Solution

REAL NUMBERS WS 4

Class 10 - Mathematics

Section A

1.
(b) an irrational number
Explanation: Rational Numbers say $\frac{4}{9}$, $\frac{p}{q}$, $\sqrt{4}$, fraction, whole numbers, terminating decimal, repeating decimal, perfect square, can be expressed as a ratio of two integers provided the denominator is not equal to zero
Irrational Numbers $\sqrt{2}$,$\sqrt{5}$,$\sqrt{7}$, π not a fraction, decimal does not repeat, decimal does not end, non-perfect square, we cannot express as a ratio but both can be expressed as decimal numbers
The difference between a rational and an irrational number is always an irrational number.
e.g. rational - irrational = irrational say 2 - $\sqrt{2}$ = irrational

2. **(a)** an irrational number
Explanation: $\left(\sqrt{3}+\sqrt{5}\right)^2$ = $\left(\sqrt{3}\right)^2+\left(\sqrt{5}\right)^2+2\times\sqrt{3}\times\sqrt{5}$
= $3+5+2\sqrt{15}$
= $8+2\sqrt{15}$
Here, $\sqrt{15}=\sqrt{3}\times\sqrt{5}$
Since $\sqrt{3}$ and $\sqrt{5}$ both are an irrational number. Therefore, $\left(\sqrt{3}+\sqrt{5}\right)^2$ is an irrational number.

3. **(a)** an irrational number
Explanation: Let 2 - $\sqrt{3}$ be rational number
2 - $\sqrt{3}$ = $\frac{p}{q}$ where p and q are composite numbers
$\sqrt{3}$ = $\frac{p}{q}$ + 2
$\sqrt{3}$ = $\frac{(p+2q)}{q}$
since p, q are integers, so $\frac{(p+2q)}{q}$ is rational
$\therefore\sqrt{3}$ is an irrational number
it shows our supposition was wrong
hence 2-$\sqrt{3}$ is an irrational number.

4. **(a)** an irrational number
Explanation: Let a be rational and $\sqrt{b}$ is irrational.
If possible let $a+\sqrt{b}$ be rational.
Then $a+\sqrt{b}$ is rational and a is rational.
$\Rightarrow\left[\left(a+\sqrt{b}\right)-a\right]$ is rational [Difference of two rationals is rational]
$\Rightarrow\sqrt{b}$ is rational.
This contradicts the fact that $\sqrt{b}$ is irrational.
The contradiction arises by assuming that $a+\sqrt{b}$ is rational.
Therefore, $a+\sqrt{b}$ is irrational.

5.
(d) a rational number or an irrational number
Explanation: The sum of two irrational numbers can be either a rational number or an irrational number.
e.g $5\sqrt{3}$ + $3\sqrt{2}$ = $5\sqrt{3}$ + $3\sqrt{2}$ sum is irrational
(2 + $6\sqrt{7}$) + (- $6\sqrt{7}$) = 2 sum is rational
Hence sum can be either rational or irrational

6. **(a)** Option (iv)
Explanation: 3.141141114 is an irrational number because it is a non-repeating and non-terminating decimal.

7.
(b) a rational number
Explanation: It can be expressed in $\frac{p}{q}$ form

$2.35 = \frac{235}{100}$

so, 2.35 is a rational number

8. **(a)** a rational number

Explanation: $(1 + \sqrt{2}) + (1 - \sqrt{2}) = 1 + \sqrt{2} + 1 - \sqrt{2} = 1 + 1 = 2$ And 2 is a rational number.

Therefore the given number is rational number.

9.

(c) a rational number

Explanation: Clearly, 1.732 is a terminating decimal.

Hence, it is a rational number.

10.

(d) both rational and irrational number

Explanation: The difference between two distinct irrational numbers can be either a rational number or an irrational number.

e.g difference between pi and $(\pi - 3)$ is equal to 3 which is rational

$\sqrt{2}$ and $\sqrt{2} + 1$ both are irrational but their difference is 1 which is rational

Similarly, $\sqrt{2}$ and $\sqrt{3}$ are irrational and their difference ($\sqrt{3} - \sqrt{2}$) is also irrational

11.

(d) an irrational number

Explanation: $(2 + \sqrt{2})$ is an irrational number.

If it is rational, then the difference of two rational is rational.

$\therefore (2 + \sqrt{2}) - 2 = \sqrt{2}$ = irrational, which is a contradiction.

Hence, $(2 + \sqrt{2})$, is an irrational number.

12.

(b) an irrational number

Explanation: an irrational number

13.

(b) π

Explanation: π

14. **(a)** both rational and irrational number

Explanation: The product of a rational number and an irrational number can be either a rational number or an irrational number.

e.g $\sqrt{5} \times \sqrt{2} = \sqrt{10}$ which is irrational

but $\sqrt{8} \times \sqrt{2} = \sqrt{16} = 4$ which is a rational number

Thus, the product of two irrational numbers can be either rational or irrational

similarly, the product of rational and irrational numbers can be either rational or irrational

$5 \times \sqrt{2} = 5\sqrt{2}$ which is irrational.

but $0 \times \sqrt{3} = 0$ which is rational.

Section B

15. Let us assume that $11 + 3\sqrt{2}$ be a rational number.

$\Rightarrow 11 + 3\sqrt{2} = \frac{a}{b}$, where a and b are integers, $b \neq 0$

$\Rightarrow \sqrt{2} = \frac{a - 11b}{3b}$

RHS is a rational number but LHS is irrational.

$\therefore$ Our assumption was wrong. Hence, $11 + 3\sqrt{2}$ is an irrational number.

16. By the definition of a composite number, we know if a number is composite, then it means it has factors other than 1 and itself.

Therefore, for the given expression

$2 \times 3 \times 5 + 5 = 5(6 + 1) = 5 \times 7$

It is a composite number

Also, $5 \times 7 \times 11 + 7 \times 5$

$= 5 \times 7(11 + 1) = 35 \times (11 + 1)$

$= 35 \times 12$

It is a composite number.

17. Let us consider $\sqrt{3}+\sqrt{5}$ is a rational number that can be written as
$\sqrt{3}+\sqrt{5}$ = a [where a is rational]
$\Rightarrow \sqrt{5}=a-\sqrt{3}$
Squaring both sides, we get
$(\sqrt{5})^2=(a-\sqrt{3})^2$
⇒ 5 = $(a)^2+(\sqrt{3})^2-2(a)(\sqrt{3})$
⇒ $2a\sqrt{3}$ = a^2 + 3 - 5
⇒ $2a\sqrt{3}$ = a^2 - 2
$\Rightarrow \sqrt{3}=\frac{a^2-2}{2a}$
As a^2 – 2, 2a are rational numbers .
So $\frac{a^2-2}{2a}$ is also rational but $\sqrt{3}$ is not rational which contradicts our consideration.
Since a rational number cannot be equal to an irrational number. Our assumption that √3 + √5 is a rational is wrong .
So, $\sqrt{3}+\sqrt{5}$ is irrational.

18. If possible let a = 6 + $\sqrt{7}$ be a rational number.
Squaring a^2 = (6 + $\sqrt{7}$)^2
a^2 = 36 + 7 + 12$\sqrt{7}$
$\sqrt{2}$ = $\frac{a^2-43}{12}$ ...(1)
Since a is a rational number the expression $\frac{a^2-43}{12}$ is also rational number.
$\Rightarrow \sqrt{7}$ is a rational number
This is a contradiction. Hence, 6 + $\sqrt{7}$ is irational.
Hence proved.

19. Let us assume that 10 + 2$\sqrt{3}$ is a rational number
10 + 2$\sqrt{3}$ = $\frac{p}{q}$; q $\neq$ 0 and p, q are integers
$\Rightarrow \sqrt{3}$ = $\frac{p-10q}{2q}$
RHS is rational but LHS is irrational
$\therefore$ Our assumption is wrong. Hence 10 + 2$\sqrt{3}$ is an irrational number.

20. The given number is $\frac{2+3\sqrt{2}}{7}$
Let us assume that $\frac{2+3\sqrt{2}}{7}$ is rational no.
so $\frac{2+3\sqrt{2}}{7}=\frac{p}{q}$
$\Rightarrow \frac{3\sqrt{2}}{7}=\frac{p-2q}{q}$
$\Rightarrow \frac{\sqrt{2}}{7}=\frac{p-2q}{3q}$
$\Rightarrow \sqrt{2}$ = 7($p-\frac{2q}{3q}$)
Clearly R.H.S is rational and L.H.S is irrational, which is impossible.
Hence our assumption is false.
So, $\frac{2+3\sqrt{2}}{7}$ is an irrational number.

21. Let us assume that $5\sqrt{2}$ is rational. Then, there exist positive co-primes a and b such that
$5\sqrt{2}=\frac{a}{b}$
$\sqrt{2}=\frac{a}{b}-5$
$\sqrt{2}=\frac{a-5b}{b}$
As a-5b and b are integers .
So, $\frac{a-5b}{b}$ is rational number .
But $\sqrt{2}$ is not rational number .
Since a rational number cannot be equal to an irrational number. Our assumption that $5\sqrt{2}$ is rational wrong.
Hence, $5\sqrt{2}$ is irrational.

22. Let given no be rational
so, $\frac{3+\sqrt{7}}{5}=\frac{p}{q}$
$\Rightarrow 3+\sqrt{7}=\frac{5p}{q}$
$\Rightarrow \sqrt{7}=\frac{5p}{q}-3$

$\Rightarrow \sqrt{7} = \frac{5p-3q}{q}$

Since, p and q are rational numbers, so is $\frac{5p-3q}{q}$. It means that $\sqrt{7}$ is also rational, but we know that $\sqrt{7}$ is irrational this contradicts our assumption that $\frac{3+\sqrt{7}}{5}$ is rational , hence it is an irrational number.

23. Let us assume that 4 + 2$\sqrt{3}$ is a rational number

 4 + 2$\sqrt{3}$ = $\frac{p}{q}$; q $\neq$ 0 and p, q are integers

 $\Rightarrow \sqrt{3}$ = $\frac{p-4q}{2q}$

 RHS is rational but LHS is irrational

 $\therefore$ Our assumption was wrong. Hence 4 + 2$\sqrt{3}$ is an irrational number.

24. We will solve this by contradiction method i.e.,

 Assume $5 - 2\sqrt{3} = \frac{p}{q}$ be a rational number.

 $\therefore 5 - \frac{p}{q} = 2\sqrt{3}$ or $\frac{5q-p}{2q} = \sqrt{3}$,

 Since p,q are integers, therefore $\frac{5q-p}{2q}$ is a rational number, which is a contradiction,since $\sqrt{3}$ is an irrational number.

 Therefore, our supposition is wrong and hence, 5 - 2$\sqrt{3}$ is irrational.

25. Let us assume that 3 - 2$\sqrt{5}$ is a rational number.

 $\therefore$ 3 - 2$\sqrt{5}$ = $\frac{p}{q}$, q $\neq$ 0, p and q are integers

 $\Rightarrow \sqrt{5}$ = $\frac{3q-p}{2q}$

 Now RHS is rational but LHS is irrational

 $\therefore$ Our assumption was wrong ; 3 - 2$\sqrt{5}$ is an irrational number.

26. Let 4 - 5$\sqrt{2}$ is a rational number

 $\Rightarrow$ 4 - 5$\sqrt{2}$ = x, where x is a rational

 $\sqrt{2}$ = $\frac{4-\mathrm{x}}{5}$

 irrational = rational

 which is contradiction. This contradiction has arisen because of our wrong assumption. hence 4 - 5$\sqrt{2}$ is irrational

27. Let us assume that $2 - 3\sqrt{5}$ is rational. Then, there exist positive co-primes a and b such that

 $2 - 3\sqrt{5} = \frac{a}{b}$

 $3\sqrt{5} = 2 - \frac{a}{b}$

 $3\sqrt{5} = \frac{\mathrm{2b-a}}{\mathrm{b}}$

 $\sqrt{5} = \frac{\mathrm{2b-a}}{\mathrm{3b}}$

 We observe that $\frac{2b-a}{3b}$ is a rational number.

 It shows that $\sqrt{5}$ is a rational number.

 This contradicts the fact that $\sqrt{5}$ is an irrational number

 This contradiction has raised because we assumed that $2 - 3\sqrt{5}$ is a rational number

 Hence, our assumption is wrong ,and $2 - 3\sqrt{5}$ is an irrational number.

28. Let us assume that $5 + 2\sqrt{7}$ is not an irrational number.

 $\therefore$ $5 + 2\sqrt{7}$ is a rational number p i.e. $5 + 2\sqrt{7}$ = p

 $\Rightarrow \sqrt{7} = \frac{p-5}{2}$

 This is a contradiction as RHS is rational but LHS is irrational.

 Hence $5 + 2\sqrt{7}$ can not be rational, so irrational.

29. Let us assume that 2 + $3\sqrt{3}$ is a rational number

 2 + $3\sqrt{3}$ = $\frac{p}{q}$; p, q are integers and q $\neq$ 0

 $\Rightarrow \sqrt{3}$ = $\frac{p-2q}{3q}$

 RHS is rational but LHS is irrational

 $\therefore$ Our assumption is wrong. Hence 2 + $3\sqrt{3}$ is an irrational number.

30. Let 5 - 3$\sqrt{2}$ be rational

 $\therefore$ 5 - 3$\sqrt{2}$ = $\frac{p}{q}$, p & q are integers, q $\neq$ 0, HCF (p, q) = 1

 $5 - \frac{p}{q} = 3\sqrt{2}$

 $\frac{15q-p}{3q} = \sqrt{2}$

 Rational = Irrational

which is a contradiction.

Hence, 5 - $3\sqrt{2}$ is irrational.

31. Let $\sqrt{5}$ is a rational no $\sqrt{5} = \frac{a}{b}$, where a and b are co-primes

Squaring both sides

5 = $\frac{a^2}{b^2}$

$5b^2 = a^2$

5 divides a^2 ($\because$ 5 divides $5b^2$)

5 divides a ...(i)

a = 5c for some integer 'c'

squaring both sides

$a^2 = 25c^2$

$5b^2 = 25c^2$ ($\because a^2 = 5b^2$)

$b^2 = 5c^2$

5 divides b^2 ($\because$ 5 divides $5c^2$)

5 divides b ...(ii)

From (i) and (iii), we can conclude 5 is also a common factor of a and b

But this contradicts the fact that a and b are co-primes.

It means our assumption is incorrect

Hence, $\sqrt{5}$ is an irrational no.

32. Let us assume, to the contrary, that is $\frac{1}{\sqrt{2}}$ rational.

So, we can find coprime integers a and b $(\neq 0)$ such that

$\frac{1}{\sqrt{2}} = \frac{a}{b} \Rightarrow \sqrt{2} = \frac{b}{a}$

Since, a and b are integers, $\frac{b}{a}$ is rational, and so is $\sqrt{2}$ rational.

But this contradicts the fact that is $\sqrt{2}$ irrational.

So, we conclude that is $\frac{1}{\sqrt{2}}$ irrational.

33. Let $\sqrt{3}$ be a rational number.

$\sqrt{3} = \frac{p}{q}$ p, q are coprime, $q \neq 0$

$3q^2 = p^2 \Rightarrow 3 \mid p^2 \Rightarrow 3 \mid p$ Let p = 3 m

$3q^2 = 9m^2 \Rightarrow q^2 = 3m^2 \Rightarrow 3 \mid q^2 \Rightarrow 3 \mid q$

$\therefore$ 3 is common factor of p and q

Contraction to our assumption

Hence $\sqrt{3}$ is irrational No.

34. Let us assume that 7 - $2\sqrt{3}$ is a rational number

$\Rightarrow 7 - 2\sqrt{3} = \frac{a}{b}$, where a and b are integers, $b \neq 0$

$\Rightarrow \sqrt{3} = \frac{7b - a}{2b}$

RHS is a rational number but LHS is irrational.

$\therefore$ Our assumption was wrong. Hence, 7 - $2\sqrt{3}$ is irrational.

35. Let us assume that $\sqrt{2} + \sqrt{3}$ is a rational number

Let $\sqrt{2} + \sqrt{3} = \frac{a}{b}$ Where a and b are co-prime positive integers

On squaring both sides, we get

$(\sqrt{2} + \sqrt{3})^2 = \frac{a^2}{b^2}$

$2 + 3 + 2\sqrt{6} = \frac{a^2}{b^2}$

$5 + 2\sqrt{6} = \frac{a^2}{b^2}$

$2\sqrt{6} = \frac{a^2}{b^2} - 5$

$2\sqrt{6} = \frac{a^2 - 5b^2}{b^2}$

$\sqrt{6} = \frac{a^2 - 5b^2}{2b^2}$

Now $\frac{a^2 - 5b^2}{2b^2}$ is a rational number.

This shows that $\sqrt{6}$ is a rational number.
But this contradicts the fact that $\sqrt{6}$ is an irrational number.
This contradiction has raised because we assume that $\left(\sqrt{2}+\sqrt{3}\right)$ is a rational number.
Hence, our assumption is wrong and $\left(\sqrt{2}+\sqrt{3}\right)$ is an irrational number.

36. Let us assume, to the contrary, that $3\sqrt{2}$ is rational.
That is, we can find coprimes a and b (b ≠ 0) such that $3\sqrt{2}=\frac{a}{b}$
Rearranging, we get $\sqrt{2}=\frac{a}{3b}$ ...(i)
Since 3, a and b are integers, $\frac{a}{3b}$ is rational, and so (i) shows that $\sqrt{2}$ is rational.
But this contradicts the fact that $\sqrt{2}$ is irrational.
So, we conclude that $3\sqrt{2}$ is irrational.

37. To Prove: 5+3$\sqrt{2}$ is irrational number
Proof: If possible let us assume 5 + 3$\sqrt{2}$ is a rational number.
$\implies$ 5 + 3$\sqrt{2}$ = $\frac{p}{q}$ where q $\neq$ 0 and p and q are coprime integers.
$\implies 3\sqrt{2}=\frac{p}{q}-5$
$\implies 3\sqrt{2}=\frac{p-5q}{q}$
$\implies \sqrt{2}=\frac{p-5q}{3q}$
$\implies \sqrt{2}=\frac{integer}{integer}$
⇒ $\sqrt{2}$ is a rational number.
This contradicts the given fact that $\sqrt{2}$ is irrational.
Hence 5 + 3$\sqrt{2}$ is an irrational number.

38. 7 × 11 × 13 + 13
Take 13 common there we get
= 13(7 x 11 +1)
= 13(77 + 1)
= 13(78)
It is the product of two numbers and both numbers are more than 1. So, it is a composite number.

39. Let $3\sqrt{3}$ - 7 be a rational number.
$3\sqrt{3}$ - 7 = x, where x is rational
⇒ $\sqrt{3}$ = $\frac{\text{x}+7}{3}$
irrational = rational
This is a contradiction. This contradiction has arisen because of our wrong assumption. Hence $3\sqrt{3}$ - 7 is irrational.

40. Let us assume that 7 + 4$\sqrt{5}$ is rational
7 + 4$\sqrt{5}$ = $\frac{p}{q}$; q $\neq$ 0 and p, q are integers
⇒ $\sqrt{5}$ = $\frac{p-7q}{4q}$
Clearly $\frac{p-7q}{4q}$ is rational but $\sqrt{5}$ is irrational
Our assumption was wrong ⇒ 7 + 4$\sqrt{5}$ is irrational.

41. Let us assume that 6 + √2 is a rational number.
So we can write this number as
6 + √2 = a/b
Here a and b are two co-prime numbers and b is not equal to 0
Subtract 6 both side we get
√2 = a/b – 6
√2 = (a-6b)/b
Here a and b are integers so (a-6b)/b is a rational number. So √2 should be a rational number. But √2 is an irrational number. It is a contradiction.
Hence result is 6 + √2 is a irrational number

42. Let us assume that 8 + 5$\sqrt{5}$ is a rational number
⇒ 8 + 5$\sqrt{5}$ = $\frac{\text{a}}{\text{b}}$, where a and b are integers, b $\neq$ 0.
⇒ $\sqrt{5}$ = $\frac{a-8\text{ b}}{5\text{ b}}$
RHS is a rational number but LHS is irrational.
$\therefore$ Our assumption was wrong. Hence, 8 + 5$\sqrt{5}$ is an irrational number.

43. Let us assume that 2 - $3\sqrt{5}$ is a rational number

2 - $3\sqrt{5}$ = $\frac{p}{q}$; q $\neq$ 0 and p, q are integers

$\Rightarrow \sqrt{5} = \frac{2q-p}{q}$

RHS is rational but LHS is irrational

$\therefore$ Our assumption was wrong. Hence 2 - $3\sqrt{5}$ is an irrational number.

44. Let us assume √3 be a rational, then as every rational can be represented in the form p/q where q≠0

Let √3=p/q where p,q have no common factor.

Now squaring on both sides we get $3=p^2/q^2$

$\implies 3\times q^2=p^2$

Which means 3 divides p^2 which implies 3 divides p

Hence we can write p=3$\times$k, where k is some constant.

This gives $3\times q^2=9\times k^2$

$q^2=3\times k^2$

Which means 3 divides q^2 which implies 3 divides q.

3 divides p and q which means 3 is a common factor for p and q.

And this is a contradiction for our assumption that p and q have no common factor…

Hence we can say our assumption that √3 is rational is wrong…

And therefore √3 is an irrational…

45. Let us assume, to the contrary, that $3\sqrt{2}$ is rational. Then, there exist co-prime positive integers a and b such that

$3\sqrt{2}=\frac{a}{b}$

$\Rightarrow \quad \sqrt{2}=\frac{a}{3b}$

$\Rightarrow \quad \sqrt{2}$ is rational $\left[\because 3, a \text{ and } b \text{ are integers } \therefore \frac{a}{3b} \text{ is a rational number}\right]$

This is a contradiction. Hence our assumption is wrong.

So, $3\sqrt{2}$ is an irrational number.

46. Let us assume that $2\sqrt{3}-1$ is a rational. number

Then, there exist positive co-primes a and b such that

$2\sqrt{3}-1=\frac{a}{b}$

$2\sqrt{3}=\frac{a}{b}+1$

$2\sqrt{3}=\frac{\mathrm{a+b}}{\mathrm{b}}$

$\sqrt{3}=\frac{a+b}{2b}$

Here $\frac{\mathrm{a+b}}{\mathrm{b}}$ is a rational number ,so $\sqrt{3}$ is a rational number

This contradicts the fact that $\sqrt{3}$ is an irrational number

Hence $2\sqrt{3}-1$ is irrational

47.
  i. 0.05918 is a rational number as decimal expansion is terminating.
  ii. $1.010010001\ldots$ is an irrational number as decimal expansion is non-terminating non-recurring (non-repeating).
  iii. $\sqrt{\frac{9}{27}}=\sqrt{\frac{1}{3}}=\frac{1}{\sqrt{3}}$,it is an irrational number.
  iv. $\sqrt{\frac{12}{75}}=\sqrt{\frac{4}{25}}=\frac{2}{5}$, which is a rational number.

48. Let us assume $\sqrt{2}$ be a rational number and its simplest form be $\frac{a}{b}$, a and b are coprime positive integers and b $\neq$ 0.

So $\sqrt{2}$ = $\frac{a}{b}$

$\Rightarrow a^2 = 2b^2$

Thus a^2 is a multiple of 2

$\Rightarrow$ a is a multiple of 2.

Let a = 2 m for some integer m

$\therefore b^2 = 2m^2$

Thus b^2 is a multiple of 2

$\Rightarrow$ b is a multiple of 2

Hence 2 is a common factor of a and b.

This contradicts the fact that a and b are coprimes

Hence $\sqrt{2}$ is an irrational number.

49. Consider $\sqrt{p}+\sqrt{q}$ is rational and can be represented as $\sqrt{p}+\sqrt{q}$ = a

$\Rightarrow (\sqrt{p}) = a - \sqrt{q}$

$\Rightarrow (\sqrt{p})^2 = (a - \sqrt{q})^2$ (squaring both sides)

⇒ p = a^2 + $\left(\sqrt{q}\right)^2$ - 2 a $\sqrt{q}$

⇒ p = a^2 + q - 2 a $\sqrt{q}$

⇒ 2a $\sqrt{q}$ = a^2 + q - p

$\Rightarrow \sqrt{q} = \frac{a^2+q-p}{2a}$

As q is prime so $\sqrt{q}$ is not rational but $\frac{a^2+q-p}{2a}$ is rational because a, p, q are non-zero integers which contradicts our consideration.

Hence, $\sqrt{p}+\sqrt{q}$ is irrational where p and q are primes.

50. We will solve it by contradiction method i.e., assume that $5-\sqrt{3}$ is rational.

Then, there exist co-prime positive integers a and b such that

$5-\sqrt{3} = \frac{a}{b}$

$\Rightarrow 5 - \frac{a}{b} = \sqrt{3}$

$\Rightarrow \frac{5b-a}{b} = \sqrt{3}$

$\Rightarrow \frac{5b-a}{b} = \sqrt{3}$

Since a, b are integers, therefore $\sqrt{3}$ is a rational number which is a contradiction.

So, our assumption is incorrect.

Hence, $5-\sqrt{3}$ is an irrational number.

51. Let us assume that $\frac{2}{\sqrt{7}}$ is rational. Then, there exist positive co-primes a and b such that

$\frac{2}{\sqrt{7}} = \frac{a}{b}$

$\sqrt{7} = \frac{2b}{a}$

As 2b and a are rational numbers.

Then $\frac{2b}{a}$ is rational number.

But $\sqrt{7}$ is not a rational number.

Since a rational number cannot be equal to an irrational number . Our assumption that $\frac{2}{\sqrt{7}}$ is rational number is wrong .

Hence $\frac{2}{\sqrt{7}}$ is an irrational number

52. Let us assume that $\frac{3}{2\sqrt{5}}$ is rational. Then, there exist positive co-primes a and b such that

$\frac{3}{2\sqrt{5}} = \frac{a}{b}$

$\sqrt{5} = \frac{3b}{2a}$

As 3b and 2a are integers .

So, $\frac{3b}{2a}$ is rational number .

But$\sqrt{5}$ is not rational number .

Since a rational number cannot be equal to an irrational number . Our assumption that $\frac{3}{2\sqrt{5}}$ is a rational number is wrong .

Hencem $\frac{3}{2\sqrt{5}}$ is irrational number.

53. Let us assume that $5-2\sqrt{3}$ is a rational number.

Then, there must exist positive co primes a and b such that

$\Rightarrow\ 5-2\sqrt{3} = \frac{\mathrm{a}}{\mathrm{b}}$

$\Rightarrow -2\sqrt{3} = \frac{\mathrm{a}}{\mathrm{b}} - 5$

$\Rightarrow\ 2\sqrt{3} = 5 - \frac{\mathrm{a}}{\mathrm{b}}$

$\Rightarrow\ 2\sqrt{3} = \frac{5\mathrm{b}-\mathrm{a}}{\mathrm{a}}$

$\Rightarrow\ \sqrt{3} = \frac{5\mathrm{b}-\mathrm{a}}{2\mathrm{a}}$

The right side $\frac{5\mathrm{b}-\mathrm{a}}{2\mathrm{a}}$ is a rational numbers so $\sqrt{3}$ is a rational number

This contradicts the fact that $\sqrt{3}$ is an irrational number

Hence our assumption is incorrect and $5-2\sqrt{3}$ is an irrational number.

54. Let us assume that 5 + 3$\sqrt{2}$, is a rational number.

Then there exist co primes a and b such that

5 + $3\sqrt{2}=\frac{a}{b}$

$3\sqrt{2}=\frac{a}{b}$-5

$=\frac{a-5b}{b}$

So $\sqrt{2}=\frac{a-5b}{3b}$ -------(i)

$\frac{a-5b}{3b}$ is rational so this shows that $\sqrt{2}$ is rational

But $\sqrt{2}$ is irrational.

∴(i) presents a contradiction.,

Hence 5 + $3\sqrt{2}$ is an irrational number.

55. Given

$\sqrt{3}$ is an irrational number

Let 5 + $2\sqrt{3}$ is a rational number

∴ we can write 5 + $2\sqrt{3}$ = $\frac{p}{q}$, where p and q are integers

$\Rightarrow$ $2\sqrt{3}$ = $\frac{p}{q}-5=\frac{p-5q}{q}$

$\sqrt{3}$ = $\frac{p-5q}{2q}$

Here, $\frac{p-5q}{2q}$ is a rational number

So, $\sqrt{3}$ is also a rational number.

But it is given that $\sqrt{3}$ is irrational number.

$\Rightarrow$ our assumption was wrong

$\Rightarrow$ 5 + $2\sqrt{3}$ is an irrational number.

56. Let 7 - $2\sqrt{2}$ = m, where m is a rational number

$\sqrt{2}$ = $\frac{7-\mathrm{m}}{2}$

Irrational = Rational

$\Rightarrow$ LHS $\neq$ RHS

It means out assumption is wrong.

Hence, 7 - $2\sqrt{2}$ is irrational

57. We will prove this by contradiction i.e., let us assume that $3+\sqrt{2}$ is rational number. Then, there exist positive co-primes a and b such that

$3+\sqrt{2}=\frac{a}{b}$

$\sqrt{2}=\frac{a}{b}-3$

$\sqrt{2}=\frac{a-3b}{b}$

Since a,b are integers, therefore $\sqrt{2}$ is a rational number which is a contradiction.

Hence, the given number i.e., $3+\sqrt{2}$ is irrational.