

Solution
REAL NUMBERS WS 3
Class 10 - Mathematics
Section A

1.

(c) A is true but R is false.
Explanation: Here, reason is not true.
 $\sqrt{9} = \pm 3$, which is not an irrational number.
 A is true but R is false.
2.

(a) Both A and R are true and R is the correct explanation of A.
Explanation: As we know that square root of every prime number is an irrational number. So, both assertion and reason are correct and reason explains assertion.
3.

(d) A is false but R is true.
Explanation: A is false but R is true.
4. $(3 - \sqrt{7})(3 + \sqrt{7}) = (3)^2 - (\sqrt{7})^2$
 $= 9 - 7 = 2$ which is rational.
5. $288 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$
6. we have $7 - 2\sqrt{2}$ since
 $\sqrt{2}$ is an irrational number and we know that product of rational \times irrational = irrational
 also difference of rational and irrational is irrational.
 therefore $7 - 2\sqrt{2}$ is irrational
7. $\sqrt{21} = \sqrt{3} \times \sqrt{7}$ is an irrational number because $\sqrt{3}$ and $\sqrt{7}$ are irrational being square roots of prime numbers.
8. $(1 + \sqrt{5}) - (4 + \sqrt{5}) = -3$, which is a rational number.
9. $\sqrt[3]{3}$ is an irrational number because 3 is a prime number.
10. $-\sqrt{0.4} = -\frac{2}{\sqrt{10}}$, which is a quotient of a rational and an irrational number and so it is an irrational number.
11. π is an irrational number because it is a non-repeating and non-terminating decimal.
12. Let $3\sqrt{5}$ be rational.
 Therefore, it can be expressed as $\frac{a}{b}$ where a and b are two integer such that $b \neq 0$
 $3\sqrt{5} = \frac{a}{b} \Rightarrow \sqrt{5} = \frac{a}{3b}$
 $\Rightarrow \sqrt{5}$ is rational, this is a contradiction because $\frac{a}{3b}$ is rational.
 $\therefore 3\sqrt{5}$ is irrational.
13. 0.5918 is a terminating decimal expansion.
 Hence, it is a rational number.
14. 3.1416 is a rational number because it is a terminating decimal.
15. since we know that square root of every prime is an irrational no.
 now sq. root of 3, is also an irrational no. as 3 is prime.
 also product of rational and irrational no. is an irrational no. and the ratio of irrational and rational no. is again an irrational no.
 Therefore $\frac{2\sqrt{3}}{5}$ is an irrational number.
16. since we know that 7, is a prime number
 as it has only 2, factors i.e 1 and itself
 also we know that square root of every prime number is an irrational number.
 therefore $\sqrt{7}$ is an irrational number.
17. $\frac{\sqrt{12}}{\sqrt{75}} = \sqrt{\frac{4}{25}} = \frac{2}{5}$, which is a rational number.
18. No, sometimes rational, sometimes irrational.
 For example, $\sqrt{5} \times \sqrt{5} = 25$ and $\sqrt{3} \times \sqrt{5} = \sqrt{15}$.

Section B

19. We assume that $2 + \sqrt{5}$ is a rational number.
 So it can be written as $\frac{a}{b}$, where a, b are co-prime integers and b is not zero.

The new equation will be as below:

$$\implies 2 + \sqrt{5} = \frac{a}{b}$$

So, we will get $\sqrt{5} = \frac{a}{b} - 2$

As we know that 2 and $\frac{a}{b}$ are rational numbers, their difference will be rational number only.

on the other hand, $\sqrt{5}$ is an irrational number and it can not be written as $\frac{a}{b}$.

So, this contradicts the fact that $\sqrt{5}$ is a rational number.

Therefore our assumption is wrong and $2 + \sqrt{5}$ is irrational.

20. We can prove $\frac{1}{\sqrt{2}}$ irrational by contradiction.

Let us suppose that $\frac{1}{\sqrt{2}}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$)

Such that

$$\frac{1}{\sqrt{2}} = \frac{a}{b}$$

$$\implies \sqrt{2} = \frac{b}{a} \dots\dots\dots(1)$$

R.H.S of (1) is rational but we know that is $\sqrt{2}$ irrational.

It is not possible which means our supposition is wrong.

Therefore, $\frac{1}{\sqrt{2}}$ can not be rational.

Hence, it is irrational.

21. Let $\frac{1}{2+\sqrt{3}}$ be a rational number.

A rational number can be written in the form of $\frac{p}{q}$ where p, q are integers.

$$\frac{1}{2+\sqrt{3}} = \frac{p}{q}$$

$$\implies \sqrt{3} = \frac{q-2p}{p}$$

p, q are integers then $\frac{q-2p}{p}$ is a rational number.

Then $\sqrt{3}$ is also a rational number.

But this contradicts the fact as $\sqrt{3}$ is an irrational number.

So, our supposition is false.

Therefore, $\frac{1}{2+\sqrt{3}}$ is an irrational number.

22. Suppose \sqrt{p} be rational \implies it can be written in the form of $\frac{a}{b}$.

$$\sqrt{p} = \frac{a}{b} \text{ (where } a \text{ and } b \text{ are co-prime)}$$

On squaring both sides, we get

$$p = \frac{a^2}{b^2}$$

a^2 has a factor p .

$$pb^2 = a^2 \dots\dots\dots(i)$$

a also has a factor p .

$$\text{So } a = pc$$

$$pb^2 = a^2$$

$$a^2 = p^2c^2$$

Put the value of a^2 in equation (i),

$$pb^2 = p^2c^2$$

b^2 has a factor p ,

$\therefore b$ has a factor p .

a and b have common factor p .

But we assume that a and b are co-prime

\therefore our assumption is wrong.

\sqrt{p} must be an irrational number, (p is a prime number.)

\sqrt{q} is also an irrational number (q is a prime number.)

Sum of two irrational numbers is irrational

$\therefore \sqrt{p} + \sqrt{q}$ is irrational number.

23. Let us assume, to the contrary, that $2\sqrt{3}$ is rational number.

Then, there must exist co-primes number between a and b ($b \neq 0$) such that

$$2\sqrt{3} = \frac{a}{b} \implies \sqrt{3} = \frac{a}{2b} .$$

Since a and b are integers, so $\frac{a}{2b}$ is rational number because division of two rational number is always a rational number .

Thus, $\sqrt{3}$ is also rational.

But, this contradicts the fact that $\sqrt{3}$ is irrational.

So, our assumption is incorrect.

Hence, $2\sqrt{3}$ is irrational.

24. Let us assume, to the contrary, that is $3 + 2\sqrt{5}$ rational.

That is, we can find coprime integers a and b ($b \neq 0$) such that

$$3 + 2\sqrt{5} = \frac{a}{b} \text{ Therefore, } \frac{a}{b} - 3 = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{b} = 2\sqrt{5}$$

$$\Rightarrow \frac{a-3b}{2b} = \sqrt{5} \Rightarrow \frac{a}{2b} - \frac{3}{2}$$

Since a and b are integers,

We get $\frac{a}{2b} - \frac{3}{2}$ is rational, also so $\sqrt{5}$ is rational.

But this contradicts the fact that $\sqrt{5}$ is irrational.

This contradiction arose because of our incorrect assumption that $3 + 2\sqrt{5}$ is rational.

So, we conclude that $3 + 2\sqrt{5}$ is irrational.

25. Suppose $\sqrt{3} = \frac{a}{b}$, where a and b are co-prime integers, $b \neq 0$

Squaring both sides,

$$\Rightarrow 3 = \frac{a^2}{b^2}$$

Multiplying with b on both sides,

$$\Rightarrow 3b = \frac{a^2}{b}$$

$$\text{LHS} = 3 \times b = \text{Integer}$$

$$\text{RHS} = \frac{a^2}{b} = \frac{\text{Integer}}{\text{Integer}} = \text{Rational Number}$$

$$\Rightarrow \text{LHS} \neq \text{RHS}$$

\therefore Our supposition is wrong.

$$\Rightarrow \sqrt{3} \text{ is irrational.}$$

Suppose $15 + 17\sqrt{3}$ is a rational number.

$$\therefore 15 + 17\sqrt{3} = \frac{a}{b}, \text{ where } a \text{ and } b \text{ are co-prime, } b \neq 0$$

$$\Rightarrow 17\sqrt{3} = \frac{a}{b} - 15$$

$$\sqrt{3} = \frac{a-15b}{17b}$$

$$\frac{a-15b}{17b} \text{ is rational number,}$$

$\sqrt{3}$ is irrational.

$$\therefore \sqrt{3} \neq \frac{a-15b}{17b}$$

\therefore Our supposition is wrong.

$$\Rightarrow 15 + 17\sqrt{3} \text{ is irrational.}$$

26. If possible let $3 + \sqrt{2}$ is rational number, and we take another rational number 3 for our calculation.

$$\Rightarrow (3 + \sqrt{2}) - 3 = \sqrt{2} \text{ (difference of two rational number is a rational number)}$$

$$\therefore \sqrt{2} \text{ is rational}$$

This contradicts the fact that $\sqrt{2}$ is irrational

Since the contradiction arises by assuming that $3 + \sqrt{2}$ is rational.

Hence, $3 + \sqrt{2}$ is irrational.

27. Let assume that $\sqrt{5} + \sqrt{3}$ is rational

Therefore it can be expressed in the form of $\frac{p}{q}$, where p and q are integers and $q \neq 0$

$$\text{Therefore we can write } \sqrt{5} = \frac{p}{q} - \sqrt{3}$$

$$(\sqrt{5})^2 = \left(\frac{p}{q} - \sqrt{3}\right)^2$$

$$5 = \frac{p^2}{q^2} - \frac{2p\sqrt{3}}{q} + 3$$

$$5 - 3 = \frac{p^2}{q^2} - \frac{2p\sqrt{3}}{q}$$

$$\frac{p^2}{q^2} - 2 = \frac{2p\sqrt{3}}{q}$$

$$\frac{p^2 - 2q^2}{qp} = \sqrt{3}$$

$\frac{p^2-2q^2}{qp}$ is a rational number as p and q are integers. This contradicts the fact that $\sqrt{3}$ is irrational, so our assumption is incorrect.

Therefore $\sqrt{5} + \sqrt{3}$ is irrational.

28. Let us assume that $3 + 7\sqrt{2}$ is a rational number. \Rightarrow

$$3 + 7\sqrt{2} = \frac{p}{q}, p, q \text{ are integers and } q \neq 0$$

$$\Rightarrow \sqrt{2} = \frac{p-3q}{7q}$$

RHS is rational but LHS is irrational

\therefore Our assumption was wrong

Hence, $3 + 7\sqrt{2}$ is an irrational number

29. Let us assume that $2 - \sqrt{3}$ is rational.

Then, there exist positive co-primes a and b such that

$$2 - \sqrt{3} = \frac{a}{b}$$

$$\sqrt{3} = 2 - \frac{a}{b}$$

As 2 and $\frac{a}{b}$ are rational number .

So, $\sqrt{3}$ is also rational number .

But $\sqrt{3}$ is not rational number .

Since a rational number cannot be equal to an irrational number. Our assumption that $2 - \sqrt{3}$ is rational wrong .

Hence $2 - \sqrt{3}$ is irrational

30. Let first we consider, $\sqrt{2}/3$ be rational. We can write $\sqrt{2}/3$ as

$$\frac{1}{3} \times \sqrt{2}$$

We know that product of two rational number is always a rational number.

$$\frac{\sqrt{2}}{3} \times 3 = \sqrt{2}$$

But

$\sqrt{2}$ is irrational.

\therefore Here the Contradiction arises by assuming that $\frac{\sqrt{2}}{3}$ is rational. Actually it is irrational.

Hence, $\frac{\sqrt{2}}{3}$ is irrational.

31. If possible, let $\sqrt{6}$ be rational and let its simplest form be $\frac{a}{b}$ then, a and b are integers having no common factor other than 1, and $b \neq 0$.

$$\text{Now, } \sqrt{6} = \frac{a}{b}$$

$$\Rightarrow 6 = \frac{a^2}{b^2} \text{ [on squaring both sides]}$$

$$\Rightarrow 6b^2 = a^2 \text{(i)}$$

$$\Rightarrow 6 \text{ divides } a^2 \text{ [}\therefore 6 \text{ divides } 6b^2\text{]}$$

$$\Rightarrow 6 \text{ divides } a$$

Let $a = 6c$ for some integer c

putting $a = 6c$ in (i), we get

$$a^2 = 36c^2$$

$$6b^2 = 36c^2 \text{ [}6b^2 = a^2\text{]}$$

$$\Rightarrow b^2 = 6c^2$$

$$\Rightarrow 6 \text{ divides } b^2 \text{ [}\therefore 6 \text{ divides } 6c^2\text{]}$$

$$\Rightarrow 6 \text{ divides } b \text{ [}\therefore 6 \text{ divides } b^2 = 6 \text{ divides } b\text{]}$$

Thus, 6 is a common factors of a and b

But, this contradicts the fact that a and b have no common factor other than 1

The contradiction arises by assuming that $\sqrt{6}$ is rational.

Hence $\sqrt{6}$ is irrational.

32. Let us assume that $5 - \sqrt{3}$ is rational number

$$\therefore 5 - \sqrt{3} = \frac{p}{q}; q \neq 0 \text{ and } p, q \text{ are integers}$$

$$\Rightarrow \sqrt{3} = \frac{5q-p}{q}$$

RHS is rational but LHS is irrational

\therefore Our assumption was wrong

$\therefore 5 - \sqrt{3}$ is an irrational number

33. Suppose that $(2 + \sqrt{3})$ is rational number.

\Rightarrow there must exist co-prime numbers between a and b ($b \neq 0$) such that

$$(2 + \sqrt{3}) = \frac{a}{b}$$

$$\Rightarrow \sqrt{3} = \frac{a}{b} - 2 \Rightarrow \sqrt{3} = \frac{a-2b}{b}$$

Since a and b are integers, so $\frac{a-2b}{b}$ is rational number

$\Rightarrow \sqrt{3}$ is also rational number .

But, this contradicts the fact that $\sqrt{3}$ is irrational.

\therefore our assumption is incorrect.

$\therefore (2 + \sqrt{3})$ is irrational.

34. Let $3 - \sqrt{5} = \frac{p}{q}$

$\therefore 3 - \sqrt{5} = \frac{p}{q}$ (where p and q are integers, co-prime and $q \neq 0$)

$$\Rightarrow 3 - \frac{p}{q} = \sqrt{5}$$

$$\Rightarrow \frac{3q-p}{q} = \sqrt{5}$$

$(3q - p)$ and q are integers, so $\left(\frac{3q-p}{q}\right)$ is a rational number, but $\sqrt{5}$ is an irrational number. This contradiction arises because of our wrong assumption.

So $(3 - \sqrt{5})$ is an irrational number

35. Let $5 + 3\sqrt{2}$ is rational. It can be written in the form $\frac{p}{q}$.

$$(5 + 3\sqrt{2}) = \frac{p}{q}$$

$$3\sqrt{2} = \frac{p}{q} - 5$$

$$3\sqrt{2} = \frac{p-5q}{q}$$

$$\sqrt{2} = \frac{p-5q}{3q}$$

As $p - 5q$ and $3q$ are integers .

So, $\frac{p-5q}{3q}$ is rational number .

But $\sqrt{2}$ is not rational number .

Since a rational number cannot be equal to an irrational number. Our assumption that $5 + 3\sqrt{2}$ is rational wrong.

Hence, $5 + 3\sqrt{2}$ is irrational.

36. Let us assume that $\sqrt{3}$ is a rational number.

Then, as we know a rational number should be in the form of p/q where p and q are the co-prime numbers.

So,

$$\sqrt{3} = p/q \text{ where P and Q are co-prime}$$

$$\sqrt{3}q = p$$

Now, squaring both sides we get

$$3q^2 = p^2 \dots(i)$$

So, 3 is a factor of p^2 and hence of p.

Let $p = 3m$ where m is an integer

Using this value of p in (i) we get

$$3q^2 = 9m^2$$

$$q^2 = 3m^2$$

So,

if 3 factors of q^2

Then, 3 is also factor of q

So it contradicts our assumption that p & q are co-prime. Hence our assumption is wrong .

Therefore, $\sqrt{3}$ is an irrational number

37. Let $3 + \sqrt{5}$ is a rational number.

$$3 + \sqrt{5} = \frac{p}{q}, q \neq 0$$

$$3 + \sqrt{5} = \frac{p}{q}$$

$$\Rightarrow \sqrt{5} = \frac{p}{q} - 3$$

$$\Rightarrow \sqrt{5} = \frac{p-3q}{q}$$

Now in RHS $\frac{p-3q}{q}$ is rational

This shows that $\sqrt{5}$ is rational

But this contradicts the fact that $\sqrt{5}$ is irrational, This is because we assumed that $3 + \sqrt{5}$ is a rational number.

$\therefore 3 + \sqrt{5}$ is an irrational number.

38. We have, $\frac{1}{\sqrt{3}} = \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$ (i)

If possible, let $\frac{1}{\sqrt{3}}$ be rational

Then, from (i) it follows that $\frac{\sqrt{3}}{3}$ is rational.

Let $\frac{\sqrt{3}}{3} = \frac{a}{b}$ where a and b are non-zero integers having no common factor other than 1.

Now, $\frac{\sqrt{3}}{3} = \frac{a}{b}$

$\Rightarrow \sqrt{3} = \frac{3a}{b}$ (ii)

But 3a and b are non-zero integers

$\therefore \frac{3a}{b}$ is rational. So, $\sqrt{3}$ is rational.

This contradicts the fact that $\sqrt{3}$ is irrational

The Contradiction arises by assuming that $\frac{1}{\sqrt{3}}$ is rational.

Hence $\frac{1}{\sqrt{3}}$ is irrational.

39. Let $3 + 5\sqrt{2}$ be rational and have only common factor 1.

Let, $3 + 5\sqrt{2} = \frac{a}{b}$

$5\sqrt{2} = \frac{a}{b} - 3$

$\sqrt{2} = \frac{a-3b}{5b}$

If $\frac{a-3b}{5b}$ is rational so, $\sqrt{2}$ is also rational number but it is not true as $\sqrt{2}$ is an irrational number.

So it is contradiction to our assumption,

Therefore, $3 + 5\sqrt{2}$ is an irrational number.

40. Let take that $3 + 2\sqrt{5}$ is a rational number.

So we can write this number as

$3 + 2\sqrt{5} = \frac{a}{b}$

Here a and b are two co-prime numbers and b is not equal to 0.

Subtract 3 both sides we get,

$2\sqrt{5} = \frac{a}{b} - 3$

$2\sqrt{5} = \frac{a-3b}{b}$

Now divide by 2 we get

$\sqrt{5} = \frac{a-3b}{2b}$

Here a and b are an integer so $\frac{a-3b}{2b}$ is a rational number so $\sqrt{5}$ should be a rational number but $\sqrt{5}$ is an irrational number so it contradicts the fact.

Hence the result is $3 + 2\sqrt{5}$ is an irrational number

Now its square will again contain an irrational number.

Hence the given number is an irrational number.

41. We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$)

such that

$7\sqrt{5} = \frac{a}{b}$

$\Rightarrow \sqrt{5} = \frac{a}{7b}$ (1)

R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

42. We have to prove that $\sqrt{2}$ is an irrational number.

Let $\sqrt{2}$ be a rational number.

$\therefore \sqrt{2} = \frac{p}{q}$

where p and q are co-prime integers and $q \neq 0$

On squaring both the sides, we get,

$$\text{or, } 2 = \frac{p^2}{q^2}$$

$$\text{or, } p^2 = 2q^2$$

$\therefore p^2$ is divisible by 2.

p is divisible by 2.....(i)

Let $p = 2r$ for some integer r

$$\text{or, } p^2 = 4r^2$$

$$2q^2 = 4r^2 \quad [\because p^2 = 2q^2]$$

$$\text{or, } q^2 = 2r^2$$

or, q^2 is divisible by 2.

$\therefore q$ is divisible by 2.....(ii)

From (i) and (ii)

p and q are divisible by 2, which contradicts the fact that p and q are co-primes.

Hence, our assumption is wrong.

$\therefore \sqrt{2}$ is irrational number.

43. i. We can prove $7\sqrt{5}$ irrational by contradiction.

Let us suppose that $7\sqrt{5}$ is rational.

It means we have some co-prime integers a and b ($b \neq 0$) such that

$$7\sqrt{5} = \frac{a}{b}$$

$$\Rightarrow \sqrt{5} = \frac{a}{7b} \dots (1)$$

R.H.S of (1) is rational but we know that $\sqrt{5}$ is irrational.

It is not possible which means our supposition is wrong.

Therefore, $7\sqrt{5}$ cannot be rational.

Hence, it is irrational.

ii. We will prove $6 + \sqrt{2}$ irrational by contradiction.

Let us suppose that $(6 + \sqrt{2})$ is rational.

It means that we have co-prime integers a and b ($b \neq 0$) such that

$$6 + \sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

$$\Rightarrow \sqrt{2} = \frac{a-6b}{b} \dots (1)$$

a and b are integers.

It means **L.H.S** of (1) is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. $(6 + \sqrt{2})$ cannot be rational.

Hence, $(6 + \sqrt{2})$ is irrational.

44. Let $\sqrt{6} + \sqrt{2}$ be rational number

$$\sqrt{6} + \sqrt{2} = \frac{p}{q}$$

$$\sqrt{2} = \frac{p}{q} - \sqrt{6}$$

$$\sqrt{2} = \frac{p - q\sqrt{6}}{q}$$

$$2q^2 = p^2 + 6q^2 - 2\sqrt{6}q$$

$$2q^2 - p^2 - 6q^2 = -2\sqrt{6}q$$

$$\sqrt{6} = \frac{2q^2 - p^2 - 6q^2}{-2q}$$

as $\frac{2q^2 - p^2 - 6q^2}{-2q}$ is in $\frac{p}{q}$ form it is rational number, so $\sqrt{6}$ should be rational number but in general $\sqrt{6}$ is irrational.

So our assumption is wrong.

Therefore given number is irrational.

45. We will prove $6 + \sqrt{2}$ irrational by contradiction.

Let us suppose that $(6 + \sqrt{2})$ is rational.

It means that we have co-prime integers a and b ($b \neq 0$)

Such that

$$6 + \sqrt{2} = \frac{a}{b}$$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 6$$

$$\Rightarrow \sqrt{2} = \frac{a-6b}{b} \dots\dots\dots (1)$$

a and b are integers.

It means L.H.S of (1) is rational but we know that $\sqrt{2}$ is irrational. It is not possible.

Therefore, our supposition is wrong. $(6 + \sqrt{2})$ cannot be rational.

Hence, $(6 + \sqrt{2})$ is irrational.

46. Let the $\frac{2+\sqrt{3}}{5}$ be a rational number.

Therefore it can be written in form of p/q (q not = 0 and p, q are co. Prime)

$$\frac{2+\sqrt{3}}{5} = \frac{p}{q}$$

$$\sqrt{3} = \frac{5p}{q} - 2$$

$$\sqrt{3} = \frac{5p-2q}{q}$$

Here LHS i.e, $\sqrt{3}$ is an irrational and RHS is a rational number. This is not possible. Hence our supposition is wrong. So the given number $\frac{2+\sqrt{3}}{5}$ Is irrational.

47. Let $\sqrt{5}$ is a rational number.

$$\sqrt{5} = \frac{a}{b}, \text{ (a, b are co-primes and } b \neq 0)$$

$$\text{or, } a = b\sqrt{5}$$

On squaring both the sides, we get

$$a^2 = 5b^2 \dots\dots(1)$$

Hence 5 is a factor of a^2

so 5 is a factor of a

Let a = 5c, (c is some integer)

$$\therefore a^2 = 25c^2$$

From equation(1) putting the value of a^2 gives

$$5b^2 = 25c^2$$

$$\text{or } b^2 = 5c^2$$

so 5 is a factor of b^2

or 5 is a factor of b

Hence 5 is a common factor of a and b

But this contradicts the fact that a and b are co-primes.

This is because we assumed that $\sqrt{5}$ is rational

$\therefore \sqrt{5}$ is irrational.

48. Let us prove $\sqrt{5}$ irrational by contradiction.

Let us suppose that $\sqrt{5}$ is rational. It means that we have co-prime integers a and b ($b \neq 0$)

$$\text{Such that } \sqrt{5} = \frac{a}{b}$$

$$\Rightarrow b \sqrt{5} = a$$

Squaring both sides, we get

$$\Rightarrow 5b^2 = a^2 \dots (1)$$

It means that 5 is factor of a^2

Hence, 5 is also factor of a by Theorem. ... (2)

If, 5 is factor of a, it means that we can write $a = 5c$ for some integer c.

Substituting value of a in (1),

$$5b^2 = 25c^2$$

$$\Rightarrow b^2 = 5c^2$$

It means that 5 is factor of b^2 .

Hence, 5 is also factor of b by Theorem. ... (3)

From (2) and (3), we can say that 5 is factor of both a and b.

But, a and b are co-prime.

Therefore, our assumption was wrong. $\sqrt{5}$ cannot be rational. Hence, it is irrational.

49. Let us assume on the contrary that $\sqrt{2} + \sqrt{5}$ is a rational number. Then, there exist co-prime positive integers a and b such that

$$\sqrt{2} + \sqrt{5} = \frac{a}{b}$$

$$\begin{aligned} \Rightarrow \frac{a}{b} - \sqrt{2} &= \sqrt{5} \\ \Rightarrow \left(\frac{a}{b} - \sqrt{2}\right)^2 &= (\sqrt{5})^2 \text{ [Squaring both sides]} \\ \Rightarrow \frac{a^2}{b^2} - \frac{2a}{b}\sqrt{2} + 2 &= 5 \\ \Rightarrow \frac{a^2}{b^2} - 3 &= \frac{2a}{b}\sqrt{2} \\ \Rightarrow \frac{a^2 - 3b^2}{2ab} &= \sqrt{2} \\ \Rightarrow \sqrt{2} \text{ is a rational number [} \therefore \frac{a^2 - 3b^2}{2ab} \text{ is rational as a and b are integers]} \end{aligned}$$

This contradicts the fact that $\sqrt{2}$ is irrational. So, our assumption is wrong.
Hence, $\sqrt{2} + \sqrt{5}$ is irrational.

50. Let us assume, to the contrary, that $5 - \sqrt{3}$ is rational.

That is, we can find coprime numbers a and b ($b \neq 0$) such that $5 - \sqrt{3} = \frac{a}{b}$

Therefore, $5 - \frac{a}{b} = \sqrt{3}$

Rearranging this equation, we get $\sqrt{3} = 5 - \frac{a}{b} = \frac{5b - a}{b}$

Since a and b are integers, we get $5 - \frac{a}{b}$ is rational, and so $\sqrt{3}$ is rational.

But this contradicts the fact that $\sqrt{3}$ is irrational

This contradiction has arisen because of our incorrect assumption that $5 - \sqrt{3}$ is rational.

So, we conclude that $5 - \sqrt{3}$ is irrational.

51. Let us assume that $4 - 5\sqrt{2}$ is rational. Then, there must exist positive co-primes between a and b such that

$$\begin{aligned} 4 - 5\sqrt{2} &= \frac{a}{b} \\ \Rightarrow 5\sqrt{2} &= \frac{a}{b} - 4 \\ \Rightarrow \sqrt{2} &= \frac{\frac{a}{b} - 4}{5} \\ \Rightarrow \sqrt{2} &= \frac{a - 4b}{5b} \end{aligned}$$

Since a, b are integers, $\therefore \frac{a - 4b}{5b}$ is a rational number. Therefore, it follows that $\sqrt{2}$ is a rational number, which is a contradiction as $\sqrt{2}$ is an irrational number.

\therefore Our supposition is wrong.

Hence $4 - 5\sqrt{2}$ is irrational.

52. \therefore Given that $\sqrt{2}$ is an irrational number.

Let us assume that $7\sqrt{2}$ is a rational number.

Then, $7\sqrt{2} = \frac{a}{b}$, where a and b have no common factors other than 1.

$$(7\sqrt{2})b = a$$

$$\sqrt{2} = \frac{a}{7b}$$

Since, a, 7, and b are integers, so, $\frac{a}{7b}$ is a rational number. This means $\sqrt{2}$ is rational. But this contradicts the fact that $\sqrt{2}$ is irrational.

So, our assumption was wrong. Therefore, $7\sqrt{2}$ is an irrational number.

53. Let $\sqrt{5} + \sqrt{3}$ be rational number equal to $\frac{a}{b}$. there exist co-prime integers a and b such that

$$\begin{aligned} \sqrt{5} + \sqrt{3} &= \frac{a}{b} \\ \Rightarrow \sqrt{5} &= \frac{a}{b} - \sqrt{3} \\ \Rightarrow (\sqrt{5})^2 &= \left(\frac{a}{b} - \sqrt{3}\right)^2 \text{ [Squaring both sides] we get,} \\ \Rightarrow 5 &= \frac{a^2}{b^2} - \frac{2a\sqrt{3}}{b} + 3 \\ \Rightarrow 2 &= \frac{a^2}{b^2} - \frac{2\sqrt{3}a}{b} \\ \sqrt{3} &= (a^2 - 2b^2)\frac{b}{2ab} \end{aligned}$$

Since a, b are integers, therefore $(a^2 - 2b^2)\frac{b}{2ab}$ is a rational number which is a contradiction as $\sqrt{3}$ is an irrational number.

Hence, $\sqrt{5} + \sqrt{3}$ is irrational.

54. Let us preassume that $3\sqrt{2}$ is a rational number.

In that case, $3\sqrt{2}$ can be written as $\frac{p}{q}$, where p and q are co-prime integers and q is not zero.

$$\begin{aligned} \text{So, } \frac{p}{q} &= \frac{3\sqrt{2}}{1} \\ \Rightarrow \frac{p}{3q} &= \frac{\sqrt{2}}{1} \end{aligned}$$

Since, p is an integer and $3q$ is also an integer where $3q$ is not zero.

So, $\frac{p}{3q}$ is a rational number but the equalient number $\sqrt{2}$ should also be a rational number.

But this contradicts the fact that $\sqrt{2}$ is an irrational number.

so, this assumption is wrong and $3\sqrt{2}$ is an irrational number.

55. Let's assume on the contrary that $3 + \sqrt{2}$ is a rational number. Then, there exist co prime positive integers a and b such that $3 + \sqrt{2} = \frac{a}{b}$

$$\Rightarrow \sqrt{2} = \frac{a}{b} - 3$$

$$\Rightarrow \sqrt{2} = \frac{(a-3b)}{b}$$

$$\Rightarrow \sqrt{2} \text{ is rational [} \because a \text{ and } b \text{ are integers } \therefore \frac{(a-3b)}{b} \text{ is a rational number]}$$

This contradicts the fact that $\sqrt{2}$ is irrational.

So, our assumption is incorrect.

Hence, $3 + \sqrt{2}$ is an irrational number.

Section C

56. Suppose that $\sqrt{p} + \sqrt{q}$ is a rational number equal to $\frac{a}{b}$, where a and b are integers having no common factor.

$$\text{Now, } \sqrt{p} + \sqrt{q} = \frac{a}{b}$$

$$\Rightarrow \sqrt{p} = \frac{a}{b} - \sqrt{q} \text{ (squaring both side)}$$

$$\Rightarrow (\sqrt{p})^2 = \left(\frac{a}{b} - \sqrt{q}\right)^2$$

$$\Rightarrow p = \frac{a^2}{b^2} - 2\left(\frac{a}{b}\right)\sqrt{q} + q$$

$$\Rightarrow 2\left(\frac{a}{b}\right)\sqrt{q} = \frac{a^2}{b^2} + q - p$$

$$\Rightarrow 2\frac{a}{b}\sqrt{q} = \frac{a^2 + b^2(q-p)}{b^2}$$

$$\Rightarrow \sqrt{q} = \frac{a^2 + b^2(q-p)}{2ab}$$

$$\Rightarrow \sqrt{q} \text{ is a rational number. (because sum of two rational numbers is always rational)}$$

This is a contradiction as \sqrt{q} is an irrational number.

Hence, $\sqrt{p} + \sqrt{q}$ is an irrational number.

57. Suppose $\sqrt[3]{6}$ be rational number and $\sqrt[3]{6} = \frac{a}{b}$ where a and b are co-prime and $b \neq 0$

$$\Rightarrow (\sqrt[3]{6})^3 = \frac{a^3}{b^3}$$

$$\Rightarrow 6 = \frac{a^3}{b^3}$$

$$\Rightarrow 6 \cdot b^3 = a^3$$

$$\Rightarrow a^3 \text{ is divisible by } 6 \Rightarrow a \text{ is divisible by } 6.$$

$$\text{Let } a = 6c$$

$$6b^3 = (6c)^3$$

$$\Rightarrow b^3 = 36c^3$$

$$\Rightarrow b^3 \text{ is divisible by } 6 \Rightarrow b \text{ is divisible by } 6.$$

$$\Rightarrow a \text{ and } b \text{ have a common factor i.e, } 6$$

$$\Rightarrow a \text{ and } b \text{ are not co-prime which is a contradiction}$$

$$\therefore \sqrt[3]{6} \text{ is an irrational.}$$

58. If possible, let $\sqrt{11}$ be rational and let its simplest form be $\frac{a}{b}$.

Then, a and b are integers having no common factor other than 1, and $b \neq 0$.

$$\text{Now, } \sqrt{11} = \frac{a}{b}$$

$$\Rightarrow 11 = \frac{a^2}{b^2} \text{ [on squaring both sides]}$$

$$\Rightarrow 11b^2 = a^2 \dots\dots\dots (i)$$

$$\Rightarrow 11 \text{ divides } a^2 \text{ So, } 11 \text{ divides by } a$$

Let $a = 11c$ for some positive integer c .

Putting $a = 11c$ in (i), we get

$$11b^2 = (11c)^2$$

$$11b^2 = 121c^2$$

$$\Rightarrow b^2 = 11c^2$$

$\Rightarrow 11$ divides b^2 [$\because 11$ divides $11c^2$]

$\Rightarrow 11$ divides b [$\because 11$ is prime and 11 divides $b^2 \Rightarrow 11$ divides b].

Thus, 11 is a common factor of a and b .

But, this contradicts the fact that a and b have no common factor other than 1 .

The contradiction arises by assuming that $\sqrt{11}$ is rational.

Hence, $\sqrt{11}$ is irrational.

59. Let $(4 + 3\sqrt{2})$ be a rational number

Then both $(4 + 3\sqrt{2})$ and 4 are rational.

$\Rightarrow (4 + 3\sqrt{2} - 4) = 3\sqrt{2}$ is rational [Since difference of two rational numbers is rational]

$\Rightarrow 3\sqrt{2}$ is rational.

on multiplying with $\frac{1}{3}$ we get

$\Rightarrow \frac{1}{3}(3\sqrt{2})$ is rational. (Since product of two rational numbers is rational)

$\Rightarrow \sqrt{2}$ is rational.

This contradicts the fact that $\sqrt{2}$ is irrational,

This contradicts because we assumed that $(4 + 3\sqrt{2})$ is rational. So our assumption is wrong

Hence, $(4 + 3\sqrt{2})$ is irrational.

60. let us assume that $\sqrt{3}$ be a rational number.

$\sqrt{3} = \frac{a}{b}$, where a and b are integers and co-primes and $b \neq 0$

Squaring both sides, we have

$$\frac{a^2}{b^2} = 3$$

or, $a^2 = 3b^2$ -----(i)

a^2 is divisible by 3 .

Hence a is divisible by 3(ii)

Let $a = 3c$ (where c is any integer)

squaring on both sides we get

$$(3c)^2 = 3b^2$$

$$9c^2 = 3b^2$$

$$b^2 = 3c^2$$

so b^2 is divisible by 3

hence, b is divisible by 3(iii)

From equation(ii) and (iii), we have

3 is a factor of a and b which is contradicting the fact that a and b are co-primes.

Thus, our assumption that $\sqrt{3}$ is rational number is wrong.

Hence, $\sqrt{3}$ is an irrational number.

Let us assume that $7 - 2\sqrt{3}$ is a rational number

$7 - 2\sqrt{3} = \frac{p}{q}$ ($q \neq 0$ and p and q are co-primes)

$$7 - 2\sqrt{3} = \frac{p}{q}$$

$$-2\sqrt{3} = \frac{p}{q} - 7$$

$$2\sqrt{3} = 7 - \frac{p}{q}$$

$$2\sqrt{3} = \frac{7q-p}{q}$$

$$\sqrt{3} = \frac{7q-p}{2q}$$

Here $7q-p$ and $2q$ both are integers, hence $\sqrt{3}$ is a rational number.

But this contradicts the fact that $\sqrt{3}$ is an irrational number.

This contradict is due to our assumption that $7 - 2\sqrt{3}$ is rational.

Hence, $7 - 2\sqrt{3}$ is irrational.

61. Let $\sqrt{p} + \sqrt{q}$ is rational number

A rational number can be written in the form of $\frac{a}{b}$

$$\sqrt{p} + \sqrt{q} = \frac{a}{b}$$

Squaring on both side, we get

$$p + q + 2\sqrt{pq} = \left(\frac{a}{b}\right)^2$$

$$\sqrt{pq} = \frac{1}{2} \left[\left(\frac{a}{b}\right)^2 - p - q \right]$$

Now p and q are prime positive number.

So, \sqrt{p} and \sqrt{q} is irrational number .

Also \sqrt{pq} is irrational number .

Since a rational number cannot be equal to an irrational number . Our assumption that $\sqrt{p} + \sqrt{q}$ is rational wrong .

So, $\sqrt{p} + \sqrt{q}$ is an irrational number.

62. We do not know whether $\frac{a}{b} < \frac{a+2b}{a+b}$ or, $\frac{a}{b} > \frac{a+2b}{a+b}$.

Therefore, to compare these two numbers, let us compute $\frac{a}{b} - \frac{a+2b}{a+b}$

We have,

$$\frac{a}{b} - \frac{a+2b}{a+b} = \frac{a(a+b) - b(a+2b)}{b(a+b)} = \frac{a^2 + ab - ab - 2b^2}{b(a+b)} = \frac{a^2 - 2b^2}{b(a+b)}$$

$$\therefore \frac{a}{b} - \frac{a+2b}{a+b} > 0$$

$$\Rightarrow \frac{a^2 - 2b^2}{b(a+b)} > 0$$

$$\Rightarrow a^2 - 2b^2 > 0$$

$$\Rightarrow a^2 > 2b^2$$

$$\Rightarrow a > \sqrt{2b}$$

$$\text{and, } \frac{a}{b} - \frac{a+2b}{a+b} < 0$$

$$\Rightarrow \frac{a^2 - 2b^2}{b(a+b)} < 0$$

$$\Rightarrow a^2 - 2b^2 < 0$$

$$\Rightarrow a^2 < 2b^2$$

$$\Rightarrow a < \sqrt{2b}$$

Thus, $\frac{a}{b} > \frac{a+2b}{a+b}$, if $a > \sqrt{2b}$ and $\frac{a}{b} < \frac{a+2b}{a+b}$, if $a < \sqrt{2b}$.

So, we have the following cases:

CASE I When $a > \sqrt{2b}$

In this case, we have

$$\frac{a}{b} > \frac{a+2b}{a+b} \text{ i.e., } \frac{a+2b}{a+b} < \frac{a}{b}$$

We have to prove that

$$\frac{a+2b}{a+b} < \sqrt{2} < \frac{a}{b}$$

We have,

$$a > \sqrt{2b}$$

$$\Rightarrow a^2 > 2b^2 \text{ [Adding } a^2 \text{ on both sides]}$$

$$\Rightarrow 2a^2 + 2b^2 > (a^2 + 2b^2) + 2b^2 \text{ [Adding } 2b^2 \text{ on both sides]}$$

$$\Rightarrow 2(a^2 + b^2) + 4ab > a^2 + 4b^2 + 4ab \text{ [Adding } 4ab \text{ on both sides]}$$

$$\Rightarrow 2(a^2 + 2ab + b^2) > a^2 + 4ab + 4b^2$$

$$\Rightarrow 2(a+b)^2 > (a+2b)^2$$

$$\Rightarrow \sqrt{2}(a+b) > a+2b$$

$$\Rightarrow \sqrt{2} > \frac{a+2b}{a+b} \text{(i)}$$

Again,

$$a > \sqrt{2b}$$

$$\Rightarrow \frac{a}{b} > \sqrt{2} \text{(ii)}$$

From (i) and (ii), we get

$$\frac{a+2b}{a+b} < \sqrt{2} < \frac{a}{b}$$

CASE II When $a < \sqrt{2b}$

In this case, we have

$$\frac{a}{b} < \frac{a+2b}{a+b}$$

We have to show that $\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$

We have,

$$a < \sqrt{2}b$$

$$\Rightarrow a^2 < 2b^2$$

$$\Rightarrow a^2 + a^2 < a^2 + 2b^2 \text{ [Adding } a^2 \text{ on both sides]}$$

$$\Rightarrow 2a^2 + 2b^2 < a^2 + 2b^2 + 2b^2 \text{ [Adding } 2b^2 \text{ on both sides]}$$

$$\Rightarrow 2a^2 + 2b^2 < a^2 + 4b^2$$

$$\Rightarrow 2a^2 + 4ab + 2b^2 < a^2 + 4ab + 4b^2 \text{ [Adding } 4ab \text{ on both sides]}$$

$$\Rightarrow 2(a+b)^2 < (a+2b)^2$$

$$\Rightarrow \sqrt{2}(a+b) < a+2b$$

$$\Rightarrow \sqrt{2} < \frac{a+2b}{a+b} \dots(\text{iii})$$

$$\Rightarrow a < \sqrt{2}b \Rightarrow \frac{a}{b} < \sqrt{2} \dots(\text{iv})$$

From (iii) and (iv), we get

$$\frac{a}{b} < \sqrt{2} < \frac{a+2b}{a+b}$$

Hence, $\sqrt{2}$ lies between $\frac{a}{b}$ and $\frac{a+2b}{a+b}$.

63. Let us assume that $4 + \sqrt{3}$ is a rational number equal to a/b where a and b are two integers

$$\Rightarrow 4 + \sqrt{3} = \frac{a}{b}$$

$$\Rightarrow \sqrt{3} = \frac{a}{b} - 4$$

we know that subtraction of two rational number is always a rational number but we get $\sqrt{3}$ as rational number which contradict the fact as $\sqrt{3}$ is an irrational number.

Hence, $4 + \sqrt{3}$ is a irrational number.

64. We have to prove that $\sqrt{5} + \sqrt{7}$ is irrational number

Let us assume that $\sqrt{5} + \sqrt{7}$ is a rational number

so $\sqrt{5} + \sqrt{7} = \frac{a}{b}$ (a and b are integer, b is not equal to 0)

$$\sqrt{5} = \frac{a}{b} - \sqrt{7} = \text{integer/integer} - \text{rational} = \text{rational}$$

Since a,b and 5 are integers,

This shows that $\sqrt{5}$ is rational

But this contradicts the fact that $\sqrt{5}$ is irrational

Thus our Assumption is wrong

Hence $\sqrt{5} + \sqrt{7}$ is irrational

65. Suppose $\sqrt{2}$ is a rational number. That is , $\sqrt{2} = \frac{p}{q}$ for some $p \in \mathbb{Z}$ and $q \in \mathbb{Z}$. We can assume the fraction is in lowest form, That is

p and q shares no common factors.

$$\text{Then } \sqrt{2}q = p$$

Squaring both side we get,

$$2q^2 = p^2$$

So p^2 is a multiple of 2,

let's assume $p = 2m$

$$\text{Then, } 2q^2 = (2m)^2$$

$$2q^2 = 4m^2$$

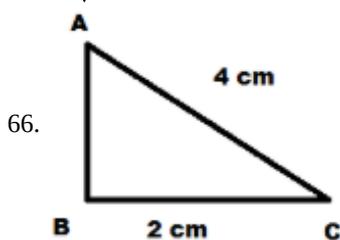
$$\text{Or } q^2 = 2m^2$$

So q^2 is a multiple of 2,

\therefore q is multiple of 2

Thus p and q shares a common factor. This is contradiction.

$\Rightarrow \sqrt{2}$ is an irrational number.



It is given that $BC = 2\text{cm}$, $AC = 4\text{ cm}$.

We have to prove that the area of $\triangle ABC$ is irrational.

In $\triangle ABC$ we know that

$$AC^2 = AB^2 + BC^2$$

$$(4)^2 = AB^2 + (2)^2$$

$$AB^2 = 16 - 4$$

$$= 12$$

$$AB = \sqrt{12} = 2\sqrt{3} \text{ cm}$$

$$\text{So area of } \triangle ABC = \frac{1}{2} \times BC \times AB$$

$$= \frac{1}{2} \times 2 \times 2\sqrt{3} = 2\sqrt{3} \text{ cm}^2 = \text{irrational (As } \sqrt{3} \text{ is irrational)}$$

So area of $\triangle ABC$ is irrational.

Section D

67. State True or False:

(i) **(b)** False

Explanation: False

(ii) **(b)** False

Explanation: False. The sum of two irrational numbers, in some cases, will be irrational. However, if the irrational parts of the numbers have a zero sum (cancel each other out), the sum will be rational.

$$4\sqrt{5} + 3\sqrt{2} = 4\sqrt{5} + 3\sqrt{2} \text{ which is irrational.}$$

$$(2 + 6\sqrt{7}) + (-6\sqrt{7}) = 2 \text{ which is rational}$$

(iii) **(a)** True

Explanation: True, because it cannot be expressed in the fractional form of a/b , where both a and b are integers and b is non-zero

(iv) **(b)** False

Explanation: False

(v) **(a)** True

Explanation: True

(vi) **(b)** False

Explanation: False

(vii) **(a)** True

Explanation: True