 2^2 \times 3 = 12$$

Therefore, in each room 12 participants can be seated.

64. The Number of room will be minimum if each room accomodates maximum number of participants. Therefore, the number of participants in each room must be the HCF of 60, 84 and 108. The prime factorisations of 60, 84 and 108 are as under.

$$60 = 2^2 \times 3 \times 5, 84 = 2^2 \times 3 \times 7 \text{ and } 108 = 2^2 \times 3^3$$

$$\therefore \text{HCF of } 60, 84 \text{ and } 108 \text{ is } 2^2 \times 3 = 12$$

Therefore, in each room 12 participants can be seated.

$$\therefore \text{Number of rooms required} = \frac{\text{Total number of participants}}{12}$$

$$= \frac{60+84+108}{12}$$

$$= \frac{252}{12}$$

$$= 21$$

65. Prime factorisation of 60 = $2 \times 2 \times 3 \times 5$

Prime factorisation of 84 = $2 \times 2 \times 3 \times 7$

Hence, LCM of 60, 84 = $2 \times 2 \times 3 \times 5 \times 7 = 420$

And HCF of 60, 84 = $2 \times 2 \times 3 = 12$

Now, LCM \times HCF = $420 \times 12 = 5040$

Also, $60 \times 84 = 5040$

i.e., HCF \times LCM = Product of the two numbers

66. Product of numbers = HCF \times LCM

$$\Rightarrow 1080 = 30 \times \text{LCM}$$

$$\therefore \text{LCM} = \frac{1080}{30} = 36$$

Section E

67. By applying Euclid's division lemma

$$592 = 252 \times 2 + 88$$

Since remainder $\neq 0$, apply division lemma on divisor 252 and remainder 88

$$252 = 88 \times 2 + 76$$

Since remainder $\neq 0$, apply division lemma on divisor 88 and remainder 76

$$88 = 76 \times 1 + 12$$

Since remainder $\neq 0$, apply division lemma on divisor 76 and remainder 12

$$76 = 12 \times 6 + 4$$

Since remainder $\neq 0$, apply division lemma on divisor 12 and remainder 4

$$12 = 4 \times 3 + 0.$$

Therefore, $H.C.F. = 4$.

HCF of 2 numbers can be expressed as the liner combination the two numbers

$$\Rightarrow 4 = 76 - 12 \times 6$$

$$\begin{aligned}
&= 76 - 88 - 76 \times 1 \times 6 \\
&= 76 - 88 \times 6 + 76 \times 6 \\
&= 76 \times 7 - 88 \times 6 \\
&= 252 - 88 \times 2 \times 7 - 88 \times 6 \\
&= 252 \times 7 - 88 \times 14 - 88 \times 6 \\
&= 252 \times 7 - 88 \times 20 \\
&= 252 \times 7 - 592 - 252 \times 2 \times 20 \\
&= 252 \times 7 - 592 \times 20 + 252 \times 40 \\
&= 252 \times 47 - 592 \times 20 \\
&= 252 \times 47 + 592 \times (-20)
\end{aligned}$$

Hence obtained.

68. We have to explain why the numbers $8 \times 7 \times 6 \times 5 \times 4 + 5 \times 4$ and $11 \times 13 \times 15 + 11$ are composite numbers.

Prime numbers can be divided by 1 and only itself, whereas composite numbers have factors other than 1 and itself.

$$8 \times 7 \times 6 \times 5 \times 4 + 5 \times 4 = 6720 + 20$$

$$= 20 \times (336 + 1)$$

$$= 20 \times 337$$

The given expression has 20 and 337 as its factors other than 1 and number itself.

Therefore, it is a composite number.

Now

$$11 \times 13 \times 15 + 11 = 11 \times (13 \times 15 + 1)$$

$$= 11 \times 196$$

$$= 11 \times 2 \times 2 \times 7 \times 7$$

$$= 2 \times 2 \times 7 \times 7 \times 11$$

The given expression has 2, 7 and 11 as its factors other than 1 and number itself.

Therefore, it is a composite number.

69. Fundamental theorem of arithmetic:

"Every composite number can be expressed (factorized) as a product of primes, and this factorization is unique, apart from the order in which the prime factors occur."

$$2520 = 8 \times 9 \times 5 \times 7 = 2^3 \times 3^2 \times 5 \times 7$$

$$10530 = 2 \times 81 \times 5 \times 13 = 2 \times 3^4 \times 5 \times 13$$

$$\therefore \text{LCM} = 2^3 \times 3^4 \times 5 \times 7 \times 13 = 294840$$

70. Let three consecutive numbers be x , $(x + 1)$ and $(x + 2)$

$$\text{Let } x = 6q + r \quad 0 \leq r < 6$$

$$\therefore x = 6q, 6q + 1, 6q + 2, 6q + 3, 6q + 4, 6q + 5$$

$$\text{product of } x(x + 1)(x + 2) = 6q(6q + 1)(6q + 2)$$

if $x = 6q$ then which is divisible by 6

$$\text{if } x = 6q + 1$$

$$= (6q + 1)(6q + 2)(6q + 3)$$

$$= 2(3q + 1) \cdot 3(2q + 1)(6q + 1)$$

$$= 6(3q + 1) \cdot (2q + 1)(6q + 1)$$

which is divisible by 6

$$\text{if } x = 6q + 2$$

$$= (6q + 2)(6q + 3)(6q + 4)$$

$$= 3(2q + 1) \cdot 2(3q + 1)(6q + 4)$$

$$= 6(2q + 1) \cdot (3q + 1)(6q + 1)$$

Which is divisible by 6

$$\text{if } x = 6q + 3$$

$$= (6q + 3)(6q + 4)(6q + 5)$$

$$= 6(2q + 1)(3q + 2)(6q + 5)$$

which is divisible by 6

$$\text{if } x = 6q + 4$$

$$= (6q + 4)(6q + 5)(6q + 6)$$

$$= 6(6q + 4)(6q + 5)(q + 1)$$

which is divisible by 6

$$\text{if } x = 6q + 5$$

$$= (6q + 5)(6q + 6)(6q + 7)$$

$$= 6(6q + 5)(q + 1)(6q + 7)$$

which is divisible by 6

\therefore the product of any three natural numbers is divisible by 6.

71. The number of physics books is 192, the number of chemistry books is 240 and the number of mathematics books is 168.

Here, we have to find the HCF of 192, 240 and 168 because the HCF will be the largest number which divides 192, 240 and 168 exactly.

$$192 = 2^6 \times 3$$

$$240 = 2^4 \times 3 \times 5$$

$$168 = 2^3 \times 3 \times 7$$

Now, the HCF of 192, 240 and 168 is $= 2^3 \times 3 = 24$

There must be 24 books in each stack

$$\therefore \text{Number of stacks of physics books} = \frac{192}{24} = 8$$

$$\text{And number of stacks of chemistry books} = \frac{240}{24} = 10$$

$$\text{And number of stacks of mathematics books} = \frac{168}{24} = 7$$

72. It is given that on dividing the polynomial $4x^4 - 5x^3 - 39x^2 - 46x - 2$ by the polynomial $g(x)$, the quotient is $x^2 - 3x - 5$ and the remainder is $-5x + 8$. We have to find the polynomial $g(x)$.

Now, we know that

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

$$4x^4 - 5x^3 - 39x^2 - 46x - 2 = g(x)(x^2 - 3x - 5) + (-5x + 8)$$

$$\text{or, } 4x^4 - 5x^3 - 39x^2 - 46x - 2 + 5x - 8 = g(x)(x^2 - 3x - 5)$$

$$\text{or, } 4x^4 - 5x^3 - 39x^2 - 41x - 10 = g(x)(x^2 - 3x - 5)$$

$$g(x) = \frac{4x^4 - 5x^3 - 39x^2 - 41x - 10}{(x^2 - 3x - 5)}$$

$$\begin{array}{r} 4x^2 + 7x + 2 \\ x^2 - 3x - 5 \overline{) 4x^4 - 5x^3 - 39x^2 - 41x - 10} \\ \underline{4x^4 - 12x^3 - 20x^2} \\ 7x^3 - 19x^2 - 41x - 10 \\ \underline{7x^3 - 21x^2 - 35x} \\ 2x^2 - 6x - 10 \\ \underline{2x^2 - 6x - 10} \\ 0 \end{array}$$

$$\text{Hence, } g(x) = 4x^2 + 7x + 2$$

73. Let us first find the HCF of 210 and 55. Applying Euclid's division lemma on 210 and 55, we get

$$210 = 55 \times 3 + 45 \dots (i)$$

$$\therefore 55 \overline{) 210} \left(3 \right. \\ \underline{165} \\ 45$$

Since, the remainder $45 \neq 0$. So, we now apply division lemma on the divisor 55 and the remainder 45 to get

$$55 = 45 \times 1 + 10 \dots (ii)$$

$$\therefore 45 \overline{) 55} \left(1 \right. \\ \underline{45} \\ 10$$

We consider the divisor 45 and the remainder 10 and apply division lemma to get

$$45 = 4 \times 10 + 5 \dots (iii)$$

$$\begin{array}{r} \therefore 10 \overline{)45} \quad (4 \\ \underline{40} \\ 5 \end{array}$$

We consider the divisor 10 and the remainder 5 and apply division lemma to get

$$10 = 5 \times 2 + 0 \dots(\text{iv})$$

We observe that the remainder at this stage is zero. So, the last divisor i.e. 5 is the HCF of 210 and 55.

$$\therefore 5 = 210 \times 5 + 55y$$

$$\Rightarrow 55y = 5 - 210 \times 5 = 5 - 1050$$

$$\Rightarrow 55y = -1045$$

$$\Rightarrow y = \frac{-1045}{55} = -19.$$

74. On applying the Euclid's division lemma to find HCF of 152, 272, we get

$$\begin{array}{r} 152 \overline{)272} \quad (1 \\ \underline{152} \\ 120 \end{array}$$

$$\begin{array}{r} 120 \overline{)152} \quad (1 \\ \underline{120} \\ 32 \end{array}$$

$$272 = 152 \times 1 + 120$$

Here the remainder = 0.

Using Euclid's division lemma to find the HCF of 152 and 120, we get

$$152 = 120 \times 1 + 32$$

Again the remainder = 0.

Using division lemma to find the HCF of 120 and 32, we get

$$\begin{array}{r} 32 \overline{)120} \quad (3 \\ \underline{96} \\ 24 \end{array}$$

$$120 = 32 \times 3 + 24$$

Similarly,

$$\begin{array}{r} 8 \overline{)24} \quad (3 \\ \underline{24} \\ 0 \end{array}$$

$$32 = 24 \times 1 + 8$$

$$24 = 8 \times 3 + 0$$

HCF of 272 and 152 is 8.

$272 \times 8 + 152x = \text{H.C.F. of the numbers}$

$$\Rightarrow 8 = 272 \times 8 + 152x$$

$$\Rightarrow 8 - 272 \times 8 = 152x$$

$$\Rightarrow 8(1 - 272) = 152x$$

$$\Rightarrow x = \frac{-2168}{152} = \frac{-271}{19}$$

75. LCM of rational number = $\frac{\text{LCM of numerators}}{\text{HCF of denominators}}$

Numbers are $\frac{25}{10}, \frac{5}{10}, \frac{175}{1000}$

Now, $25 = 5 \times 5$; $5 = 5 \times 1$; $175 = 5 \times 5 \times 7$

LCM of (25, 5, 175) = $5 \times 5 \times 7 = 175$

Also,

$10 = 2 \times 5$; $1000 = 2 \times 2 \times 2 \times 5 \times 5 \times 5$

HCF of (10, 10, 1000) = 10

LCM of (2.5, 0.5, 0.175) = $\frac{175}{10} = 17.5$