Solution

REAL NUMBERS WS 2

Class 10 - Mathematics

1. 170
   Explanation:
   Given that two tankers contain 850 litres and 680 litres of petrol respectively. We have to find the maximum capacity of a container which can measure the petrol of either tanker in exact number of times
   Clearly, the maximum capacity of the container is the HCF of 850 and 680 in litres. So, Let us find the HCF of 850 and 680 by Euclid's algorithm.
   Clearly, HCF of 850 and 680 is 170.
   Hence, capacity of the container must be 170 litres.
2. 179
   Explanation:
   Two positive integers are 76254 and 6265 .
   By applying division lemma
   76254 = 6265$\times$12 + 1074
   6265 = 1074$\times$5 + 895
   1074 = 895$\times$1 + 179
   895 = 179$\times$5 + 0
   Therefore, HCF of 76254 and 6265 is 179.
3. 24
   Explanation:
   Given numbers are 438 and 606 deducting the remainder 6 from both numbers
   438 - 6 = 432
   606 - 6 = 600.
   Therefore HCF of 432 and 606 is the largest number dividing both of them
   Prime factorization of 432 and 600 are:
   $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$
   $600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3 \times 5^2$
   HCF(432, 600) = $2^3 \times 3$ = 24.
   The largest number which divides 438 and 606 leaving remainder 6 is 24.
4. 154
   Explanation:
   In order to get the result we have to find the HCF of 2002 and 2618. There prime factors are,
   2002 = 2 $\times$ 7 $\times$ 11 $\times$ 13 and
   2618 = 2 $\times$ 7 $\times$ 11 $\times$ 17
   Hence HCF = 2 $\times$ 7 $\times$ 11 = 154
5. 36
   Explanation:
   $\text{H. C. F} = \frac{\text{Product of numbers}}{\text{LCM}}$
   $\text{H.C.F} = \frac{396 \times 576}{6336}$
   $\text{H. CF} = \frac{228096}{6336}$
   H.C.F = 36
6. 5.84
   Explanation:
   By prime factorization method we have,
   7 = 7 $\times$ 1
   8 = 2 $\times$ 2 $\times$ 2
   11 = 11 $\times$ 1
   and, 12 = 2 $\times$ 2 $\times$ 3
   Therefore, LCM of 7, 8,11,12 = $2 \times 2 \times 2 \times 3 \times 7 \times 11$ = 1848

∴ Bells will toll together after every 1848 seconds.

∴ In next 3 hrs, number of times the bells will toll together = $\frac{3\times3600}{1848}$=5.84 times

7. 9831

   Explanation:

   LCM of (4, 7, 13) = 364

   Largest 4 digit number = 9999

   On dividing 9999 by 364 we get remainder as 171

   Greatest number of 4 digits divisible by 4, 7 and 13 = (9999 - 171) = 9828

   Hence, required number = (9828 + 3) = 9831

   Therefore 9831 is the number.

8. 999720

   Explanation:

   The greater number of 6 digits is 999999.

   LCM of 24, 15, and 36 is 360.

   $999999 = 360 \times 2777 + 279$

   Required number is = 999999 - 279 = 999720

9. 23

   Explanation:

   AS 4025 > 1656 So applying Euclid's division algorithm on 4025 and 1656 we get

   4025 = 1656 $\times$ 2 + 713

   1656 = 713 $\times$ 2 + 230

   713 = 230 $\times$ 3 + 23

   230 = 23 $\times$ 10

   Hence, HCF(1656, 4025) = 23

10. 23

    Explanation:

    According to question we have to find the least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3.

    Take LCM of 5, 6, 4 and 3 which is equal to 60.

    Now on dividing 2497 by LCM of 5, 6, 4 and 3 that is by 60, we get a reminder of 37.

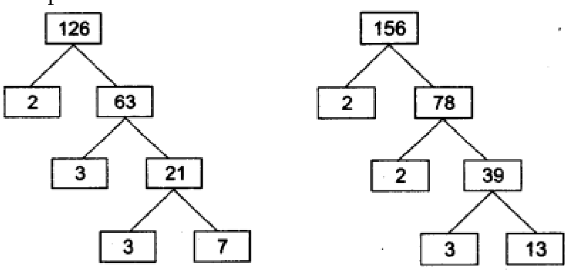
    Therefore the number which should be added to 2497 is : 60 - 37 = 23.

    Hence 23 is the required number.

11. 3276

    Explanation:

    The prime factor tree of 126 and 156 is

    $\therefore 126 = (2 \times 3 \times 3 \times 7) = \left(2 \times 3^2 \times 7\right)$

    and $156 = (2 \times 2 \times 3 \times 13) = \left(2^2 \times 3 \times 13\right)$.

    ∴ HCF(126, 156) = product of common terms with lowest power

    HCF $= \left(2^1 \times 3^1\right) = (2 \times 3)$

    HCF = 6

    and LCM(126, 156) = product of prime factors with highest power

    HCF $= \left(2^2 \times 3^2 \times 7 \times 13\right) = (4 \times 9 \times 7 \times 13)$

    LCM = 3276.

    ∴ HCF (126, 156) = 6

    and LCM (126, 156) = 3276

12. 24
    Explanation:
    The prime factorisation of 48 = $2 \times 2 \times 2 \times 2 \times 3 = 2^4 \times 3$
    and prime factorisation of 72 = $2 \times 2 \times 2 \times 3 \times 3 = 2^3 \times 3^2$
    Now, HCF(48, 72) = $2^3 \times 3 = 24$
13. HCF of two or more numbers is the greatest common factor which can divide all the numbers exactly.
    On applying Euclid's division lemma on 120 and 105 we get
    $120 = 105 \times 1 + 15$.
    Since remainder ≠ 0, apply division lemma on divisor 105 and remainder 15
    $105 = 15 \times 7 + 0$.
    Therefore, H.C.F. of 105 and 120 = 15.
14. Given numbers are 438 and 606 deducting the remainder 6 from both numbers
    438 - 6 = 432
    606 - 6 = 600.
    Therefore HCF of 432 and 606 is the largest number dividing both of them
    Prime factorization of 432 and 600 are:
    $432 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 = 2^4 \times 3^3$
    $600 = 2 \times 2 \times 2 \times 3 \times 5 \times 5 = 2^3 \times 3 \times 5^2$
    HCF(432, 600) = $2^3 \times 3$
    = 24.
    The largest number which divides 438 and 606 leaving remainder 6 is 24.
15. Clearly, the maximum number of columns = HCF (612, 48).

2	612
2	306
3	153
3	51
	17

2	48
2	24
2	12
2	6
	3

    Now, 612 =$2 \times 2 \times 3 \times 3 \times 17 = (2^2 \times 3^2 \times 17)$
    and 48 $= 2 \times 2 \times 2 \times 2 \times 3 = (2^4 \times 3)$.
    $\therefore$ HCF (612, 48) $= (2^2 \times 3) = (4 \times 3)$
    $\therefore$ HCF (612, 48)= 12.
    $\therefore$ Maximum number of columns in which they can march = 12.
16. 18180 = $2^2 \times 3^2 \times 5 \times 101$
    7575 = $3 \times 5^2 \times 101$
    LCM = $2^2 \times 3^2 \times 5^2 \times 101 = 90900$
    HCF = $3 \times 5 \times 101 = 1515$
17. The Highest Common Factor H C F of two or more numbers is the highest number that divides the numbers exactly.
    By applying Euclid’s division lemma for 190 and 100
    $190 = 100 \times 1 + 90$.
    Since remainder ≠ 0, apply division lemma on divisor 100 and remainder 90
    $100 = 90 \times 1 + 10$.
    Since remainder ≠ 0, apply division lemma on divisor 90 and remainder 10
    $90 = 10 \times 9 + 0$.
    Here remainder is zero
    Hence HCF (190,100)=10
18. Given integers are 408 and 1032 where 408 < 1032
    By applying Euclid’s division lemma, we get $1032 = 408 \times 2 + 216$.
    Since the remainder $\neq 0$, so apply division lemma again on divisor 408 and remainder 216, we get the relation as
    $408 = 216 \times 1 + 192$.
    Since the remainder $\neq 0$, so apply division lemma again on divisor 216 and remainder 192
    $216 = 192 \times 1 + 24$.
    Since the remainder $\neq 0$, so apply division lemma again on divisor 192 and remainder 24

192 = 24 × 8 + 0.

Now the remainder has become 0. Therefore, the H.C.F of 408 and 1032 = 24.

Therefore,

24 = 1032 m - 408 $\times$ 5

1032 m = 24 + 408 $\times$ 5

1032 m = 24 + 2040

1032 m = 2064

$m = \frac{2064}{1032}$

Therefore, m = 2.

19. 6 = 2 $\times$ 3

$72 = 8 \times 9 = 2^3 \times 3^2$

$120 = 8 \times 15 = 2^3 \times 3 \times 5$

HCF(6,72,120) =2$\times$3 = 6

LCM (6,12,120)= $2^3 \times 3^2 \times$ 5= 360

20. First subtracting the remainders

122 - 5 = 117

150 - 7 = 143

115 - 11 = 104

Now prime factors of 117,143 and 104 are

117=$3^2 \times$ 13

143= 11 $\times$ 13

104= $2^3 \times$ 13

The HCF of 104, 117 and 143 is 13

The largest number which divides 122, 150 and 115 leaving 5, 7 and 11 respectively as remainders is 13

21. Given number,

7 $\times$ 9 $\times$ 13 $\times$ 15 + 15 $\times$ 14

= 15(7 $\times$ 9 $\times$ 13 + 14)

Clearly, this number is a product of two numbers other than 1 and has factors other than 1, and itself.

Therefore, it is a composite number.

22. HCF of two or more numbers is the biggest common factor which divides all the said numbers exactly.

Now, by applying Euclid's Division lemma to 240 and 6552 we get

6552 = 240 $\times$ 27 + 72.

Since remainder ≠ 0, apply division lemma on divisor 240 and remainder 72 gives

240 = 72 $\times$ 3 + 24.

Since remainder ≠ 0, apply division lemma on divisor 72 and remainder 24 gives

72 = 24 $\times$ 3 + 0.

Here we get the remainder =0

Hence, HCF of 6552 and 240 is 24.

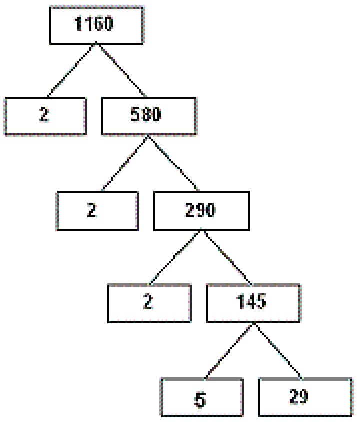
23. According to question we have to find the least number which should be added to 2497 so that the sum is exactly divisible by 5, 6, 4 and 3.

Take LCM of 5, 6, 4 and 3 which is equal to 60.

Now on dividing 2497 by LCM of 5, 6, 4 and 3 that is by 60, we get a reminder of 37.

Therefore the number which should be added to 2497 is: 60 - 37 = 23.

Hence 23 is the required number.

24. 

25. LCM of (4,7,13) = 364

Largest 4 digit number = 9999

On dividing 9999 by 364 we get remainder as 171

Greatest number of 4 digits divisible by 4, 7 and 13 = (9999 – 171) = 9828

Hence, required number = (9828 + 3) = 9831

Therefore 9831 is the number.

26. The prime factorization of 42, 49 and 63 are:

42 = 2 $\times$ 3 $\times$ 7

49 = 7 $\times$ 7

63 = 3 $\times$ 3 $\times$ 7

HCF(42, 49, 63) = 7

So the greatest possible length of each plank = 7m

Sum of of the length of three pieces of timber

= 42 + 49 + 63 = 154 m

Total no. of planks = $\frac{154}{7}$ = 22

27. $HCF \times LCM = a \times b$

$\Rightarrow \quad 23 \times 1449 = 161 \times b$

$\Rightarrow \quad b = \frac{23 \times 1449}{161} = 207$

$\therefore$ Other number is 207

28. Given numbers are 234 and 111

Using Euclid’s division lemma , we get

234 = 111 $\times$ 2 + 12

Now 111 = 12 $\times$ 9 + 3

and 12 = 3 $\times$ 4 + 0

HCF = 3

As per given condition

3 = 234x + 111y

$\Rightarrow$ 3 - 234x = 111y

$\Rightarrow \frac{3-234x}{111} = y$

Taking x = - 9, we get

$\frac{3-234(-9)}{111}$ = y

Therefore, y = 19.

29. Let p(x) = $(x-1)(x-2)$

and q(x) = $(x-2)(x-7)$

$\therefore$ LCM = $(x-1)(x-2)(x-7)$

30. 336 and 54

336 = 2 × 2 × 2 × 2 × 3 × 7 = 2^4 × 3 × 7

54 = 2 × 3 × 3 × 3 = 2 × 3^3

HCF = 2 × 3 = 6

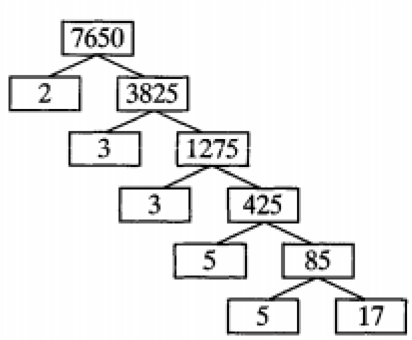
LCM = 2^4 × 3^3 × 7 = 3024

Product of two numbers 336 and 54 = 336 × 54 = 18144

HCF × LCM = 6 × 3024 = 18144

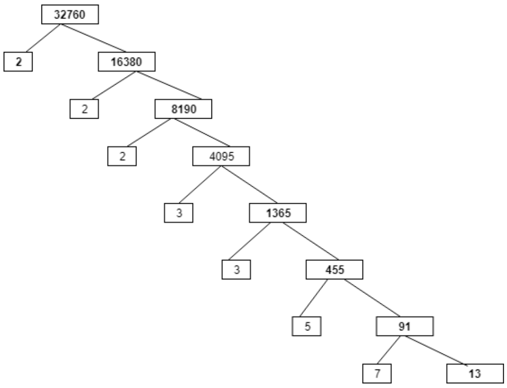
Hence, product of two numbers = HCF × LCM`

31. Using factor tree we have



Prime factors of $7650 = 2 \times 3 \times 3 \times 5 \times 5 \times 17$ = $2 \times 3^2 \times 5^2 \times 17$

32.



91 = 7 $\times$ 13, So, P = 7, Q = 13

$O = \frac{4095}{1365} = 3$

N = 2 $\times$ 8190 = 16380

The composite number =M = 16380 $\times$ 2 = 32760

33. The prime factorization of 2520 are

2520 = $2^3 \times 3^2 \times 5 \times 7$----------(1)

It is given that

2520 = $2^3 \times 3^p \times q \times 7$-----------(2)

On comparing equation (1) and (2) we get

p = 2 and q = 5

34. Let the larger number be x and the smaller number be y. Them,

$x = x^2 - \mathrm{y}_2^2 = 45 \ldots\ldots (i)$

$\text{ and } y^2 = 4x \ldots.. (\text{ ii })$

substituting y^2 = 5x in (i), we have

$x^2 - 4x = 45$

$\Rightarrow x^2 - 4x = 45 = 0$

$\Rightarrow x^2 - 9x + 5x - 45 = 0$

$\Rightarrow x(x-9) + 5(x-9) = 0$

$\Rightarrow (x-0)(x+5) = 0$

$\Rightarrow$ either $x - 9 = 0$ or $x + 5 = 0$

$\Rightarrow x = 9$ or $x = -5$

$\therefore$ smaller number is $\sqrt{36}$ or $\sqrt{-20}$ [From (ii)]

$\Rightarrow$ smaller number is $= \pm 6 [\because \sqrt{-20}$ is not real]

$\therefore$ Larger number = 9 and smaller number $= \pm 6$

35. 26 = 2 $\times$13

91 = 7 $\times$13

HCF = 13

LCM = 2 $\times$7 $\times$13 = 182

36. Let us assume that $4 + \sqrt{2}$ is rational. Then, there exist positive co-primes a and b such that

$4 + \sqrt{2} = \frac{a}{b}$

$\sqrt{2}=\frac{a}{b}-4$
$\sqrt{2}=\frac{a-4b}{b}$
As a - 4b and b are integers.
So, $\frac{a-4b}{b}$ is a rational number .
But $\sqrt{2}$ is not rational number .
Since a rational number cannot be equal to an irrational number . Our assumption that $4+\sqrt{2}$ is a rational number is wrong .
Hence, $4+\sqrt{2}$ is irrational.

37. Two positive integers are 1001 and 385.
By applying Euclid’s division lemma
1001 = 385 $\times$ 2 + 231
385 = 231$\times$1 + 154
231 = 154$\times$1 + 77
154 = 77$\times$2 + 0
HCF = 77
Hence HCF of 1001 and 385 is 77.
38. LCM of 12, 16, 24 = 48
Required number is 48 + 7 = 55.
39. 44 = $2^2 \times 11$
96 = $2^5 \times 3$
404 = $2^2 \times 101$
HCF = 2^2 = 4
LCM = $2^5 \times 11 \times 3 \times 101$
= 106656
40. 26 = 2 $\times$ 13
65 = 5 $\times$ 13
117 = 3 $\times$ 3 $\times$ 13
$\therefore$ HCF of 26, 65 and 117 = 13
and LCM of 26, 65 and 117 = 13 $\times$ 3 $\times$ 3 $\times$ 2 $\times$ 5 = 1170
41. $\mathrm{P(x)=22x(x+1)^2=2\times 11\times x\times (x+1)^2}$
$\text{ and } Q(x)=36\mathrm{x}^2\left(2\mathrm{x}^2+3\mathrm{x}+1\right)$
$=2^2\times 3^2\times \mathrm{x}^2\left(2\mathrm{x}^2+2\mathrm{x}+\mathrm{x}+1\right)$
$=2^2\times 3^2\times \mathrm{x}^2\times [2\mathrm{x}(\mathrm{x}+1)+1(\mathrm{x}+1)]$
$=2^2\times 3^2\times \mathrm{x}^2\times (\mathrm{x}+1)(2\mathrm{x}+1)$
$\therefore LCM=2\times 11\times x\times (x+1)\times (x+1)\times 2\times 3^2\times x\times (2x+1)$
$=2^2\times 3^2\times 11\times \mathrm{x}^2\times (\mathrm{x}+1)^2(2\mathrm{x}+1)=396\mathrm{x}^2(\mathrm{x}+1)^2(2\mathrm{x}+1)$
42. It can be observed that Ravi takes lesser time than Sonia for completing 1 round of the circular path. As they are going in the same direction, they will meet again at the same time when Ravi will have completed 1 round of that circular path with respect to Sonia. And the total time taken for completing this 1 round of circular path will be the LCM of time taken by Sonia and Ravi for completing 1 round of circular path respectively i.e., LCM of 18 minutes and 12 minutes.
18 = 2 × 3 × 3 And, 12 = 2 × 2 × 3
LCM of 12 and 18 = 2 × 2 × 3 × 3 = 36
Therefore, Ravi and Sonia will meet together at the starting point after 36 minutes.
43. 510 and 92
510= 2 × 3 × 5× 17
92 = 2 × 2 × 23
HCF = 2
LCM = 2 × 2 × 3 × 5 × 17 × 23 = 23460
Product of two numbers 510 and 92 = 510 × 92 = 46920
HCF × LCM = 2 × 23460 = 46920
Hence, product of two numbers = HCF × LCM
44. The smallest number divisible by 520 and 468 = LCM(520,468)

Prime factors of 520 and 468 are :

$520 = 2^3 \times 5 \times 13$

$468 = 2 \times 2 \times 3 \times 3 \times 13$

Hence LCM(520,468) = $2^3 \times 3^2 \times 5 \times 13 = 8 \times 9 \times 5 \times 13 = 4680$

Now the smallest number which when increased by 17 is exactly divisible by both 520 and 468.

=LCM(520,468)-17

=4680-17

=4663

45. We have,

$96 = 2^5 \times 3$ and $404 = 2^2 \times 101$

$\therefore \quad \text{HCF} = 2^2 = 4$

Now, $\text{HCF} \times \text{LCM} = 96 \times 404$

$\Rightarrow \quad \text{LCM} = \frac{96 \times 404}{\text{HCF}} = \frac{96 \times 404}{4} = 96 \times 101 = 9696$

46. $510 = 2 \times 3 \times 5 \times 17$

$92 = 2 \times 2 \times 23$

HCF of 510 and 92 = 2

LCM of 510 and 92 = $2 \times 2 \times 3 \times 5 \times 17 \times 23 = 23460$

47. HCF of two or more positive integers is the largest positive integer that divides all the numbers exactly.

Now,

By applying Euclid's Division lemma to 70 and 30, we get

$70 = 30 \times 2 + 10$.

Since remainder ≠ 0, apply division lemma on divisor 30 and remainder 10, we have

$30 = 10 \times 3 + 0$.

Therefore, H.C.F. of 70 and 30 = 10.

48. The capacity of a container will be equal to HCF of 504 and 735

Let's find out the prime factorization of 504 and 735 by division

2	504
2	252
2	126
3	63
3	21
	7

5	735
3	147
7	49
	7

We get

$504 = (2^3 \times 3^2 \times 7)$

and $735 = (5 \times 3 \times 7^2)$.

Therefore; HCF (504, 735) $= (3 \times 7) = 21$.

Therefore, the capacity of a container which can measure the milk of either tank in exact number of times

= 21 liter

49. $\frac{175}{15} = 11.667$

Hence 175 is not divisible by 15

But LCM of two numbers should be divisible by their HCF.

$\therefore$ Two numbers cannot have their HCF as 15 and LCM as 175.

50. Since $3 \times 5 \times 7 + 7 = (3 \times 5 + 1) \times 7 = (15 + 1) \times 7 = 16 \times 7$.

Hence, it is a composite number.

51. By prime factorisation, we get

2	108
2	54
3	27
3	9
	3

2	120
2	60
2	30
3	15
	5

2	252
2	126
3	63
3	21
	7

108 = $\left(2^2 \times 3^3\right)$
120 = $\left(2^3 \times 3 \times 5\right)$
252 = $\left(2^2 \times 3^2 \times 7\right)$
HCF (108, 120, 252) = product of common terms with lowest power
$= \left(2^2 \times 3\right) = (4 \times 3)$
HCF = 12
LCM (108, 120, 252) = product of prime factors with highest power
$= \left(2^3 \times 3^3 \times 5 \times 7\right)$
LCM = 7560
$\therefore$ HCF (108, 120, 252) = 12
and LCM (108, 120, 252) = 7560.

52. 48 = 2 $\times$ 2 $\times$ 2 $\times$ 2 $\times$ 3
72 = 2 $\times$ 2 $\times$ 2 $\times$ 3 $\times$ 3
108 = 2 $\times$ 2 $\times$ 3 $\times$ 3 $\times$ 3
Therefore, L.C.M of 48, 72, 108 is
(2 $\times$ 2 $\times$ 2 $\times$ 3 $\times$ 3 $\times$ 3)
= 432
So, time when they change again = 432 seconds
But we need to find time after 7 am So, first we convert 432 seconds into minutes.
Time = 432 second
= $\frac{432}{60}$ minutes
$\therefore$ Time = 7 minutes 12 seconds
Thus,
Required time = 7am + 7 minutes 12 seconds
= 7 : 07 : 12 am

53. $\text{Let } \mathrm{P(x)} = 2\mathrm{x}^4 - 2\mathrm{y}^4$
$= 2\left[\left(\mathrm{x}^2\right)^2 - \left(\mathrm{y}^2\right)^2\right]$
$= 2\left(x^2+y^2\right)\left(x^2-y^2\right)$ Using identity $a^2-b^2=(a+b)(a-b)$
$= 2\left(\mathrm{x}^2+\mathrm{y}^2\right)(\mathrm{x}+\mathrm{y})(\mathrm{x}-\mathrm{y})$

and $Q(x) = 3x^3 + 6x^2y - 3xy^2 - 6y^3$
$= 3x^2(x+2y) - 3y^2(x+2y)$
$= (x+2y)\left(3x^2-3y^2\right)$
$= 3(x+2y)\left(x^2-y^2\right)$
$= 3(x+2y)(x+y)(x-y)$
$\therefore HCF = (x+y)(x-y) = x^2-y^2$ Using Identity $a^2-b^2=(a+b)(a-b)$

54. Let a and b are numbers and HCF = x
Then LCM = 14x
Now sum of HCF and LCM
x+14x =600
15x = 600
x = 40
Hence HCF=40 and LCM=14$\times$40
Given a=280 and b=?
We know that
a $\times$ b = HCF $\times$ LCM
So b $= \frac{40\times14\times40}{280} = 2\times40 = 80$
Hence the other number = 80

55. Given that,
At intervals of 6, 12, and 18 minutes, three bells ring.
We know that,
Three bells ring at interval of 6, 12 and 18 minutes

Let us find the L.C.M of 6, 12 and 18.
$6 = 2 \times 3$
$12 = 2 \times 2 \times 3$
$18 = 2 \times 3 \times 3$
LCM(6, 12, 18) = $2 \times 2 \times 3 \times 3$ = 36
Therefore, all the three bells will rang together again at 6 : 36 am.

56. Clearly, the required number divides (245 - 5) = 240 and (1037 - 5) = 1032 exactly.
So, the required number is HCF of (240, 1032).

2	240
2	120
2	60
2	30
3	15
	5

2	1032
2	516
2	258
3	129
	43

Now $240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5 = \left(2^4 \times 3 \times 5\right)$
and $1032 = 2 \times 2 \times 2 \times 3 \times 43 = \left(2^3 \times 3 \times 43\right)$
$\therefore$ HCF (240, 1032)= $\left(2^3 \times 3\right)$
HCF (240, 1032) = 24
Hence, the largest number which divides 245 and 1037, leaving remainder 5 is 24.

57. Least common factor (LCM) = 78
Greatest divisor is also a HCF = 13
As, we know , HCF $\times$ LCM = a $\times$ b (a and b are two natural no. of which LCM and HCF is given)
Therefore, required product of no. = 13 $\times$ 78 = a $\times$ b =1014 or $2\times3\times13\times13$
Since 13 is common in both no. as its HCF is 13
Required no.= 13×3 = 39 and 13 $\times 2$ = 26
$\therefore$ Pairs of natural numbers are 78 and 13 or 26 and 39.

58. On applying division lemma on 1288 and 575 we get
1288 = 575 $\times$ 2+ 138........(1)
Since remainder ≠ 0, apply division lemma on divisor 575 and remainder 138
575 = 138 $\times$ 4 + 23..........(2)
Since remainder ≠ 0, apply division lemma on divisor 138 and remainder 23
138 = 23 $\times$ 6 + 0.
Hence HCF(1288,575) = 23
Now let 1288 = H and 575 = K
Then equation (1) and (2) can be written as:
H = 2k+138
or 138 = H-2K ....(3)
and K = 138 $\times$ 4 + 23
or 23 = K -138 $\times$ 4
Substituting the value of 138 from (3) we get
23=K-4$\times$ (H-2K)
=K-4H+8K
=9K-4H
Now substituting the values of H and K we get
23=9$\times$575 - 4$\times$1288

59. According to question we have to find the least number which when divided by 20, 25, 35 and 40 leaves remainders 14, 19, 29 and 34 respectively.
Take the LCM of 20, 25, 35 and 40 i.e.,
$20 = 2 \times 2 \times 5$
$25 = 5 \times 5$
$35 = 1 \times 5 \times 7$
$40 = 2 \times 2 \times 2 \times 5$
Now LCM of 20, 25, 35 & 40 = $2 \times 2 \times 5 \times 5 \times 7 \times 2$ = 1400

If the number 1400 is divided by 20, 25, 35, 40 it leaves a remainder 14, 19, 29, 34.
i.e. 6 less than the divisor in each case
Hence, the required number = 1400 - 6 = 1394.

60. **Prime Number:** A number which have exactly two factors 1 and the number itself.
Composite Number: A number having more than two factors.
$7 \times 11 \times 13 + 13 = 13(7 \times 11 + 1)$
$= 13 \times 78$
The resulting number have more then 2 factors.
Hence, it is composite.

61. Let n = 4q + 1 (an odd integer)
$\therefore \quad n^2 - 1 = (4q+1)^2 - 1$
$= 16q^2 + 1 + 8q - 1 \quad$ Using Identity $(a+b)^2 = a^2 + 2ab + b^2$
$= 16q^2 + 8q$
$= 8\left(2q^2 + q\right)$
= 8m, which is divisible by 8.

62. 12, 15 and 21
$12 = 2^2 \times 3$
$15 = 3 \times 5$
$21 = 3 \times 7$
HCF = 3
LCM = $2^2 \times 3 \times 5 \times 7 = 420$

63. In order to get the result we have to find the HCF of 2002 and 2618. There prime factors are,
$2002 = 2 \times 7 \times 11 \times 13$ and
$2618 = 2 \times 7 \times 11 \times 17$
Hence HCF = $2 \times 7 \times 11 = 154$

64. $HCF(306, 1314) = 18$
$LCM(306, 1314) = ?$
Let, a = 306
b = 1314
LCM (a, b) $\times$ HCF (a,b) = a $\times$ b
or, LCM (a, b) $\times$ 18 = 306 $\times 1314$
or LCM(a,b) = $\frac{306 \times 1314}{18}$ =22338
Therefore, $LCM(306, 1314) = 22338$

65. Let *a* be any positive integer and *b* = 3.
Then *a* = 3*q* + *r* for some integer *q* ≥ 0
And *r* = 0, 1, 2 because 0 ≤ *r* < 3
Therefore, *a* = 3*q* or 3*q* + 1 or 3*q* + 2
Or,
$a^2 = (3q)^2$ or $(3q + 1)^2$ or $(3q + 2)^2$
$a^2 = 9q^2$ or $9q^2 + 6q + 1$ or $9q^2 + 12q + 4$
$= 3 \times (3q^2)$ or $3(3q^2+2q) + 1$ or $3(3q^2+4q)+1$
$= 3k_1$ or $3k_2+1$ or $3k_3+1$
Where k_1, k_2, and k_3 are some positive integers

66. We have to find the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.
Let assume that x be the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.
So, it means
x divides 85 - 1 = 84
and
x divides 72 - 2 = 70
So, from this we concluded that
= x divides 84 and 70
= x = HCF (84, 70)

Now, to find HCF(84, 70), we use method of prime factorization.
Prime factors of 84 = $2 \times 2 \times 3 \times 7$
Prime factors of 70 = $2 \times 5 \times 7$
So,
= HCF (84, 70) = 2×7 = 14
= x = 14
Hence, 14 is the greatest number which divides 85 and 72 leaving remainder 1 and 2 respectively.

67. $18 = 2 \times 3 \times 3 = 2 \times 3^2$
$24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$
$\therefore \quad HCF = 2 \times 3 = 6$
LCM =$2\times2\times2\times3\times3$
= 72
68. 8, 9 and 25
8 = 2 × 2 × 2 = 2^3
9 = 3 × 3 = 3^2
25 = 5 × 5 = 5^2
HCF = 1
LCM = $2^3 \times 3^2 \times 5^2$ = 1800
69. Maximum number of burfis in each stack = HCF (420, 150).
420=150 $\times$2 +120
150= 120 $\times$1 +30
120=30$\times$4 + 0
Hence HCF(420, 150) .
$\therefore$ maximum number of burfis in each stack = 30.
$\therefore$ Number of stacks of badam burfis = $\frac{150}{30} = 5$
Number of stacks of kaju burfis = $\frac{420}{30} = 14$
70. The prime factorization of 90 and 140 are as follows
$90 = 2 \times 3 \times 3 \times 5 \ = 2 \times 3^2 \times 5$
$144 = 2 \times 2 \times 2 \times 2 \times 3 \times 3 = 2^4 \times 3^2$
Hence HCF (90,144) = 2×3^2 = 18
and LCM (90,144) = $2^4 \times 3^2 \times 5$ = 720
71. Let us first find the factors of 40, 36 and 126
40 = $2^3 \times 5$
36 = $2^2 \times 3^2$
126 = $2 \times 3 \times 3 \times 7$
Now, L.C.M of 40, 36 and 126 = $2^3 \times 3^2 \times 5 \times 7$
L.C.M of 40, 36 and 126 = 2520
H.C.F of 40, 36 and 126 = 2
72. L.C.M. of 60 and 62 seconds is 1860 seconds
$\frac{1860}{60} = 31$ minutes
They will beep together at 10:31 a.m.
73. The prime factorisation of 120 = 2 × 2 × 2 × 3 × 5 = 2^3 × 3 × 5
and prime factorisation of 144 = 2 × 2 × 2 × 2 × 3 × 3 = $2^4 \times 3^2$
Now, LCM(120,144) = $2^4 \times 3^2 \times 5$ = 720
and HCF(120,144) = $2^3 \times 3$ = 24
74. 26 and 91
26= 2 × 13
91 = 7 × 13
HCF = 13
LCM = 2 × 7 × 13 = 182
Product of two numbers 26 and 91 = 26 × 91 = 2366

HCF × LCM = 13 × 182 = 2366

Hence, product of two numbers = HCF × LCM

75. Two positive integers are 687897 and 81445.

By applying Euclid's division lemma

$687897 = 81445 \times 8 + 36337$

$81445 = 36337 \times 2 + 8771$

$36337 = 8771 \times 4 + 1253$

$8771 = 1253 \times 7 + 0$

$\therefore$ HCF = 1253